Leh Wi Lan

Improving Secondary Education in Sierra Leone

UKaid from the British people

CAMBRIDGE EDUCATION

FACILITATION GUIDE

Improving Teaching and Learning

SSS Teacher Professional Development (TPD): Mathematics (Term 3)

Introduction

Teacher professional development (TPD) is most effective when several elements are combined to support teachers. Training and support should be as close to the school as possible so teachers have opportunities to practice their learning in their own context. They can work with their peers to share success and challenges and reflect on their problems, devising contextually relevant solutions. Teachers also need some form of external support so that they are introduced to new ideas, ways of working and can refresh their subject knowledge and ensure that it is up to date.

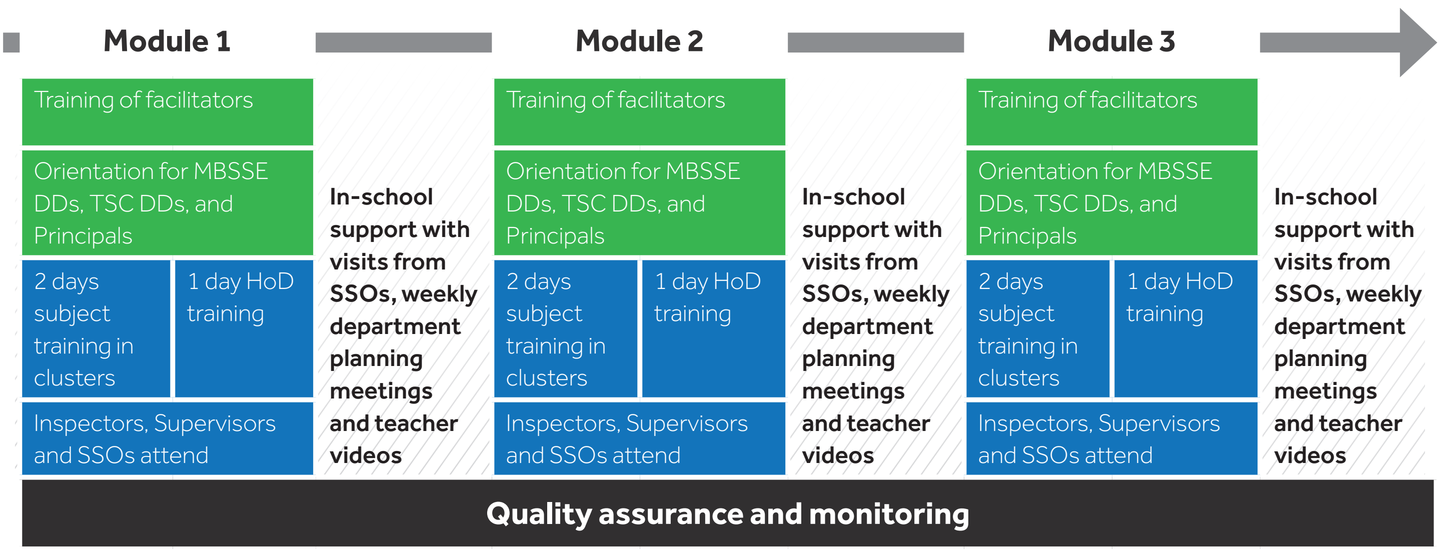
This is especially important in a context like Sierra Leone where Junior Secondary Schools (JSS) and Senior Secondary Schools (SSS) vary greatly in terms of access to resources and distribution of qualified teachers. Within government schools there are large numbers of teachers who are: not qualified for secondary level, qualified but not government approved, qualified to teach, but not in the subject they teach, and volunteer teachers with no prior training or qualification.

Teacher professional development demands a variety of activities so that all teachers, whatever their circumstances and environment, can access structured quality professional development which supports their professional growth and helps deliver quality education.

Between 2017 and 2020 Leh Wi Lan implemented a TPD strategy to support all JSS and SSS English and maths teachers and Heads of Department (HoD), from approximately 1600 government-assisted schools.

The strategy included subject content training in maths and English as well as academic leadership training for HoD. Teachers received training through termly face to face cluster sessions, led by national facilitators who were subject specialists. After each face-to-face cluster training there was in-school support for teachers through regular visits from Leh Wi Lan School Support Officers[1] where they conducted lesson observations and gave feedback to teachers. Instructional videos were also created providing step by step explanations of specific concepts and examples of good classroom practice.

The diagram below shows how the TPD programme worked over a given academic year:



This facilitator manual is part of the Leh Wi Lan TPD programme. It contains subject training for maths teachers in SSS. It is designed to be delivered over one academic year and each module links directly to SSS English lessons being taught in the upcoming term (in this case term 3). It should be used alongside the MBSSE Lesson Plan Manual and Pupil Handbook. Whilst focusing primarily on subject content, the materials have been designed to develop key pedagogical skills including gender-responsive pedagogy, using teaching and learning aids, inclusive learning, reflection and problem solving.

These materials were produced by Cambridge Education, in collaboration with TSC, and delivered as part of the UK-aid funded Leh Wi Lan project for training teachers in government assisted junior and senior secondary schools in Sierra Leone. These training materials are in draft. They can be shared and adapted for use as long as they are not used for commercial purposes.

[1] 200 School Support Officers conducted regular visits to government assisted secondary schools nationwide. Each covered approximately 8-10 schools and supported the English and maths teachers by conducting lesson observations and giving feedback to enhance teaching and learning. They were supported by Leh Wi Lan. This MBSSE School Quality Assurance Officers now perform this supportive supervision role

Handout 1.1: TPD Term 3 Workshop time-table

<table>
<tr><th>Date / Time</th><th>09:00</th><th>11:00</th><th>11:30</th><th>13:30</th><th>14:30</th><th>16:30</th><th>17:00</th></tr>
<tr><td>Day 1</td><td>Session 1: Involving all learners in lessons</td><td rowspan="2">Break</td><td>Session 2: Relations
Relations and Types of Relations; Mapping, Including Domain and Range; Involving all learners in the lesson</td><td rowspan="2">Lunch</td><td colspan="2">Session 3: Quadratic Functions
Completing the Square and Perfect Squares; The Quadratic Formula</td><td rowspan="3">Closure</td></tr>
<tr><td>Day 2</td><td>Session 4: Special angles
Special Angles (30°, 45°, 60°) and applications; Bearings</td><td>Session 5: Geometry
Circle theorems; Volume, Area & Surface Area of pyramids</td><td>Session 6: Taking our learning back to school</td><td><u>Teacher self-assessment</u></td></tr>
<tr><td>Day 3</td><td>HoD Session A: Introduction to lesson observation cycle</td><td colspan="2">HoD Session B: Preparations for lesson observation</td><td>Lunch</td><td>HoD Session C: Lesson observation and form completion</td><td>HoD Session D: Practice: Feedback & Practice/Plan</td></tr>
</table>

Facilitator Standards

Well prepared:
Arrives early
Has charts written and materials organised so they are ready to give out.
Refers to training notes but doesn't read them constantly
Demonstrates strong familiarity of the lesson plan structure and content.
Time management:
Manages time
Session and activities start and end on time.
Understanding SSS:
Exhibits knowledge of the current context of Senior Secondary Schools - uses examples that are relevant to the context
Subject Knowledge:
Clearly explains how to do the content of the lesson plans, using a variety of examples to add depth.
Participatory:
Gives opportunities for participants to work together
Gives time for participants to think of how to tackle a problem before explaining
Supportive:
Listens to the participants and acts on their comments
Accepts answers and asks questions to help participants, rather than telling the answers
Inclusive:
Ask questions to a range of participants
Uses gender responsive language and interaction
Finds ways to support those participants who don't understand
Uses group, pair and individual work and moves round to support all participants
Enjoyable:
Greets the participants, creates a friendly atmosphere.
This session is quick and active with a positive approach.

Session 1: Involving all learners in lessons 90 minutes

Session objectives

By the end of the session, participants will be able to:

- describe how to use the video showing good classroom practice
- explore creative ways of interpreting the steps in the LPM
- demonstrate diverse ways of involving all learners in the scripted lessons

Materials

Chart 1.1	Session objectives /reflection on pedagogy	Introduction
Chart 1.2	Activities for both teachers and students	Introduction
Chart 1.3	Lesson openings	Activity 2
Chart 1.4	Learner centred web	Activity 3
Handout 1.1	TPD Term 3 Workshop Timetable	Introduction

Session outline

Introduction	Reflections on pedagogy	20 minutes
Activity 1	Good classroom practice	20 minutes
Activity 2	Let’s get creative	30 minutes
Activity 3	Involving all learners in lessons (child centred principles)	25 minutes
Session review		5 minutes

Introduction Reflections on pedagogy 20 minutes

1. Welcome participants to the first day of Term 3 Teacher Professional Development training and ask two of the participants (Christian and Muslim) to open in prayers.
2. Have new participants do a quick introduction of themselves.
3. Agree on rules for the workshop as quickly as possible
4. Give the workshop timetable (**Handout 1.1**) to participants, spend only 1 minute on it.
5. Display **Chart 1.1** and go through the learning outcomes.
6. Use Margolis Wheel to reflect on pedagogical skills learnt in Term 1 on how children learn and Term 2 on familiarising themselves with the LPM. Go around to listen to discussions by participants. Try to discuss:
    a. identify effective ways in which children learn (Doing activities themselves; Using a variety of materials; Working with, and helping others; Building new learning on what is already known; Focusing on understanding and use of skills; Working at their individual levels of ability; Using what is learned practically; Having time to practice new skills)
    b. mention skills explored in Term 2 in familiarising themselves with the LPM (following steps of the plans; observing lesson timing; linking teaching activities to objectives; identifying preparations required; demonstrating mastery of the content; practicing; working with others; observing each other; checking that learning is happening; linking pupils' handbook to LPM)
    c. describe which of the skills you were able to carry out after the last two workshops and how.
7. Display **Chart 1.2** Activities for both teachers and students.
8. Ask participants in groups to identify which of the activities in Chart 1.2 will help all children to learn, satisfying child-centred learning principles. (Answers: Working at their individual levels of ability; Using a variety of materials; Working with, and helping others; Focusing on understanding and use of skills; Building new learning on what is already known; Having time to practise new skills; Using what is learned practically; Doing activities themselves).
9. Play BINGO and ask groups who are not able to shout BINGO to mention the activities left unmarked and address them together as whole group.
10. Agree that if the activities do not show all children learning at the same time, they are teacher-centred activities and limited time must be spent on those.

Activity 1 Good classroom practice 20 minutes

1. Tell participants that good classroom practice has evidence of all learners participating in a lesson. Remind them that our visits to a number of schools revealed no visible evidence that most children are learning.
2. Ask participants to discuss in pairs what likely evidence they would see in a lesson – if they were observers - to show that children are learning.
3. Take responses, which could include children talking to one another, spending most of the lesson time working on tasks, correct answers in books, explaining their work/answers, asking questions to further their understanding and helping one another.
4. Tell participants that they will be watching a video on good classroom practice and that you will only play it for 5 minutes so they must be attentive and be alert. Ask them to write down examples of children actively learning and what the teacher is doing to involve all children.
5. Play the clip and ask participants to turn to their partner and share examples of children actively involved in their learning. Allow 1 minute for discussions.
6. Take feedback from participants and write responses on flip chart paper.
7. Agree together what the teacher does in the video to ensure all children are participating. (E.g. walking around to encourage pupils, helping learners who need support)
8. Give groups 3 minutes to brainstorm other ways they can help all children actively participate and learn in lessons. (Grouping, pairing, discussing, questions, explaining, drama, etc)
9. Show participants how to access the clips using SD cards on their own devices. Explain that they should study the videos to see examples of good teaching. This can help them improve. The SSOs will support them to use the video clips well.
10. Take comments from participants and wrap up.

Activity 2 Let's get creative 30 minutes

1. Explain that lessons in the LPMs are scripted and interpretations may vary slightly from teacher to teacher but the variations may be so insignificant in such a way that it would disrupt the uniformity intended.
2. The variations we would likely see is dependent on how creative a teacher is in connecting with her children to ensure information is passed to all of them in an interactive manner.
3. The only interactions seen so far in lessons are children responding to teacher's questions, mainly closed questions.
4. In as much as we want the lessons delivered as in the LPMs, teachers can get creative by improving on the connections and relationships with their students.

5. Ask participants to read the lesson openings on **Chart 1.3** (Term 3 L135) and spend 20 seconds on each step thinking about creative ways to deliver them to pupils in an attempt to involve all learners.
6. Let participants turn to their partners to share their plans.
7. After 1 minute, take brief responses from 5 pairs spread across the room
8. Always stress the importance of connecting with the target audience within the allotted time and ensuring that most learners are actively doing something.
9. Demonstrate what you planned to do too and check with them if they agree it is creative. (math a. SOH, CAH, TOA on index cards. As I flash the cards, you have 10 seconds to write what it means on a sheet of paper and display it on your forehead when I shout 'show me'; For English, quickly assign a topic to groups and give them 1 minute to write as many words as possible relating to the topic. The winner is the group with most relevant words).
10. Ask groups to pick any term 3 lesson, go to teaching and learning section, agree on creative way of delivering the first 3 steps and all to get prepared to deliver to the whole group. Warn them that you will randomly pick the presenters yourself. This should get everyone ready.
11. Presenters get 2 minutes to present and get feedback from whole group about their creativity. Ask them to identify why the activities were good and how they were encouraged to take part.
12. Thank everyone for their participation and reiterate the feel-good effect of the connections to both teacher and students.

Activity 3 Involving all learners in lessons (child centred principles) 25 minutes

1. Ask participants: What is Lesson One for teachers who want students to successfully grasp, retain, and apply new material?
2. Explain that you need to "arouse their interest." Teachers need to find ways to make learning "relevant, authentic, and valuable" in students' lives.
3. In this activity, we will be exploring what learner involvement looks like in practical terms.
4. Display **Chart 1.4** Learner centred web depicting everything (students do a **variety of activities** where they can use their experiences, **all** students are involved in **doing** the activities, the students **work together** in groups or pairs and help each other, students use a variety of **materials** that appeal to them to help them learn, students of **all abilities** take part in activities, students do activities where they **have to think and use**

knowledge, not just memorise and recall knowledge, students have some level of **choice** in learning, students can **self-assess** and adjust) revolving around pupils.

5. Ask participants what their observations are. Learner is in the middle of the web while all other things are spread around it, more like our star and its planets.
6. **Child**-**centred learning**, also known as child or learner-centred education, broadly encompasses **methods** of teaching that shift the focus of instruction from the teacher to the **student**. It focuses on skills and practices that enable lifelong learning and independent problem-solving
7. For this to happen teachers intentionally plan around students' interests, experiences and abilities giving them choice and voice.
8. Regroup participants. (You could do this by playing Titanic game. Facilitator: Titanic is moving, participants: it is moving 3 times; Facilitator: Titanic is about to capsize and the lifeboats can only carry 3, 4, 5 passengers)
9. Use **Chart 1.5** Teaching Scenarios to give different scenarios seen from our school visits. Ask participants to work in groups and state the problem with the style of delivery, if any, plan how to improve on it.

Group 1: A teacher with 150 pupils introduce his new lesson by asking pupils what they learned in previous lessons to (Lesson 91 Opening) - *Whole class questioning should be quick. Pupils should be given time to think so it isn't the same pupils putting their hands up to answer each time. You could give them a couple of minutes to discuss the answer with a partner, then take contributions from different pairs around the room. (see questioning in video)*

Group 2: Two pupils (a boy and a girl) were called out in a competition to solve a problem on the board while the rest of the class was cheering in support of their representatives *– only 2 pupils can be said to be learning*

Group 3: Teacher used instructional materials where all students could see her to demonstrate a concept during a lesson *– good but all pupils should have materials to work with, not just watch the teacher. Teacher needs to prepare sufficient materials to go around (see making of protractor in video clip)*

Group 4: Students watched a 10 minutes video clip on impact of modern slavery on economy. 5 pupils came out to dramatize a scene. Students later answered some questions in their exercise books *– assign different tasks to groups on same topic to enable others participate or have others discuss group's success and areas of improvement or ask all pupils to work in groups to dramatize the scene, then watch one or two.*

Group 5: Teacher spent most time, during teaching and learning solving 5 examples to aid students' comprehension *– to shorten the direct instruction time, assign 3 of those to groups to try while you support or show one example then ask all pupils to work in pairs to* *solve the examples - walk round the class and explain it individually to those pairs who are struggling. This way you can help with their specific problems and not just assume that all pupils understand the same things.*

Group 6: Students were given same group tasks. One student was very active and did all the work. She did the presentation, with the other group members contributing little – *support by explaining group dynamics to students or think carefully about your grouping. Could you group together the larger personalities who tend to dominate and then put the quieter pupils in another group?*

Group 7: Teacher walked around to observe while pupils were doing their tasks in groups or pairs *– walking around must be focused on supporting pupils especially below average ones. The teacher should listen to each group and where discussion is slow, ask them questions to move their learning forward, or re explain the task in a different way if it looks like they don't understand how to start.*

10. Select any quiet member of the team to present briefly. After each presentation get feedback from the whole group to know if they were excited about the delivery and if there's a better way to deliver it.

Session review **5 minutes**

- Show participants Chart 1.1 again. Read each objective and ask what we did to achieve it.

Materials

Chart 1.1 Session Objectives

By the end of the session, participants will be able to:

- describe how to use the video showing good classroom practice
- explore creative ways of interpreting the steps in the LPM
- demonstrate diverse ways of involving all learners in the scripted lessons

Chart 1.2 Activities for both teachers and students

1. Doing activities themselves;
2. Using a variety of materials;
3. Listening to the teacher explain;
4. Working with, and helping others;
5. Chanting after the teacher;
6. Building new learning on what is already known;
7. Working individually after listening to the teachers presentation;
8. Focusing on understanding and use of skills;
9. Working at their individual levels of ability;
10. Using what is learned practically;
11. Using only textbooks and the blackboard;
12. Having time to practice new skills;
13. Focusing on memorising facts;
14. Everyone working at the same level of ability;
15. The teacher uses materials;
16. Doing the same activity for a long time;
17. watching pupils do examples on the blackboard;
18. listening to the same pupils answering questions.

Chart 1.3 (Term 3 L135)

Math Opening *(2 minutes)*

a. Write on the board: SOHCAHTOA
b. Ask volunteers to explain what this means. Ask them to write the associated trigonometric ratios on the board. (Answer: $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$)
c. Explain: This lesson is on solving triangles using the trigonometric ratios.

English Opening *(5 minutes)*

a. Have pupils brainstorm their vocabulary for each of the following topics:
- Environment (biodegradable, carbon emissions, climate change, endangered)
- Science and technology (geology, artificial intelligence, computerised, data, digital information)
- Building and construction (electrification, foundation, carpenter, cement, mason, crane)

b. Tell pupils that in today's lesson they will review general vocabulary connected to environment, science and technology, and building and construction.

Chart 1.4 Learner centred web

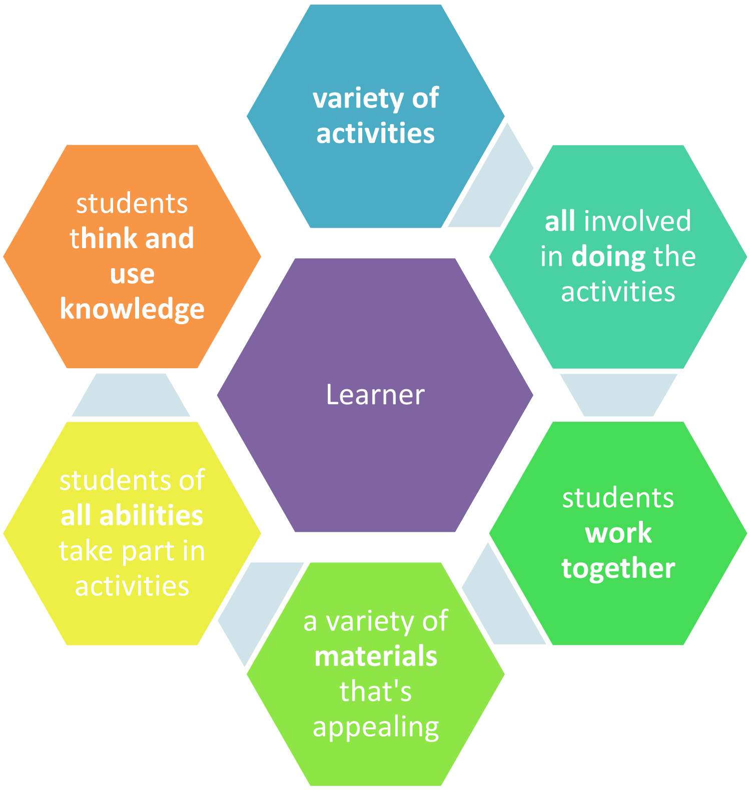


Chart 1.5 Teaching Scenarios

Group 1: A teacher with 150 pupils introduce his new lesson by asking pupils what they learned in previous lessons to (Lesson 91 Opening)

Group 2: Two pupils (a boy and a girl) were called out in a competition to solve a problem on the board while the rest of the class was cheering in support of their representatives

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Group 5: Teacher spent most time, during teaching and learning solving 5 examples to aid students' comprehension

Group 6: Students were given same group tasks. One student was very active and did all the work. She did the presentation, with the other group members contributing little

Group 7: Teacher walked around to observe while pupils were doing their tasks in groups or pairs

Session 2 Relations **90 minutes**

Learning Outcomes

By the end of the session participants will be able to:

- define relations, name and identify the different types of relations
- describe mappings between sets and give examples
- define and identify functions
- find the range of a function from its domain
- identify different ways of involving all participants in their learning

Materials

Chart 2.1	Learning Outcomes
Chart 2.2	Types of Relations
Chart 2.3	Exercise 1
Chart 2.4	Exercise 2
Chart 2.5	Exercise 3
Chart 2.6	Group discussion and presentation
Chart 2.7	Exercise 4

Activity Outline

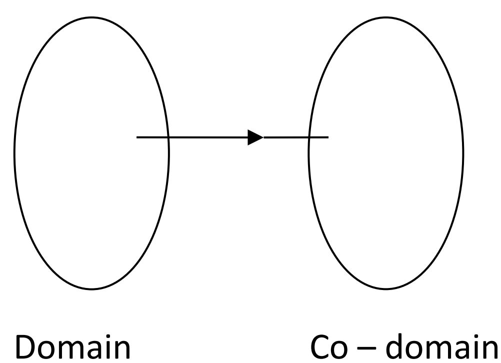
Introduction		**2 minutes**
Activity 1	**Meaning and types of Relations**	**20 minutes**
Activity 2	**Mappings**	**25 minutes**
Activity 3	**Finding the rule of a Mapping**	**20 minutes**
Activity 4	**Functions**	**20 minutes**
Summary		**3 minutes**

Introduction **2 minutes**

1. Tell participants that relation between people or sets is a broad term and that in this session, some restrictions in certain relations will be looked at.
2. Ask a volunteer to name a relation he/she has with a person or thing.
3. Agree that this refers to links or attributes that connect them to the people.

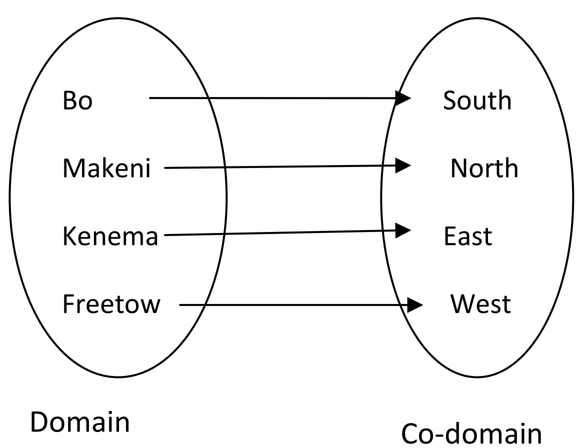
Activity 1 Meaning and types of Relations **20 minutes**

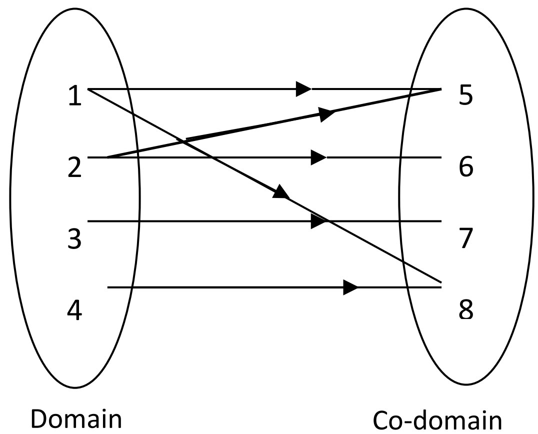
1. Ask participants to discuss in pairs their understanding of the meaning of relations (in math).
2. Take responses from a couple of pairs and agree that 'A relation is a connection between two sets the domain and the co – domain'. It is a way in which people or things are connected.



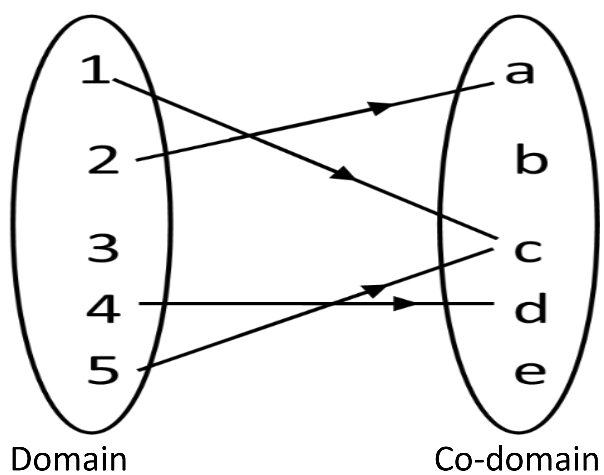
3. Say that normally in a relation, there are no restrictions but simple rules or laws.

E.g.





4. Ask groups to spend 1 minute to create own relations with domain and co-domain
5. Allow a couple of groups to share
6. Sometimes in a relation, not every element in the domain has an image in the Co – domain. Also, not every element in the Co – domain has an association with the domain.
7. Display the relation below on a chart quickly draw on the board



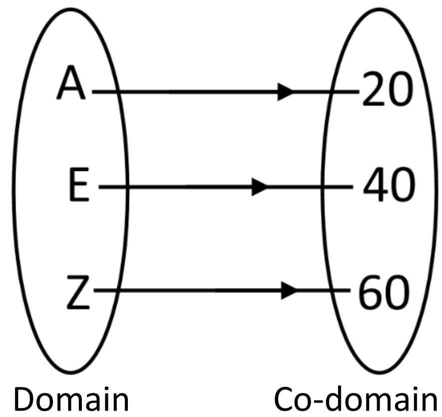
8. Ask them, 'Which has/have no relations?'
9. Agree that element 3 has no relation in the Co – domain and the elements b and e in the Co – domain have no relation in the domain.

Types of Relations

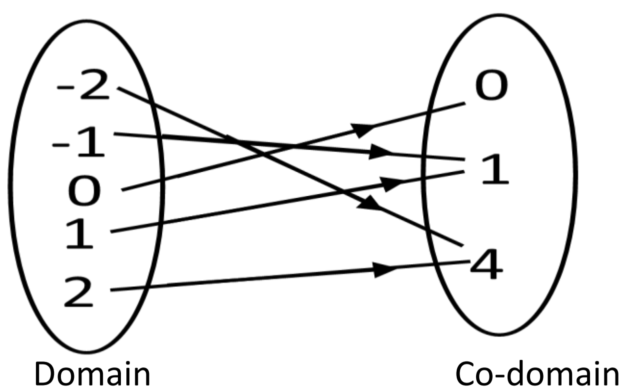
10. Ask participants to name and give examples of the different types of relations

11.There are four different types of relations.

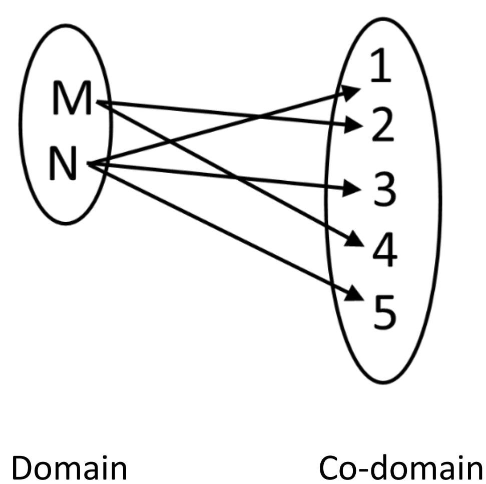
- ONE – TO – ONE



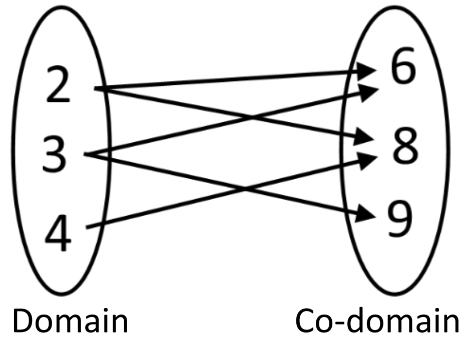
- MANY – TO – ONE



- ONE – TO – MANY



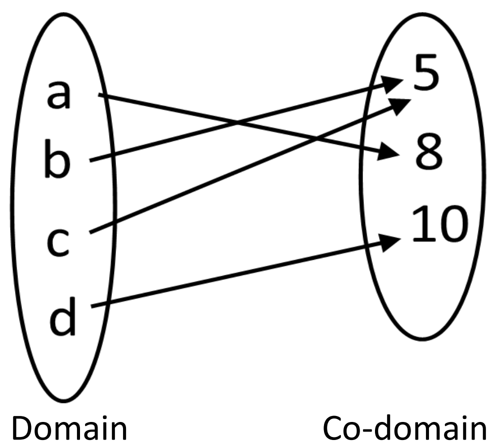
- MANY – TO – MANY



Activity 2 **Mappings** **25 minutes**

1. Ask participants to define mapping
2. Say: A mapping is a relation in which every element in the domain is related or mapped to only one element in the Co – domain and every element in the Co – domain has a relation with an element in the domain.

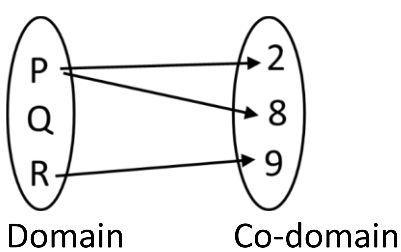
e.g.



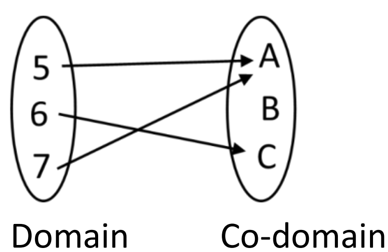
3. Give Exercise 1 to participants in pairs to state whether each of the relations is a mapping or not and give reasons for your answers.

<u>Exercise 1</u>

i



ii



iii (0,4), (1,7), (1,5), (2,9) iv (4,2), (9,3), (16,4), (25,5)

Answers:

i) is not a mapping because the element P in the domain is related to two elements in the co-domain and element Q in the domain has no relation in the co-domain.

ii) is not a mapping because the element B in the co-domain has no relation with elements in the domain.

iii) is not a mapping because the element 1 in the domain is related to two elements (5 and 7) in the co- domain.

iv) is a mapping because every element in the domain is related to only one element in the co-domain

How to describe a mapping

4. Ask participants to name the ways mappings are described, and take quick answers from around the room
5. Say there are different ways of describing a mapping. Below are some of the ways:

i. Mapping diagrams as shown in the examples above.

ii. Set of ordered pairs
e.g. (0, 2), (1, 6), (2, 9)

iii. Algebraic relations stating the rule or formula
e.g. $y = 4x + 2$ $or\ x \rightarrow 3x + 5$
$or\ f(x) = 3x + 5$

iv. Graphs $y = 3x + 5$

Activity 3 Finding the rules of mappings 20 minutes

1. Discuss with participants the different methods used in finding the rule for a mapping.
2. You can describe a mapping by stating its rule. The rule serves as a definition.
3. The rule can be found by:

i. Inspection

ii. The method of constant difference for linear mappings and

iii. The method of ratio for exponential mappings

Method of inspection

- Find the rules for the mappings:

a. 1 2 3 4

↓ ↓ ↓ ↓

2 4 6 8

$x \rightarrow 2x$

b. 1 2 3 4

↓ ↓ ↓ ↓

3 5 7 9

$x \rightarrow 2x + 1$

Method of constant difference

- A mapping is said to be linear if the difference between consecutive terms of both the domain and co – domain is constant.

The rule is $y = ax + b$ for linear mappings.

$a = \frac{constant\ difference\ of\ Co-domain}{constant\ difference\ of\ domain} = \frac{\Delta y}{\Delta x}$. b can be found by putting an element and its image in the rule.

E.g. Find the rule for the mapping

1 2 3 4

↓ ↓ ↓ ↓

2 4 6 8

$a = \frac{\Delta y}{\Delta x} = \frac{2}{1} = 2$

$y = 2x + b$

Substitute the element 2 and its image into the rule to get b

$4 = 4 + b \; \rightarrow b = 0$

The rule is therefore $y = 2x$

Exercise 2

A. Find the rule for the following mappings:

a. 1 2 3 4

↓ ↓ ↓ ↓

6 12 18 24

b. 1 2 3 4

↓ ↓ ↓ ↓

5 7 9 10

c. 2 4 6 8

↓ ↓ ↓ ↓

11 19 27 35

d. -1 0 1 2

↓ ↓ ↓ ↓

-2 3 8 13

e.

-2	-1	0	1	2
↓	↓	↓	↓	↓
11	7	3	-1	-5

B. Find the missing element represented by A or B in the mapping

a.

X	1	2	3	4	5
↓	↓	↓	↓	↓	↓
Y	10	9	8	7	A

b.

X	2	5	8	11
↓	↓	↓	↓	↓
y	8	23	38	B

Answers

A. a) $y = 6x$ b) $y = 2x + 3$ c) $y = 4x + 3$ d) $y = 5x + 3$ e) $y = -4x + 3$

B. a) 6 b) 53

iii. **Method of constant ratio for exponential mappings.**

4. Ask participants to discuss in pairs the meaning of exponential mapping. Get a couple of responses from volunteers
5. Say, 'A mapping is said to be exponential if the ratio between constant elements in the co – domain is constant.
6. Give participants in their groups a minute to explain how to find the rule for exponential mapping and take quick responses from them

7. The rule for such mappings is of the form $y = ar^{x-b}$, where a is the first term of the elements in the co – domain; b is the first term of the elements in the domain.
8. Discuss with participants how to find the rule for the following mapping:

i.

X	1	2	3	4
↓	↓	↓	↓	↓
Y	5	15	45	135

9. In this mapping, there is a constant ratio between consecutive terms of the co-domain, it is therefore an exponential mapping.

$$y = ar^{x-b}, \quad a = 5, r = 3, b = 1$$

The relation is therefore given by:

$$y = 5(3)^{x-1}$$

Exercise 3

Find the rule for the following mappings (participants in pairs):

i.

1	2	3
↓	↓	↓
-4	8	-16

ii.

X	-2	-1	0	1	2
↓	↓	↓	↓	↓	↓
Y	2	4	8	16	32

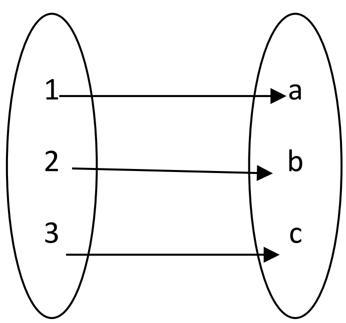
Answers

i) $y = -4(-2)^{x-1}$ ii) $y = 2(2)^{x+2}$

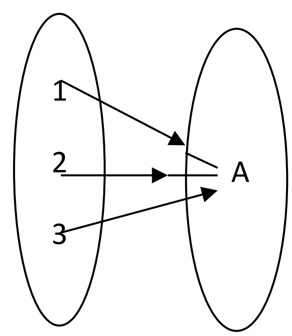
Activity 4 Functions 20 minutes

1. Discuss with participants the meaning of a function
2. Technical definition of a function is a relation from a set of inputs to a set of possible outputs where each input is related to exactly one output. It is often written as "f(x)" where x is the input value.
3. A function is a relation/mapping between two sets: the domain and the Co – domain, such that every element of the domain has only one element (image) in the Co – domain.
4. Ask groups to determine whether the following are functions or not and give reasons.

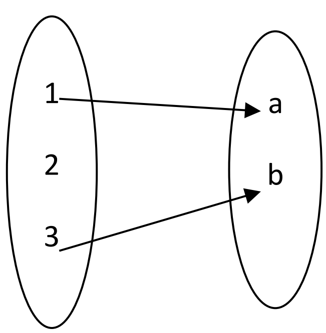
i. Domain Co-domain



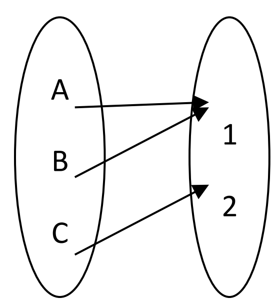
ii



iii.



iv.



v. (0, 1), (2, 6), (5, 9), (0, 12)

vi. Domain Co-domain

<u>**Answers**</u>

i) is a function: every element of domain is related to only one member of the co-domain.

ii) is a function: every element of domain is related to only one member of the co-domain.

iii) is not a function: the element 2 in the domain is not related to any member of the co-domain.

iv) is not a function: the element 0 has two images in the co-domain, 1 and 12.

v) is not a function: every member of the domain is related to two members of the co-domain.

How to find the range of a function

5. Get participants in pairs to explain how to find the range of a function given the domain. Take brief responses to clarify.
6. Give an example: Given the function $f(x) = 1 + 3x$, find the range of the domain (-1, 0, 1, 2)
7. Solve: To find the range, find the image of each element in the domain. The range is the same as the co-domain.

$f(-1) = 1 + 3(-1) = -2, \quad f(0) = 1 + 3(0) = 1, \; f(1) = 1 + 3(1) = 4,$
$f(2) = 1 + 3(2) = 7.$
The range is therefore $(-2, 1, 4, 7)$

Exercise 4

Ask participants to do this exercise on their own – walk around the room to see who can do it and help those who are struggling.

Then ask them to exchange books with a partner and compare answers. Where the answers are different ask them to discuss why they are different and come to one answer.

Ask for feedback from one or two pairs.

1. Write the following in function notation
   a. $p = 5t - 3$
   b. $m = 3x + 4$

   c. $g = \frac{4x+1}{3x-2}$
2. Given the function $f: x \rightarrow \frac{3x+3}{2x-1}$
a. Find the images of the following elements of the domain: {-2, -1, 0, 1}
b. What element of the domain corresponds to the following images

   i. 2 ii. $2\frac{1}{2}$

Answers

2. a $(-\frac{3}{5}, 0, -3, 6)$ b i) 5 ii) $\frac{11}{5}$

Summary **3 minutes**

- Ask participants to say the different techniques you used during the lesson to make sure that they were all involved in their learning.
- List them on the board, then ask participants to say how and why they would use that technique with their class.
- Go through the learning outcomes with the participants and thank the for their attention and participation

Session 3 Quadratic functions and equations 90 minutes

Learning outcomes

By the end of the session participants will be able to

- write a quadratic expression as a complete square, plus or minus a constant
- solve a quadratic equation by completing the square

Identify ways of involving participants in their learning

Materials

Chart 3.1 Learning Outcomes

Flash cards with equations from Exercise 1 written on them

Exercises 2, 3, 4 should be written in advance

Activity outline

Introduction		**2 minutes**
Activity 1	**Completing the squares**	**50 minutes**
Activity 2	**Quadratic formula**	**35 minutes**
Summary		**3 minutes**

Background to facilitator

In this session we consider how quadratic expressions can be written in an equivalent form using the technique known as completing the square. This technique has applications in a number of areas, but we will see an example of its use in solving a quadratic equation. In order to master the techniques explained here it is vital that you undertake plenty of practice exercises so that all this becomes second nature. To help you to achieve this, the unit includes a substantial number of such exercises.

Introduction: **2 minutes**

In this unit we will look at a process called completing the square. It can be used to write a quadratic expression in an alternative form. Later in the unit we will see how it can be used to solve a quadratic equation.

Activity 1: Completing the square **50 minutes**

A. Simple equations

1. Take participants through some simple equations
2. Get the participants involved as much as possible (through questioning, pair chats, group agreement) in solving the examples
3. Consider the quadratic equation x^2 = 9.
4. We can solve this by taking the square root of both sides: x = 3 or – 3, remembering that when we take the square root there will be two possible answers, one positive and one negative. This is often written in the briefer form x = ±3. This process for solving x^2 = 9 is very straightforward, particularly because:
   - 9 is a 'square number', or 'complete square'. This means that it is the result of squaring another number, or term, in this case the result of squaring 3 or −3.
   - x^2 is a complete square - it is the result of squaring x. So simply square-rooting both sides solves the problem.
5. Consider the equation x^2 = 5.
6. Again, we can solve this by taking the square root of both sides: x = $\sqrt{5}$ or – $\sqrt{5}$
7. In this example, the right-hand side of x^2 = 5, is not a square number. But we can still solve the equation in the same way. It is usually better to leave your answer in this exact form, rather than use a calculator to give a decimal approximation.
8. Suppose we wish to solve the equation $(x-7)^2$ = 3
9. Again, we can solve this by taking the square root of both sides. The left-hand side is a complete square because it results from squaring x − 7.

x − 7 = $\sqrt{3}$ or – $\sqrt{3}$

By adding 7 to each side we can obtain the values for x:

x = 7 + $\sqrt{3}$ or 7 − $\sqrt{3}$

10. We could write this in the briefer form x = 7 ± $\sqrt{3}$.
11. Suppose we wish to solve $(x + 3)^2$ = 5
12. Again the left-hand side is a complete square. Taking the square root of both sides: x + 3 = $\sqrt{5}$ or − $\sqrt{5}$ By subtracting 3 from each side we can obtain the values for x: x = −3 + $\sqrt{5}$ or − 3 − $\sqrt{5}$
13. Assign one of the problems below (on flash cards) to each group to solve
14. Instruct them to pass the flash card to another group when they are done
15. Ask them to attempt as many problems as possible in 3 minutes

Exercise 1

Solve the following quadratic equations

a) x^2 = 25;
b) x^2 = 10;
c) x^2 = 2;
d) $(x + 1)^2$ = 9;
e) $(x + 3)^2$ = 16;
f) $(x - 2)^2$ = 100;
g) $(x - 1)^2$ = 5;
h) $(x + 4)^2$ = 2

B. The basic technique

1. Now suppose we wanted to try to apply the method used in the three previous examples to x^2+ 6x = 4. In each of the previous examples, the left-hand side was a complete square. This means that in each case it took the form $(x + a)^2$ or $(x - a)^2$.
2. This is not the case now and so we cannot just take the square-root. What we try to do instead is rewrite the expression so that it becomes a complete square - hence the name completing the square.
3. Observe that complete squares such as $(x + a)^2$ or $(x - a)^2$ can be expanded as follows:

Key point

complete squares: $(x + a)^2$ = (x + a)(x + a) = x^2+2ax + a^2

$(x - a)^2$ = (x − a)(x − a) = x^2 − 2ax + a^2

4. We will use these expansions to help us to complete the square in the following examples.
5. Consider the quadratic expression x^2+ 6x − 4
6. We compare this with the complete square x^2+2ax + a^2
7. Clearly the coefficients of x^2 in both expressions are the same. We would like to match up the term 2ax with the term 6x. To do this note that 2a must be 6, so that a = 3. Recall that $(x + a)^2$ = x^2+2ax + a^2

   Then with a = 3

   $(x + 3)^2$ = x^2+6x + 9

8. This means that when trying to complete the square for x^2+ 6x – 4, we can replace the first two terms, x^2+ 6x, by $(x + 3)^2$ – 9.
9. So x^2+6x − 4 = by $(x + 3)^2$ − 9 − 4 = by $(x + 3)^2$− 13 We have now written the expression x^2+6x − 4 as a complete square plus or minus a constant. We have completed the square. It is important to note that the constant term, 3, in brackets is half the coefficient of x in the original expression.
10. Suppose we wish to complete the square for the quadratic expression x^2- 8x + 7. We want to try to rewrite this so that it takes the form of a complete square plus or minus a constant. We compare x^2- 8x + 7 with the standard form x^2- 2ax + a^2
11. The coefficients of x^2 are the same. To make the coefficients of x the same we must choose **a** to be 4.

    Recall that $(x - a)^2$ = x^2 − 2ax + a^2

    Then with a = 4, $(x - 4)^2$ = x^2 − 8x + 16

    This means that when trying to complete the square for x^2 − 8x +7 we can replace the first two terms, x^2 − 8x, by $(x - 4)^2$ − 16. So x^2 − 8x +7 = $(x - 4)^2$− 16 + 7 = $(x - 4)^2$− 9

12. We have now written the expression x^2 − 8x +7 as a complete square plus or minus a constant. We have completed the square. Again note that the constant term, −4, in brackets is half the coefficient of x in the original expression.
13. Suppose we wish to complete the square for the quadratic expression x^2 + 5x + 3. This means we want to try to rewrite it so that it has the form of a complete square plus or minus a constant.

14. In the examples we have just worked through we have seen how this can be done by comparing with the standard forms $(x+a)^2$ and $(x-a)^2$.
15. We would like to be able to 'complete the square' without writing down all the working we did in the previous examples.
16. The key point to remember is that the number in the bracket of the complete square is half the coefficient of x in the quadratic expression.
17. So with x^2 + 5x + 3 we know that the complete square will be $(x+\frac{5}{2})^2$. This has the same x^2 and x terms as the given quadratic expression but the constant term is different.
18. We must balance the constant term by a) subtracting the extra constant that our complete square has introduced, that is $(\frac{5}{2})^2$, and b) putting in the constant term from our quadratic, that is 3.
19. Putting this together we have x^2 + 5x + 3 = $(x+\frac{5}{2})^2 - (\frac{5}{2})^2$ + 3
20. To finish off we just combine the two constants $-(\frac{5}{2})^2$ + 3 = $-\frac{25}{4}+\frac{12}{4}$ = - $\frac{13}{4}$

    And so x^2 + 5x + 3 = $(x+\frac{5}{2})^2 - \frac{13}{4}$

21. We have now written the expression x^2 + 5x + 3 as a complete square plus or minus a constant. We have completed the square. Again note that the constant term, $\frac{5}{2}$, in brackets is half the coefficient of x in the original expression. The explanation given above is really just an outline of our thought process; when we complete the square in practice we would not write it all down. We would probably go straight to equation (1). The ability to do this will come with practice.

Exercise 2 (As in Exercise 1)

- Ask the participants to work in pairs to answer the following questions
- Ask participants to raise their hand if they feel really confident with the method and would be happy to explain it to someone else.
- Pair someone with their hands up with someone who did not raise their hand and ask themt o work together to answer the questions.
- Move around the room and support individuals who are finding it hard or are showing that they have misunderstood the method.

Complete the square for the following quadratic expressions

a) x^2 + 2x + 2;
b) x^2 + 2x + 5;
c) x^2 + 2x – 1;
d) x^2+ 6x + 8;
e) x^2 − 6x + 8;
f) x^2+ x + 1;
g) x^2– x – 1;
h) x^2+ 10x – 1;
i) x^2+ 5x + 4;
j) x^2+ 6x + 9;
k) x^2– 2x + 6;
l) x^2– 3x + 1

C. Cases in which the coefficient of x^2 is not 1.

1. Ask if anyone is confident to explain this case to the group.
2. Put the participants in groups and assign one confident person to each group to explain the method.
3. Move around the groups and check that they are explaining it accurately and support them if they are not.
4. We now know how to complete the square for quadratic expressions for which the coefficient of $x^2$is 1. When faced with a quadratic expression where the coefficient of x^2 is not 1 we can still use this technique but we put in an extra step first - we factor out this coefficient.
5. Suppose we wish to complete the square for the expression 3x^2 − 9x + 50. We begin by factoring out the coefficient of x^2, in this case 3. It does not matter that 3 is not a factor of 50; we can still do this by writing the expression as 3 $(x^2 - 3x + \frac{50}{3})$
6. Now the expression in brackets is a quadratic with coefficient of x^2 equal to 1 and so we can proceed as before. The number in the complete square will be half the coefficient of x, so we will use $(x + \frac{3}{2})^2$. Then we must balance up the constant term just as we did before by subtracting the extra constant we have introduced, that is $(\frac{3}{2})^2$, and putting in the constant from the quadratic expression, that is $\frac{50}{3}$.

$$3\{x^2 - 3x + \frac{50}{3}\} = 3\{(x + \frac{3}{2})^2 - \left(\frac{3}{2}\right)^2 + \frac{50}{3}\}$$

7. The arithmetic to tidy up the constants is a bit messy: $-\left(\frac{3}{2}\right)^2 + \frac{50}{3} = -\frac{9}{4} + \frac{50}{3} = -\frac{27}{12} + \frac{200}{12} = \frac{173}{12}$
8. So, putting all this together $x^2 - 3x + \frac{50}{3} = (x + \frac{3}{2})^2 + \frac{173}{12}$
9. And finally, $3(x^2 - 3x + \frac{50}{3}) = 3\{(x + \frac{3}{2})^2 + \frac{173}{12}\}$ and we have completed the square.
10. This is the 'completing the square' form for a quadratic expression for which the coefficient of x^2 is not 1.

- Remove the confident person from each group and put them in a group together.
- Ask each group to work together to answer the sums in Exercise 3
- Ask groups to swap books and check each other's answers

Exercise 3

Completing the square for the following quadratic expressions

a) $2x^2 + 4x - 8$
b) $5x^2 + 10x + 15$
c) $3x^2 - 27x + 9$
d) $2x^2 + 6x + 1$
e) $3x^2 - 12x + 2$
f) $15 - 10x - x^2$
g) $24 + 12x - 2x^2$
h) $9 + 6x - 3x^2$

Summary of the process

It will be useful if you can get used to doing this process automatically. The method can be summarised as follows:

Key point

1. factor out the coefficient of x^2 - then work with the quadratic expression which has a coefficient of x^2 equal to 1

2. check the coefficient of x in the new quadratic expression and take half of it - this is the number that goes into the complete square bracket

3. balance the constant term by subtracting the square of the number from step 2, and putting in the constant from the quadratic expression

4. the rest is arithmetic that may often involve fractions

D. Solving a quadratic equation by completing the square

1. Let us return now to a problem posed earlier. We want to solve the equation x^2 + 6x = 4.
2. We write this as x^2 + 6x − 4 = 0. Note that the coefficient of x^2 is 1 so there is no need to take out any common factor. Completing the square for quadratic expression on the left-hand side:

$x^2 + 6x - 4 = 0$

$(x + 3)^2 - 9 - 4 = 0$ (1)

$(x + 3)^2 - 13 = 0$ (2)

$(x + 3)^2 = 13$

$x + 3 = \pm\sqrt{13}$

$x = -3 \pm \sqrt{13}$

3. We have solved the quadratic equation by completing the square. To produce equation (1) we have noted that the coefficient of x in the quadratic expression is 6 so the number in the 'complete square' bracket must be 3; then we have balanced the constant by subtracting the square of this number, 3^2, and putting in the constant from the quadratic, −4. To get equation (2) we just do the arithmetic which in this example is quite straightforward.

Exercise 4

Ask participants to work with a partner to discuss and answer these questions

If necessary pair someone who can do it easily with someone you have identified as finding it difficult

Use completing the square to solve the following quadratic equations

a) $x^2 + 4x - 12 = 0$;
b) $x^2 + 5x - 6 = 0$;
c) $10x^2 + 7x - 12 = 0$;
d) $x^2 + 4x - 8 = 0$;
e) $10 + 6x - x^2 = 0$
f) $2x^2 + 8x - 25 = 0$

Give your answers either as fractions or in the form p ± $\sqrt{q}$

Answers

1. a) ±5 b) ± $\sqrt{10}$ c) ± $\sqrt{2}$ d) 2, -4 e) 1, -7
f) 12, -8 g) 1 ± $\sqrt{5}$ h) −4 ± $\sqrt{2}$

2. a) $(x+1)^2$ + 1 b) $(x+1)^2$ + 4 c) $(x+1)^2$ − 2
d) $(x+3)^2$ − 1 e) $(x-3)^2$ − 1 f) $(x+\frac{1}{2})^2+\frac{3}{4}$
$g)$ $(x-\frac{1}{2})^2-\frac{5}{4}$ h) $(x+5)^2$ − 26 i) $(x+\frac{5}{2})^2+\frac{9}{4}$
j) $(x+3)^2$ k) $(x-1)^2$+ 5 l) $(x-\frac{3}{2})^2-\frac{5}{4}$

3. a) 2 [$(x+1)^2$ – 5} b) 5 [$(x+1)^2$ + 2] c) 3 [$(x-\frac{9}{2})^2-\frac{69}{4}$]
d) 2 $\{(x+\frac{3}{2})^2-\frac{7}{4}\}$ e) 3 $\{(x-2)^2-\frac{10}{3}\}$ f) – $\{(x+5)^2$ – 40}
g) −2 $\{(x-3)^2$ – 21} h) −3 $\{(x-1)^2$ – 4

4. a) 2, −6 b) 1, −6 c) $-\frac{3}{2},\frac{4}{5}$ d) −2 ± $\sqrt{12}$ e) 3 ± $\sqrt{19}$
f) −2 ± $\sqrt{\frac{33}{2}}$

Activity 2: **Quadratic formula** **35 minutes**

Involve the participants as much as possible by asking open and close ended questions to individuals, pairs or groups.

1. This activity explores solving quadratic equations using a formula
2. Consider the general quadratic equation ax2 + bx + c = 0.
3. There is a formula for solving this: x = $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. It is so important that you should learn it.

Key point

Formula for solving $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. We will illustrate the use of this formula in the following example.
5. Suppose we wish to solve $x^2 - 3x - 2 = 0$. Comparing this with the general form $ax^2 + bx + c = 0$ we see that a = 1, b = −3 and c = −2.
6. These values are substituted into the formula. $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - (4.1.-2)}}{2.1}$

$$= \frac{3 \pm \sqrt{9+8}}{2} = \frac{3 \pm \sqrt{17}}{2}$$

7. These solutions are exact.
8. Suppose we wish to solve $3x^2 = 5x - 1$.
9. First we write this in the standard form as $3x^2 - 5x + 1 = 0$ in order to identify the values of a, b and c. We see that a = 3, b = −5 and c = 1.
10. These values are substituted into the formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - (4.3.1)}}{2.3} = \frac{5 \pm \sqrt{25-12}}{6} = \frac{5 \pm \sqrt{13}}{6}$$

11. Again there are two exact solutions. Approximate values could be obtained using a calculator.

Exercise 5

Use the quadratic formula to solve the following quadratic equations.

a) $x^2 - 3x + 2 = 0$

b) $4x^2 - 11x + 6 = 0$

c) $x^2 - 5x - 2 = 0$

d) $3x^2 + 12x + 2 = 0$

e) $2x^2 = 3x + 1$

f) $x^2 + 3 = 2x$

g) $x^2 + 4x = 10$

h) $25x^2 = 40x - 16$

Answers

a) 1, 2 b) 2, $\frac{3}{4}$ c) $\frac{5\pm\sqrt{33}}{2}$ d) $\frac{-12\pm\sqrt{120}}{6}$ e) $\frac{3\pm\sqrt{17}}{4}$ f) No real roots g) $-2 \pm \sqrt{14}$ h) $\frac{4}{5}$ repeated.

Session summary **3 minutes**

Ask participants to identify the ways in which you involved them in their learning during the session. Add these methods to the list you made in the previous session.
Ask participants to think about how they could be applied in their lessons back at school
Take participants through the learning outcomes again.

Materials

Chart 3.1 Learning outcomes
Flash Cards with equations from Exercise 1 written on them:

a) $x^2 = 25$; b) $x^2 = 10$; c) $x^2 = 2$; d) $(x+1)^2 = 9$; e) $(x+3)^2 = 16$; f) $(x-2)^2 = 100$; g) $(x-1)^2 = 5$; h) $(x+4)^2 = 2$

Exercise 2

a) $x^2 + 2x + 2$; b) $x^2 + 2x + 5$; c) $x^2 + 2x - 1$; d) $x^2 + 6x + 8$; e) $x^2 - 6x + 8$; f) $x^2 + x + 1$; g) $x^2 - x - 1$; h) $x^2 + 10x - 1$; i) $x^2 + 5x + 4$; j) $x^2 + 6x + 9$; k) $x^2 - 2x + 6$; l) $x^2 - 3x + 1$

Exercise 3

a) $2x^2 + 4x - 8$
b) $5x^2 + 10x + 15$
c) $3x^2 - 27x + 9$
d) $2x^2 + 6x + 1$
e) $3x^2 - 12x + 2$
f) $15 - 10x - x^2$
g) $24 + 12x - 2x^2$
h) $9 + 6x - 3x^2$

Exercise 4

a) $x^2 + 4x - 12 = 0$; b) $x^2 + 5x - 6 = 0$; c) $10x^2 + 7x - 12 = 0$; d) $x^2 + 4x - 8 = 0$; e) $10 + 6x - x^2 = 0$ f) $2x^2 + 8x - 25 = 0$

Exercise 5

a) $x^2 - 3x + 2 = 0$ b) $4x^2 - 11x + 6 = 0$ c) $x^2 - 5x - 2 = 0$ d) $3x^2 + 12x + 2 = 0$ e) $2x^2 = 3x + 1$ f) $x^2 + 3 = 2x$ g) $x^2 + 4x = 10$ h) $25x^2 = 40x - 16$

Session 4 Special Angles 90 minutes

Learning outcomes

By the end of the session, participants will be able to

- derive and apply the trigonometric ratios of some special angles,
- define and interpret bearings in terms of
  - I. Angles in three digits
  - II. Compass direction
- Solve bearing related problems.
- Identify different ways of involving participants in their learning

Materials

Chart 4.1	Learning Outcomes
Chart 4.2	Figures 1, 2 & 3
Chart 4.3	Table for special angles
Chart 4.4	Figures 4 & 5
Chart 4.5	Fig 6
Chart 4.6	Fig 7
Chart 4.7	Fig 8
Chart 4.8	Fig 9

Activity Outline

Introduction	**2 minutes**
Activity 1: Deriving trigonometric ratios of some special angles	**30 minutes**
Activity 2: Applications of trigonometric ratios of special angles	**25 minutes**
Activity 3: Meaning of bearing and its representation	**30 minutes**
Summary	**3 minutes**

Important note to facilitators

Understanding of basic concepts is the key to learning mathematics.

Facilitators are therefore encouraged to spend more time in discussing basic concepts in every activity and tell them to engage the teachers more when they are facilitating, rather than teaching or lecturing.

The only way you will understand the concepts or techniques of solving problems in mathematics is to do a lot of practice. Facilitators are therefore encourage to tell teachers to **constantly** refer students to their handbooks at the end of each lesson.

Session introduction **2 minutes**

1. Tell participants they are already familiar with the construction of some special angles.
2. Ask them to name some of these special angles.
3. Agree: $30°, 45°, 60°, 90°$
4. Tell them in this session, we are going to be looking at how to derive values for these special angles including 0.

Review

5. Ask participants to work in pairs and write down the names and definitions of the basic trigonometric ratios.
6.

$\text{Sine} = \frac{\text{Opp}}{\text{hyp}}$, Cosine$= \frac{adj}{hyp}$, Tangent$= \frac{opp}{adj}$, Cotangent$= \frac{adj}{opp}$

Cosecant$= \frac{hyp}{opp}$, Secant$= \frac{hyp}{adj}$

Activity 1: Deriving trigonometric ratios of some special angles **30 minutes**

Deriving Trigonometric Ratios of 45° *:*

1. Display Chart 4.2 (figures 1, 2 & 3)

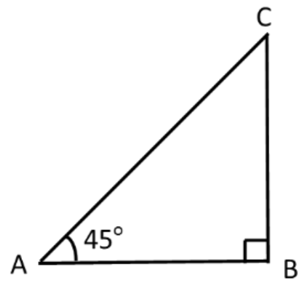


Fig. 1

2. From fig 1, ask participants in pairs to find the value of the remaining $< C$ and what that means.

From ΔABC right-angled at B, if $< A = 45°$, then $< C = (90 - 45)° = 45°$. This means $|AB| = |BC|$. If $|AB| = x$, then $|BC| = x$.

3. Ask participants how to find |AC| knowing |AB|=|BC|

Agree that we apply Pythagoras theorem to find |AC|.

$|AC|^2 = |AB|^2 + |BC|^2$, that is, $|AC|^2 = x^2 + x^2 = 2x^2$, That is $|AC| = x\sqrt{2}$.

4. From fig 1, knowing all of the three sides of ΔABC, ask participants how to find the trigonometric ratios of $45°$. Give pairs 2 minutes to do this

From ΔABC, $\sin A = \sin 45° = \frac{BC}{AC} = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}}$, Similarly, $\cos 45° = \frac{AB}{AC} = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}}$,

$\tan 45° = \frac{BC}{AB} = \frac{x}{x} = 1$, $\cot 45° = \frac{AB}{BC} = \frac{x}{x} = 1$

$\csc 45° = \frac{AC}{BC} = \frac{x\sqrt{2}}{x} = \sqrt{2}$, $\sec 45° = \frac{AC}{AB} = \frac{x\sqrt{2}}{x} = \sqrt{2}$

Deriving Trigonometric Ratios of $30°$ and $60°$:

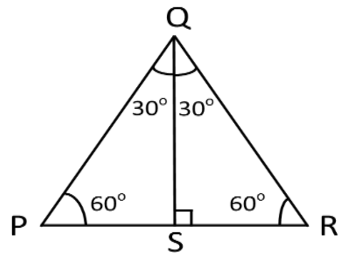


Fig. 2

1. Point to Figure 2.
2. Ask participants how they will derive trig ratios for $30°$ and $60°$.
3. By considering the equilateral ΔPQR, all of the three sides are equal and all of the three angles are equal, each measuring $60°$.
4. By drawing a perpendicular from $< Q$ to PR, a right-angled triangle at S is obtained.

5. Ask participants how to get the three sides of the ΔPQS so as to derive the trig ratios of $30°$ and $60°$:
6. Since ΔPQR is equilateral, all of the sides are equal, let each side be equal to 2a.
7. From the right-angled triangle PQS, PQ=2a, PS= a, then QS is obtained by using Pythagoras theorem:
   $QS^2 = PQ^2 - PS^2 = (2\text{a})^2 - a^2$
   $\text{QS}^2 = 3\text{a}^2$, then $QS = a\sqrt{3}$

8. Ask participants how to derive the trig ratios of $30°$ and $60°$, knowing the three sides of ΔPQS.
9. Using the definitions of the six basic trigonometric ratios, let participants in pairs write the 6 trig ratios for $30°$:
   From ΔPQS in fig 2,
   $\sin 30° = \frac{\text{PS}}{\text{PQ}} = \frac{a}{2a} = \frac{1}{2}$, $\cos 30° = \frac{QS}{PQ} = \frac{\text{a}\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$
   $\tan 30° = \frac{PS}{QS} = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$ $\cot 30° = \frac{QS}{PS} = \frac{\text{a}\sqrt{3}}{a} = \sqrt{3}$
   $\csc 30° = \frac{PQ}{PS} = \frac{2\text{a}}{a} = 2$ $\sec 30° = \frac{PQ}{QS} = \frac{2\text{a}}{\text{a}\sqrt{3}} = \frac{2}{\sqrt{3}}$

10. Similarly, the trigonometric ratios for $60°$ are obtained by using the definitions of the ratios in ΔPQS.

$\sin 60° = \frac{\text{QS}}{\text{PQ}} = \frac{\text{a}\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{PS}{PQ} = \frac{\text{a}}{\text{2a}} = \frac{1}{2}$

$\tan 60° = \frac{QS}{PS} = \frac{\text{a}\sqrt{3}}{a} = \sqrt{3}$

$\cot 60° = \frac{PS}{QS} = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\csc 60° = \frac{PQ}{QS} = \frac{\text{2a}}{\text{a}\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\sec 60° = \frac{PQ}{PS} = \frac{\text{2a}}{a} = 2$

Deriving Trigonometric Ratios of $0°$ and $90°$:

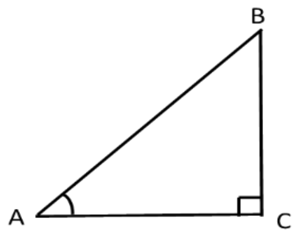


Fig. 3

1. Point to Figure 3.
2. Tell participants to consider the right –angled triangle, ABC, and to explain what happens when $< A$ gets smaller and smaller until it becomes $0°$.
3. The side BC decreases until it eventually becomes 0 and the side AB will be the same as AC.
4. Ask participants to explain how to derive the trigonometric ratios of $0°$ when the $< A$ becomes 0.
5. Use the definitions of the six basic trigonometric ratios.
6. From fig 3, when $< A$ is $0°$, the side AB=AC, and BC=0.

$\sin 0° = \frac{\text{BC}}{\text{AB}} = \frac{0}{\text{AB}} = 0.$

$\cos 0° = \frac{AC}{AB} = \frac{AB}{AB} = 1$

$\tan 0° = \frac{BC}{AC} = \frac{0}{AC} = 0$

$\cot 0° = \frac{AC}{BC} = \frac{AC}{0} = \infty$

$\csc 0° = \frac{AB}{BC} = \frac{\text{AB}}{0} = \infty$

$\sec 0° = \frac{AB}{AC} = \frac{\text{AC}}{AC} = 1$

7. Ask participants to explain what happens, when $< A$ increases and increases until it is $90°$.
8. The side BC increases until it is equal to AB and AC becomes 0.
9. Ask participants to explain how to derive the trigonometric ratios of $90°$ under this situation.
10. Use the definitions of the basic six trigonometric ratios.
11. From fig 3, when $< A$ is $90°$, the side BC=AB, and AC=0.

$\sin 90° = \frac{\text{BC}}{\text{AB}} = \frac{\text{AB}}{\text{AB}} = 1.$

$\cos 90° = \frac{AC}{AB} = \frac{0}{AB} = 0$

$\tan 90° = \frac{BC}{AC} = \frac{\text{AB}}{0} = \infty$

$\cot 90° = \frac{AC}{BC} = \frac{0}{BC} = 0$

$\csc 90° = \frac{AB}{AB} = \frac{\text{AB}}{\text{AB}} = 1$

$\sec 90° = \frac{AB}{AC} = \frac{\text{AB}}{0} = \infty$

Activity 2: Applications of trig ratios of some special angles **25 minutes**

1. Fill in the table completely from the derived values of the special angles given. Assign one special angle to each member of a group to complete the trig ratios. Allow them time to run their responses with a partner then with the whole group before completing their table on chart paper

	$0°$	$30°$	$45°$	$60°$	$90°$
sin					
cos					
tan					
cot					
sec					
cosec					

2. Write the following question on the board and lead the way in solving it:

Without using calculators or log tables find the value of $\cos 45° + \sin 60°$, leaving your answer in surd form

Solution:
Ask participants to give the values of $\sin 45°$ and $\cos 60°$, then substitute these values into

$$\cos 45° + \sin 60° = \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} = \frac{2+\sqrt{6}}{2\sqrt{2}}$$

Group Work
Arrange the participants into groups and assign each of the following questions to a group leaving answers in surd form:
1. $\sin 30° + \cos 30°$
2. $1 + \sec 30° + \cot 60°$
3. $\sin 60° \cos 30° + \csc 60° + \cot 30°$
4. $2\tan^2 30° + \cot 30° - \sin 60°$
5. $\frac{3\tan 45°}{1+\csc 60°}$
6. A man $1.5m$ tall stands $20m$ away from the foot of a tree and observes the angle of elevation of the top of the tree to be $60°$. How high is the tree leaving your answer in surd form.

Answers
1). $\frac{1+\sqrt{3}}{2}$ 2). $\frac{3+\sqrt{3}}{\sqrt{3}}$ 3). $\frac{11+4\sqrt{3}}{4}$ 4). $\frac{4+3\sqrt{3}}{6}$ 5). $\frac{3\sqrt{3}}{2+\sqrt{3}}$ 6). $(1.5 + 20\sqrt{3})\ m$

Activity 3: Meaning of bearing and its representation **30 minutes**

1. Discuss with the participants, the meaning of bearing.
2. Display Chart 4.4 (figures 4 & 5)
3. The <u>true bearing</u> to a point is the angle measured in degrees in a clockwise direction from the North Pole to the line joining the centre of the compass with the point

Note: Three figures are used to give bearings.

Fig 4

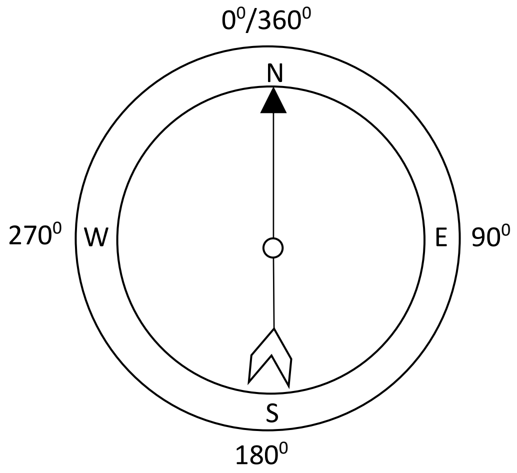
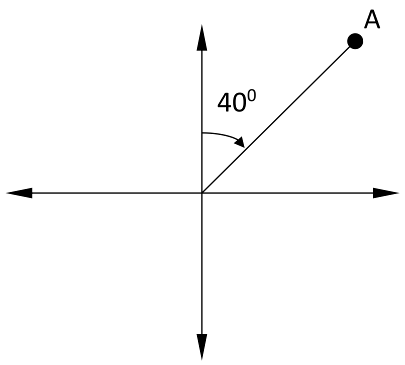


Fig 5

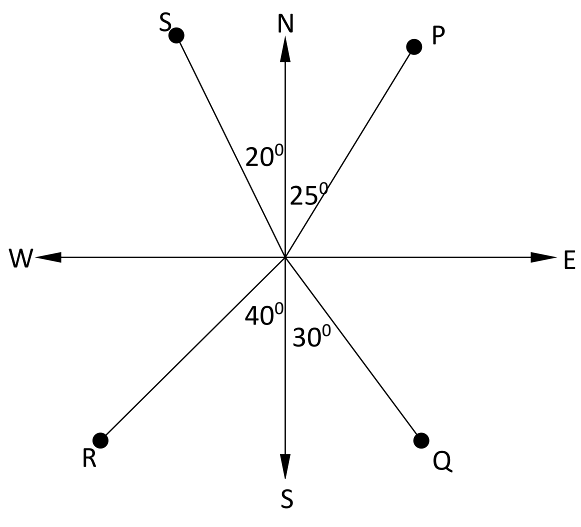


The bearing of A is 040^0.

Example B1

4. Display Chart 4.5 (fig 6)

Fig 6



Arrange the participants in groups and assign each group to give the bearing of one of the points PQRS.

Answers

(i) Measure from the North, the line OP makes an angle of 25^0. So the bearing of the point P is 025^0

(ii) Measure from the North, the line OQ makes an angle of $(180 – 30)^0$. So the bearing of the point Q is 150^0.

(iii) Measure from the North, the line OR makes an angle of $(180 + 40)^0$. So the bearing of the point R is 220^0.

(iv) Measure from the North, the line OS makes an angle of $(360 - 20)^0$. So the bearing of the point S is 340^0.

5. Ask participants, if there are other ways of representing bearings
6. The conventional bearing simply called **DIRECTION** is another way of representing bearings
7. The conventional bearing of a point is stated as the number of degrees east or west of the north – south line.
8. To state the direction of a point, write

(i) N or S which is determined by the angle being measured

(ii) The angle between the north or south line and the point, measured in degrees.

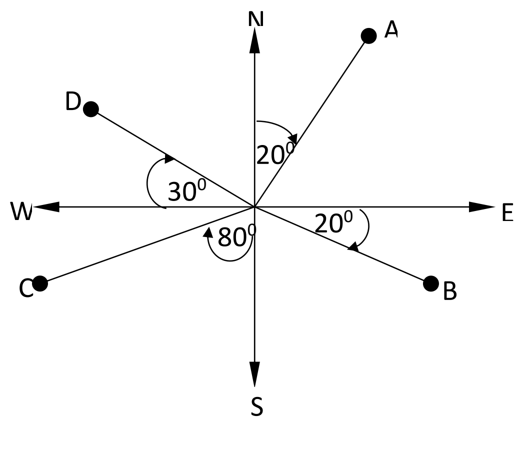
(i) E or W which is determined by the location of the point relative to the north – south line.

Example B2

9. Write the directions of the various points on the diagram

Display Chart 4.6 (fig 7)

Fig 7



10. Discuss with participants how to write the direction of the points ABCD.
11. Ask for volunteers to give the answers.

Answers

(I) The direction of A is $N20^0E$

(II) The direction of B is S(<SOB)E

= S(90 – 20)E = $S70^0E$

(iii) The direction of C is $S80^0W$

(iv) The direction of D is N(<NOD)W

= N $(90 - 30)^0$ W

= $N60^0W$

Activity 4: Bearing of a point relative to another point.

1. Discuss with participants that bearings are used to represent the direction of one point relative to another point.

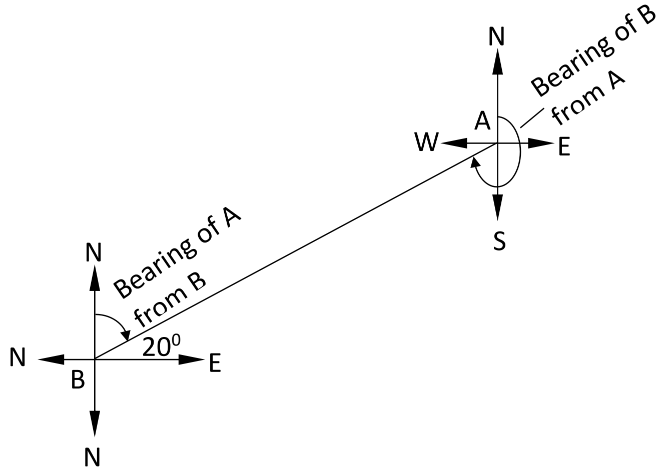
Example B3

2. Display Chart 4.7 (fig 8)
3. From the diagram, ask participants to write the bearing of

(a) A from B

(b) B from A

Fig 8



(a) From the diagram, the bearing of A from B is the angle $N\hat{B}A$.

$N\hat{B}A = 90 – 20 = 70^0$

Therefore the bearing of A from B is 070^0

(b) The bearing of B from A is 180 + $B\hat{A}S$

Angle B$\hat{A}S$ = 70^0 fig 9

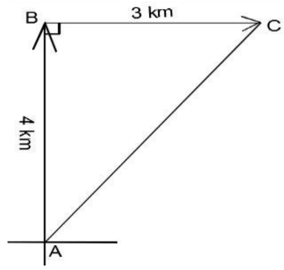
Therefore the bearing of B from A is 180+70 = 250^0

Example B4

4. Write the following problem on the board:

Hawa walked 4 km from point A to B in the north direction, then 3 km from point B to C in the east direction. Draw a diagram and find the bearing from point A to point C.

5. Ask participants to work with seatmates to draw the diagram.



6. Invite a volunteer to draw it on the board and allow others to compare with theirs

7. Solve the problem with the participants together on the board:

8. The direction of A to C is the angle $< A$ in ΔBAC.

To find the angle A, use the trig ratio, $\tan A = \frac{3}{4} = 0.75$

$A = \tan^{-1}(0.75)$
$< \mathrm{A} = 36.87°$

The bearing of A to C or C from A is $\mathbf{036.87}°$ or $\mathbf{N36.87°E}$.

Summary **3 minutes**

Review the Learning Outcomes and answer any questions from participants.

Session 5: Circle Theorems, Pyramids & Prisms 90 minutes

Learning outcomes

By the end of the session, participants will be able to:

- state the eight basic circle theorems and apply them to circle related problems.
- find the total surface area of pyramids
- find the volume of pyramids.
- find the total surface area and volume of prisms.

Materials

Chart 5.1	Learning outcomes
Chart 5.2	Figure1 (circle theorems)
Chart 5.3	Figure 2 (questions on circle)
Chart 5.4	Figure 3 (pyramid diagrams)
Chart 5.5	Figure 4 (pyramid diagrams)
Chart 5.6	Figure 5 (pyramid diagrams)
Chart 5.7	Figure 6 (cuboid)

Activity Outline		**Time (minutes)**
Introduction		**2**
Activity 1:	**Stating the eight basic theorems of circle**	**25**
Activity 2:	**Finding the surface area of a pyramid.**	**25**
Activity 3:	**Finding the volume of a pyramid.**	**20**
Activity 4:	**Finding the total surface area and volume of prisms.**	**15**
Summary		**3**

Session introduction **2 minutes**

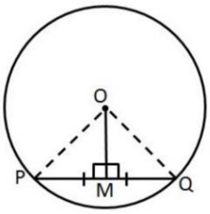
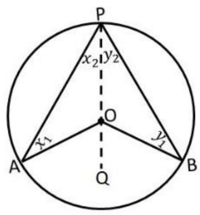
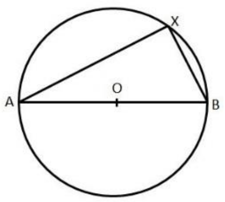
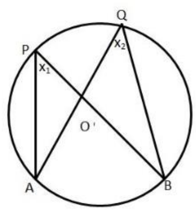
- Tell participants they are already familiar with circles and solid figures.
- Ask volunteers to name some solid figures.
- Expected answers:
  Cone, cylinder, sphere, pyramid etc.
- Tell them in this session, we are going to be looking at how to find the total surface area and volume of solid figures and the application of circle theorems to circle related problems.

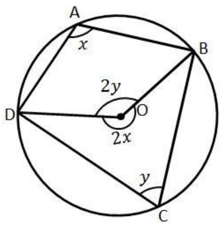
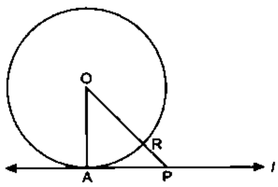
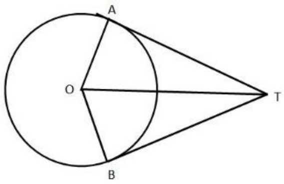
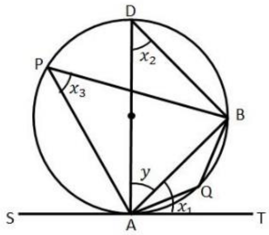
Activity 1 Stating the eight basic theorems of circle **25 minutes**

Review

1. Ask participants to give the meaning of a circle and name its parts.
2. A circle is the locus of a point which moves such that its distance from a fixed point is a constant.
3. The parts of a circle are: centre, radius and circumference.
4. Remind participants that there are eight basic circle theorems.
5. Assign 1 to each group to state with diagram on chart paper
6. Display Chart: 5.2 Fig 1 (cover the theorems and display them one by one after groups present theirs).

Fig 1

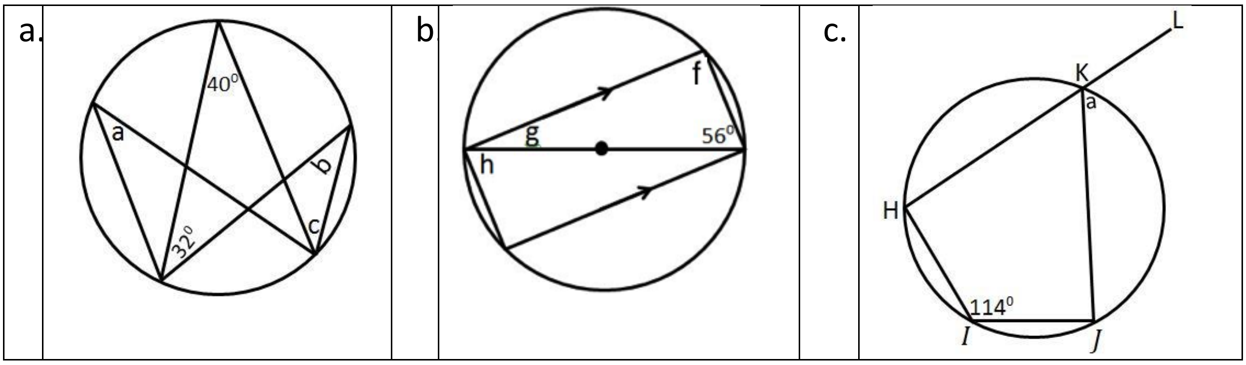
Circle Theorem 1: A straight line from the centre of a circle that bisects a chord is at right angles to the chord. $\lvert PM \rvert = \lvert QM \rvert$ and OM ⊥ PQ.		**Circle Theorem 2:** The angle subtended at the centre of a circle is twice that subtended at the circumference. ∠AOB=2×∠APB.	
Circle Theorem 3: The angle in a semi-circle is a right angle. $\angle AXB$=90°		**Circle Theorem 4:** Angles subtended at the circumference by a chord or arc in the same segment of a circle are equal. $\angle APB$=$\angle AQB$.	

Circle Theorem 5: The opposite angles of a cyclic quadrilateral are supplementary. $\angle BAD + \angle BCD = 180°$ and $\angle ABC + \angle ADC = 180°$.		**Circle Theorem 6:** The angle between a tangent and a radius is equal to 90°. OA $\perp$ l.	
Circle Theorem 7: The lengths of the two tangents from a point to a circle are equal. $\lvert TA \rvert = \lvert TB \rvert$. Also, ∠AOT=∠BOT and ∠ATO=∠BTO.		**Circle Theorem 8:** The angle between a chord and a tangent at the end of the chord equals the angle in the alternate segment. ∠TAB = ∠APB and ∠SAB = ∠AQB. This is the alternate segment theorem.	

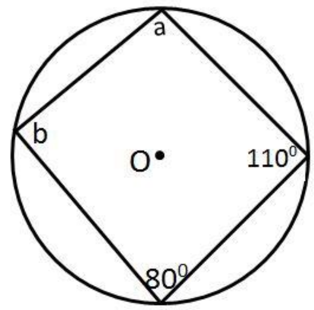
Applying Circle theorems to circle related problems.

1. Write the following questions on the board:
2. Display Chart 5.3 (Figure 2)
3. Find the unknown angles for each of the circles shown below. Point O is the centre of the circle. Give reasons for your answers.

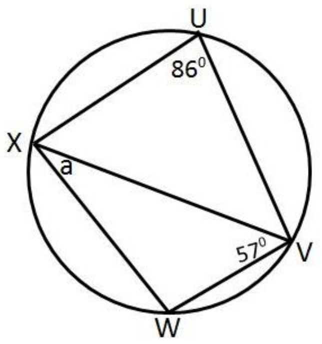
Fig 2



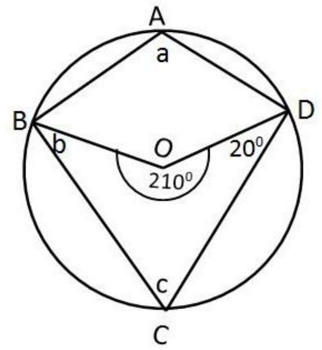
d.



e.



f.



4. Arrange participant into various groups and assign a question to each group.
5. Walk around to check for understanding and make sure everyone is contributing positively in arriving at answers.
6. Discuss with participants that, in some problems you might use more than one theorem in arriving at the answer.
7. Invite a volunteer from each group to come to the board and write the solutions.

Solution

a.	a	$=$	$40°$	∠s in the same segment
	a	$=$	b	∠s in the same segment
	b	$=$	$40°$	
	c	$=$	$32°$	∠s in the same segment
b.	f	$=$	$90°$	∠ in a semi-circle
	g	$=$	$180 - 90 - 56$	∠ in a triangle
	g	$=$	$34°$	
	h	$=$	$56°$	alternate ∠s
c.	a	$=$	$\angle HIJ$	
	a	$=$	$114°$	
d.	a	$=$	$180 - 80$	∠s in a cyclic quadrilateral
	a	$=$	$100°$	
	b	$=$	$180 - 110$	∠s in a cyclic quadrilateral

$b = 70°$

e. $\angle VWX + 86 = 180$ opposite ∠s of a cyclic quadrilateral

$\angle VWX = 180 - 86$

$\angle VWX = 94°$

$a + \angle VWX + 57 = 180°$ ∠s in a triangle

$a + 94 + 57 = 180°$

$a = 180 - 94 - 57$

$a = 29°$

f. $2a = 210$ ∠ at the centre = 2∠ at the circumference

$a = \frac{1}{2} \times 210$

$a = 105°$

$c + 105 = 180$ opposite ∠s of a cyclic quadrilateral

$c = 180 - 105$

$c = 75°$

$b + 20 + 75 + 210 = 360$ sum of ∠s in a quadrilateral

$b = 360 - 20 - 75 - 210$

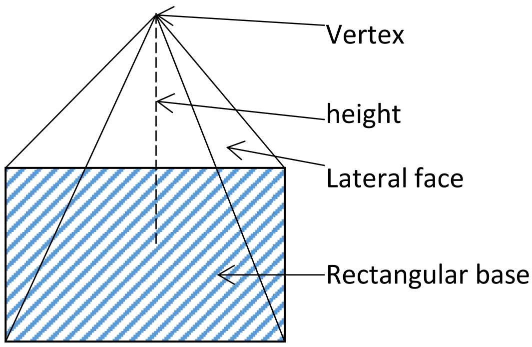
$b = 55°$

Activity 2 Finding the surface area of a pyramid 25 minutes

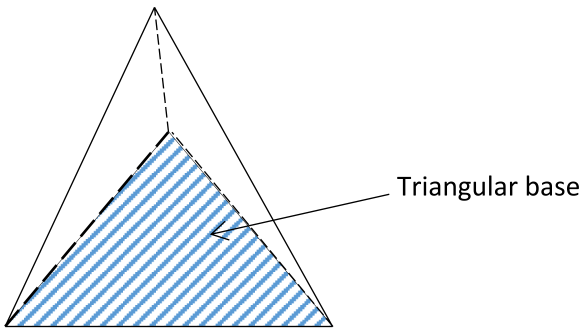
1. Ask participants to discuss in pairs what a pyramid is, and to name its parts.
2. Agree that a pyramid is a structure whose outer surfaces are triangular and converge to a single point at the top, making the shape roughly a pyramid in the geometric sense. The base of a pyramid can be trilateral, quadrilateral, or any polygon shape. As such a pyramid has at least three outer triangular surfaces (at least four faces including the base). The square pyramid, with square base and four triangle outer surfaces, is a common version.

3. Display Chart 5.4 (fig 3)

Fig 3



Rectangular Pyramid

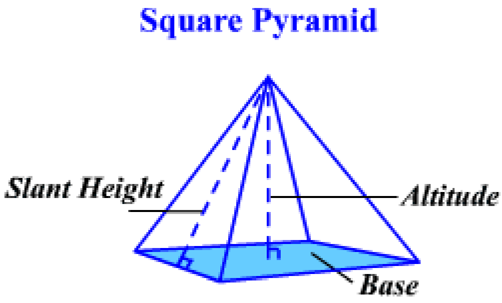


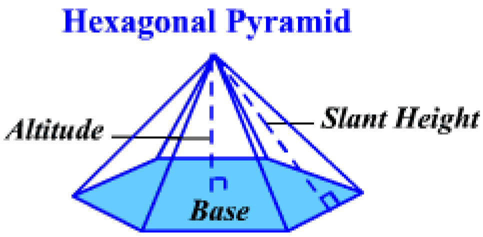
Triangular Pyramid

4. A regular pyramid has a base that is a regular polygon and a vertex that is above the centre of the polygon. A pyramid is named after the shape of its base.
5. A rectangular pyramid has a rectangular base. A triangular pyramid has a triangle base.

6. Ask volunteers to explain how to find the surface area of a pyramid
7. Display chart 5.5 (Fig 4)

Fig4.





8. Surface area of any pyramid = area of base + area of each of the lateral faces.
9. Surface area of a regular pyramid = area of base $+\frac{1}{2}\times$perimeter of base × slant height.
10. The **lateral** surface area **of a regular** pyramid is the sum of the areas of its lateral faces.
11. The **total surface area of a regular pyramid** is the sum of the areas of its lateral faces and its base= area of base $+\frac{1}{2}\times$perimeter of base × slant height.

Examples:

Display the following questions on chart paper:

1. Find the total surface area of a regular pyramid with a square base if each edge of the base measures $15m$, the slant height of a side is $14m$.

2. Find the total surface area of a pyramid with a rectangular base if the length of the base is $20m$ and width $13m$ and the slant height of a side is $10m$.

Solution:

Arrange the participants into groups and assign a question to each group.

1. This is a regular pyramid with a square base. Refer participants to figure 4.

Total Surface Area	=	area of base $+\frac{1}{2}\times$perimeter of base × slant height.
	=	$15\times15+\frac{1}{2}\times(4\times15)\times13\ \mathrm{m}^2$
	=	$(225+390)\ m^2$
	=	$615\ m^2$

3. Ask a volunteer to give the name of the pyramid. (A rectangular pyramid because the base is a rectangular).

4. Ask for volunteers to explain how to find the total surface area of this type of pyramid.

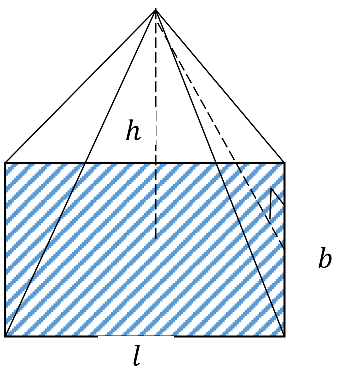
Total Surface area= area of base + sum of the areas of the four sides.

Total Surface Area	=	area of base +sum of the areas of the four sides
	=	$20\times13+2\times(\frac{1}{2}\times20\times10)$ $+2\times(\frac{1}{2}\times13\times10)$
	=	$(260+200+130)\ m^2$
	=	$590\ m^2$

Activity 3 Finding the volume of a pyramid 20 minutes

1. Discuss with participants how to find the volume of a pyramid.
2. Display Chart: 5.6 Fig 5

Fig 5



Answers:

Write the formula for the volume of a pyramid.
Volume (V) is equal to one third of the **base area** multiplied by **the height** $(\boldsymbol{h})$**.**

The volume of a rectangular-based pyramid as in figure 5, for example is given by
V =$\frac{1}{3}(l \times b) \times h$.

Examples

Write the following questions on the board

1) Find the volume of a rectangular pyramid whose base measures 4m × 6m and height of pyramid 20m.

2) Find the volume of a pyramid with a square base if each edge of the base measures $18m$ and height of pyramid is $23m$.
3) Find the volume of a pyramid whose base measures 140cm ×260cm and height of pyramid 200cm

4) Find the volume of a pyramid with a square base if each edge of the base measures $8m$ and the slant height of a side is $12m$

Solutions

Arrange the participants into groups and assign a question from 1 to 3 to each group.
Ask for volunteers from each group to write the answers on the board.

1).

$$\begin{aligned}\text{Volume of pyramid} &= \frac{1}{3} \times Base\ area \times height \\ &= \frac{1}{3} \times (4 \times 6) \times 20 \\ &= 160\ m^2\end{aligned}$$

2).

Volume of pyramid $= \frac{1}{3} \times Base\ area \times height$

$= \frac{1}{3} \times (18 \times 18) \times 23$

$= 2484\ m^2$

3.)

Volume of pyramid $= \frac{1}{3} \times Base\ area \times height$

$= \frac{1}{3} \times (140 \times 260) \times 200$

$= 2{,}426{,}666.67\ cm^2$

4.) Lead the way in solving question 4, with the participants

Ask Participants what to do in solving this question

Expected answers

Since the slant height is given and not the real height, you will have to find the real height of the pyramid.

Refer Participants to chart 5.5.

The real height can be calculated from the triangle which has the slant height as the hypotenuse of the right-angled triangle, then,

$h^2 = s^2 - \left(\frac{1}{2} length\ of\ theside\ of\ the\ base\right)^2$, where s is the slant height.

$= 12^2 - \left(\frac{1}{2} \times 8\right)^2$

$= 128$

$h = \sqrt{128}$

11.83

Volume of pyramid $= \frac{1}{3} \times (8 \times 8) \times 11.83$

$= 252.37\ m^2$

Activity 4 Finding the total surface area and volume of prisms 15 minutes

1. Ask participants the meaning of a prism and to give examples.
2. Expected answers
3. A prism is a solid with uniform Cross – sectional area, called the base. The base is parallel to a cross – sectional area exactly the same as the base.
4. Examples of prisms are:

(i) Cuboid (rectangle prism, the base is a rectangle)

(ii) Cube (the base is a square)

(iii) Cylinder (the base is a circle) etc

5. Discuss with participants, how to find the total surface area of a prism.

Expected answers.

Total surface Area of a prism = sum of areas of the parallel faces + Area of the sides.

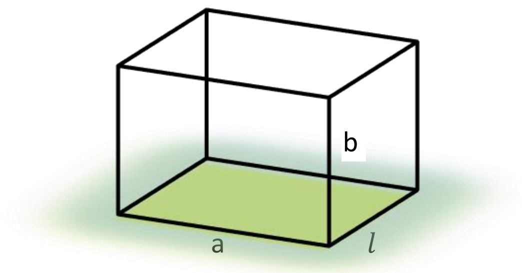
Area of sides = perimeter of the base × height

Exercise

Display Chart 5.7 Fig. 6

Find the total surface area of the figure below.

Fig 6.



Solution

Ask for volunteers to explain how to find the total surface area of the figure

Expected answer

The figure is a prism

Total surface Area	$=$	sum of areas of the parallel faces + Area of the sides.
	$=$	$2(al) + 2(a + l)b$
	$=$	$2ab + 2al + 2bl$

Finding the volume of a prism

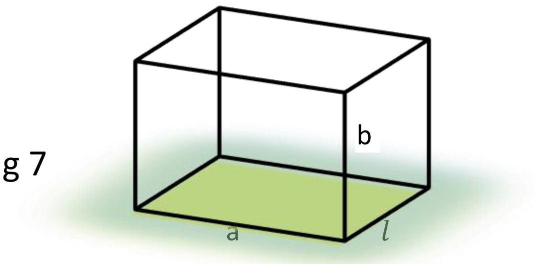
- Discuss with participants, how to find the volume of a prism
- The volume of a prism is given by: Volume = Area of base × height

Exercise

Find the volume of the figure below

Refer participants to Chart 5.7 Fig 7

Fig 7



Ask for volunteers to explain how to find the volume of the figure

Expected answers:

The figure is a Prism

The volume of a prism is given by: Volume = Area of base × height

Volume$= (a \times l) \times b = abl$

Summary **3 minutes**

Review the objectives and answer any questions participants have.

Materials

Chart 5.1	Learning outcomes
Chart 5.2 – 5.7	Fig 1 - 7

Session 6 Taking our Learning Back to School 90 Minutes

Session Objectives

By the end of the session, participants will be able to:

- Review and agree lessons and learning from the workshop
- Discuss and agree steps for implementing learning from the workshop in their various schools
- Explore various ways of involving all pupils in the teaching-learning process in their schools.

Materials

Chart 6.1	Session objectives	Introduction
Chart 6.2	Planning for Group Work	Activity 2

Session Outline

Session Introduction		10 minutes
Activity 1	Learning from TPD Workshop	30 minutes
Activity 2	Taking Learning Back to School	40 minutes
Session Review		10 minutes

Background for Facilitators

The purpose of this last session is to help the participants to realise and appreciate that the TPD workshop is not an end in itself, but an initiative to make them more competent at delivering lessons in their schools, so that pupils can learn better and meet set standards. Consequently, this session will lead participants to review learning from the workshop so far, and together discuss and agree what steps they will take back in their schools to ensure that all pupils are helped by teachers to learn optimally using various strategies that have been shared in the workshop to ensure they involve all pupils in the teaching-learning process.

All through this workshop we've tried to model some of the ideas for involving all pupils in the teaching-learning process. One clear idea has been to show teachers that since pupils, like other human beings, learn in different ways, teachers will have to organise pupils' learning experience in different ways. For example, a lesson might include pupils working individually, in pairs, in small groups or as a whole class. The LPM recognises and promotes these different ways of learning, but in practice, teachers appear to be struggling with helping pupils to work effectively in groups.

This could be one of the ideas that teachers take back to their schools. So, Activity of this session should prioritise how teachers organise children to work in groups Hopefully, SSOs and HoDs could follow up with implementation of this idea in schools during their support and monitoring activities.

To ensure everybody is involved, participants should be put in different groups to prepare for the allotted activity and present to their colleagues.

Session /Workshop Introduction 10 Minutes

1. Welcome participants and commend them for making it to the last session of the workshop.
2. Display Chart 6.1 and ask two participants from different parts of the training room to read out each of the two session objective.
3. Inform them that the session will focus on what they do with their new knowledge and skills when they return to their schools.

Activity 1 Learning from TPD Workshop 30 Minutes

1. Explain to participants that the energy and resources expended on this workshop will only be of benefit if as teachers they go back to their schools and implement whatever they might have learnt.
2. Ask: Have the two days of this workshop being a good use of your time or not?
3. Take a few responses and inform the participants that this activity will help them to identify and explain the new knowledge and skills they have learnt that will help them to be better at delivering lessons in their various seasons.
4. First ask: What new ideas/things have you learnt in this workshop that you're eager to go back to your school and apply when schools resume for the third term?
5. Hand each person a couple of handout notes to write:
   (a) 3 ideas/things on English language content;
   (b) 2 ideas/things on lessons delivery (pedagogy).
6. Allow about 5 minutes for the exercise, while you and your co-facilitator move around to ensure everybody understands what to do and support any struggling individuals.
7. Ascertain that everybody has finished and ask each participant to pair up with their closest neighbour. Let each pair sit together and talk about their learning and discuss what they might do to implement the ideas back in their schools. They should also consolidate their ideas by removing any overlaps and be ready to share their new list of ideas in a group with other pairs.
8. Put the pairs in groups in such a way that a group is no more than 4 or six. Ask each group to further discuss and consolidate the content knowledge and pedagogy ideas of their pairs into a group list by removing overlapping ideas. Have them also discuss how they might implement their ideas back in their schools and any challenges they envisage.
9. You and your co-facilitator should listen in to groups' discussions to capture any ideas, especially challenges you might want to highlight during plenary, e.g. checking that children are planning, preparing for lessons ahead of the lesson, classes are too large for grouping, and lesson time is too short.
10. Conduct a plenary and reinforce the idea that the proof of the benefit of this work is improved teaching and learning in every school. Respond to any concerns or questions participants might have and move to the next activity.

Activity 2 Helping Pupils to Work Effectively in Groups 40 Minutes

1. Explain that this last activity will enable the participants to practise one of the ideas they identified in activity 1--helping pupils to work effectively in groups.

2. It might be necessary to ensure that participants have a shared understanding of grouping pupils in the teaching-learning process.
3. So, ask: What is grouping and why is it important in the teaching-learning process?
4. Take responses from a few participants and conclude that grouping means to put children in small groups to sit together, discuss and carry out assigned tasks/activities as a group during a lesson. It is important because it provides opportunities for pupils to learn from, support and help one another. It helps children to improve their interaction and communication skills. Research on grouping children for learning reveals that many children learn certain concepts and ideas faster from their peers than from teachers.
5. Emphasise that grouping is not just about seating children together, but making sure that they interact with each other by doing activities together as part of the lesson—discuss, find answers to problems and report or present their findings as a group.
6. It may also be useful to probe what participants already know about the different ways pupils could be grouped.
7. Take feedback from a number of participants and explain that there are two main ways of grouping pupils to work together. The first way is **heterogeneous** grouping which means grouping pupils of different ability levels together—high, medium, low. The essence of this is to make sure that the pupils can really help and support one another. The second is **homogenous** grouping which puts together pupils of similar levels of ability. For example, pupils who are yet to master a particular concept, say in maths or English, could be grouped together so the teacher could work more closely with them, while pupils who have mastered the concept might be doing something else in their own groups.
8. Ask participants to discuss in their table groups which type of grouping they prefer and why.
9. Take feedback from groups across the tables, and discuss the three questions in **Chart 6.2** that need to guide how pupils are grouped.
10. Finally, ask different groups to discuss and agree practical ways they plan to implement grouping of pupils for learning in their various schools. Give them flip charts to write their ideas.
11. Hold a plenary to discuss the points raised by the groups and inform them that SSOs and their HoDs will follow up to ensure that they're using group work to help all their pupils to learn optimally.

Session Summary 10 Minutes

- Show participants chart 6.1 again.
- Read through each objective. For each one, ask what they did to try to achieve it.
- Explain that it's up to them to go back to their schools and implement the ideas and strategies they have learnt in the workshop.

Chart 6.1 Session Objectives:

By the end of the session, participants will be able to:

- Review and agree lessons and learning from the workshop
- Discuss and agree steps for implementing learning from the workshop in their various schools

Chart 6.2 Planning for Group Work

- When planning for group work, consider what you want your students to get out of it.
- Do you want your pupils of higher ability levels to help those with lower ability levels? (Just be careful here and know your pupils. Make sure they will *all* benefit from this.)
- Do you want to have pupils of lower ability levels grouped together so you can work with them in a smaller group setting?
- Do you simply want your pupils to get to know each other and start building community in your class?
- Your purpose should drive your groups.

TPD Teacher self-assessment

Name:

District:

Subject taught:

1. Number of TPD trainings attended this academic year:

   1
   2
   3

2. One thing that I have improved in my teaching this year is:

3. One example of improved learning in my classroom is:

4. Put the following in order of how much they have helped improve your teaching, where number one is the intervention that has helped you improve your teaching the most and number helped you the least.

Intervention	Insert number 1, 2, 3, or 4	How did this intervention help you?	What could be added for further improvement?
SSO coaching			
LPM			
Pupil Handbook			
TPD training			