**FURTHER MATHEMATICS/MATHEMATICS (ELECTIVE)**

**AIMS OF THE SYLLABUS**

The aims of the syllabus are to test candidates’

(i) development of further conceptual and manipulative skills in Mathematics;

(ii) understanding of an intermediate course of study which bridges the gap between Elementary Mathematics and Higher Mathematics;

(iii) acquisition of aspects of Mathematics that can meet the needs of potential

Mathematicians, Engineers, Scientists and other professionals.

(iv) ability to analyse data and draw valid conclusion

(v) logical, abstract and precise reasoning skills.

**EXAMINATION SCHEME**

There will be two papers, Papers 1 and 2, both of which must be taken.

**PAPER 1**: will consist of forty multiple-choice objective questions, covering the entire syllabus. Candidates will be required to answer all questions in 1hours for 40 marks. The questions will be drawn from the sections of the syllabus as follows:

 Pure Mathematics - 30 questions

 Statistics and probability - 4 questions

 Vectors and Mechanics - 6 questions

**PAPER 2:** will consist of two sections, Sections A and B, to be answered in 2 hours for 100 marks.

Section A will consist of eight compulsory questions that areelementary in type for 48 marks. The questions shall be distributed as follows:

Pure Mathematics - 4 questions

Statistics and Probability - 2 questions

Vectors and Mechanics - 2 questions

Section B will consist of seven questions of greater length and difficulty put into three parts:Parts I, II and III as follows:

 Part I: Pure Mathematics - 3 questions

 Part II: Statistics and Probability - 2 questions

 Part III: Vectors and Mechanics - 2 questions

Candidates will be required to answer four questions with at least one from each part for 52 marks.

**DETAILED SYLLABUS**

In addition to the following topics, more challenging questions may be set on topics in the General Mathematics/Mathematics (Core) syllabus.

In the column for CONTENTS, more detailed information on the topics to be tested is given while the limits imposed on the topics are stated under NOTES.

Topics which are marked with asterisks shall be tested in Section B of Paper 2 only.

**KEY:**

\* Topics peculiar to Ghana only.

\*\* Topics peculiar to Nigeria only

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| **Topics** | **Content** | **Notes** |
| **I**. Pure Mathematics 1. Sets
2. Surds
3. Binary Operations
4. Logical Reasoning
5. Functions
6. Polynomial Functions
7. Rational Functions
8. Indices and Logarithmic Functions
9. Permutation

And Combinations.(10) Binomial        Theorem(11) Sequences  and Series1. Matrices and Linear Transformation
2. Trigonometry
3. Co-ordinate

 Geometry 1. Differentiation
2. Integration

**II**. Statistics and  Probability1. Statistics
2. Probability

**III**. Vectors and      Mechanics1. Vectors
2. Statics
3. Dynamics
 | (i) Idea of a set defined by a  property, Set notations and their meanings. (ii) Disjoint sets, Universal set and  complement of set (iii) Venn diagrams, Use of sets  And Venn diagrams to solve problems.(iv) Commutative and Associative  laws, Distributive properties over union and intersection.Surds of the form , a and a+b where a is rational, b is a positive integer and n is not a perfect square.Properties:Closure, Commutativity, Associativity and Distributivity, Identity elements and inverses.(i) Rule of syntax:  true or false statements, rule of logic applied to      arguments, implications and      deductions.(ii) The truth table(i) Domain and co-domain of a function.(ii) One-to-one, onto, identity and      constant mapping;(iii) Inverse of a function.(iv) Composite of functions.(i) Linear Functions, Equations and  Inequality(ii) Quadratic Functions, Equations      and Inequalities(ii) Cubic Functions and Equations(i) Rational functions of the form Q(x) = ,g(x) ≠ 0. where g(x) and f(x) are polynomials. e.g. f:x →(ii) Resolution of rational  functions into partial fractions.(i) Indices(ii) Logarithms(i) Simple cases of arrangements (ii) Simple cases of selection of      objects.Expansion of (a + b)n. Use of (1+x)n ≈1+nx for any rational n, where x is sufficiently small. e.g (0.998)1/3(i) Finite and Infinite sequences.(ii) Linear sequence/Arithmetic  Progression (A.P.) and  Exponential sequence/Geometric  Progression (G.P.)(iii) Finite and Infinite series.(iv) Linear series (sum of A.P.) and  exponential series (sum of  G.P.)\*(v) Recurrence Series(i) Matrices  (ii) Determinants(iii) Inverse of 2 x 2 Matrices(iv) Linear Transformation(i) Trigonometric Ratios and Rules(ii) Compound and Multiple       Angles.(iii) Trigonometric Functions and       Equations(i) Straight Lines(ii) Conic Sections(i) The idea of a limit(ii) The derivative of a function(iii)Differentiation of polynomials (iv) Differentiation of trigonometric  Functions(v) Product and quotient rules.  Differentiation of implicit  functions such as ax2 + by2 = c\*\*(vi) Differentiation of  Transcendental Functions(vii) Second order derivatives and  Rates of change and small       changes (x), Concept of        Maxima and Minima (i) Indefinite Integral(ii) Definite Integral(iii) Applications of the Definite  Integral(i) Tabulation and Graphical  representation of data(ii) Measures of location(iii) Measures of Dispersion(iv)Correlation(i) Meaning of probability.(ii) Relative frequency.(iii) Calculation of Probability using       simple sample spaces. (iv) Addition and multiplication of       probabilities.(v) Probability distributions.(i) Definitions of scalar and vector  Quantities.(ii) Representation of Vectors.(iii) Algebra of Vectors.(iv) Commutative, Associative and  Distributive Properties.(v) Unit vectors.(vi) Position Vectors.(vii) Resolution and Composition of Vectors.(viii) Scalar (dot) product and its application.\*\*(ix) Vector (cross) product and its application.(i) Definition of a force.(ii) Representation of forces.(iii) Composition and resolution of coplanar forces acting at a      point.(iv) Composition and resolution of general coplanar forces on        rigid bodies.(v) Equilibrium of Bodies.(vi) Determination of Resultant.(vii) Moments of forces.(viii) Friction.(i) The concepts of motion(ii) Equations of Motion (iii) The impulse and momentum  equations: \*\*(iv) Projectiles. | (x : x is real), ∪, ∩, { },∉, ∈, , , U (universal set) and A’ (Complement of set A).More challenging problems involving union, intersection, the universal set, subset and complement of set.Three set problems. Use of De Morgan’s laws to solve related problemsAll the four operations on surdsRationalising the denominator of surds such as , , .Use of properties to solve related problems.Using logical reasoning to determine the validity of compound statements involving implications and connectivities. Include use of symbols: Pp q, p ∧ q, p ⇒ qUse of Truth tables to deduce conclusions of compound statements. Include negation.The notation e.g. *f : x* → 3x+4;*g : x* → x2 ; where x ∈***R***.Graphical representation of a function ; Image and the range.Determination of the inverse of a one-to-one function e.g. If f: x →sx + , the inverse relation f-1: x →x - is also a function.Notation: fog(x) =f(g(x)) Restrict to simple algebraic functions only.Recognition and sketching of graphs of linear functions and equations.Gradient and intercepts forms of linear equations i.e.ax + by + c = 0; y = mx + c; + = k. Parallel and Perpendicular lines. Linear Inequalities e.g. 2x + 5y ≤ 1, x + 3y ≥ 3Graphical representation of linear inequalities in two variables. Application to Linear Programming.Recognition and sketching graphs of quadratic functions e.g. f: x → ax2 +bx + c, where a, b and c Є R.Identification of vertex, axis of symmetry, maximum and minimum, increasing and decreasing parts of a parabola. Include values of x for which f(x) >0 or f(x) < 0.Solution of simultaneous equations: one linear and one quadratic. Method of completing the squares for solving quadratic equations. Express f(x) = ax2 + bx + c in the form f(x) = a(x + d)2 + k, where k is the maximum or minimum value. Roots of quadratic equations – equal roots (b2 - 4ac = 0), real and unequal roots (b2 – 4ac > 0), imaginary roots (b2 – 4ac < 0); sum and product of roots of a quadratic equation e.g. if the roots of the equation 3x2 + 5x + 2 = 0 are and β, form the equation whose roots are and . Solving quadratic inequalities.Recognition of cubic functions e.g. f: x → ax3 + bx2 +cx + d. Drawing graphs of cubic functions for a given range. Factorization of cubic expressions and solution of cubic equations. Factorization of a3 ± b3. Basic operations on polynomials, the remainder and factor theorems i.e. the remainder when f(x) is divided by f(x – a) = f(a). When f(a) is zero, then (x – a) is a factor of f(x).g(x) may be factorised into linear and quadratic factors (Degree of Numerator less than that of denominator which is less than or equal to 4).The four basic operations.Zeros, domain and range, sketching not required.Laws of indices.Application of the laws of indices to evaluating products, quotients, powers and nth root.Solve equations involving indices.Laws of Logarithms. Application of logarithms in calculations involving product, quotients, power (log an), nth roots (log , log a1/n).Solve equations involving logarithms (including change of base).Reduction of a relation such as y = axb, (a,b are constants) to a linear form: log10y = b log10x+log10a.Consider other examples such aslog abx = log a + x log b;log (ab)x = x(log a + log b)  = x log ab\*Drawing and interpreting graphs of logarithmic functions e.g. y = axb. Estimating the values of the constants a and b from the graphKnowledge of arrangement and selection is expected. The notations: nCr, and nPr for selection and arrangement respectively should be noted and used. e.g. arrangement of students in a row, drawing balls from a box with or without replacements. npr = n! (n-r)!nCr= n!r!(n-r)!Use of the binomial theorem for positive integral index only.Proof of the theorem **not** required.e.g. (i) u1, u2,…, un. (ii) u1, u2,….Recognizing the pattern of a sequence. e.g. (i) Un = U1 + (n-1)d, where d is the common difference.(ii) Un= U1 rn-1 where r is the common ratio.(i) U1 + U2 + U3 + … + Un(ii)U1 + U2 + U3 + …. (i) Sn = (U1+Un)  (ii) Sn = [2a + (n – 1)d] (iii) Sn= U1(1-rn) , r<1 l - r (iv) Sn=U1(rn-1) , r>l. r – 1 (v) Sum to infinity (S) =  r < 1Generating the terms of a recurrence series and finding an explicit formula for the sequence e.g. 0.9999 =  + + + + ....Concept of a matrix – state the order of a matrix and indicate the type.Equal matrices – If two matrices are equal, then their corresponding elements are equal. Use of equality to find missing entries of given matricesAddition and subtraction of matrices (up to 3 x 3 matrices).Multiplication of a matrix by a scalar and by a matrix (up to 3 x 3 matrices)Evaluation of determinants of 2 x 2 matrices.\*\*Evaluation of determinants of 3 x 3 matrices.Application of determinants to solution of simultaneous linear equations.e.g. If A = , then A-1 = Finding the images of points under given linear transformationDetermining the matrices of given linear transformation. Finding the inverse of a linear transformation (restrict to 2 x 2 matrices). Finding the composition of linear transformation. Recognizing the Identity transformation. (i) reflection in the     x - axis(ii) reflection in the     y - axis(iii) reflection in the line      y = x(iv) for anti-clockwise rotation through θ about the origin.(v) , the general matrix for reflection in a line through the origin making an angle θ with the positive x-axis.\*Finding the equation of the image of a line under a given linear transformationSine, Cosine and Tangent of general angles (0o≤θ≤360o).Identify trigonometric ratios of angles 30O, 45O, 60o without use of tables.Use basic trigonometric ratios and reciprocals to prove given trigonometric identities.Evaluate sine, cosine and tangent of negative angles. Convert degrees into radians and vice versa.Application to real life situations such as heights and distances, perimeters, solution of triangles, angles of elevation and depression, bearing(negative and positive angles) including use of sine and cosine rules, etc,Simple cases only.sin (A B),cos (A B),tan(A B).Use of compound angles in simple identities and solution of trigonometric ratios e.g. finding sin 75o, cos 150oetc, finding tan 45o without using mathematical tables or calculators and leaving your answer as a surd, etc.Use of simple trigonometric identities to find trigonometric ratios of compound and multiple angles (up to 3A).Relate trigonometric ratios to Cartesian Coordinates of points (x, y) on the circle x2 + y2 = r2. f:x →sin x, g: x → a cos x + b sin x = c.Graphs of sine, cosine, tangent and functions of the form asinx + bcos x. Identifying maximum and minimum point, increasing and decreasing portions. Graphical solutions of simple trigonometric equations e.g. asin x + bcos x = k.Solve trigonometric equations up to quadratic equations e.g. 2sin2x – sin x – 3 =0, for 0o ≤ x ≤ 360o.\*Express f(x) = asin x + bcos x in the form Rcos (x ) or Rsin (x ) for 0o ≤ ≤ 90oand use the result to calculate the minimum and maximum points of a given functions.Mid-point of a line segmentCoordinates of points which divides a given line in a given ratio.Distance between two points;Gradient of a line;Equation of a line: (i) Intercept form; (ii) Gradient form;Conditions for parallel and perpendicular lines.Calculate the acute angle between two intersecting lines e.g. if m1 and m2 are the gradients of two intersecting lines, then tan θ = . If m1m2 = -1, then the lines are perpendicular. \*The distance from an external point P(x1, y1) to a given line ax + by + c using the formulad = ||.Loci of variable points which move under given conditionsEquation of a circle: (i) Equation in terms of  centre, (a, b), and          radius, r,  (x - a)2+(y - b)2 = r2; (ii) The general form:x2+y2+2*gx*+2*fy*+c = 0, where (-*g*, -*f*) is the centre and radius, r = . Tangents and normals to circlesEquations of parabola in rectangular Cartesian coordinates (y2 = 4ax, include parametric equations (at2, at)).Finding the equation of a tangent and normal to a parabola at a given point.\*Sketch graphs of given parabola and find the equation of the axis of symmetry.(i) Intuitive treatment of limit.  Relate to the gradient of a curve. e.g. f*1*(x) = .(ii) Its meaning and its  determination from first principles (simple cases      only). e.g. axn + b, n ≤ 3, (n ∈*I* )e.g. axm – bxm - 1+ ...+k, where m Є *I* , k is a constant.e.g. sin x, y = a sin x b cos x. Where a, b are constants.including polynomials of the form (a + b*x*n)m.e.g. y = eax, y = log 3x,  y = ln x(i) The equation of a tangent to a curve at a point.(ii) Restrict turning points to maxima and minima.(iii)Include curve sketching (up  to cubic functions) and linear kinematics.(i) Integration of polynomials of the form axn; n ≠ -1. i.e.∫xn dx = + c, n ≠ -1.(ii) Integration of sum and difference of polynomials.  e.g. ∫(4x3+3x2-6x+5) dx\*\*(iii)Integration of polynomials of the form axn; n = -1.  i.e. ∫ x -1 dx = ln xSimple problems on integration by substitution.Integration of simple trigonometric functions of the form .(i) Plane areas and Rate of  Change. Include linear kinematics.  Relate to the area under a curve.(ii)Volume of solid of revolution(iii) Approximation restricted to trapezium rule.Frequency tables.Cumulative frequency tables.Histogram (including unequal class intervals).Cumulative frequency curve (Ogive) for grouped data.Central tendency: mean, median, mode, quartiles and percentiles. Mode and modal group for grouped data from a histogram.Median from grouped data.Mean for grouped data (use of an assumed mean required).Determination of:(i) Range, Inter- Quartile and  Semi inter-quartile range from an Ogive.(ii) Mean deviation, variance      and standard deviation for  grouped and ungrouped data. Using an assumed mean or true mean.Scatter diagrams, use of line of best fit to predict one variable from another, meaning of correlation; positive, negative and zero correlations,.Spearman’s Rank coefficient.Use data without ties. \*Equation of line of best fit by least square method. (Line of regression of y on x).Tossing 2 dice once; drawing from a box with or without replacement. Equally likely events, mutually exclusive, independent and conditional events.Include the probability of an event considered as the probability of a set.(i) Binomial distribution P(x=r)=nCrprqn-r , where  Probability of success = p, Probability of failure = q*,*p + q = 1 and n is the     number of trials. Simple     problems only.\*\*(ii) Poisson distribution  P(x) = , where λ = np, n is large and p is small.Representation of vector in the form a**i** + b**j**.Addition and subtraction, multiplication of vectors by vectors, scalars and equation of vectors. Triangle, Parallelogram and polygon Laws.Illustrate through diagram, Illustrate by solving problems in elementary plane geometry e.gcon-currency of medians and diagonals.The notation:***i*** for the unit vector 1 and 0***j*** for the unit vector 0 1along the x and y axes respectively. Calculation of unit vector (â) along a i.e. â = .Position vector of A relative to O is .Position vector of the midpoint of a line segment. Relate to coordinates of mid-point of a line segment.\*Position vector of a point that divides a line segment internally in the ratio (λ : μ). Applying triangle, parallelogram and polygon laws to composition of forces acting at a point. e.g. find the resultant of two forces (12N, 030o) and (8N, 100o) acting at a point.\*Find the resultant of vectors by scale drawing.Finding angle between two vectors.Using the dot product to establish such trigonometric formulae as(i) Cos (a ± b) =cos a cos b sin a sin b(ii) sin (a ± b)= sin a cos b ± sin *b* cos*a*(iii) c2 = a2 + b2 - 2ab cos C(iv) =.Apply to simple problems e.g. suspension of particles by strings.Resultant of forces, Lami’s theoremUsing the principles of moments to solve related problems.Distinction between smooth and rough planes. Determination of the coefficient of friction.The definitions of displacement, velocity, acceleration and speed.Composition of velocities and accelerations.Rectilinear motion.Newton’s laws of motion.Application of Newton’s LawsMotion along inclined planes (resolving a force upon a plane into normal and frictional forces).Motion under gravity (ignore air resistance).Application of the equations of motions: V = u + at, S = ut + ½ at 2;v2 = u2 + 2as.Conservation of Linear Momentum(exclude coefficient of restitution).Distinguish between momentum and impulse.Objects projected at an angle to the horizontal. |

1. **UNITS**

Candidates should be familiar with the following units and their symbols.

**( 1 ) Length**

1000 millimetres (mm) = 100 centimetres (cm) = 1 metre(m).

1000 metres = 1 kilometre (km)

**( 2 ) Area**

10,000 square metres (m2) = 1 hectare (ha)

**( 3 ) Capacity**

1000 cubic centimeters (cm3) = 1 litre (l)

**( 4 ) Mass**

1. milligrammes (mg) = 1 gramme (g)

 1000 grammes (g) = 1 kilogramme( kg )

1. ogrammes (kg) = 1 tonne.

**( 5) Currencies**

 The Gambia – 100 bututs (b) = 1 Dalasi (D)

 Ghana - 100 Ghana pesewas (Gp) = 1 Ghana Cedi ( GH¢)

 Liberia - 100 cents (c) = 1 Liberian Dollar (LD)

 Nigeria - 100 kobo (k) = 1 Naira (N)

 Sierra Leone - 100 cents (c) = 1 Leone (Le)

 UK - 100 pence (p) = 1 pound (£)

 USA - 100 cents (c) = 1 dollar ($)

French Speaking territories 100 centimes (c) = 1 Franc (fr)

Any other units used will be defined.

1. **OTHER IMPORTANT INFORMATION**

**( 1) Use of Mathematical and Statistical Tables**

Mathematics and Statistical tables, published or approved by WAEC may be used in the examination room. Where the degree of accuracy is not specified in a question, the degree of accuracy expected will be that obtainable from the mathematical tables.

1. **Use of calculators**

The use of non-programmable, silent and cordless calculators is allowed. The calculators must, however not have a paper print out **nor be capable of receiving/sending any information. Phones with or without calculators are not allowed.**

1. **Other Materials Required for the examination**

Candidates should bring rulers, pairs of compasses, protractors, set squares etc required for papers of the subject. They will **not** be allowed to borrow such instruments and any other material from other candidates in the examination hall.

Graph papers ruled in 2mm squares will be provided for any paper in which it is required.

**( 4) Disclaimer**

In spite of the provisions made in paragraphs 2 (1) and (2) above, it should be noted that some questions may prohibit the use of tables and/or calculators.