Leh Wi Lan

Improving Secondary Education in Sierra Leone

UKaid from the British people

CAMBRIDGE EDUCATION

FACILITATION GUIDE

Improving Teaching and Learning

SSS Teacher Professional Development (TPD): Mathematics (Term 2)

Introduction

Teacher professional development (TPD) is most effective when several elements are combined to support teachers. Training and support should be as close to the school as possible so teachers have opportunities to practice their learning in their own context. They can work with their peers to share success and challenges and reflect on their problems, devising contextually relevant solutions. Teachers also need some form of external support so that they are introduced to new ideas, ways of working and can refresh their subject knowledge and ensure that it is up to date.

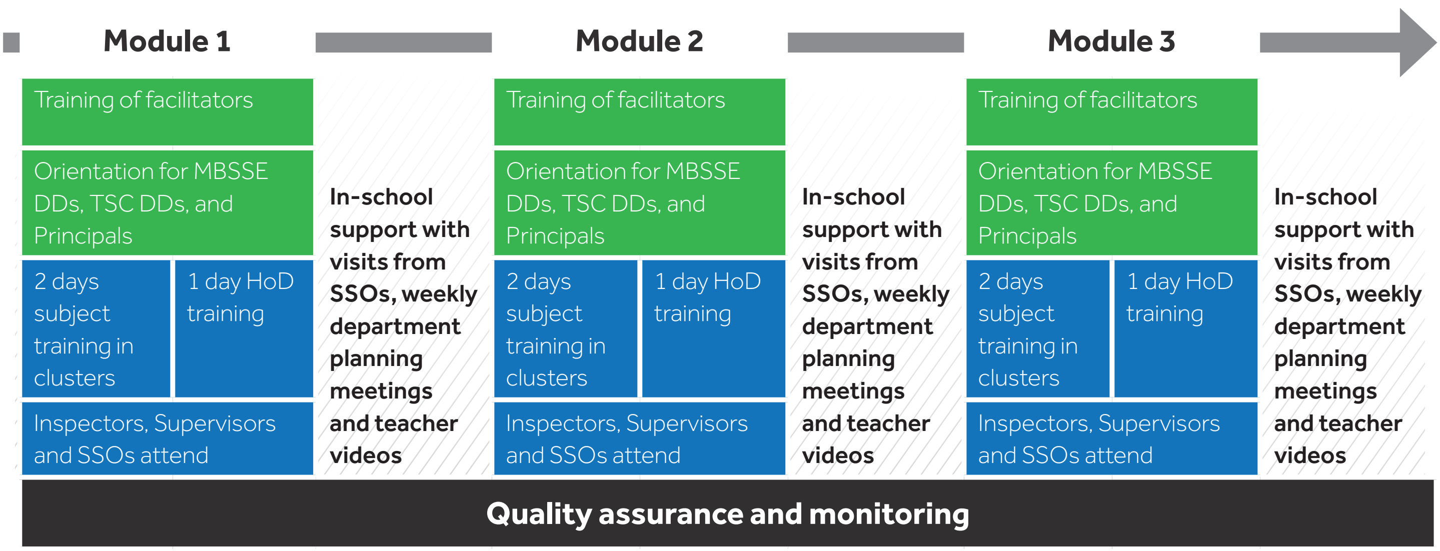
This is especially important in a context like Sierra Leone where Junior Secondary Schools (JSS) and Senior Secondary Schools (SSS) vary greatly in terms of access to resources and distribution of qualified teachers. Within government schools there are large numbers of teachers who are: not qualified for secondary level, qualified but not government approved, qualified to teach, but not in the subject they teach, and volunteer teachers with no prior training or qualification.

Teacher professional development demands a variety of activities so that all teachers, whatever their circumstances and environment, can access structured quality professional development which supports their professional growth and helps deliver quality education.

Between 2017 and 2020 Leh Wi Lan implemented a TPD strategy to support all JSS and SSS English and maths teachers and Heads of Department (HoD), from approximately 1600 government-assisted schools.

The strategy included subject content training in maths and English as well as academic leadership training for HoD. Teachers received training through termly face to face cluster sessions, led by national facilitators who were subject specialists. After each face-to-face cluster training there was in-school support for teachers through regular visits from Leh Wi Lan School Support Officers[1] where they conducted lesson observations and gave feedback to teachers. Instructional videos were also created providing step by step explanations of specific concepts and examples of good classroom practice.

The diagram below shows how the TPD programme worked over a given academic year:



This facilitator manual is part of the Leh Wi Lan TPD programme. It contains subject training for maths teachers in SSS. It is designed to be delivered over one academic year and each module links directly to SSS English lessons being taught in the upcoming term (in this case term 2). It should be used alongside the MBSSE Lesson Plan Manual and Pupil Handbook. Whilst focusing primarily on subject content, the materials have been designed to develop key pedagogical skills including gender-responsive pedagogy, using teaching and learning aids, inclusive learning, reflection and problem solving.

These materials were produced by Cambridge Education, in collaboration with TSC, and delivered as part of the UK-aid funded Leh Wi Lan project for training teachers in government assisted junior and senior secondary schools in Sierra Leone. These training materials are in draft. They can be shared and adapted for use as long as they are not used for commercial purposes.

[1] 200 School Support Officers conducted regular visits to government assisted secondary schools nationwide. Each covered approximately 8-10 schools and supported the English and maths teachers by conducting lesson observations and giving feedback to enhance teaching and learning. They were supported by Leh Wi Lan. This MBSSE School Quality Assurance Officers now perform this supportive supervision role

Senior Secondary School Teacher Professional Development

Training schedule

Date / Time	09:00	11:00	11:30	13:30	14:30	16:30	17:00
Day 1 (HoD)	HoD Session 1: Role of Heads of Department	Break	HoD Session 2: Monitoring use of LPMs and PHs	Lunch	HoD Session 3: Supporting teachers to use LPM and PH		Closure
Day 2 (teachers)	Session 1: Welcome and introduction		Session 2: Logarithms		Session 3: Transformations		
Day 3 (teachers)	Session 4: Surds		Session 5: Sequence and series		Session 6: Back to school	Teacher self-assessment	

Facilitator Standards

Well prepared:
Arrives early
Has charts written and materials organised so they are ready to give out.
Refers to training notes but doesn't read them constantly
Demonstrates strong familiarity of the lesson plan structure and content.
Time management:
Manages time
Session and activities start and end on time.
Understanding SSS:
Exhibits knowledge of the current context of Senior Secondary Schools - uses examples that are relevant to the context
Subject Knowledge:
Clearly explains how to do the content of the lesson plans, using a variety of examples to add depth.
Participatory:
Gives opportunities for participants to work together
Gives time for participants to think of how to tackle a problem before explaining
Supportive:
Listens to the participants and acts on their comments
Accepts answers and asks questions to help participants, rather than telling the answers
Inclusive:
Ask questions to a range of participants
Uses gender responsive language and interaction
Finds ways to support those participants who don't understand
Uses group, pair and individual work and moves round to support all participants
Enjoyable:
Greets the participants, creates a friendly atmosphere.
This session is quick and active with a positive approach.

Session 1: Welcome and introduction to Term 2 Training 90 minutes

Session objectives

By the end of the session, participants will be able to:

- Identify participants and facilitators at the workshop
- Review and share learning from term 1 TPD workshop
- Discuss and agree objectives and expectations and time-table for Term 2 TPD workshop

Materials

Chart 1.1	Session objectives	Introduction
Chart 1.2	Good morning song	Introduction
Chart 1.3	Ground rules	Introduction
Chart 1.4	TPD Term 2 Workshop Objectives	Activity 3
Handout 1.1	TPD Term 2 Workshop timetable	Activity 3

Flip charts and markers, post-it notes, masking tapes

Session outline

Session/Workshop introduction		15 minutes
Activity 2	Learning from Term 1 Workshop	40 minutes
Activity 3	Introduction to Term 2 Training Programme	25 minutes
Session review		10 minutes

Session /workshop introduction 15 minutes

1. Welcome participants to the first day of their second teacher professional development workshop and ask two of the participants (Christian and Muslim) to open in prayers.
2. Sing “Good morning” song with participants (see “Good Morning Song” chart)
3. Introduce the facilitators and key MEST and district staff members, asking a government staff (district deputy director) to make brief remarks to declare the workshop open.
4. Show participants Chart 1.1, with the objectives covered up (i.e. showing only the title). Ask a participant to explain what ‘session objectives’ are. (They tell us what we are aiming to achieve by the end of the session). Ask another participant to explain why it is useful and important to have session objectives and to share them with participants (To enable everyone be on the same page in knowing our ‘destinations’ and to help us evaluate ourselves by the end of the session).
5. Show the session objectives, one by one, asking a participant to read each one.
6. Explain that you will work towards the first objective right now, which is about getting to know participants and facilitators at this workshop.
7. Take participants to a space where they can stand in a circle and stand as one of them.
8. Remind participants of your name, and ask someone to tell you what letter it begins with. Explain that you will think of an adjective to describe yourself that starts with the same letter as your name. You will also think of an action to go with your adjective.
9. Step into the middle of the circle and say ‘I am ______ ______’, doing your action at the same time as you say your name. Repeat.
10. Step back to your place in the circle, and explain that, when someone introduces himself or herself, everyone else needs to welcome him or her – so they should say ‘Welcome, _______ ________’, repeating your name, adjective and the action that goes with it.
11. Explain that you will show them what to do one more time – and then they will introduce themselves in turn. Each person will step into the circle, introduce himself/herself, and then be welcomed by everyone else. Explain that they need to think of an adjective to go with their name, and an action to go with it before it comes to their turn. They should try to think of an adjective different from everyone else’s!
12. Step into the circle and introduce yourself again, reminding them that they need to welcome you. Then encourage participants to introduce himself/herself, going around the circle. Help individuals who are struggling to think of a good word or action for their introduction, and make sure the group has heard and used their ‘Workshop name’ to welcome each person.

13. Round off the introduction by displaying Chart 1.3 as the ground rules that participants and facilitators will follow during the workshop ask
14. Thank participants for their work on this and ask them to return to their seats.

Activity 1 Learning from Term 1 TPD Workshop 40 minutes

1. Ask the participants to show by a raise of their hands if they were present during the very first workshop in September.
2. You might also wish to take note of participants who did not attend the first workshop and find out why.
3. Explain that those who did not attend the first workshop will have the time to catch up but that this activity is meant for those who attended the first workshop to share what they learnt from that workshop and whether their participating in that workshop helped improve lesson delivery and the learning of children in their schools.
4. Tell participants that that you will lead them to review the workshop under three headings: what went well, what didn't go so well, lessons learnt and what has changed in their practice because of the workshop.
5. Hand each participant 4 post- it notes to write just one statement under each heading personalising it to 'one thing I liked about the workshop', 'one thing I didn't like about the workshop', and 'one thing that has changed in my practice as a teacher because of the workshop'.
6. Allow about 3 minutes for participants to think and write their responses on the post- it notes.
7. Find comfortable spaces in the training room and display on A4 paper bold headings: What we liked about the workshop; What we didn't like about the workshop; and What changed in our practice because of the training.
8. Ask each participant to stick each of their responses below the headings that apply and go back to their seats.

9. Acknowledge their responses to the first two questions above and then focus on identifying what has changed in their practice as teachers because of the training using the questions below as guide. Remind them that since, the training is to help teachers improve the teaching-learning process, so pupils can learn better, we need to know whether the training supported you to do this. Ask them to find a partner and discuss the first question for 2 minutes. Ask them to change partners and discuss the second question for 2 minutes. Repeat until every question has been discussed.

- Have you used the lesson plans? If not, why not?
- what did you find challenging? How did you manage to overcome the challenge?
- What did you find easy/straightforward?
- What did the pupils enjoy?
- Was there anything they/you could understand better because of the lesson plans
- What support did you get to help you implement the plans? Who from?
- What more support would you like?
- Have you used the Pupil Handbook?
- How are you using the Pupil Handbook in the classroom?
- What do you find challenging about using the Pupil Handbook ?
- Do you need more support for using the Pupil Handbook?

10 Move around the room and listen to some of the responses, taking note of anything interesting you hear that could be shared with the group.

11 Take feedback at the end encouraging different people to share one or two experiences /thoughts about their implementation of the lesson plans.

12 Please hand the post-it notes of the responses to the LWL representative for further analysis and action, if required.

12 Remind participants that the training would amount to a waste of time and resources if it does not lead to improvement in the way they as teachers deliver lessons and the learning of the children eventually.

14.Take any questions participants might have and move to the last activity for the session.

Activity 2 Introduction to Term 2 Training Programme 25 minutes

1. Explain that this last activity will enable the participants to preview this 3-day teacher professional development workshop based on the LPM for the second term of the 2018/19 academic session.
2. Tell participants, “having shared what you learned from the last training, I’ll give each of you an opportunity to share your expectations from this current workshop.”
3. Explain that by expectations, you mean what they should be able to do better as teachers of English or mathematics by the end of the workshop—in terms of subject content knowledge, teaching skills, attitude/behaviour.
4. Hand each participant post-it notes to write 3 expectations from the workshop—what they expect to learn or gain from the workshop that will make them better teachers of English or Mathematics back in their schools.
5. Give them 3 minutes to think, write and stick their expectations on any convenient corner of the training room.
6. Lead the participates to analyse and discuss the participants’ expectations, bringing out ideas that relate to content knowledge, teaching skills, and attitudes/behaviour.
7. You might wish to work with them to consolidate all the ideas into a maximum of 6 covering all three areas—knowledge, skills and attitudes/behaviours and ask a volunteer to copy the agreed expectations neatly on flip chart paper.
8. Tell them that at the end of the workshop, they’ll have a chance to check whether their expectations have been met.
9. Display Chart 1.2 with Term 2 TPD workshop objectives and discuss by going through each of the objectives. For each one, agree what it means and when they will be addressing it, explaining that some of the objectives might not apply to them, except if they are also subject head of departments.
10. Give out Handout 1.1 – the workshop timetable.
11. Go through the timetable and ask them to identify where we are on the programme.
12. Take a few responses and agree with them that we are doing the first session of the 2-day workshop.
13. Ask participants how many sessions there are, and how long each one lasts. Talk through the timings of each day and ask participants to write these into their booklets.
14. Tell them that where space is available, both English and maths teachers will do this session together and thereafter separate to their subject sessions and might also get back together to do Session 6. Emphasise that while sessions 1 and 6 are general, sessions 2-5 are specific to English or maths teachers.

15. Inform them that sessions A-C are only for English and maths teachers who are also heads of departments for English and maths in their schools.

Session review 10 minutes

- Show participants chart 1.1 again. Read through each objective.
- For each one, ask what they did to try to achieve each one.
- Explain that, as they've just seen, there is a lot to be covered during the workshop; this means we need to manage time very well, which underscores the need for participants to arrive early each day and for each of the sessions.
- Tell them that, when they come to the end of a session and look at the session objectives, and feel that they haven't achieved the session objectives, they should speak up because it's a shared responsibility between the participants and the facilitators to ensure that participants' expectations are met.
- Make sure they understand that if they feel that they haven't achieved an objective, they should talk to you or the co- facilitator during the break.
- Thank them for their focus during this session and remind them what time to come back from the break. You could appoint a timekeeper to help them all remember what time to return.

Chart 1.1 Session Objectives

By the end of the session, participants will be able to:

- Identify participants and facilitators at the workshop
- Review and share learning from term 1 TPD workshop
- Discuss and agree expectations and time-table for Term 2 TPD workshop

Chart 1.2 Good morning song

Good morning, good morning, good morning to you (2 times)

Our day is beginning there's so much to do

Good morning, good morning, good morning to you

Chart 1.3 Ground rules

- Manage time effectively
- Be punctual
- Take turns in talking and making contributions
- Respect other participants' opinions
- Switch off/ put phones on silent mode
- Minimize movements

Chart 1.4 TPD Term 2 Workshop Objectives

By the end of the Workshop, all SSS teachers will be able to:

- Familiarise themselves with SSS Term 2 English and mathematics lesson plans
- Make more effective use of English and mathematics lesson plan manuals
- Improve their content knowledge of SSS English and mathematics
- Improve their teaching skills
- Acquire new teacher attitudes and behaviour
- Deliver English and mathematics lessons more competently

Handout 1.1: TPD Term 2 Workshop time-table

Date /Time	09:00	11:00	11:30	13:30	14:30	16:30	17:00
Day One (HoD)	Session A: Roles of Head of Department (HOD)	Break	Session B: Monitoring use of LPM an PH	Lunch	Session C: Supporting teachers to use the LPM and PH		Closure
Day 2 (teachers)	Session 1: Introduction and Welcome		Session 2: English/Math		Session 3: English/Math		
Day 3 (teacher)	Session 4: English/ Math		Session 5: English/ Math		Session 6: Back to School	Teacher Self Assessment	

Session 2 Logarithms 2 ***90 minutes***

Learning outcomes

By the end of the session participants will be able to deliver lessons on logarithms:

- following steps of the plans and
- observing lesson timing

> Important note to facilitators:
>
> Please ensure that all participants are actively engaged (pairs, groups, individual and whole group) to demonstrate good practice. Avoid lecturing and get the teachers to do most of the tasks. Remember, they are math teachers.
>
> Logarithms appear in all sorts of calculations in engineering and science, business and economics.
> Before the days of calculators, they were used to assist in the process of multiplication by replacing the operation of multiplication by addition. Similarly, they enabled the operation of division to be replaced by subtraction. They remain important in other ways, one of which is that they provide the underlying theory of the logarithm function. This has applications in many fields, for example, the decibel scale in acoustics.
> In order to master the techniques explained in this session, it is vital that you do plenty of practice exercises so that they become second nature.
> After this presentation on this topic you should be able to:
> • explain what is meant by a logarithm
> • state and use the laws of logarithms
> • solve simple equations requiring the use of logarithms.

Materials

Chart 2.1	Learning Outcomes
Chart 2.2	Implementing the lesson plans
Handout 2.1	TGM1-T
Handout 2.2	TGM1-T2 lesson 049

Activity outline

Introduction		10 minutes
Activity 1	Review of logarithms	20 minutes
Activity 2	The laws of logarithms	30 minutes
Activity 3	Timing and steps in implementing the LPM	25 minutes
Summary		5 minutes

Session Introduction 10 minutes

- Welcome participants to the session
- Display Chart 2.1 (Learning outcomes) and go through it with the participants.
- Tell participants that in this session we are going to be looking at logarithms in more detail. However, to deal with logarithms we need to revise indices. This is because logarithms and indices are closely related, and in order. To understand logarithms a good knowledge of indices is required.
- Write $16 = 2^4$ on the board and get the participants to say that the number 4 is the power.
- Ask what other names they are called. Sometimes we call it an exponent. Sometimes we call it an index. In the expression 2^4, the number 2 is called the base.
- Write $64 = 8^2$ and get the participants to label the right side of the equation. (In this example 2 is the power, or exponent, or index. The number 8 is the base).
- Ask, “Why do we study logarithms?” (a rhetorical question) and say in order to motivate our study of logarithms, consider the following: we know that $16 = 2^4$. We also know that $8 = 2^3$
- Suppose that we wanted to multiply 16 by 8. One way is to carry out the multiplication directly using long-multiplication and obtain 128. But this could be long and tedious if the numbers were larger than 8 and 16. Can we do this calculation another way using the powers?
- Give participants 1 minute to write in their notepad. Ask them to raise their answers up by saying “show me”
- Write solution on the board $16 \times 8 = 2^4 \times 2^3 = 2^7$ using the rules of indices which tell us to add the powers 4 and 3 to give the new power, 7. What was a multiplication sum has been reduced to an addition sum.

- Similarly, if we wanted to divide 16 by 8:
- Write 16 ÷ 8 and get individuals to solve using powers. (It can be written $2^4 \div 2^3 = 2^1 = 2$ using the rules of indices which tell us to subtract the powers 4 and 3 to give the new power, 1).
- If we had a look-up table containing powers of 2, it would be straightforward to look up 2^7 and obtain $2^7 = 128$ as the result of finding 16 × 8.
- Any relationships among operations?
- Notice that by using the powers, we have changed a multiplication problem into one involving addition (the addition of the powers, 4 and 3).
- Historically, this observation led John Napier (1550-1617) and Henry Briggs (1561-1630) to develop logarithms as a way of replacing multiplication with addition, and also division with subtraction.

Activity 1: Review of logarithms 20 minutes

- Consider the expression $16 = 2^4$ (write on the board). Remember that 2 is the base, and 4 is the power. An alternative, yet equivalent, way of writing this expression is ------? Take responses from 2 individuals ($\log_2 16 = 4$).
- Get participants to state this as 'log to base 2 of 16 equals 4'.
- Give participants a minute to compare the logarithm form and the index form. We see that the logarithm is the same as the power or index in the original expression.
- It is the base in the original expression which becomes the base of the logarithm.
- The two statements $16 = 2^4$ and $\log_2 16 = 4$ are equivalent statements. If we write either of them, we are automatically implying the other.
  Eg. Write $64 = 8^2$ then get the equivalent statement from the participants using logarithms ($\log_8 64 = 2$).
- Write $\text{Log}_3 27 = 3$ and ask for the equivalent statement using powers. ($3^3 = 27$).
- So the two sets of statements, one involving powers and one involving logarithms are equivalent.
- In the general case we have:

> Key Point
>
> if $x = a^n$ then equivalently $\log_a x = n$

- Let us develop this a little more. Because 10 = 10^1 we can write the equivalent logarithmic form $\log_{10} 10 = 1$.
- Similarly, the logarithmic form of the statement $2^1 = 2$ is $\log_2 2 = 1$.
- Ask pairs to write the log form for any base a in general (a = a^1 and so $\log_a a = 1$).

Key Point

$\log_a a = 1$

- We can see from the examples above that indices and logarithms are very closely related. In the same way that we have rules or laws of indices, we have laws of logarithms. These are developed in the following sections.

1. Write the following using logarithms instead of powers

a) $8^2 = 64$ b) $3^5 = 243$ c) $10^{-3} = 0.001$ d) $27^{2/3} = 9$

2. Determine the value of the following logarithms

a) $\log_3 9$ b) $\log_2 32$ c) $\log_4 64$ d) $\log_3 81$

Answer key

1a. $\log_8 64 = 2$ b. $\log_3 243 = 5$ c. $\log_{10} 0.001 = -3$ $\log_{27} 9 = \frac{2}{3}$

2a. 2; b. 5 c. 3 d. 4

Activity 2: The laws of logarithms 30 minutes

- Suppose x = a^n and y = a^m then the equivalent logarithmic forms are --- and ---? ($\log_a x = n$ and $\log_a y = m$) (1)
- Using the first rule of indices, xy = a^n x a^m = a^{n+m}
- Ask pairs to write the logarithmic form of the statement (xy = a^{n+m} is $\log_a xy = n + m$). But $n = \log_a x$ and $m = \log_a y$ from (1) and so putting these results together we have $\log_a xy = \log_a x + \log_a y$.
- So, if we want to multiply two numbers together and find the logarithm of the result, we can do this by adding together the logarithms of the two numbers. This is the first law.

Key Point

$\log_a xy = \log_a x + \log_a y$

The second law of logarithms

- Get groups to prove the second law of logarithms (division)
- Move round to support then write proof on the board
- As before, suppose x = a^n and y = a^m with equivalent logarithmic forms are $\log_a x = n$ and $\log_a y = m$ (2)
- Consider x ÷ y = $\frac{x}{y} = a^n \div a^m = a^{n-m}$ using the rules of indices.
- In logarithmic form $\log_a \frac{x}{y} = n - m$, which from (2) can be written $\log_a \frac{x}{y} = \log_a x - \log_a y$

> Key Point
>
> $\log_a \frac{x}{y} = \log_a x - \log_a y$

The third law of logarithms

- Give groups 2 minutes to prove the third law (logarithm of a power)
- Suppose x = a^n, or equivalently $\log_a x = n$. Suppose we raise both sides of x = a^n to the power m: $x^m = (a^n)^m$. Using the rules of indices we can write this as $x^m = a^{nm}$
- Thinking of the quantity x^m as a single term, the logarithmic form is $\log_a x^m = nm = m\log_a x$
- This is the third law. It states that when finding the logarithm of a power of a number, this can be evaluated by multiplying the logarithm of the number by that power.

> Key Point
>
> $\log_a x^m = m \log_a x$

The logarithm of base expressed as an index

- $\boldsymbol{Write\ \log_{b^n} a = \frac{1}{n}\log_b a}$ 3
- From (3) above, let $\log_{b^n} a = x$ 4
- Then $(b^n)^x = a$
- Taking logs on both sides to base b, gives: $\log_b (b^n)^x = \log_b a$
- From the third law of logarithm, $nx \log_b b = \log_b a$ 5
- But $\log_b b = 1$

- From (5) above, $nx = \log_b a$
- Solving for x, gives $x = \frac{1}{n}\log_b a$
- From (4), therefore, $\log_{b^n} a = \frac{1}{n}\log_b a$

Give participants the following example: find the value of $\log_8 2$

$$\log_8 2 = \log_{2^3} 2 = \frac{1}{3}\log_2 2 = \frac{1}{3}$$

The logarithm of 1

- Recall that any number raised to the power zero excluding zero itself is 1: $a^0 = 1$, $a \neq 0$. The logarithmic form of this is $\log_a 1 = 0$

> Key Point
>
> $\log_a 1 = 0$. The logarithm of 1 in any base is 0, $a \neq 0$.

- Suppose we wish to find $\log_2 512$. This is the same as being asked 'what is 512 expressed as a power of 2?'
- Now 512 is in fact 2^9 and so $\log_2 512 = 9$.

Example

- Suppose we wish to find $\log_8 \frac{1}{64}$ This is the same as being asked 'what is $\frac{1}{64}$ expressed as a power of 8?'
- Now $\frac{1}{64}$ can be written 64^{-1}. Noting also that $8^2 = 64$ it follows that $\frac{1}{64} = 64^{-1} = (8^2)^{-1} = 8^{-2}$ using the rules of indices. So $\log_8 \frac{1}{64} = -2$.

Exercises

1. Each of the following expressions can be simplified to logN.

Determine the value of N in each case. We have not explicitly written down the base. You can assume the base is 10, but the results are identical whichever base is used.

a) log 3 + log 5 b) log 16 – log 2 c) 3 log 4

d) 2 log 3 – 3 log 2 e) log 236 + log 1 f) log 236 – log 1

g) 5 log 2 + 2 log 5 h) log 128 – 7 log 2 i) log 2 + log 3 + log 4

j) log 12 − 2 log 2 + log 3 k) 5 log 2 + 4 log 3 − 3 log 4 l) log 10 + 2 log 3 − log 2

Answer Key

a) $\log 15$ b) $\log 8$ c) $\log 64$ d) $\log \frac{9}{8}$ e) $\log 236$ f) $\log 236$ g) $\log 800$ h) $\log 1 = 0$

i) $\log 24$ J) $\log 9$ k) $\log \frac{81}{2}$ l) $\log 45$

2. Express as a sum or difference of logarithms:

a) $\log bc$ b) $\log a^2bc^3$ c) $\log(\frac{ab^2}{c^3})$ d) $\log 6$ e) $\log 30$ f) $\log 60$

g) $\log \frac{32}{81}$

Answer Key:

a) $\log b + \log c$ b) $2\log a + \log b + 3\log c$ c) $\log a + 2\log b - 3\log c$

d) $\log 2 + \log 3$ e) $\log 2 + \log 3 + \log 5$ f) $2\log 2 + \log 3 + \log 5$

g) $5\log 2 - 4\log 3$

3. Find the value of the following:

a) $\log_{25} 5$ b) $\log_{\sqrt{2}} 8$ c) $(\log 32)/(\log 128)$

Answer Key:

a) $\frac{1}{2}$ b) 6 c) $\frac{5}{7}$

Using logarithms to solve equations

- We can use logarithms to solve equations where the unknown is in the power. Suppose we wish to solve the equation 3^x= 5. We can solve this by taking logarithms of both sides. Whilst logarithms to any base can be used, it is common practice to use base 10, as these are readily available on your calculator. So, log 3^x = log 5

- Now using the laws of logarithms, the left-hand side can be re-written to give x log 3 = log 5. This is more straightforward. The unknown is no longer in the power. Straightaway x = $\frac{log5}{log3}$ If we wanted, this value can be found from a calculator.

Example

- Solve 3^x = 5^{x-2}. Again, notice that the unknown appears in the power. Take logs of both sides, log 3^x = log 5^{x-2}
- Now use the laws of logarithms. X log 3 = (x-2) log 5
- Notice now that the x we are trying to find is no longer in a power. Multiplying out the brackets, x log 3 = x log 5 – 2 log 5
- Rearrange this equation to get the two terms involving x on one side and the remaining term on the other side, 2 log 5 = x log 5 – x log 3
- Factorise the right-hand side by extracting the common factor of x
  2 log 5 = x(log 5 – log 3) = x log($\frac{5}{3}$)

  x = $\frac{2 \log 5}{\log\left(\frac{5}{3}\right)}$ using the laws of logarithms.

- If we wanted, this value can be found from a calculator.

Activity 3: Implementing the lesson plans (timing and steps) 25 minutes

- Tell participants that parts of what teachers need to do to implement the lesson plans will be explored in this activity.
- Display Chart 2.2 Implementing the lesson plans with points covered
- Display the first 2 points and get participants to discuss in pairs what they understand by the points
  1. following steps of the plans
  2. observing lesson timing
- Agree that the LPMs have 4 steps and it is important they are able to name the steps (Opening, Teaching and Learning, Practice and Closing)

- Assign sample lessons to participants in pairs with focus on sets and notation. Give them about 2 minutes to check that the steps are as stated above.

TGM1-T2
Lesson 58: Define and Describe Sets and Elements of a Set 36
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- Get them to look at the trend in time allocation to them
- Ask what the impact of spending more or less time would have on pupils' learning
- Give them 5 minutes in their groups to agree on strategies they would use to ensure time is adequately used for Teaching and Learning step of TGM2-T2 lesson 049 (Handout 2.2).
- Take feedback and assign different sub-steps to groups to briefly demonstrate.
- Give quick feedback as a whole group reinforcing that effort must be made to ensure that times for every step are adhered to and the best strategy is to ensure that we do not deviate from the content of the plan. Teachers tend to add a lot to what is written. Inasmuch as it shows creativity, care must be taken to ensure limited time is spent on contents that are not too relevant.

Session summary 5 minutes

Take participants through the learning outcomes again and ask if they are confident about them and thank them for their active participation.

Materials

Chart 2.1 Learning outcomes Introduction

Chart 2.2 Familiarising yourselves with the LPMs

- following steps of the plans
- observing lesson timing
- linking teaching activities to objectives
- identifying preparations required
- demonstrating mastery of the content
- practicing
- working with others
- observing each other
- checking that learning is happening
- linking pupils' handbook to LPM

Session 3 Transformations 90 minutes

Learning outcomes

By the end of the session participants will be able to deliver lessons on transformations (enlargement, anti-clockwise rotation through 90° about the origin, reflection and translation) with special focus on:

- identifying preparations required
- demonstrating mastery of the content
- linking teaching activities to objectives

Materials

Chart 3.1	Learning Outcomes
Chart 2.2	Implementing the lesson plans
Handout 3.1	TGM3-T2
Handout 3.2	TGM3-T2 lesson 058

Activity outline

Introduction		2 minutes
Activity 1	Enlargement	20 minutes
Activity 2	Rotation, reflection and translation	40 minutes
Activity 3	Implementing the LPM (preparations, content, objectives)	25 minutes
Summary		3 minutes

Important note to facilitators:

Please ensure that all participants are actively engaged (pairs, groups, individual and whole group) to demonstrate good practice. Avoid lecturing and get the teachers to do most of the tasks. Remember, they are math teachers.

Introduction 2 minutes

1. Tell participants that they will be reflecting on two areas of geometry called enlargement and transformations.
2. This session is to look at the content as well as some basic required skills needed to implement the lesson plan manuals.

Activity 1: Enlargement 25 minutes

1. Give participants a minute to answer question a on Chart 3.2 (Find 3$\mathbf{a}$ when $\mathbf{a} = \binom{3}{-2}$)
2. Take responses from a couple of participants (Answer: $\mathbf{a} = \binom{3}{-2}$, $3\mathbf{a} = \mathbf{3}\binom{3}{-2} = \binom{9}{-6}$)
3. Ask participants what they understand by the word enlargement.
4. Agree that an enlargement is a transformation which enlarges or reduces the size of an image.
5. Tell participants that this activity will enable them use scalar multiplication to enlarge any given shapes.
6. Enlargement is described by a centre of enlargement and a scale factor, k.
7. Two different formulas are given for enlargement:
   - The formula for enlargement from the origin $\boldsymbol{O}$ by a scale factor $\boldsymbol{k}$ is given by:

     $$\binom{x}{y} \longrightarrow k\binom{x}{y} = \binom{kx}{ky}$$ where k is positive or negative whole number or fraction

     $$(x, y) \longrightarrow (kx, ky)$$
   - The formula for enlargement from any point $(\boldsymbol{a}, \boldsymbol{b})$ other than the origin $\boldsymbol{O}$ by a scale factor $\boldsymbol{k}$ can be found by following the steps given below.
8. Display Chart 3.3 and quickly go through the steps:

Step 1.	Subtract the co-ordinates of the centre of rotation (a, b) from (x, y)	$\binom{x-a}{y-b}$
Step 2.	Enlarge using the given scale factor	$\binom{x-a}{y-b} \longrightarrow \binom{k(x-a)}{k(y-b)}$
Step 3.	Add the result in Step 2 to the centre of rotation to get the image point	$\binom{k(x-a)+a}{k(y-b)+b}$

Step 4. Write the co-ordinates of the image point $(k(x-a)+a,\ k(y-b)+b)$

9. Ask participants to work with colleagues to answer question b. (Find the image of $(-1,-6)$ under the enlargement with scale factor of 4 from: i. The origin ii. The point (2,4)
10. Invite volunteers to show how they worked out the formulae on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

b. Given: $(-1,-6)$, enlarge with scale factor 4

i. $\begin{pmatrix}-1\\-6\end{pmatrix} \longrightarrow 4\begin{pmatrix}-1\\-6\end{pmatrix} = \begin{pmatrix}4\times(-1)\\4\times(-6)\end{pmatrix} = \begin{pmatrix}-4\\-24\end{pmatrix}$

ii. Given: $(-1,-6)$ enlarge about the point (2,4) with scale factor 4

$\begin{pmatrix}x-a\\y-b\end{pmatrix} \longrightarrow \begin{pmatrix}-1-2\\-6-4\end{pmatrix} = \begin{pmatrix}-3\\-10\end{pmatrix}$ subtract components of the centre of rotation from given point

$\begin{pmatrix}-3\\-10\end{pmatrix} \longrightarrow 4\begin{pmatrix}-3\\-10\end{pmatrix} = \begin{pmatrix}-12\\-40\end{pmatrix}$ enlarge using given scale factor

$\begin{pmatrix}-12\\-40\end{pmatrix} \longrightarrow \begin{pmatrix}-12+2\\-40+4\end{pmatrix} = \begin{pmatrix}-10\\-36\end{pmatrix}$ add back components of the centre of rotation

11. Invite a volunteer to assess questions c. i. and ii. and tell the class what information we are given. (Answer: given: triangle PQR with points $P(1,1)$, $Q(3,1)$ and $R(1,4)$)
12. Invite another volunteer to say what we have been asked to do. (Answer: draw the triangle PQR; find the image triangle $P_1Q_1R_1$ of triangle PQR under an enlargement from the origin with scale factor 2)
13. Show on the board how to enlarge using the given information.

Solution:

c. Given: triangle PQR with points $P(1,1)$, $Q(3,1)$ and $R(1,4)$

i. All diagrams for this question can be found at the end of the question.

ii Mapping under an enlargement from the origin $\begin{pmatrix}x\\y\end{pmatrix} \longrightarrow \begin{pmatrix}kx\\kx\end{pmatrix} = \begin{pmatrix}2x\\2x\end{pmatrix}$

$$\begin{pmatrix}1\\1\end{pmatrix} \longrightarrow 2\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}2\\2\end{pmatrix}$$
$$\begin{pmatrix}3\\1\end{pmatrix} \longrightarrow 2\begin{pmatrix}3\\1\end{pmatrix} = \begin{pmatrix}6\\2\end{pmatrix}$$
$$\begin{pmatrix}1\\4\end{pmatrix} \longrightarrow 2\begin{pmatrix}1\\4\end{pmatrix} = \begin{pmatrix}2\\8\end{pmatrix}$$

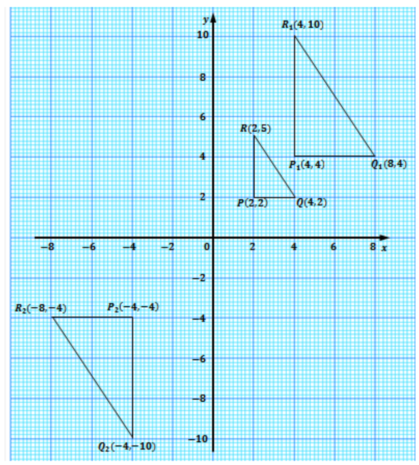
$P_1(2,2), Q_1(6,2)$ and $R_1(2,8)$

14. Ask participants to work with colleagues to complete questions c. iii.
15. Invite a volunteer to show the solution on the board. The rest of the class should check their solution and correct any mistakes.

iii.

$$\begin{pmatrix}1\\1\end{pmatrix} \longrightarrow -2\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}-2\\-2\end{pmatrix}$$
$$\begin{pmatrix}3\\1\end{pmatrix} \longrightarrow -2\begin{pmatrix}3\\1\end{pmatrix} = \begin{pmatrix}-6\\-2\end{pmatrix}$$
$$\begin{pmatrix}1\\4\end{pmatrix} \longrightarrow -2\begin{pmatrix}1\\4\end{pmatrix} = \begin{pmatrix}-2\\-8\end{pmatrix}$$

$P_2(-2,-2), Q_2(-6,-2)$ and $R_2(-2,-8)$



iv. The image $P_2Q_2R_2$ is upside down.

16. Explain: Negative scale factors give images at the opposite side of the centre of enlargement. The image is turned upside down (or inverted).
17. Ask participants to work with colleagues to complete question d.
18. Invite a volunteer to show the solution on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

d Given: $A(8,4)$, $B(8,-8)$, $C(-4,-8)$ and $D(-4,4)$

i.

$$\begin{pmatrix}8\\4\end{pmatrix} \longrightarrow \frac{1}{2}\begin{pmatrix}8\\4\end{pmatrix} = \begin{pmatrix}4\\2\end{pmatrix}$$
$$\begin{pmatrix}8\\-8\end{pmatrix} \longrightarrow \frac{1}{2}\begin{pmatrix}8\\-8\end{pmatrix} = \begin{pmatrix}4\\-4\end{pmatrix}$$
$$\begin{pmatrix}-4\\-8\end{pmatrix} \longrightarrow \frac{1}{2}\begin{pmatrix}-4\\-8\end{pmatrix} = \begin{pmatrix}-2\\-4\end{pmatrix}$$
$$\begin{pmatrix}-4\\4\end{pmatrix} \longrightarrow \frac{1}{2}\begin{pmatrix}-4\\4\end{pmatrix} = \begin{pmatrix}-2\\2\end{pmatrix}$$

$A_1(4,2), B_1(4,-4), C_1(-2,-4)$, and $D_1(-2,2)$

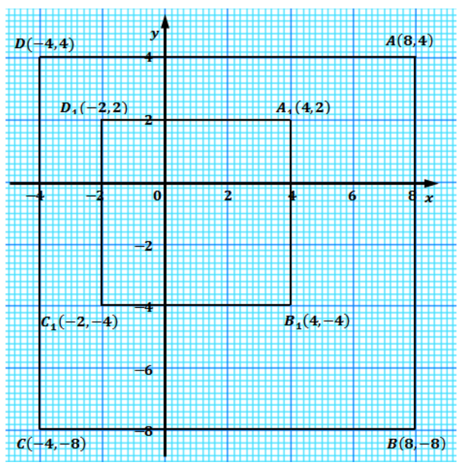
ii. The co-ordinates of $A_1B_1C_1D_1$ are half that of the original

19. Explain:
- An object (shape) under enlargement with a scale factor which is a fraction, results in a smaller image than the object. It is a reduction.
- The diagram in the next question will show this clearly.

20. Ask participants to work independently to answer question e.
21. Walk around, if possible, to check the answers and clear any misconceptions.
22. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solution:

Given: $A(8,4)$, $B(8,-8)$, $C(-4,-8)$ and $D(-4,4)$ from



Activity 2: Rotation, reflection and translation 30 minutes

1. Invite a volunteer to assess questions a. i. and ii. and tell the group what information we are given. (Answer: given: triangle PQR with $P(3,2)$, $\overrightarrow{PQ} = \binom{4}{1}$ and $\overrightarrow{QR} = \binom{-1}{2}$)
2. Invite another volunteer to say what we have been asked to do. (Answer: Draw the image triangle $P_1Q_1R_1$ of triangle PQR under an anti-clockwise rotation of 90° about the origin.)

Solution:

a. Step 1. Assess and extract the given information from the problem.

Given: $P(3,2)$, $\overrightarrow{PQ} = \binom{4}{1}$ and $\overrightarrow{QR} = \binom{-1}{2}$

Step 2. Find the co-ordinates of Q and R.

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ $\qquad$ $\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$

$\binom{4}{1} = \overrightarrow{OQ} - \binom{3}{2}$ $\qquad$ $\binom{-1}{2} = \overrightarrow{OR} - \binom{7}{3}$

$\overrightarrow{OQ} = \binom{4}{1} + \binom{3}{2}$
$= \binom{7}{3}$

$\overrightarrow{OR} = \binom{-1}{2} + \binom{7}{3}$
$= \binom{6}{5}$

$P(3,2)$, $Q(7,3)$ and R $(6,5)$.

Step 3. Draw the triangle with vertices above

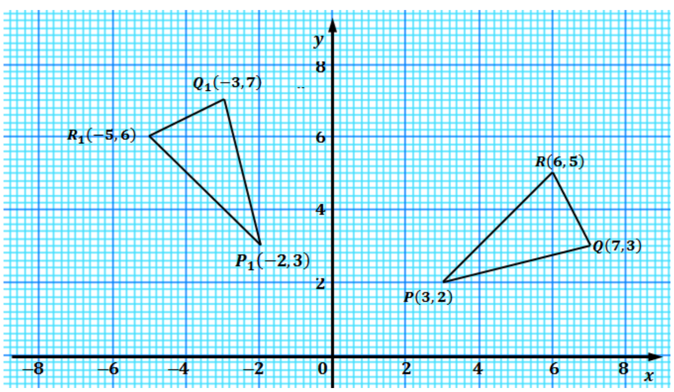
ΔPQR shown below.

Step 4. Apply the appropriate mapping formula.

$\binom{x}{y} \longrightarrow \binom{-y}{x}$
$\binom{3}{2} \longrightarrow \binom{-2}{3}$
$\binom{7}{3} \longrightarrow \binom{-3}{7}$
$\binom{6}{5} \longrightarrow \binom{-5}{6}$

Step 5. Write the answer and draw the image triangle (shown below).

$P_1(-2,3)$, $Q_1(-3,7)$ and $R_1(-5,6)$



Step 6. Use the graph to find $\overrightarrow{P_1R_1}$ and hence $|\overrightarrow{P_1R_1}|$.

$\overrightarrow{P_1R_1} = \binom{-3}{3}$

$|\overrightarrow{P_1R_1}| = \sqrt{(-3)^2 + 3^2}$

$= \sqrt{9+9} = \sqrt{18}$

$= 3\sqrt{2}$ units

3. Ask participants to work in groups to answer question b.
4. Invite volunteers to show their solution on the board. The rest of the group should check their solution and correct any mistakes.

   Solution:

   b. $A(2,1)$, $B(6,1)$ and $C(2,5)$

   The translation of A by a vector $\binom{4}{0}$ maps it to B

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} x+a \\ y+b \end{pmatrix}$$
$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 2+4 \\ 1+0 \end{pmatrix}$$
$$\longrightarrow \begin{pmatrix} 6 \\ 1 \end{pmatrix}$$

The reflection in the line $x = 4$ maps $A\ to\ B$, $k = 4$

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow \begin{pmatrix} 2k-x \\ y \end{pmatrix} = \begin{pmatrix} 2(4)-x \\ y \end{pmatrix} = \begin{pmatrix} 8-x \\ y \end{pmatrix}$$
$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 8-2 \\ 1 \end{pmatrix}$$
$$\longrightarrow \begin{pmatrix} 6 \\ 1 \end{pmatrix}$$

5. Ask participants to work independently to answer questions c. and d.
6. Walk around, if possible, to check the answers and clear up any misconceptions.
7. Invite volunteers to come to the board to show their solutions. The rest of the group should check their solutions and correct any mistakes.
   Solutions:
   c. Given: $A(1,0)$, $B(1,3)$, $C\ (4,3)$, scale factor $k = -2$, centre of enlargement $(0,0)$

$$\begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow k\begin{pmatrix} x \\ y \end{pmatrix} = -2\begin{pmatrix} x \\ y \end{pmatrix}$$
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \longrightarrow -2\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$
$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \longrightarrow -2\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$$
$$\begin{pmatrix} 4 \\ 3 \end{pmatrix} \longrightarrow -2\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -8 \\ -6 \end{pmatrix}$$

$A_1(-2,0)$, $B_1(-2,-6)$ and $C_1(-8,-6)$

Given: $B(1,3)$, $B_1(-2,-6)$,

gradient $= \frac{y_2-y_1}{x_2-x_1}$ Let $x_1 = 1$, $y_1 = 3$, $x_2 = -2$, $y_2 = -6$

$= \frac{-6-3}{-2-1}$ substitute the assigned variables

$= \frac{-9}{-3}$

$= 3$

Using point $B(1,3)$

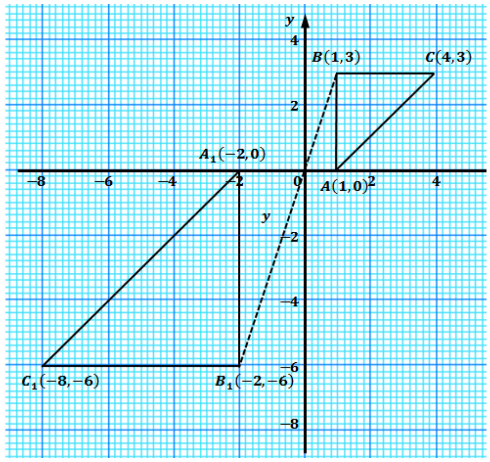
$\frac{y-3}{x-1} = 3$ standard formula for equation of a straight line

$y - 3 = 3(x - 1)$

$y - 3 = 3x - 3$

$y = 3x$

The equation of line BB_1 is $y = 3x$.



d. Given: scale factor $k = -0.5$, $AB = 6.4$ cm

$$k = \frac{\text{length of image}}{\text{length of object}}$$

$$-0.5 = \frac{\text{length of image}}{6.4}$$

$$\text{length of image} = 6.4 \times (-0.5) = -3.2$$

Since lengths cannot be negative, length of image = 3.2 cm.

8. Ask participants to discuss with partners the main differences between enlargements and the other transformations.
9. Invite volunteers to give their answer. (Example answer: Enlargements change the size of the object while the other transformations maintain the size of the object)

Activity 3: Implementing the LPM (preparations, content, objectives) 30 minutes

1. Ask participants to turn to Chart 2.2 and identify 3 other activities we will be applying in this session in the course of implementing the LPMs.

2. Get volunteers to read them out to the whole group (identifying preparations required for LPM implementation; demonstrating mastery of the content and linking teaching activities to objectives)
3. Ask participants what each means and what they aim at achieving
4. Required preparation ensures teachers get all the necessary materials they would need to help them deliver competent lessons in terms of instructional materials, games in instructions, grouping especially in the case of large classes, assessment rubrics etc.
5. Mastery of content is done when teachers engage in self-study or work with colleagues to overcome any challenges in delivering the LPM. This could be done daily or weekly to get familiar with the content.
6. Learning outcomes state what to be achieved by the students by the end of the lesson. It is important to ensure that most of the teaching activities are directly relevant to achieving the learning outcomes. Teachers need to ask themselves during preparations, 'How will these activities help me achieve the objectives?' They are also very useful in assessing self after lesson delivery
7. Also agree that these three areas help in getting ourselves ready to deliver competent lessons with confidence. They also help us to remain focused and get help as soon as possible before getting to face the pupils.
8. Get participants to turn to TGM2-T2 lesson 58
9. Give them 5 minutes to discuss in their groups how they would approach delivering this using the 3 guides they explored above
10. Assign 'opening' to 2 pairs; teaching and learning to 12 pairs and practice to 3 or 4 pairs
11. Participants may need to read all steps of the lesson to have an overall understanding of the lesson.
12. Invite pairs to demonstrate the assigned tasks. Take feedback and allow these to run to the end of the session.
13. Feedback should include questions like: Did you need any materials? How did you know materials to use? Did you struggle with the task? How did you overcome this? Where else could help come from? Did you stay right on track? Etc

Session summary 5 minutes

Take participants through the learning outcomes again.

Materials

Chart 3.1 Learning outcomes Introduction

Chart 3.2: Questions

a. Find 3$\mathbf{a}$ when $\mathbf{a} = \binom{3}{-2}$.
b. Find the image of $(-1, -6)$ under the enlargement with scale factor of 4 from:
   i. The origin ii. The point (2,4)
c. Draw on the given axes showing clearly the co-ordinates of all vertices:
   i. The triangle PQR with $P(1,1)$, $Q(3,1)$ and $R(1,4)$.
   ii. The image triangle $P_1Q_1R_1$ of triangle PQR under an enlargement from the origin with scale factor 2 where $P \longrightarrow P_1$, $Q \longrightarrow Q_1$ $R \longrightarrow R_1$.
   iii. The image triangle $P_2Q_2R_2$ of triangle PQR under an enlargement from the origin with scale factor -2 where $P \longrightarrow P_2$, $Q \longrightarrow Q_2$ $R \longrightarrow R_2$.
   iv. What do you notice about the enlargement $P_2Q_2R_2$?
d. A square has vertices $A(8,4)$, $B(8,-8)$, $C(-4,-8)$ and $D(-4,4)$.
   i. Find the co-ordinates of the vertices of the image square $A_1B_1C_1D_1$ of $ABCD$ under an enlargement from the origin with scale factor $\frac{1}{2}$ where $A \longrightarrow A_1$, $B \longrightarrow B_1$, $C \longrightarrow C_1$ and $D \longrightarrow D_1$.
   ii. What do you notice about the co-ordinates of $A_1B_1C_1D_1$?
e. Using a scale of 2 cm to 2 units on both axes, draw on a sheet of graph paper, two perpendicular axes Ox and Oy for $-4 \leq x \leq 8$ and $-8 \leq y \leq 4$.
   Draw on the same axes, showing clearly the co-ordinates of all vertices:
   i. The square $ABCD$ from question d.
   ii. The image $A_1B_1C_1D_1$ of $ABCD$ from question d.

Chart 3.4: Questions

a. Draw on the given axes, showing clearly the co-ordinates of all vertices:
   i. The triangle PQR with $P(3,2)$, $\overrightarrow{PQ} = \binom{4}{1}$ and $\overrightarrow{QR} = \binom{-1}{2}$
   ii. The image triangle $P_1Q_1R_1$ of triangle PQR under an anti-clockwise rotation of 90° about the origin where $P \longrightarrow P_1$, $Q \longrightarrow Q_1$ $R \longrightarrow R_1$.

iii. Use the graph to find $\overrightarrow{P_1R_1}$ and hence $|\overrightarrow{P_1R_1}|$ leaving the answer in surd form.

b. A triangle ABC has vertices $A(2,1), B(6,1)$ and $C(2,5)$.
Describe two different transformations that fully map A to B.

c. Using a scale of 2 cm to 2 units on both axes, draw two perpendicular axes Ox and Oy for $-8 \leq x \leq 4$ and $-8 \leq y \leq 4$.

a. Draw on the same axes, showing clearly the co-ordinates of all vertices:

i. The triangle $A(1,0)$, $B(1,3)$ and C $(4,3)$

ii. The image $\Delta A_1B_1C_1$ of ΔABC under an enlargement about the origin with scale factor -2 where $A \longrightarrow A_1$, $B \longrightarrow B_1$ and $C \longrightarrow C_1$..

b. Find the equation of the line BB_1.

d. Triangle P is mapped to triangle Q by an enlargement of scale factor -0.5. If AB is 6.4 cm long, how long is FD?

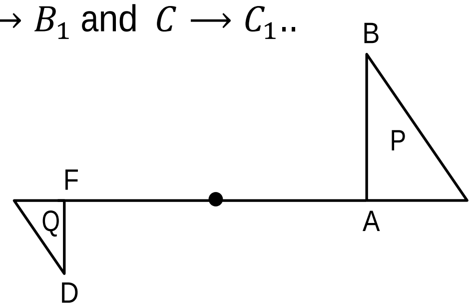


Chart 2.2 Familiarising yourselves with the LPMs

- following steps of the plans
- observing lesson timing
- linking teaching activities to objectives
- identifying preparations required
- demonstrating mastery of the content
- practicing
- working with others
- observing each other
- checking that learning is happening
- linking participants' handbook to LPM

Session 4: Surds, and other roots 90 minutes

Learning outcomes

By the end of the session participants will be able to prepare to deliver lessons on surds demonstrating:

- practicing
- working with others
- observing each other

Important note to facilitators:

Please ensure that all participants are actively engaged (pairs, groups, individual and whole group) to demonstrate good practice. Avoid lecturing and get the teachers to do most of the tasks. Remember, they are math teachers.

Roots and powers are closely related, but only some roots can be written as whole numbers. Surds are roots which cannot be written in this way. Nevertheless, it is possible to manipulate surds, and to simplify formulae involving them.

In order to master the techniques explained here it is vital that you undertake plenty of practice exercises so that they become second nature.

After reading this text, (and/or viewing the video tutorial) on this topic, you should be able to:

- understand the relationship between negative powers and positive powers;
- understand the relationship between fractional powers and whole-number powers;
- replace formulae involving roots with formulae involving fractional powers;
- understand the difference between surds and whole-number roots;
- simplify expressions involving surds;
- rationalise fractions with surds in the denominator.

Materials

Chart 4.1	Learning Outcomes
Chart 2.2	Implementing the lesson plans
Handout 4.1	TGM1-T2
Handout 4.2	TGM1-T2 lesson

Activity outline

Introduction		2 minutes
Activity 1	Powers and roots	10 minutes
Activity 2	Surds and irrational numbers	15 minutes
Activity 3	Simplifying expressions involving surds	20 minutes
Activity 4	Rationalising expressions containing surds	20 minutes
Activity 5	Implementing the LPM (practice, working with others and observing each other	20 minutes
Summary		3 minutes

Introduction

- In this unit we are going to explore numbers written as powers, and perform some calculations involving them. In particular, we are going to look at square roots of whole numbers which produce irrational numbers — that is, numbers which cannot be written as fractions. These are called surds.

Activity 1: Powers and roots 20 minutes

- We know that 2 cubed is 2 × 2 × 2, and we say that we have 2 raised to the power 3, or to the index 3. An easy way of writing this repeated multiplication is by using a 'superscript', so that we would write 2^3:
- $2^3 = 2 \times 2 \times 2 = 8$.
- Similarly, 4 cubed is 4 × 4 × 4, and equals 64. So we write
  $4^3 = 4 \times 4 \times 4 = 64$.
- But what if we have negative powers? What would be the value of 4^{-3}?
- Take responses and solve as below with them
  $4^3 = 4 \times 4 \times 4 = 64$.
  $4^2 = 4 \times 4 = 16$,
  $4^1 = 4 = 4$, and so $4^0 = 4 \div 4 = 1$ (because to get the answer you divide the previous one by 4).
- Now let groups continue the pattern to -3 as below:
  $4^{-1} = 1 \div 4 = \frac{1}{4}$
  $4^{-2} = \frac{1}{4} \div 4 = \frac{1}{16}$
  $4^{-3} = \frac{1}{16} \div 4 = \frac{1}{64}$

and $\frac{1}{64}$ = $1/4^3$. So a negative power gives the reciprocal of the number — that is, 1 over the number. Thus $4^{-2} = \frac{1}{4} \div 4 = \frac{1}{16}$ and $4^{-1} = 1 \div 4 = \frac{1}{4}$

- Similarly,

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

And $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

- A common misconception is that since the power is negative, the result must be negative: as you can see, this is not so.
- Now we know that $4^0 = 1$ and $4^1 = 4$, but what is $4^{1/2}$?
- Using the rules of indices, we know that $4^{1/2} \times 4^{1/2} = 4^1 = 4$ because $\frac{1}{2} + \frac{1}{2} = 1$. So $4^{1/2}$ equals 2, as 2 × 2 = 4. Therefore $4^{1/2}$ is the square root of 4. It is written as √4 and equals 2: $4^{1/2} = \sqrt{4} = 2$.
- Similarly, $9^{1/2} = \sqrt{9} = 3$.
- And in general, any number a raised to the power $\frac{1}{2}$ equals the square root of a:

$a^{1/2} = \sqrt{a}$.

- So the power, or index, associated with square roots is $\frac{1}{2}$. Also, in the same way that the index $\frac{1}{2}$ represents the square root, other fractions can be used to represent other roots. The cube root of the number 4 is written as $4^{1/3} = \sqrt[3]{4}$ where $\frac{1}{3}$ is the index representing cube root. Similarly, the fourth root of 5 may be written as $5^{1/4} = \sqrt[4]{5}$ and so on. The n-th root is represented by the index $\frac{1}{n}$, and the n-th root of a is written as $a^{1/n} = \sqrt[n]{a}$ So, for example, if we have $\sqrt[3]{64}$ then this equals 64 to the power $\frac{1}{3}$; and then $\sqrt[3]{64} = 64^{1/3} = (4\ x\ 4\ x\ 4)^{1/3} = 4$.
- There are some important points about roots, or fractional powers, that we need to remember. First, we can write $4^{1/2} \times 4^{1/2} = 4$; $\sqrt{4} \times \sqrt{4} = (\sqrt{4})^2 = 4$ so that the square root of 4, squared, gives you 4 back again. In fact the square root of any number, squared, gives you that number back again.

- Next, if we have a very simple quadratic equation to solve, such as $x^2 = 4$, then the solutions are x = +2 or x = −2. There are two roots, as (+2)× (+2) = 4 and also (−2)× (−2) = 4. We can write the roots as ±2. So not all roots are unique. But in a lot of circumstances we only need the positive root, and you do not have to put a plus sign in front of the square root for the positive root. By convention, if there is no sign in front of the square root then the root is taken to be positive.

- On the other hand, suppose we were given $\sqrt{-9}$. Could we work this out and get a real answer? Now $\sqrt{-9}$ = $(-9)^{1/2}$,
and so we are looking for a number which multiplied by itself gives −9. But there is no such number, because 3 × 3 = 9 and also (−3) × (−3) = 9. So you cannot find the square root of a negative number and get a real answer.

Activity 2: Surds and irrational numbers 20 minutes

- We shall now look at some square roots in more detail. Take, for example, $\sqrt{25}$: its value is 5. And the value of $\sqrt{\frac{9}{4}}$ is $\frac{3}{2}$ or $1\frac{1}{2}$. So some square roots can be evaluated as whole numbers or as fractions, in other words as rational numbers.
- But what about $\sqrt{2}$ or $\sqrt{3}$? The roots to these are not whole numbers or fractions, and so they have irrational values. They are usually written as decimals to a given approximation. For example $\sqrt{2}$ = 1.414 to 3 decimal places, $\sqrt{3}$ = 1.732 to 3 decimal places.
- When we have square roots which give irrational numbers we call them surds. So $\sqrt{2}$ and $\sqrt{3}$ are surds. Other surds are $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$, $\sqrt{10}$ and so on. Surds are often found when using Pythagoras' Theorem, and in trigonometry. So, where possible, it is useful to be able to simplify expressions involving surds. Take, for example, $\sqrt{8}$. This can be written as $\sqrt{4 \times 2}$, which we can rewrite as $\sqrt{4} \times \sqrt{2}$, in other words as $2\sqrt{2}$:
- In general, the square root of a product is the product of the square roots, and vice versa. This is useful to know when simplifying surd expressions.
- Now suppose we have been given $\sqrt{5} \times \sqrt{15}$. At first glance this cannot be simplified. But we can rewrite the expression as the square root of 5 times 15, so it is the square root of 75, and 75 can be written as 25 times 3. But 25 is a perfect square, we can use this to simplify the expression.
$\sqrt{5} \times \sqrt{15} = \sqrt{5 \times 15} = \sqrt{75} = \sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$

- But watch out if you are given $\sqrt{4+9}$, which is the square root of 13. This does not equal $\sqrt{4}$ + $\sqrt{9}$, which is 2 + 3 = 5. Now 5 cannot be the answer as that is the square root 25, not the square root of 13.

Activity 3: Simplifying expressions involving surds 20 minutes

- Knowing the common square numbers like 4, 9, 16, 25, 36 and so on up to 100 is very helpful when simplifying surd expressions, because you know their square roots straight away, and you can use them to simplify more complicated expressions. Suppose we were asked to simplify the expression $\sqrt{400} \times \sqrt{90}$
$\sqrt{400} \times \sqrt{90} = \sqrt{4 \times 100} \times \sqrt{9 \times 10} = \sqrt{4} \times \sqrt{100} \times \sqrt{9} \times \sqrt{10} = 2 \times 10 \times 3 \times \sqrt{10} = 60\sqrt{10}$, which cannot be simplified any further.
- We can also simplify the expression $\sqrt{2000}/\sqrt{50}$. We get $\frac{\sqrt{2000}}{\sqrt{50}} = \sqrt{\frac{2000}{50}} = \sqrt{40} = \sqrt{4 \times 10} = \sqrt{4} \times \sqrt{10} = 2\sqrt{10}$

- If we have a product of brackets involving surds, for example $(1+\sqrt{3})(2-\sqrt{2})$, we can expand out the brackets in the usual way:
$(1+\sqrt{3})(2-\sqrt{2}) = 2-\sqrt{2}+2\sqrt{3}-\sqrt{3}\sqrt{2} = 2-\sqrt{2}+2\sqrt{3}-\sqrt{6}$

- But what if we have this expression, $(1+\sqrt{3})(1-\sqrt{3})$? If we expand this out and simplify the answer, we get
$(1+\sqrt{3})(1-\sqrt{3}) = 1-\sqrt{3}+\sqrt{3}-\sqrt{3}\sqrt{3} = 1-\sqrt{9} = 1-3 = -2$.
- So the product $(1+\sqrt{3})(1-\sqrt{3})$ does not involve surds at all. This is an example of a general result known as the difference of two squares. This general result may be written as $a^2 - b^2 = (a+b)(a-b)$ for any numbers a and b. In our example a = 1 and b = $\sqrt{3}$, so $(1+\sqrt{3})(1-\sqrt{3}) = 1^2 - (\sqrt{3})^2 = 1-3 = -2$.
- The expansion of the difference of two squares is another useful fact to know and remember.

Conjugates of surds:

- The conjugate of the surd $a+\sqrt{b}$ is $a-\sqrt{b}$ where a and b are whole numbers.
- The product of a surd and its conjugate is a whole number, not a surd. That is,

$(a+\sqrt{b})(a-\sqrt{b}) = a^2 - \left(\sqrt{b}\right)^2 = a^2 - b$ (using a difference of two squares), which is a whole number.

- Example

The conjugate of $5 + \sqrt{3}$ is $5 - \sqrt{3}$

The product of the two is $\left(5+\sqrt{3}\right)\left(5-\sqrt{3}\right) = 5^2 - \left(\sqrt{3}\right)^2 = 25 - 3 = 22$, which is a whole number.

Note that the conjugate of $\sqrt{a}$ is $\sqrt{a}$ because $\sqrt{a} \times \sqrt{a} = \sqrt{a^2} = a$, which is a whole number.

Eg The conjugate of $\sqrt{7}$ is $\sqrt{7}$, the product of the two is 7.

- Exercise:

Find the product of the following surds and their conjugates:

a) $\sqrt{11}$ b) $1 + \sqrt{5}$ c) $2\sqrt{3} - 5$

Answer Key.

a) 11 b) -4 c) -13

Activity 4: Rationalising expressions containing surds 25 minutes

- Sometimes in calculations we obtain surds as denominators, for example $1/\sqrt{13}$. It is best not to give surd answers in this way. Instead, we use a technique called rationalisation. This changes the surd denominator, which is irrational, into a whole number.
- To see how to do this, take our example $1/\sqrt{13}$. To rationalise this, we multiply by the fraction p13/p13 which is equal to 1. When we multiply by this fraction we do not change the value of our original expression. We obtain $1/\sqrt{13} = 1/\sqrt{13} \times \sqrt{13}/\sqrt{13} = \sqrt{13}/13$
- We can also use the expansion of the difference of two squares to rationalise more complicated expressions involving surds.
- Example: Rationalise the expression $\frac{1}{1+\sqrt{2}}$

Solution

If we multiply this expression by this fraction $\frac{1-\sqrt{2}}{1-\sqrt{2}}$ we do not change its value, as the new fraction is equal to 1. When we do this, we use the formula for the difference of two squares to work out $(1 + \sqrt{2}) \times (1 - \sqrt{2})$, and that gives us $1^2 - (\sqrt{2})^2$, which is 1 – 2 = –1. So now we have a whole number in the denominator of our fraction, and we can divide through. We get

$$\frac{1}{1+\sqrt{2}} = \frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{1-\sqrt{2}}{1^2 - (\sqrt{2})^2} = \frac{1-\sqrt{2}}{-1}$$

$= -1 + \sqrt{2}$

= p2 – 1.

- Example: Rationalise the expression $\frac{1}{\sqrt{5}-\sqrt{3}}$

Solution

To rationalise this, we must multiply it by a fraction which equals 1. To choose a suitable fraction, we think of the difference of two squares. As our expression has $\sqrt{5} - \sqrt{3}$ in the denominator, the fraction to use is $(\sqrt{5} + \sqrt{3})$ over $(\sqrt{5} + \sqrt{3})$:

$$\frac{1}{\sqrt{5}-\sqrt{3}} = \frac{1}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{5-3} = \frac{\sqrt{5}+\sqrt{3}}{2} = \frac{\sqrt{5}}{2} + \frac{\sqrt{3}}{2}$$

Exercises

1. Simplify the following expressions:

(a) $\sqrt{40} \times \sqrt{200}$

(b) $\sqrt{90} \times \sqrt{600000}$

(c) $\sqrt{1000}/\sqrt{40}$

(d) $(1 - \sqrt{5})(1 + \sqrt{5})$

(e) $(\sqrt{7} + 3)(\sqrt{7} - 3)$

2. Rationalise the following expressions:

(a) $1/\sqrt{11}$

(b) $30/\sqrt{3}$

(c) $1/(1 - \sqrt{3})$

(d) $1/(\sqrt{7} + \sqrt{2})$

(e) $2/(\sqrt{5} - \sqrt{7})$

Answer Key.

1. (a) $40\sqrt{5}$ (b) $3000\sqrt{6}$ (c) 5 (d) −4 (e) −2

2. (a) $\sqrt{11}/11$ (b) $10\sqrt{3}$ (c) $-\frac{1}{2} - \sqrt{3}/2$ (d) $\sqrt{7}/5 - \sqrt{2}/5$ (e) $-\sqrt{5} - \sqrt{7}$

Activity 5: Implementing the LPM (practice, working with others and observing each other 20 minutes

- Ask participants what they could do to get external support to become more confident in LPM implementation.
- Uncover points 6, 7 and 8 on Chart 2.2 which describes what teachers need to do to implement the lesson plans.
- Allow participants 3 minutes to discuss in groups what the points mean, and describe what practical activities are involved.
- Participants write their responses on flip chart paper and market place them for 'travellers'
- Likely responses
    1. Practice: Teacher goes through lessons (1 is sufficient) to be taught weekly and demonstrates how to deliver to colleagues, HoDs, SSOs, wife, husband, in front of a mirror, recording and playing back audio or video etc)
    2. Working with others (colleagues, HoDs and SSOs): This entails running difficult topics by anyone who could assist even if they are junior to us.
    3. Observing each other: This activity is very useful especially if positive relationships exist among teachers. They could arrange for how to observe one another with feedback for improvement. Teachers could ask if they could observe teachers that are doing well in terms of good classroom practice, and invite competent teachers to step in to observe them with the aim of improving on their content and pedagogical skills
    4. Get 3 groups to role play each of the areas. Lessons should be picked from TGM3-T2
    5. Other groups take notes for feedback

- Conclude by stressing the importance of seeking for help when the needs arise. Keeping our weaknesses to ourselves would get it out in the open when we get to the class. Sharing and asking for help is the way to go.

Session summary 3 minutes

Take participants through the learning outcomes again

Materials

Chart 2.1	Learning outcomes	Introduction

Session 5 Arithmetic and geometric progressions ***90 minutes***

Learning outcomes

By the end of the session participants will be able to prepare to deliver lessons on arithmetic and geometric progressions:

- checking that learning is happening
- linking pupils' handbook to LPM

Important note to facilitators:

Please ensure that all participants are actively engaged (pairs, groups, individual and whole group) to demonstrate good practice. Avoid lecturing and get the teachers to do most of the tasks. Remember, they are math teachers.

Arithmetic and geometric progressions

This session introduces sequences and series, and gives some simple examples of each. It also explores particular types of sequence known as arithmetic progressions (APs) and geometric progressions (GPs), and the corresponding series.

In order to master the techniques explained here it is vital that you undertake plenty of practice exercises so that they become second nature.

After reading this text, and/or viewing the video tutorial on this topic, you should be able to:

- recognise the difference between a sequence and a series;
- recognise an arithmetic progression;
- find the n-th term of an arithmetic progression;
- find the sum of an arithmetic series;
- recognise a geometric progression;
- find the n-th term of a geometric progression;
- find the sum of a geometric series;
- find the sum to infinity of a geometric series with common ratio $|r| < 1$

Materials

Chart 5.1	Learning Outcomes
Chart 2.2	Implementing the lesson plans
Handout 5.1	TGM1-T2
Handout 5.2	TGM1-T2 lesson

Activity outline

Introduction		2 minutes
Activity 1:	Sequences and series	15 minutes
Activity 2:	Arithmetic progressions, sum of an arithmetic series	25 minutes
Activity 3:	Geometric progressions, sum of geometric series	25 minutes
Activity 4:	Implementing the LPM (assessing learning and handbook)	20 minutes
Summary		3 minutes

Session Introduction 2 minutes

- Welcome participants to the session
- Display Chart 5.1 (Learning Outcomes) and go through it with the participants.

Activity 1 15 minutes

Activity 1a: Sequences and series

- What is a sequence? It is a set of numbers which are written in some particular order. For example, take the numbers 1, 3, 5, 7, 9, ...
- Here, we seem to have a rule. We have a sequence of odd numbers. To put this another way, we start with the number 1, which is an odd number, and then each successive number is obtained by adding 2 to give the next odd number.
- Here is another sequence: 1, 4, 9, 16, 25, ... This is the sequence of square numbers. And this sequence, 1, −1, 1, −1, 1, −1, . . . , is a sequence of numbers alternating between 1 and −1. In each case, the dots written at the end indicate that we must consider the sequence as an infinite sequence, so that it goes on forever.

- On the other hand, we can also have finite sequences. The numbers 1, 3, 5, 9 form a finite sequence containing just four numbers. The numbers 1, 4, 9, 16 also form a finite sequence. And so do these, the numbers 1, 2, 3, 4, 5, 6, . . . , n.
- These are the numbers we use for counting, and we have included n of them. Here, the dots indicate that we have not written all the numbers down explicitly. The n after the dots tells us that this is a finite sequence, and that the last number is n.
- Here is a sequence that you might recognise: 1, 1, 2, 3, 5, 8, ...
- This is an infinite sequence where each term (from the third term onwards) is obtained by adding together the two previous terms. This is called the Fibonacci sequence.
- We often use an algebraic notation for sequences. We might call the first term in a sequence u1, the second term u2, and so on. With this same notation, we would write un to represent the n-th term in the sequence. So u1, u2, u3, . . . , un would represent a finite sequence containing n terms. As another example, we could use this notation to represent the rule for the Fibonacci sequence. We would write un = un−1 + un−2 to say that each term was the sum of the two preceding terms.

Exercise 1

(a) A sequence is given by the formula un = 3n + 5, for n = 1, 2, 3, ... Write down the first five terms of this sequence.

(b) A sequence is given by un = 1/n2, for n = 1, 2, 3, ... Write down the first four terms of this sequence. What is the 10th term?

(c) Write down the first eight terms of the Fibonacci sequence defined by un = un−1+un−2, when u1 = 1, and u2 = 1.

(d) Write down the first five terms of the sequence given by un = (−1)n+1/n.

Answer Key

$a)$ 8, 11, 14, 17, 20, − − − − − − − − − − − − − − − − − −

b) $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{25},$ − − − − − − − −

$u_{10} = \frac{1}{100}$

c) 1, 1, 2, 3, 5, 8, 13, 21, − − − − − −

d) $0, -\frac{3}{2}, -\frac{8}{3}, -\frac{15}{4}, -\frac{24}{5}, - - - - -$

Activity 1b: Series

- A series is something we obtain from a sequence by adding all the terms together. For example, suppose we have the sequence u1, u2, u3, . . ., un. The series we obtain from this is u1 + u2 + u3 + . . . + un, and we write Sn for the sum of these n terms. So although the ideas of a 'sequence' and a 'series' are related, there is an important distinction between them.
- For example, let us consider the sequence of numbers 1, 2, 3, 4, 5, 6, . . . , n. Then S1 = 1, as it is the sum of just the first term on its own. The sum of the first two terms is S2 = 1 + 2 = 3. Continuing, we get S3 = 1 + 2 + 3 = 6, S4 = 1 + 2 + 3 + 4 = 10, and so on.

Exercise 2

Write down S1, S2, . . . , Sn for the sequences (a) 1, 3, 5, 7, 9, 11; (b) 4, 2, 0,−2,−4.

Answer Key

a) $1, 4, - - - - - - n^2$

b) $4, 6, - - - - - - n(5 - n)$

Activity 2 25 minutes

Activity 2a: Arithmetic progressions

- Consider these two common sequences 1, 3, 5, 7, . . . and 0, 10, 20, 30, 40, ...
- It is easy to see how these sequences are formed. They each start with a particular first term, and then to get successive terms we just add a fixed value to the previous term. In the first sequence we add 2 to get the next term, and in the second sequence we add 10. So the difference between consecutive terms in each sequence is a constant. We could also subtract a constant instead, because that is just the same as adding a negative constant. For example, in the sequence 8, 5, 2, −1, −4, . . . the difference between consecutive terms is −3. Any sequence with this property is called an arithmetic progression, or AP for short.

- We can use algebraic notation to represent an arithmetic progression. We shall let a stand for the first term of the sequence, and let d stand for the common difference between successive terms. For example, our first sequence could be written as

  1, 3, 5, 7, 9, . . .

  1, 1 + 2, 1 + 2 × 2, 1 + 3 × 2, 1 + 4 × 2, . . . , and this can be written as a, a + d, a + 2d, a + 3d, a + 4d, . . . where a = 1 is the first term, and d = 2 is the common difference. If we wanted to write down the n-th term, we would have a + (n – 1)d, because if there are n terms in the sequence there must be (n–1) common differences between successive terms, so that we must add on (n – 1)d to the starting value a. We also sometimes write ℓ for the last term of a finite sequence, and so in this case we would have ℓ = a + (n – 1)d.

Exercise 3

(a) Write down the first five terms of the AP with first term 8 and common difference 7.

(b) Write down the first five terms of the AP with first term 2 and common difference –5.

(c) What is the common difference of the AP 11,–1,–13,–25, . . . ?

(d) Find the 17th term of the arithmetic progression with first term 5 and common difference 2.

(e) Write down the 10th and 19th terms of the Aps (i) 8, 11, 14, . . ., (ii) 8, 5, 2 . . ..

(f) An AP is given by k, 2k/3, k/3, 0, ...

(i) Find the sixth term., (ii) Find the n th term., (iii) If the 20th term is equal to 15, find k.

Answer Key

a) 8, 15, 22, 29, 36, – – – – – – – – – –

b) 2, –3, –8, –13, –18, – – – – – – – –

c) –13

d) 85

e) (i) 35, 62 (ii) –19, –46

f) (i) $-\frac{2k}{3}$, (ii) $\frac{k}{3}(4-n)$ (iii) $-\frac{45}{16}$

Activity 2b: The sum of an arithmetic series

- Sometimes we want to add the terms of a sequence. What would we get if we wanted to add the first n terms of an arithmetic progression? We would get Sn = a + (a + d) + (a + 2d) + . . . + (ℓ − 2d) + (ℓ − d) + ℓ.
- Now this is now a series, as we have added together the n terms of a sequence. This is an arithmetic series, and we can find its sum by using a trick. Let us write the series down again, but this time we shall write it down with the terms in reverse order. We get Sn = ℓ + (ℓ − d) + (ℓ − 2d) + . . . + (a + 2d) + (a + d) + a.
- We are now going to add these two series together. On the left-hand side, we just get 2Sn. But on the right-hand side, we are going to add the terms in the two series so that each term in the first series will be added to the term vertically below it in the second series. We get 2Sn = (a + ℓ) + (a + ℓ) + (a + ℓ) + . . . + (a + ℓ) + (a + ℓ) + (a + ℓ), and on the right-hand side there are n copies of (a + ℓ) so we get 2Sn = n(a + ℓ). But of course we want Sn rather than 2Sn, and so we divide by 2 to get Sn = $\frac{1}{2}$n(a + ℓ) .
- We have found the sum of an arithmetic progression in terms of its first and last terms, a and ℓ, and the number of terms n.
- We can also find an expression for the sum in terms of the a, n and the common difference d.
- To do this, we just substitute our formula for ℓ into our formula for Sn.
- From ℓ = a + (n − 1)d , Sn = $\frac{1}{2}$n(a + ℓ) we obtain Sn = $\frac{1}{2}$n(a + a + (n − 1)d)
  = $\frac{1}{2}$n(2a + (n − 1)d)

Example

Find the sum of the first 50 terms of the sequence 1, 3, 5, 7, 9, ...

Solution

This is an arithmetic progression, and we can write down a = 1, d = 2 , n = 50. We now use the formula, so that Sn = $\frac{1}{2}$n(2a + (n − 1)d)

$S_{50} = \frac{1}{2} \times 50 \times (2 \times 1 + (50 - 1) \times 2)$

$= 25 \times (2 + 49 \times 2)$

$= 25 \times (2 + 98)$

$= 2500.$

Example

Find the sum of the series 1 + 3·5 + 6 + 8·5 + . . . + 101.

Solution

This is an arithmetic series, because the difference between the terms is a constant value, 2·5.

We also know that the first term is 1, and the last term is 101. But we do not know how many terms are in the series. So we will need to use the formula for the last term of an arithmetic progression, $\ell = a + (n - 1)d$ to give us $101 = 1 + (n - 1) \times 2{\cdot}5$.

Now this is just an equation for n, the number of terms in the series, and we can solve it. If we subtract 1 from each side we get $100 = (n - 1) \times 2{\cdot}5$ and then dividing both sides by 2·5 gives us $40 = n - 1$ so that $n = 41$. Now we can use the formula for the sum of an arithmetic progression, in the version using ℓ, to give us $S_n = \frac{1}{2}n(a + \ell)$

$S_{41} = \frac{1}{2} \times 41 \times (1 + 101) = \frac{1}{2} \times 41 \times 102 = 41 \times 51 = 2091.$

Example

An arithmetic progression has 3 as its first term. Also, the sum of the first 8 terms is twice the sum of the first 5 terms. Find the common difference.

Solution

We are given that a = 3. We are also given some information about the sums S_8 and S_5, and we want to find the common difference. So we shall use the formula

$S_n = \frac{1}{2}n(2a + (n - 1)d)$ for the sum of the first n terms. This tells us that $S_8 = \frac{1}{2} \times 8 \times (6 + 7d)$ and that $S_5 = \frac{1}{2} \times 5 \times (6 + 4d)$

So, using the given fact that S8 = 2S5, we see that $\frac{1}{2} \times 8 \times (6 + 7d) = 2 \times \frac{1}{2} \times 5 \times (6 + 4d)$

$4 \times (6 + 7d) = 5 \times (6 + 4d)$

$24 + 28d = 30 + 20d$

$8d = 6$

$d = \frac{3}{4}$.

Exercise 4

(a) Find the sum of the first 23 terms of the AP 4,−3,−10, ...

(b) An arithmetic series has first term 4 and common difference $\frac{1}{2}$.

Find

(i) the sum of the first 20 terms,

(ii) the sum of the first 100 terms.

(c) Find the sum of the arithmetic series with first term 1, common difference 3, and last term 100.

(d) The sum of the first 20 terms of an arithmetic series is identical to the sum of the first 22 terms. If the common difference is −2, find the first term.

Answer Key

a) −1579

b) (i) 175 (ii) 2875

c) 1717

d) 162

Activity 3a: Geometric progressions

- We shall now move on to the other type of sequence we want to explore.

- Consider the sequence 2, 6, 18, 54, ...
- Here, each term in the sequence is 3 times the previous term. And in the sequence 1, −2, 4, −8, . . . , each term is −2 times the previous term. Sequences such as these are called geometric progressions, or GPs for short.
- Let us write down a general geometric progression, using algebra. We shall take a to be the first term, as we did with arithmetic progressions. But here, there is no common difference. Instead there is a common ratio, as the ratio of successive terms is always constant. So we shall let r be this common ratio. With this notation, the general geometric progression can be expressed as a, ar, ar^2, ar^3, ...
- So the n-th can be calculated quite easily. It is ar^{n-1}, where the power (n−1) is always one less than the position n of the term in the sequence. In our first example, we had a = 2 and r = 3, so we could write the first sequence as 2, 2 × 3, 2 × 3^2, 2 × 3^3, ...
- In our second example, a = 1 and r = −2, so that we could write it as 1, 1 × (−2), 1 × $(-2)^2$, 1 × $(-2)^3$, ...

Exercise 5

(a) Write down the first five terms of the geometric progression which has first term 1 and common ratio $\frac{1}{2}$.

(b) Find the 10th and 20th terms of the GP with first term 3 and common ratio 2.

(c) Find the 7th term of the GP 2,−6, 18, . . .

Answer Key

a) 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, − − − − − − − −

b) (i) 1536 (ii) 1572864

c) 1458

Activity 3b: The sum of a geometric series

- Suppose that we want to find the sum of the first n terms of a geometric progression. What we get is Sn = a + ar + ar^2 + ar^3, + . . . + ar^{n-1}, and this is called a geometric series. Now the trick here to find the sum is to multiply by r and then subtract:

$S_n = a + ar + ar^2 + ar^3, + \ldots + ar^{n-1}$

$rS_n = ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ so that

$S_n(1 - r) = a(1 - r^n)$

- Now divide by 1 – r (as long as r ≠ 1) to give

$S_n = \frac{a(1 - r^n)}{(1 - r)}$

<u>Example</u>

Find the sum of the geometric series 2 + 6 + 18 + 54 + . . . where there are 6 terms in the series.

<u>Solution</u>

For this series, we have a = 2, r = 3 and n = 6. So

$S_n = \frac{a(1 - r^n)}{(1 - r)}$

$S_6 = \frac{2(1 - 3^6)}{(1 - 3)} = \frac{2(1 - 729)}{(-2)} = -(-728) = 728$

<u>Example</u>

Find the sum of the geometric series 8 – 4 + 2 – 1 + . . . where there are 5 terms in the series.

<u>Solution</u>

For this series, we have a = 8, $r = -\frac{1}{2}$ and n = 5. So $S_n = \frac{a(1 - r^n)}{(1 - r)}$

$S_5 = \frac{8(1 - (-\frac{1}{2})^5)}{(1 - (-\frac{1}{2}))} = \frac{8(1 - (-\frac{1}{32})}{(\frac{3}{2})} = \frac{2 \text{ x } 8 \text{ x } \frac{33}{32})}{3} = \frac{11}{2} = 5\frac{1}{2}$

<u>Example</u>

How many terms are there in the geometric progression 2, 4, 8, . . . , 128?

Solution

In this sequence a = 2 and r = 2. We also know that the n-th term is 128. But the formula for the n-th term is ar^{n-1}. So

128 = 2 × 2^{n-1}

64 = 2^{n-1}

$2^6 = 2^{n-1}$

6 = n – 1

n = 7. So there are 7 terms in this geometric progression.

Example

How many terms in the geometric progression 1, 1·1, 1·21, 1·331, . . . will be needed so that the sum of the first n terms is greater than 20?

Solution

The sequence is a geometric progression with a = 1 and r = 1·1. We want to find the smallest value of n such that Sn > 20. Now

$S_n = \frac{a(1 - r^n)}{(1 - r)} > 20$

$\frac{1(1 - 1.1^n)}{(1 - 1.1)} > 20$

$\frac{1 - 1.1^n}{- 0.1} > 20$

$(1.1^n - 1)\ x\ 10 > 20$; $(1.1^n - 1) > 2$

$1.1^n > 3$

If we now take logarithms of both sides, we get nlog1.1 > log3 and as log 1·1 > 0 we obtain n > log 3/ log 1·1 = 11.5267 . . .

and therefore the smallest whole number value of n is 12.

Exercise 6

(a) Find the sum of the first five terms of the GP with first term 3 and common ratio 2.

(b) Find the sum of the first 20 terms of the GP with first term 3 and common ratio 1.5.

(c) The sum of the first 3 terms of a geometric series is $\frac{37}{8}$. The sum of the first six terms is $\frac{3367}{512}$. Find the first term and common ratio.

(d) How many terms in the GP 4, 3.6, 3.24, . . . are needed so that the sum exceeds 35?

Answer Key

a) 35

b) 19945.54

c) $a = 2, \quad r = \frac{3}{4}$

d) 20

Activity 4: Implementing the LPM

- Tell participants that the last two parts of what teachers need to do to implement the lesson plans are focusing on learning. How do we know that learning is happening? Any links among the LOs, LPM, Handbook and learning?
- Display Chart 2.2 Implementing the lesson plans (all points now revealed)
- Display the last 2 points and get participants to discuss in pairs what they understand by the points
    1. checking that learning is happening
    2. linking pupils' handbook to LPM
- Get participants to explore ways we could ascertain that learning is happening
- List what all students should be doing to ensure learning is happening
- What are the teachers' roles to ensure this
- Participants discuss how pupils' books could be used in class besides what was captured in the LPM
- Take responses and wrap up with the session summary

Session summary 3 minutes

Take participants through the learning outcomes again

Materials

Chart 5.1 Learning outcomes Introduction

Chart 2.2 Familiarising yourselves with the LPMs

- following steps of the plans
- observing lesson timing
- linking teaching activities to objectives
- identifying preparations required
- demonstrating mastery of the content
- practicing
- working with others
- observing each other
- checking that learning is happening
- linking pupils' handbook to LPM

Session 6: Taking our learning back to school (planning and preparation for lessons)

90 minutes

Learning outcomes

By the end of the session participants, in order to be effectively familiar with the lesson plans, will be able to:

- describe requirements to deliver any competent lessons and plan their implementation.
- explain the importance of linking all teaching activities to the lesson objectives, and strict adherence to the timings of the LPM steps.
- explain strategies they will use upon return to schools to assess learning.
- describe what resources are within their reach in the school and community to enrich their lessons.

Materials

Chart 6.1	Learning outcomes	Introduction
Chart 6.2	Familiarising yourselves with LPMs	Activity 1
Chart 6.2	Sharing learning with teachers	Activity 3
Chart 6.3	Planning the teachers' meeting	Activity 3
Handout 6.1	Two stars and a wish form	Activity 2
Resource	Standards and criteria for an effective school	

Activity outline

Introduction		5 minutes
Activity 1:	Preparing for competent lesson delivery	40 minutes
Activity 2:	Is the learning goal clear?	20 minutes
Activity 3:	Evidence of learning	20 minutes
Activity 4:	From whence does my help cometh?	20 minutes
Summary		5 minutes

Background for facilitators

Read and discuss these notes with your co-facilitator before the session

This is the final session of the Term 2 workshop**.** It is crucial that by the end of the session teachers are ready to return to their schools clear on the next steps, what they will be expected to **do**, and what support they can access in their schools and communities.

In this session you need to help teachers **not just** to do things themselves but to seek for support anytime they require one and establish routines for planning together and carrying out among themselves for improvement. There is only one workshop remaining after this one and so you need to start thinking now about how you, colleagues, HoDs, principal and SSO will continue to focus on improved learning outcomes and steps to take to ensure all learners are learning. The last workshop will focus a lot more on pedagogy, which is a tool to ensure teachers are connected to their pupils

Introduction 5 minutes

1. Welcome participants to the last workshop of SSS Term 2. Remind participants that this session is for both Language Arts and Mathematics groups, just like Session 1
2. Show **Chart 6.1 Learning outcomes** and read through them.
3. Explain that this is a very full session and there is a lot for them to prepare for, back in school. Ask participants '**What have you learnt so far in training that will make you a more competent teacher (using the lesson plans effectively)**?' Get feedback and make sure you highlight these four main areas:
    a. requirements for competent lesson delivery,
    b. tying all teaching and learning activities to instructional objectives
    c. using available resources in school to help
    d. checking understanding assessing learning.
4. Explain that this last session will prepare teachers to go back to school and use their learning in these key areas.
5. Tell participants that these areas have been mentioned in the previous sessions, but this session is to tie them all together in summary.

Activity 1 Preparing for competent lesson delivery 15 minutes

1. Display **Chart 6.2** 'Familiarising yourselves with the LPMs'
2. Ask participants to go through **Chart 6.2** and make a list of three items that are relevant to the title of this activity. (Identifying preparations required; demonstrating proficiency in the content and practicing lesson delivery)
3. Tell them that others could have a link to this title but these three are the closest.
4. Give groups 2 minutes to discuss assigned item (Preparations; Content and Practice), define what it means, what it aims at achieving and cite examples. Two groups may have same item.
5. Similar groups (groups with same item) write responses on a chart paper.

6. Responses could include the following:

Preparation is essential because it ensures that teachers can teach the lesson well. These are the main steps necessary for good preparation:

a. Read the lesson plan and make sure they understand the steps of the lesson and the concepts they need to know.

b. Get all the necessary materials they need to help them deliver competent lessons in terms of instructional materials, games in instructions, grouping especially in the case of large classes, plan for use of space, assessment rubrics, consideration for differences in learners, LPM etc.

c. Write any passages/exercises on the board in advance where possible so that they can spend the whole lesson teaching.

d. Think about how they will know the pupils are learning – identify any questions they want to ask and what are they looking for in written and oral work

e. Identify the pupil they know will struggle to understand - how can they help them during the lesson?

f. Think about how they can get pupils to check their work – could they ask them to swap books and check each others

g. Master the content by looking for difficult subject topics or activities and familiarising yourself with them. Ask others to help you.

h. Practice the lesson in advance

7. Give **Handout 6.1** to participants to go through. Briefly respond to their questions and concerns.

Activity 2 Is the learning goal clear? 40 minutes

1. Ask participants the importance of stating the instructional objectives and sharing them with the learners. *(it helps both teacher and students to remain focused on destination, helps to self-assess)*
2. Ask why the learning goals must be clearly stated / SMART. *(It helps in connecting with all teaching and learning activities and ensuring they are relevant).*
3. Ask participants to refer to Chart 6.2 and pick 3 items that are related to the title. *(following steps of the plans; observing lesson timing and linking teaching activities to objectives)*
4. Get participants to discuss the items in pairs and take brief feedback on what they mean. *(The LPMs have 4 steps Opening, Teaching and Learning, Practice and Closing; Each step has duration which should just be right for delivery and linking objectives and teaching-learning activities together prevents a detour)*
5. Assign lesson 60 to all to go through for 15 minutes in groups of four.
6. Request for 4 volunteers to teach Introduction and part of Teaching and Learning (2 for each subject)
7. Divide the participants quickly into 3 sections and assign groups the task of:
   1. checking the pace of the lesson,
   2. checking that steps are followed and
   3. checking that all activities are linked to the objectives
8. Invite the volunteers to teach while the rest of the group observes.
9. Take whole group feedback and stress the importance staying right on track as we deliver our lesson. Teachers must ensure that ample time is spent on the steps as stated and we must avoid adding contents that are not relevant to the objectives
10. To know if a teacher has delivered a competent lesson, he or she could self-assess by revisiting the learning outcomes and checking to see if they were achieved.

Activity 3 Evidence of learning 15 minutes

1. Tell participants that the next 2 items of what teachers need to do to implement the lesson plans are focusing on learning. How do we know that learning is happening? Are there any links among the LOs, LPM, PH and learning?
2. Participants identify on Chart 6.2 and call out the 2 items that are related to the activity title. (**checking that learning is happening, and linking pupils' handbook to LPM)**
3. Get participants to explore ways we could ascertain that learning is happening. Let them make a list of different ways to ensure all learners are learning and what teachers should be doing to ensure this is happening.
4. Present this in form of a table on chart paper and display on the wall.
5. Give A4 containing some activities to groups and ask them to discuss whether they show students are learning or not. (teacher writes notes on the blackboard; copying notes from the blackboard; writing or answering in own words; discussing in pairs or groups; asking teacher relevant questions; responding to open-ended questions; writing questions before answering; disagreeing with teachers or mates; classwork practice exercises; homework practice exercises; tests and check-ups; projects; teacher performing experiments for pupils to see; teacher reading stories for students to listen to)
6. Stick up A4 containing headings Learning, Not learning, Unsure
7. Ask groups to read out their A4 contents in turns and get participants to move to the relevant heading.
8. Ask them why they are where they are and have a brief discussion around their choice.
9. Let participants go back to their charts and make adjustments to ways to ensure learning is happening and state that teachers must ensure children engage more in activities that would ensure they are learning.
10. Using the pupils' handbooks could reduce the amount of time spent on activities that do not aid learning. For example, writing a long story on the board during a lesson would eat deep into the pupils' learning time.
11. Ask teachers what they could do to prevent this. *(write story on the board during free periods or write on chart paper during preparations or get pupils to read from their PH)*

Activity 4: From whence does my help cometh? 15 minutes

1. Ask participants what they could do to get external support to become more confident in LPM implementation.
2. Participants call out the items on Chart 6.2 which describe what teachers need to do to implement the lesson plans in relations to the activity title. (**working with others, and observing each other)**
3. Allow participants 3 minutes to discuss in groups what the points mean, and describe what practical activities are involved.
4. Participants write their responses on flip chart paper and market place them for 'travellers'
5. Likely responses
    1. **Working with others (colleagues, HoDs and SSOs):** This entails running difficult topics by anyone who could assist even if they are junior to us.
    2. **Observing each other:** This activity is very useful especially if positive relationships exist among teachers. They could arrange for how to observe one another with feedback for improvement. Teachers could ask if they could observe teachers that are doing well in terms of good classroom practice, and invite competent teachers to step in to observe them with the aim of improving on their content knowledge and pedagogical skills
    3. Get 2 groups to role play each of the 2 areas. Lessons should be picked from TGM3-T2 or TGL3-T2
    4. Other groups take notes for feedback
    5. Conclude by stressing the importance of seeking for help when the needs arise. Keeping our weaknesses to ourselves would get it out in the open when we get to the class. Sharing and asking for help is the way to go.

Session summary 5 minutes

Take participants through the learning outcomes again

Materials

Chart 6.1 Learning outcomes Introduction

Chart 6.2 Familiarising yourselves with the LPMs

- following steps of the plans
- observing lesson timing
- linking teaching activities to objectives
- identifying preparations required
- demonstrating mastery of the content
- practicing
- working with others
- observing each other
- checking that learning is happening
- linking pupils' handbook to LPM

Handout 6.1: Requirements for competent lesson delivery

Preparation is essential because it ensures that teachers can teach the lesson well. These are the main steps necessary for good preparation:

a. Read the lesson plan and make sure they understand the steps of the lesson and the concepts they need to know.

b. Get all the necessary materials they need to help them deliver competent lessons in terms of instructional materials, games in instructions, grouping especially in the case of large classes, plan for use of space, assessment rubrics, consideration for differences in learners, LPM etc.

c. Write any passages/exercises on the board in advance where possible so that they can spend the whole lesson teaching.

d. Think about how they will know the pupils are learning – identify any questions they want to ask and what are they looking for in written and oral work

e. Identify the pupil they know will struggle to understand - how can they help them during the lesson?

f. Think about how they can get pupils to check their work – could they ask them to swap books and check each others

Content mastery is achieved when teachers understand the lesson content in advance. We can't be expected to know everything first time so we have to take time to master the content before we teach it. This includes identifying difficult subject topics or unfamiliar activities and taking action to understand them. Some of the actions are self-study, asking HoD, working with colleagues, talking to SSO, online research, listening to students / asking for their strategies etc

Practice gives teachers the confidence to deliver lessons. Teacher picks one of the most difficult lessons to be taught weekly and demonstrates how to deliver to colleagues, HoDs, SSOs, wife, husband, in front of a mirror, recording and playing back audio or video etc).

Teacher Self-Assessment

District: Subject :

Something I learned through the training is….

Something I will do differently in my teaching ...

Something I would like more training on is...