

Leh Wi Lan

Improving Secondary Education in Sierra Leone



SSS Teacher Professional Development (TPD): Mathematics (Term 1)

FACILITATION GUIDE



Introduction

Teacher professional development (TPD) is most effective when several elements are combined to support teachers. Training and support should be as close to the school as possible so teachers have opportunities to practice their learning in their own context. They can work with their peers to share success and challenges and reflect on their problems, devising contextually relevant solutions. Teachers also need some form of external support so that they are introduced to new ideas, ways of working and can refresh their subject knowledge and ensure that it is up to date.

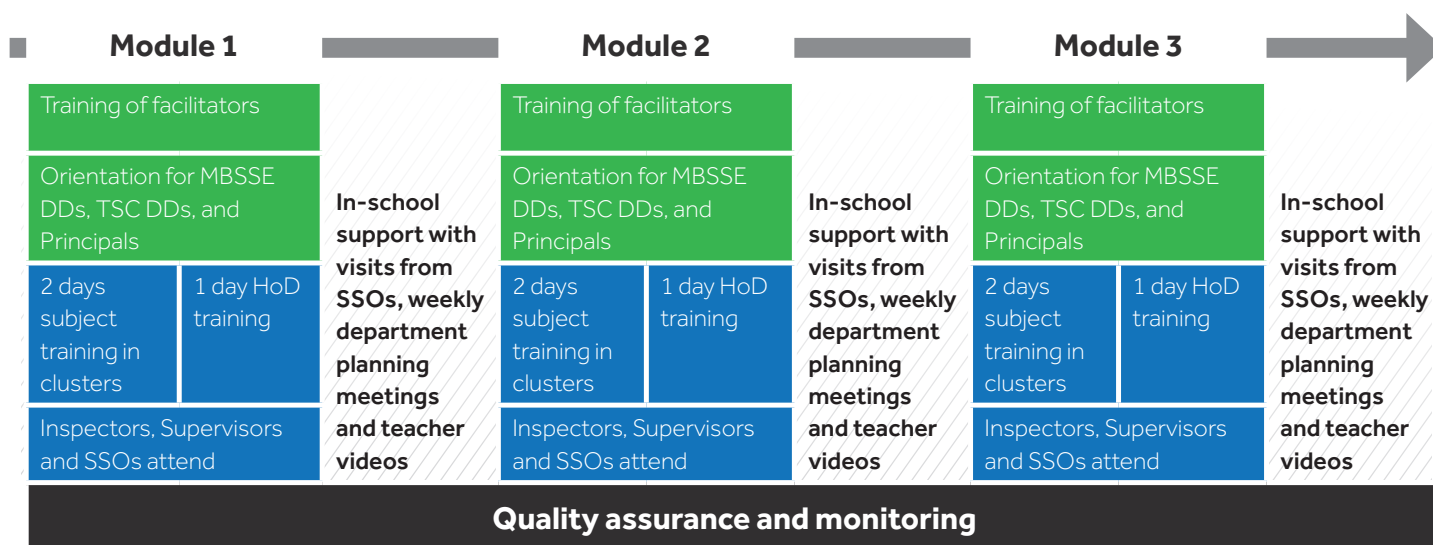
This is especially important in a context like Sierra Leone where Junior Secondary Schools (JSS) and Senior Secondary Schools (SSS) vary greatly in terms of access to resources and distribution of qualified teachers. Within government schools there are large numbers of teachers who are: not qualified for secondary level, qualified but not government approved, qualified to teach, but not in the subject they teach, and volunteer teachers with no prior training or qualification.

Teacher professional development demands a variety of activities so that all teachers, whatever their circumstances and environment, can access structured quality professional development which supports their professional growth and helps deliver quality education.

Between 2017 and 2020 Leh Wi Lan implemented a TPD strategy to support all JSS and SSS English and maths teachers and Heads of Department (HoD), from approximately 1600 government-assisted schools.

The strategy included subject content training in maths and English as well as academic leadership training for HoD. Teachers received training through termly face to face cluster sessions, led by national facilitators who were subject specialists. After each face-to-face cluster training there was in-school support for teachers through regular visits from Leh Wi Lan School Support Officers¹ where they conducted lesson observations and gave feedback to teachers. Instructional videos were also created providing step by step explanations of specific concepts and examples of good classroom practice.

The diagram below shows how the TPD programme worked over a given academic year:



This facilitator manual is part of the Leh Wi Lan TPD programme. It contains subject training for maths teachers in SSS. It is designed to be delivered over one academic year and each module links directly to SSS English lessons being taught in the upcoming term (in this case term 1). It should be used alongside the MBSSE Lesson Plan Manual and Pupil Handbook. Whilst focusing primarily on subject content, the materials have been designed to develop key pedagogical skills including gender-responsive pedagogy, using teaching and learning aids, inclusive learning, reflection and problem solving.

These materials were produced by Cambridge Education, in collaboration with TSC, and delivered as part of the UK-aid funded Leh Wi Lan project for training teachers in government assisted junior and senior secondary schools in Sierra Leone. These training materials are in draft. They can be shared and adapted for use as long as they are not used for commercial purposes.

¹ 200 School Support Officers conducted regular visits to government assisted secondary schools nationwide. Each covered approximately 8-10 schools and supported the English and maths teachers by conducting lesson observations and giving feedback to enhance teaching and learning. They were supported by Leh Wi Lan. This MBSSE School Quality Assurance Officers now perform this supportive supervision role

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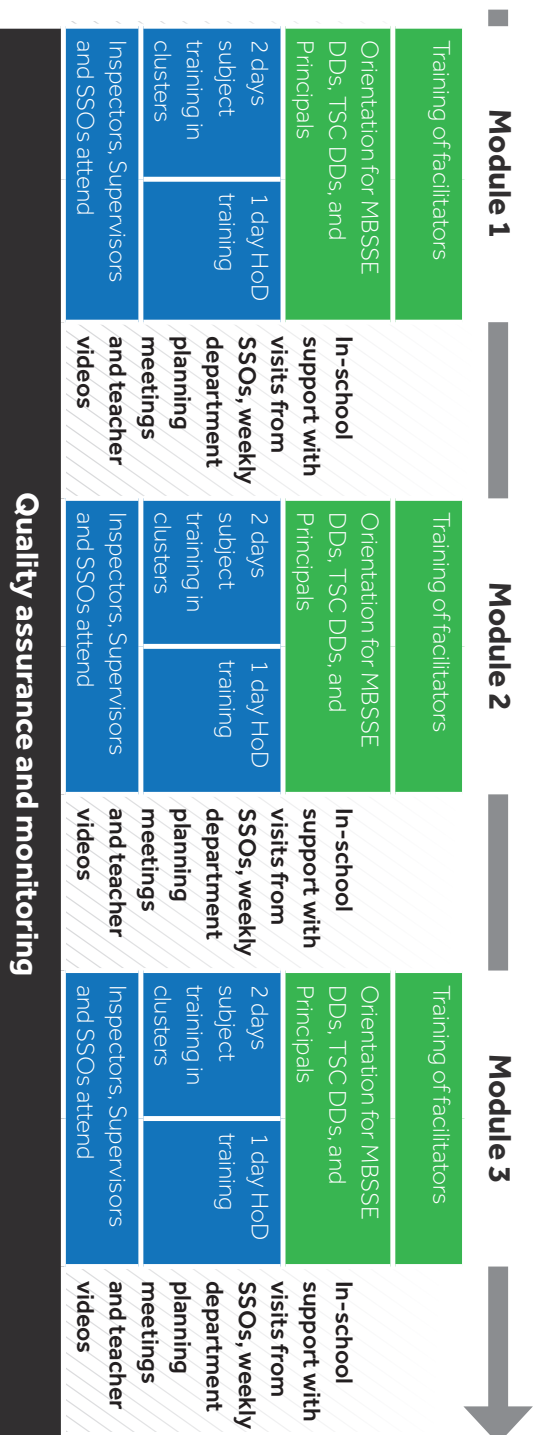
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Senior Secondary School Teacher Professional Development

Training schedule

Date / Time	08:00	09:00	11:00	11:30	13:30	14:30	16:30	17:00
Day 1	Pre-test	Session 1: Introduction (whole group)	Break	Session 2: Number and Numeration	Lunch	Session 3: Lesson plan structure		Closure
Day 2		Session 4: Numbers		Session 5: Indices and logarithms		Session 6: Lesson plan structure	Post-test	
Day 3		HoD Session 1: How schools work		HoD Session 2: Use of Lesson Plan and Pupil Handbook		HoD Session 3: Monitoring teachers		

Facilitator Standards

Well prepared:
Arrives early
Has charts written and materials organised so they are ready to give out.
Refers to training notes but doesn't read them constantly
Demonstrates strong familiarity of the lesson plan structure and content.
Time management:
Manages time
Session and activities start and end on time.
Understanding SSS:
Exhibits knowledge of the current context of Senior Secondary Schools - uses examples that are relevant to the context
Subject Knowledge:
Clearly explains how to do the content of the lesson plans, using a variety of examples to add depth.
Participatory:
Gives opportunities for participants to work together
Gives time for participants to think of how to tackle a problem before explaining
Supportive:
Listens to the participants and acts on their comments
Accepts answers and asks questions to help participants, rather than telling the answers
Inclusive:
Ask questions to a range of participants
Uses gender responsive language and interaction
Finds ways to support those participants who don't understand
Uses group, pair and individual work and moves round to support all participants
Enjoyable:
Greets the participants, creates a friendly atmosphere.
This session is quick and active with a positive approach.

Session objectives

By the end of the session, the facilitator will be able to:

- Identify development needs of the group
- Identify in which topic individuals might need extra support
- Identify a baseline for learning during the course

Materials

Handout Teacher Maths Self - Assessment

Session outline

Introduction	10 minutes
Activity 1 Teacher Self -Assessment	20 minutes

1. Ask participants why they need to know that their pupils are learning
2. Take some suggestions from the teachers and agree that the more knowledge that teachers have about their pupil's learning level the easier it is to teach them because they know what areas to concentrate their teaching for each pupil
3. Remind participants that the purpose of a professional development course is for teachers to progress in their learning. To target learning and make sure that the course meets everyone's learning needs, we need to know where you need further support
4. Explain that to do this we are going to give them a self-assessment form which helps them decide which areas they are very confident in and which areas they need further support
5. Explain that what they write on this form will be used for two purposes
 - a. To help the facilitators understand your learning needs so they can focus the training and support programme
 - b. To use as a baseline for LWL to see if the course has affected learning over the year
6. Show them the objectives and take any questions

Activity 1: Completing the Self- Assessment form

20 minutes

1. Ask the participants to find a partner who they feel comfortable to discuss their development needs.
2. Give them the teacher self -assessment form and ask them to spend a couple of minutes looking through it
3. Explain that these are all the topics they will cover during term one with SSS 1,2 3 and 4 .
4. Although it seems a long list it shouldn't take long to complete
5. Explain you will ask them to look at each topic and rate it on a scale of 1 -4. To help them decide there is an explanation of each number
6. Go through the scale explanations with them and remind them that they should be honest when rating each topic. You would not expect every answer to be a 1 or 2 and will be surprised if there are no 3s or 4s.
7. Remind them that no-one apart from the facilitator and the LWL team will see this form, as it is to help plan professional development over the next year and target learning areas

SSS TPD

8. Ask them to discuss each line with their partner, talking about what is contained in each subject area and their experience of teaching it
9. Ask them to then go through the form individually and rate the topic on each line from 1 -4, putting a mark in the box
10. Ask them to use the comments box to explain what it is they find difficult. They can also use the space at the bottom of the format if there is not enough room to write
11. After about 15 -20 minutes when everyone has completed their form, take them in and thank them
12. Explain that you will ask the HoDs to do a similar exercise with a fresh sheet during the first team meeting after this course. The purpose of that will be to discuss as a team the areas you need to focus on improving

Session 1 Welcome and introduction to the SS teacher team 120 minutes

Session objectives

By the end of the session, participants will be able to:

- identify members of the group and training team;
- identify ground rules for this training;
- describe some of the Sierra Leone Professional Standards for Teachers;
- describe Edgar Dale cone of learning and its implications on how children learn

Note Facilitators: Remind teachers that all activities in the sessions can be replicated with the students when teachers return to their schools.

Materials

Chart 1.1	Session objectives
Chart 1.2	Good morning song
Chart 1.3	Sierra Leone's vision statement
Handout 1.1	Professional Standards for Teachers
Handout 1.2	SSO Lesson Observation Form
Handout 1.3	Children Learn BEST When...
Handout 1.4	Name

Session outline

Introduction a	Participants and facilitators	8 minutes
Introduction b	Ground rules	10 minutes
Activity 1	SSS Subject Teacher	30 minutes
Activity 2	Professional Standards for Teachers	40 minutes
Activity 3	Effective learning (How children learn)	30 minutes
Session review		2 minutes

a. Introduction of participants and facilitators

8 minutes

- Welcome participants to the first day of their induction and ask any of the participants (Christian or Muslim) to open in prayers.
- Sing "Good morning" song with participants (*Chart 1.2 "Good Morning Song" to learn the song*)
- Introduce the facilitators and key MBBSE and or District members, asking a senior government staff to give a brief talk to declare the programme open.
- Show participants Chart 1.1, with the objectives covered up (i.e. showing only the title). Ask a participant to explain what 'session objectives' are. (*They tell us what we are aiming to achieve by the end of the session*). Ask another participant to explain why it is useful and important to have session objectives and to share them with participants and eventually the students (*To enable everyone be on the same page in knowing our 'destinations' and to help us evaluate ourselves by the end of the session*).
- Show the session objectives, one by one, asking a participant to read each one.
- Explain that you will work towards the first objective right now, which is about getting to know one another.
- Tell participants that some of them may already know one another but explain that they need to introduce themselves in a new way.
- Take participants to a space where they can stand in a circle, and stand as one of them.
- Remind participants of your name, and ask someone to tell you what letter it begins with. Explain that you will think of an adjective to describe yourself that starts with the same letter as your name. You will also think of an action to go with your adjective.
- Step into the middle of the circle and say, 'I am _____', doing your action at the same time as you say your name. Repeat.
- Step back to your place in the circle, and explain that, when someone introduces himself or herself, everyone else needs to welcome them – so they should say, 'Welcome, _____', repeating your name, adjective and the action that goes with it.
- Explain that you will show them what to do one more time – and then they will introduce themselves in turn. Each person will step into the circle, introduce themselves, and then be welcomed by everyone else. Explain that they need to think of an adjective to go with their name, and an action to go with it before it

comes to their turn. They should try to think of an adjective different from everyone else's!

- Step into the circle and introduce yourself again, reminding them that they need to welcome you. Then encourage participants to introduce themselves, going around the circle. Help individuals who are struggling to think of a good word or action for their introduction, and make sure the group has heard and used their new name to welcome each person.
- Thank participants for their work on this and ask them to return to their seats.

b. Ground rules

10 minutes

- Explain that it is important to have ground rules for training sessions.
- Ask pairs to discuss why they think ground rules are useful. Give them a couple of minutes to share ideas, then take feedback. *Agree that ground rules help to ensure that workshops can be conducted professionally, and that the same rules apply to everyone including the facilitators.*
- Explain that the group will set its own ground rules. Give each group 3 minutes to agree and record whatever rules they would like the whole group to follow. Let them know that this is expected of every class teacher in their schools.
- Move around supporting groups with their discussions, and particularly trying to ensure that no individuals are dominating the discussions too much. You can also use this time to make sure the 'right' rules are being suggested!
- Call the groups back together and praise them for their focused and balanced discussions.
- Ask each group in turn to propose a ground rule. If everyone is in agreement, ask a member of that group to come to the front and write it on a sheet of chart paper or chalkboard.
- Keep going until you feel the most useful ground rules have been recorded. If alternative versions of existing ground rules are proposed, encourage the whole group to come to an agreement about the best wording and make adjustments to the rules at the front of the room.
- If some key ideas are not suggested (e.g. timekeeping, keeping handsets off during sessions), you can propose them and explain why you think they are important.
- Read through the final list of rules, and explain that you will write them up or ask a volunteer to write them up neatly during the break.

- Welcome the participants and let them applaud themselves for being early and part of the programme. Teach them how to do BRAVO or ROLL n POP applause.
- Explain that the title 'Senior Secondary School Subject (Maths or English) Teacher' is not by chance: it describes who they are and what they do.
- Ask a participant to tell you the significance of the phrase 'School'. Make sure they understand that their focus is on what happens in their schools – not in MBBSE, District or anywhere else. Often the work of supporting students to succeed requires them to work with other teachers in their school especially the JSS teachers, and teachers in same cluster as their schools – but every activity should be directed towards getting children to learn and be safe. Can anyone think of such activities in schools? Take few responses to include preparing lessons to meet the needs of students, assessing students' learning, remediating to support students, communicating progress to parents, break duty, lunch supervision, managing co-curricular activities etc
- Ask a participant to tell you the significance of the word 'Subject'. A teacher takes responsibility of specific subject groups ensuring the students are learning. Most of the learning activities are planned, delivered and assessed by him or her in those areas. It is assumed that a teacher must be well versed in the area, and he or she is a specialist in delivering the subject, but that is not always the case. A subject teacher also has responsibility for ensuring children are safe in school. They might have responsibility for a club, form master or conduct assembly.
- Ask a participant to tell you the significance of the word 'Teacher'.
- Let them know that as a teacher, they should be concerned about learning and creating opportunities for all learners to have access to quality learning.
- In a school, there is a possibility of having more than one teacher in their subject area and it is therefore pertinent for all to work together as a team in preparing for delivery or sharing ideas on how to support their students.

Teachers should not be ashamed to seek for help in any topic they are not too confident in delivering. Even SSS teachers may ask for help from JSS teachers especially with the facts on ground that many of the teachers are not specialists in both math and English.

- Take participants out to play a team game (Give 3 different secret instructions to 3 groups: Group 1 - move 4 pieces of flipchart paper spread out from point A to point B; Group 2 – stack them on one another and Group 3 – stand on the pieces of paper). Then ask them all to go ahead with their task.

standards in the document would be observed and assessed sometime in the school year as they may not be seen exhibited often during lessons.

- In pairs, ask participants to agree and put a tick on each standard that is DIRECTLY linked to learning (check facilitator's version for likely answers).
- As whole group compare the checked Standards as stated in Handout 1.1 with their list (as success indicators) on chart paper displayed on the wall, and see if there are any links. Point out similarities and expunge those that have no direct links to learning and safety in the school.
- Explain that in their schools, Senior Teachers, Heads of Department and sometimes, the principals, will be conducting lesson observations. SSS will also receive visits from School Support Officers who will be supporting English and maths teachers and observing their lessons. This is to help the teachers achieve the Standards. It is a supportive activity not punitive.
- Share the Handout 1.2 Lesson Observation Form with teachers so that they can familiarise themselves with it.
- Read out each criterion from the form and ask participants to describe two observable behaviours of teachers and or students.
- Take brief feedback from teachers clarifying any questions. Explain that the SSO will give them more support and explanation during their visits.

Activity 3 How children learn 30 minutes

- Give each group Handout 1.3 – an envelope with the different ways in which children might learn. Ask the participants to discuss in their group which ones they think describe ways in which children will learn best and sort them into two groups: the best ways children learn; and less effective ways of children learning.
- Ask the members of the groups to agree on where to place each A4 assigned to them under the title 'Best' and 'Less effective'.
- Most of the statements clearly go into 1 of 2 sets – 1 set is the best ways in which children learn, and the other set is linked to more teacher centred teaching. "The teacher uses materials" and "children work individually when not listening to the teacher" are not teacher centred, but are also not part of the best ways in which children learn.
- Allow participants to correlate what they have as best ways in which children learn to the charts they produced in Activity 2 describing a quality teacher. There must be some resemblance.
- Remind participants that students are at the centre of teaching and learning. This means that everyone's job is to give them these opportunities of learning in the most effective ways. This is what it means to have 'child centred learning' and a 'child centred education system'.

Hold a brief discussion around teachers' pressure about children passing exams. Ask if they are actually passing the exams. Link this to underdevelopment in Africa be careful not to give Sierra Leone examples only. Agree that we need to change strategies. Since the LPM was designed using the curriculum, examinations have also been covered, and all we need to do as teachers is to deliver them effectively.

- Display Chart 1.3 and run the participants through the different sections of the Cone of Learning explicitly
- The cone is divided into 2 major sections: passive and active.
- Passive section comprises Verbal Receiving and Visual Receiving subsections. (We remember between 10% and 70% of what we learn after few weeks)
- Active section comprises Receiving/participating and Doing subsections. (We remember between 70% and 90% of what we learn after few weeks)
- Give 2 clear examples: 1. We tend to remember 50% of what we hear and see: (watching a movie, looking at an exhibit, watching a demonstration, and / or seeing it done on location); 2. We tend to remember 70% of what we say (participating in a discussion and / or giving a talk)
- Give out strips of paper containing a learning activity each (Handout 1.4) to each group (about 2 to a group) and display a large cone on the wall
- Give groups about 5 minutes to discuss and agree on what section of the cone the activities target.
- Groups take turns to stick their activities up on the large cone
- Discuss about five of the activities and agree on their placement.
- Ask if it's wrong to read, if only 10% of what is read is remembered. Agree that reading is important. We can use both the passive and active activities. Teachers should just ensure that more time is spent on the active activities.
- Check with a lesson from the LPM with the participants to see how much it reflects the cone. Teachers could in future modify their delivery of the LPM to apply more active learning.
- Address some questions participants may have.

Session review

2 minutes

- Show participants Chart 1.1 again. Read through each objective.
- For each one, ask what they did to try to achieve each one.
- Thank them for their focus during this session and remind them what time to come back from the break. You could appoint a timekeeper to help them all remember what time to return.

Materials

Chart 1.1 Session objectives

Chart 1.2 Good morning song

Good morning, good morning, good morning to you

Good morning, good morning, good morning to you

Our day is beginning there's so much to do

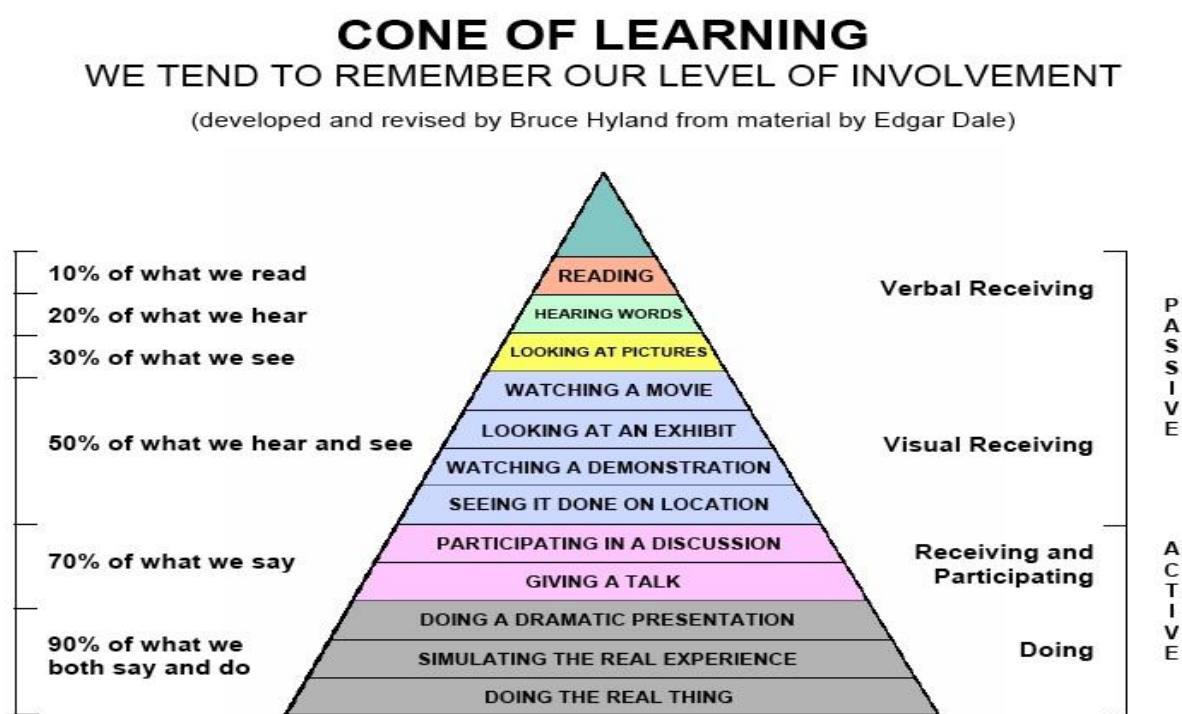
Good morning, good morning, good morning to you

Chart 1.3 Sierra Leone's vision statement

Vision: *Education that is world-class and bequeaths posterity with well-rounded individuals who excel in intellectual, skills and moral, bestriding all fields of human endeavour, from arts to humanities, mathematics, science, technology and agriculture, among others not only in Sierra Leone but all over the world.*

Mission: *To re-invent the Sierra Leonean education system through revolutionizing the quality and standards of teachers, access and equality of educational opportunities for all children, student-friendly schools and overall challenging educational goals so as to bring out the best in the students, teachers, school leaders, government and citizens.*

Chart 1.3 Cone of learning



Professional knowledge domain

- Standard 3: Know subject content ✓
- Standard 4: Know how to teach the subject ✓
- Standard 5: Know physical, socio-cultural and psychological characteristics of learners ✓
- Standard 6: Know how to assess learners ✓
- Standard 7: Know fundamentals of education governance and channels of communication

Professional practice domain

- Standard 8: Plan learning activities and programmes ✓
- Standard 9: Deliver lessons ✓
- Standard 10: Assess learning ✓
- Standard 11: Participate in and supervise extracurricular activities
- Standard 12: Safeguard human rights and lives

Professional engagement domain

- Standard 13: Engage professionally with statutory and administrative authorities
- Standard 14: Engage professionally with learners ✓
- Standard 15: Engage professionally with colleagues
- Standard 16: Engage professionally with parents and guardians
- Standard 19: Engage professionally with own development and academic leadership

Handout 1.2

SSO Lesson Observation Form (Content)

- Teacher has the correct lesson plan manual in the class? (Do not only validate that the teacher has the LPM book, but also confirm that s/he is using the one for the correct grade, subject, and term)
- *Teacher follows the lesson plan manual*
- *Teacher follows the LPM script for step 1 of the lesson. (Step 1 is opening/stating lesson objective)*
- *Teacher follows the LPM script for step 2 of the lesson. (Step 2 is introduction to new materials - "I do")*
- *Teacher follows the LPM script for step 3 of the lesson. (Step 3 is guided practice - "we do")*
- *Teacher follows the LPM script for step 4 of the lesson. (Step 4 is independent practice - "you do")*
- *Teacher follows the LPM script for step 5 of the lesson. (Step 5 is closing)*
- Teacher uses positive language, with praise exceeding reprimand.
- Teacher uses at least a teaching aid (other than textbook or blackboard).
- Teacher organizes the students in different ways during the lesson
- Teacher knows the content that the students were taught during the lesson.
- Teacher encourages all students to take part in the lesson.
- Teacher uses more than one method of assessing learning.

Handout 1.3 "Children learn BEST when....." (The table below shows the answers to the Activity. Cut the answers up, mix up the order and put them in an envelope for each group.)

<i>1 Doing activities themselves</i>	<i>2 Listening to the teacher explain</i>
<i>3 Using a variety of materials</i>	<i>4 Chanting after the teacher</i>
<i>6 Working with, and helping others</i>	<i>5 Working individually after listening to the teachers presentation</i>
<i>7 Building new learning on what is already known</i>	<i>9 Using only textbooks and the blackboard</i>
<i>8 Focusing on understanding and use of skills</i>	<i>10 Focusing on memorising facts</i>
<i>11 Working at their individual levels of ability</i>	<i>12 Everyone working at the same level of ability</i>
<i>14 Using what is learned practically</i>	<i>13 The teacher uses materials</i>
<i>15 Having time to practice new skills</i>	<i>16. Doing the same activity for a long time</i>

Handout 1.4 Learning activities - One set cut up for each group.

Students take it in turn to read parts of a story	Teacher defines fractions
Students copy from the chalkboard	Students read about fractions
Students go outside to see examples of 3D shapes	Teacher show students cut-up pieces of fruit to show fractions
Have a discussion about the meaning of the text	Students play a verb game to demonstrate verbs
Students in their groups, write own song about fraction rules	Pictures of fractions are shown in class
Students act out a story	Students listen to the teacher and recite the answer
Teacher defines fractions and explains the different types	Students read a passage by themselves

Session 2

Numbers and numeration

120 minutes

Learning outcomes

By the end of the session participants will be able to:

- Solve problems involving fractions, decimals, and percentages
- Explain significant figures, estimation, percentage error, degree of accuracy, and process of approximation

Materials

Chart 2.1 Learning Outcomes

Chart 2.2 Area of lawn

Chart 2.3 Group questions

Handout 2.1 Art Centre (1)

Handout 2.2 Art Centre (2)

Activity outline

Introduction		5 minutes
Activity 1	Comparison using Fractions, Decimals and Percentages	40 minutes
Activity 2	Significant figures and estimation	35 minutes
Activity 3	Percentage error and degree of accuracy	35 minutes
Summary		5 minutes

Session Introduction

5 minutes

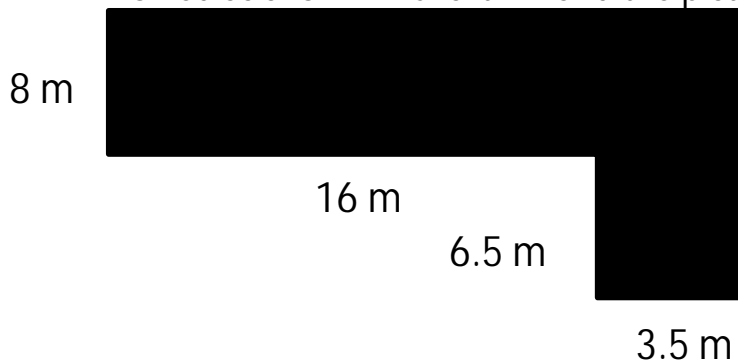
- Welcome participants to the session
- Explain that a primary goal of this session is to help teachers learn some new and useful strategies to become more confident in handling numbers.
- Tell them that they can apply these strategies to gain familiarity with fractions, decimals, percentages, statistical terms etc and to solve real-life problems.
- Explain that rational numbers are everywhere and knowing how to manage them is key as humans use them in all their daily endeavours.
- Participants will extend their understanding of fractions to the concepts of ratio and rate by exploring fractions as points on a number line, as measurements of distance, and in relative order.
- Display Chart 2.1 (Learning Outcomes) and go through it with the participants.

Activity 1: Comparison using Fractions, Decimals and Percentages

40 minutes

- Explain that some real life examples will be used in these examples. It is important to be able to solve worded problems as well as complete calculations.
- Say that it is hoped that some difficulties experienced in solving problems on the topics will be addressed.
- If anyone feels less confident about any topic not covered, facilitator and the whole group address such but care must be taken not to spend too much time on it.
- Explain that topics not covered should be addressed by the HoDs and SSOs when they return to their schools.
- Read the following background to the first exercise: Art museums own more pieces than they can display at one time. This means that art must be stored when it is not hanging in a gallery. A museum curator chooses which works to exhibit.
- Read out the following, writing the key figures on a flip chart paper: An Art Centre in Freetown held an exhibit entitled *50/50*. For the exhibit, the public voted on which pieces they wanted the museum to display, and curators chose the remaining pieces.
 - Question: What do you think 50/50 refers to in the title? (*Answer: The public selected half of the artwork and curators selected the other half*).
- Give Handout 2.1 to participants.
- Read out the following, writing the key figures on a flip chart paper: This is another Art Center in Bo. The picture (Handout 2.1) shows the public's part of the exhibit. The curators' part is covered up.
 - How many works of art do you estimate were in the exhibit? (About 50 -100; seeing only one wall of the exhibit there could be more.)

- Give Handout 2.2 to groups. The picture shows the public art on the facing wall and the curators art on the right.
 - Ask: How does this picture change your estimate above? (The information should greatly reduce the estimated number of works of art in the exhibit)
 - Estimate the percentage of the exhibit chosen by the public, and curators. (75% and 25% respectively.)
 - Use the percentages to calculate how many artworks the public and curators chose if there were 200 pieces in the exhibit. (150 and 50).
- What title would you choose for this exhibit using percentages and ratio? 75/25, 3:1.
- Explain that the next exercise would assist in connecting it all together.
- Display Chart 2.2 – Area of a Lawn
- Briefly give the background: The Kenema Boys and Girls Club, KBGC, offers after-school tutoring and activities for girls and boys.
- Read out the following, writing the key figures on a flip chart paper: One group of boys and girls planned to operate a lawn-mowing service to raise money for the Club. They decided to set the price for any lawn by measuring the area to be mowed as shown in Chart 2.2 and the picture displayed below.



- What is the area in square meters of the lawn shown? $8(16+3.5) + 6.5(3.5)$, or 178.75 sq.m. Estimate might be $8(20) + 7(4) = 188$ sq.m
- The boys and girls who had experience mowing lawns thought the job was worth about \$25. What is the price in dollars per square meter? $\$25/178.75 = \0.14
- Using the price per square meter in bullet 2 above, what should the boys and girls charge to mow a lawn with an area of 200 square meters? $200(\$0.14) = \28 . No estimation is necessary here. The exact answer is $200 \times 25/178.75 = \$27.97$
- Assign 1 or 2 questions to each group from Chart 2.3 to solve
- Take whole group feedback

Activity 2: Significant figures and estimation

35 minutes

- Say: In science many of the numbers that we deal with are a direct result of measurements. When the measuring device, whether it be a ruler or a balance, is read, a number is recorded. This number contains one or more known digits and one estimated digit. These digits are referred to as significant figures. The number of significant figures usually depends upon the measuring device. For example, a centigram balance provides two decimal places while an analytical balance can be read to four.
- Ask: What is the difference between the values of 3, 3.0 and 3.00? (*Answer: actual value for 3 is between 2.5 and 3.5; 3.0 is between 3 and 3.5; 3.00 is between 3.00 and 3.05*).
- There are rules for determining the number of significant figures:
 - Non-zero numbers are always significant. 72.3 has 3 significant figures
 - Zeros between non-zero numbers are always significant. 60.5 has 3 sf
 - Zeros before (to the left of) non-zero numbers are not significant. 0.0253 has 3 sf
 - All zeros after (to the right of) non-zero numbers are significant if there is a decimal point in the number. 123.00 has 5 sf, 12,000 has 2 sf, 120.0 has 4 sf, 12,000. has 5 sf.
- Write 35g on chart paper. Ask participants to write the number of sf boldly on given flash card.
- Ask participants to raise their answers up.
- Ask a couple to give reasons for their answers.
- Now repeat for the figures below. Write the number one by one and ask participants to write the number of sf on a flash card.
 - 35g – 2
 - 3.57m – 3
 - 3.507km – 4
 - 0.0035kg – 2
 - 2406 l – 4
 - .00004m – 1
 - 240.00g – 5
 - 20.04080 – 7
- Explain that sometimes we need to round a number to a certain number of sf. Write the examples below and ask participants to answer them with you.
 - 3.515014 to 5 sf = 3.5150
 - 3.5150 to 3 sf = 3.52
 - 3.52 to 1 sf = 4

- 3430 to 2 sf = 3400
- Now ask participants to work in pairs and round all numbers to 4 sfs. Encourage participants to work together and explain to one another if needed.
 - 84791 = 84790
 - 38.5432 = 38.54
 - 256.75 = 256.8
 - 4.9356 = 4.936
 - 0.00054818 = 0.0005482 or 5.482×10^{-4}
 - 136758 = 136800 or 1.368×10^5
 - 308659000 = 308700000 or 3.087×10^8
 - 2.0142 = 2.014

Activity 3: Percentage error and degree of accuracy

35 minutes

- Read out the following, writing the key figures on a flip chart paper: A group of children picked up rubbish in their community. They picked up 5324 pieces one day and 2148 pieces the next day. Estimate to one s.f. how many pieces of trash did they pick up in total.
- Give participants 1 minute to solve the problem. Remind them to round the numbers before applying operations.
- Invite a volunteer to give the answer and explain their reasoning. ($5000 + 2000 = 7000$ pieces of trash).
- Tell participants that we will be discussing percentage error and degree of accuracy in this activity. Percentage error helps us to understand how accurate our estimations are while degree of accuracy refers to how many significant figures (s.f.) or decimal places a number has. Generally, using more digits makes a number more accurate.
- Estimated measurements are not exact, they are approximate. This means that we always have some error.
- Percentage error tells us how close to the exact value our estimated value is.
- A smaller percentage error means our estimate is more accurate, and a larger percentage error means our estimate is less accurate.
- When using measurements, estimating with smaller units of measure gives us a smaller percentage error. For example, estimating with centimetres is more accurate than estimating with metres. The percentage error will be smaller.
- Percentage error is calculated from another amount called simply "error". We calculate error first, then percentage error.

- Error is found by first finding the range of numbers that a rounded quantity could fall between. For example, we measure the width of a room and find that it is 2.5 metres to 2 s.f. The actual width could lie anywhere between 2.45 and 2.55. Any number in this range when rounded to 2 s.f. would give us 2.5. The error of this measurement is the difference between the estimated amount and the minimum and maximum possible amounts.
- Write the following on the board and explain how to find the error in this case:
Error: $2.55 - 2.5 = 0.05$ m. or $2.45 - 2.5 = -0.05$ m. Error is $+0.05$ m. or -0.05 m.
- Percentage error is found by finding error as a percentage of the estimated measurement. Write the formula shown below on the board:
Percentage error = $\frac{\text{error}}{\text{actual}} \times 100\%$

Remind participants that in the previous example, the error is 0.05 m. and the measurement is 2.5 m.

Substitute these values into the formula and solve:

$$\text{Percentage error} = \pm 0.05\text{m} / 2.5\text{m} \times 100\% = \pm 2\%$$

Write the following on the board to explain degree of accuracy:

Rounding 23547:	23550	23600	24000	20000
	4 s.f.	3 s.f.	2 s.f.	1 s.f.
	more accurate		less accurate	

- The degree of accuracy that you should give in an answer depends on the degree of accuracy in the problem. The answer should not include more significant figures than the number of significant figures requested in the problem.
- It is not necessary to round to the given number of significant figures at each step or in the middle of a calculation. Numbers in a middle step of your calculation may have more significant figures than those in the problem and answer.
- Write the following example problem on the board: Calculate the area of a circle with a 6.5 m radius
- Ask participants to raise their hand to give the formula for the area of a circle. Write it on the board: $A = \pi r^2$
- Ask participants to raise their hand to give the value of pi to two decimal places. (Answer: 3.14)
- Solve the problem on the board.

Solution:

$$\begin{aligned} A &= \pi r^2 \\ &= (3.14)(6.5 \text{ m.})^2 \\ &= 38.465 \text{ m}^2 \end{aligned}$$

Session summary

5 minutes

Take participants through the learning outcomes again and ask if they are confident about them and thank them for their active participation.

Materials

Chart 2.1 Learning outcomes

Introduction

Chart 2.2 – Area of a Lawn

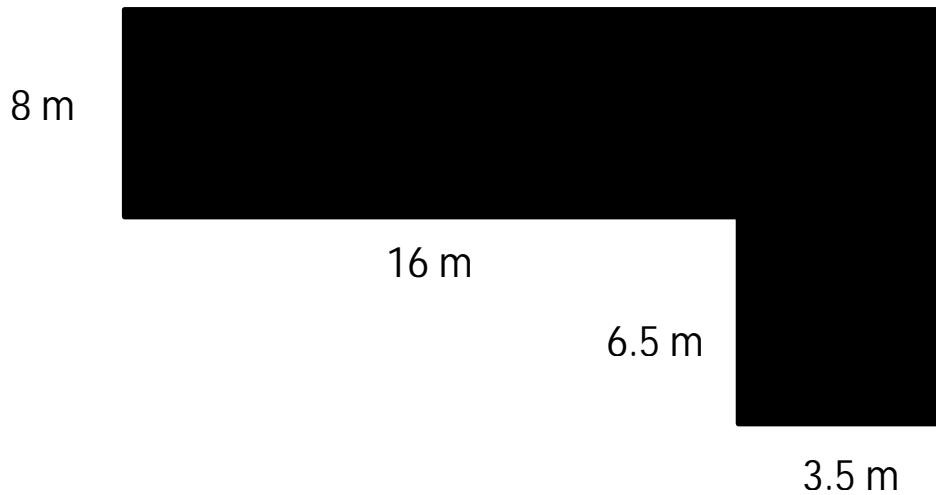
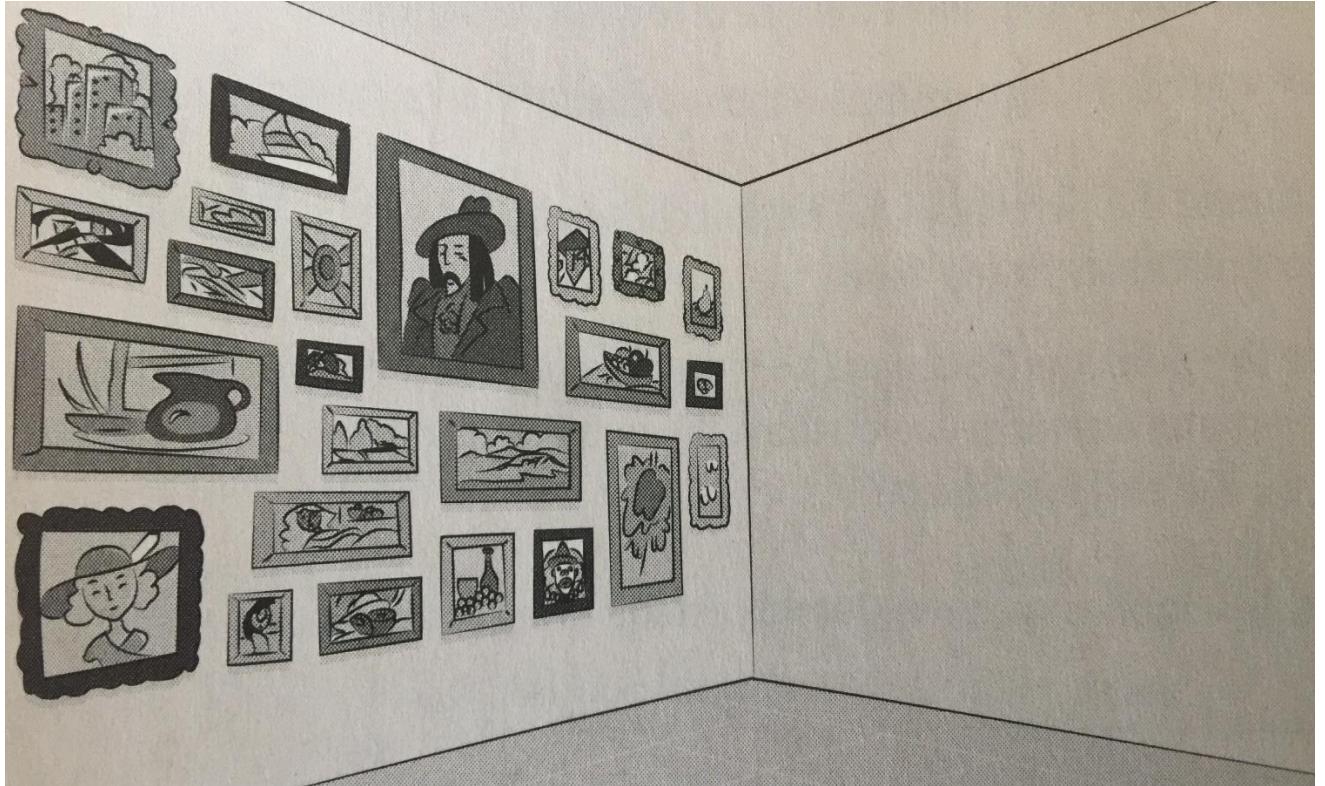


Chart 2.3 Group questions (1 per participant)

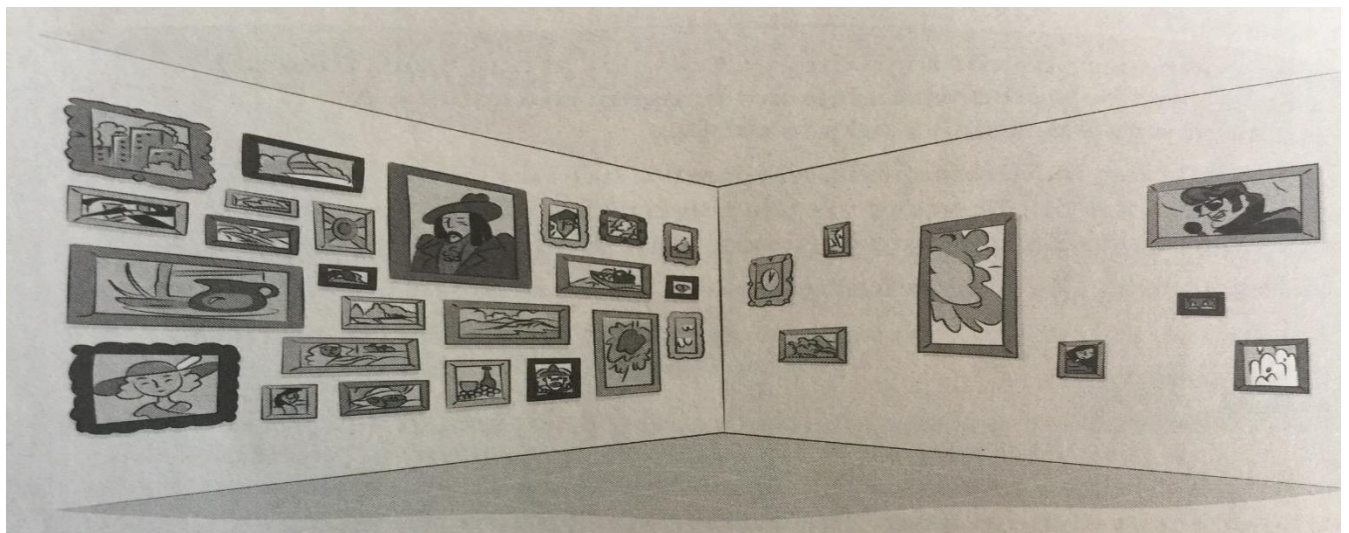
A company pays maths tutors \$5.75 per hour to tutor SSS students.

1. If a tutor works 3.4 hours on Monday and 2.7 hours on Thursday, what will the tutor earn for the week.
2. If the company has a budget of \$250 per week, how many tutoring hours can be provided?
3. Abe scores 21 out of 30 in his test. Write this as a percentage.
4. In the next test Abe scores 24 out of 30. Describe his improvement as a percentage.

Handout 2.1 Art Centre (1)



Handout 2.2 Art Centre (2)



Session 3: Introduction to the Lesson Plan Manuals

150 minutes

Learning outcomes

By the end of the session participants will be able to:

- Describe the structure of the Lesson Plan Manuals (LPMs) and connect the LPMs to Pupil Handbooks
- Explain how each step of the LPM is used in the classroom for effective learning
- Prepare and practice lessons from the LPM

Materials

Chart 3.1 Learning outcomes

Handout 3.1 Statements on the lesson plan (cut up for each group)

Flip chart paper and marker

Lesson Plan Manuals SSS1-3 term 1

Activity outline

Introduction	5 minutes
Activity 1: Describing the structure of the Lesson plan	35 minutes
Activity 2: Steps of the Lesson Plan Manuals and Pupil Handbooks	35 minutes
Activity 3: Prepare and practice using LPMs	70 minutes
Session summary	5 minutes

Background note for facilitators

The production of Lesson Plans with the accompanying Pupils' Handbook are essential educational resources for the promotion of quality education in senior secondary schools in Sierra Leone.

The Lesson Plans and Pupil Handbooks give teachers the support they need to cover each element of the national curriculum, as well as prepare pupils for the West African Examinations Council's (WAEC) examinations. The practice activities in the Pupils' Handbooks are designed to support self-study by pupils, and to give them additional opportunities to learn independently. In total, we have produced 516 lesson plans and 516 practice activities – one for each lesson, in each term, in each year for each class. This is a remarkable achievement in a matter of months.

These plans have been written by experienced Sierra Leoneans together with international educators. They have been reviewed by officials of my Ministry to ensure that they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognised techniques to deliver the best teaching.

Session introduction

5 minutes

- Welcome participants to this part of the training.
- Explain that the Ministry through DFID, IRC and Leh Wi Lan has developed Lesson Plan Manuals (LPM) that will help teachers to teach Mathematics and English in Senior Secondary classes 1 – 3 / 4 in a clear, participatory and enjoyable way.
- The LPM and support system through SSOs was implemented in all government JSS schools last year.
- The success prompted the need to have same done for SSS so teachers who have limited resources will have something to work with.
- Provision was also made for students to have Handbooks to support their learning.
- It is important for teachers to receive training and support to help them use the materials and support their students to use the learning resources.
- This session will focus on helping participants to understand the key parts of the lesson plans – different elements in Mathematics and English, the structure of the lesson plans and the ways they are best used. References may be made to Pupil Handbooks during the training.
- It is hoped that teachers would be trained every term to cover the contents and in addition pedagogical skills would be provided by HoDs, principals and SSOs in school.

Activity 1: Exploring the Lesson Plan Manuals

35 minutes

- Let participants come out and form a circle.
- Play titanic game to form groups.
- Give each group a Lesson Plan Manual or some pages from the Manual if the book is not available.
- Ask each group to look at the pages. What do they notice? Ask them to take note of important sections of the Lesson Plan.
- Ask the groups to write their observations on a chart paper
- Ask groups to display their work on the wall next to them (market place).
- Let groups go around and look at all the work on display. Ask them to see if there are similarities and differences in the different displayed presentations.
- Share observations during plenary.

- Agree heading boxes are similar as they display lesson title, theme, lesson number, class, learning outcome and preparation
- Agree that the plans have 4 steps: opening, teaching and learning, practice and closing.
- Differences could include the timings, topics, objectives and lesson numbers.
- Agree that M1-L031 means Mathematics SSS1, Lesson 31
- Explain that teachers following the steps would be adjudged competent teachers initially. Other criteria would be used in near future in addition to the effective use of the lesson plans
- Remind participants of the SSO observation form and that this refers to how well they use the lesson plans.
- Participants take a look at corresponding lesson in the Pupil Handbook (if available) for M1-L031 and discuss the link.
- Take questions to address concerns especially around likely challenges.

Activity 2: Describing the structure of the Lesson Plan Manuals 35 minutes

- Give each group a set of statements cut into strips from Handout 3.1 or written on A4 paper.
- Let groups discuss the statements for 10 minutes
- Ask groups to decide which statements are true and which are false.
- Place 'True' label on one wall and 'False' on another. Let 1 group leader read out a statement and give participants 10 seconds to move to True, or False.
- Ask participants why they chose their answer.
- Let another group read out one of their statements and repeat until all the statements have been read out.
- Encourage participants to discuss and agree the correct answers.

Activity 3: Model teaching (prepare and practice lessons) 70 minutes

During the lunch break before this activity:

- Assign each teacher a lesson from SSS Mathematics Term 1
- Provide blank chart paper and markers so that they can adequately prepare for their model lesson
- Tell teachers that they should prepare for the entire lesson but the facilitators may decide which step they will teach. They will not have time for the full lesson so will just deliver a section or two.
- Tell them the rubrics for success is ability to follow the lesson plan manual as scripted.

- Give participants about 20 minutes to prepare their lessons.
- Call them back and say: you will deliver the model lessons that you have prepared. Why might we be taking the time to deliver model lessons? What is the purpose of this activity? (Allow a few participants to share their response with the group). There are, in fact, 2 reasons for the model lessons:
 - Practicing lesson observations: Since we are going to be observed by our SSOs, HoDs and principals, it is important to practice the process. This would make self-assessment easy for us and we would see lesson observations as an activity that helps us overcome our challenges. We would be using the terms '2 stars (strengths) and 1 wish (key lever)'. We have got a chance to practice and develop these skills in a safe environment before we actually go back to our schools.
 - Familiarity with the LPM: As you recall, we need to be experts in the LPMs. Teaching from the LPM today and actively listening to others deliver lessons from the LPM will make us all more familiar with the lessons. We must know the LPM inside and out!
- Call participants up one at a time (in sequential order with regards to the LPM) to deliver their lesson. Each participant should teach only for 10 minutes (depending on the number of participants in the group).
- The participants who are not teaching at the time should take observation notes in their notebooks and actively participate as "students" in the lesson.
- Ask some teachers to start at the very beginning of the lesson (Opening) and ask others to start in teaching and learning, practice and closing. This will enable the group to see different parts of the lesson structure.
- After each participant has delivered his/her lesson, de-brief as a group pairs to come to an agreement on the 2 stars and 1 wish.
- Remind participants that they should be professional when using 2 stars and a wish. We should be supportive and respectful when giving and receiving feedback.

Session summary

10 minutes

- In this session we have looked at the structure of the Lesson Plan Manuals for SSS 1-3, Term 1. Remind participants that their understanding of the different elements mentioned in the lesson plan are very essential for them to be able to use the lesson plan effectively when they are supporting schools and teachers.
- Above all share with the participants that planning and preparation is key to being able to teach effectively.

- Conclude that these lesson plans are based on the National Curriculum and the West Africa Examination Council syllabus guidelines, and meet the requirements established by the Ministry of Basic and Senior Secondary Education.
- Read the lesson plan before you start the lesson. Look ahead to the next lesson, and see if you need to tell pupils to bring materials for next time.
- Make sure you understand the learning outcomes, and have teaching aids and other preparation ready – each lesson plan shows these using the symbols on the right.
- Follow the suggested time allocations for each part of the lesson. If time permits, extend practice with additional work.
- Lesson plans have a mix of activities for the whole class and for individuals or in pairs.
- Use the board and other visual aids as you teach.
- Interact with all pupils in the class – including the quiet ones, including girls as well as boys and any children with disabilities.
- Congratulate pupils when they get questions right! Offer solutions when they don't, and thank them for trying.

Handout 3.1 Statements on the lesson plan (cut up for each group)



- Teachers cannot write in the lesson plans.
- Students may write in their handbooks.
- SL lesson plan manual should have a lifespan of three years
- The plans contain activities which are fun and interesting for both the pupils and the teachers
- The lesson plans guide you in a structure of teaching throughout the week, term and year.
- These plans need proper preparations by the teachers when the lesson starts.
- The plans are all in English, which means that you can't use any local language while teaching
- Following these plans will help teach the whole curriculum for their year group.
- The lesson plans are taken from the curriculum and each lesson lasts according to the time table.
- Each lesson has a different structure to be followed
- During a daily lesson the lesson plan only uses whole class teachings
- The different activities in the lesson plans help your lesson to be participatory.
- After studying the lesson plan for 1 week, I know everything about it and don't need further preparation
- There is no difference between the Mathematics and the English lesson plans

Answer sheet

- Teachers cannot write in the lesson plans. True
- Students may write in their handbooks. False
- SL lesson plan manual should have a lifespan of three years. True
- The plans contain activities which are fun and interesting for both the pupils and the teachers. True
- The lesson plans guide you in a structure of teaching throughout the week, term and year. True

- These plans need proper preparations by the teachers when the lesson starts. False
- The plans are all in English, which means that you can't use any local language while teaching. False
- Following these plans will help teach the whole curriculum for their year group. True
- The lesson plans are taken from the curriculum and each lesson lasts according to the time table. True
- Each lesson has a different structure to be followed. False
- During a daily lesson the lesson plan only uses whole class teachings. False
- The different activities in the lesson plans helps your lesson to be participatory. True
- After studying the lesson plan for 1 week, I know everything about it and don't need further preparation. False
- There is no difference between the Mathematics and the English lesson plans. False

The Lesson Plans below can be used if books are not available.

Lesson Title: Order of operations (BODMAS)	Theme: Numbers and Numeration	
Lesson Number: M1-L031	Class: SSS 1	Time: 40 minutes
 Learning Outcome By the end of the lesson, pupils will be able to apply the order of operations (BODMAS) to solve mathematical problems.	 Preparation None	

Opening (3 minutes)

- Write on the board: $10 - 3 \times 3$
- Give pupils 1 minute to find the answer. They may share ideas with seatmates.
- Allow pupils to call out their answers. Some pupils may say 21 (incorrect), while other pupils say 1 (correct). Encourage all answers.
- Invite a volunteer who answered 1 to solve the problem on the board. (Answer: multiply first: $3 \times 3 = 9$, then subtract: $10 - 9 = 1$)
- Explain to pupils that today's lesson is on applying BODMAS to solve mathematical problems. We must follow the correct order of operations to get the correct answer.

Teaching and Learning (20 minutes)

- Ask pupils to tell us what the letters of BODMAS stand for? (Answer: Bracket, Of, Division, Multiplication, Addition and Subtraction)
- Explain:
 - When working problems which have more than one operation (of, \times , $+$, $-$ and \div), we use BODMAS.
 - This tells us the order in which we should work the operations in a Maths problem.
 - The term "of" represents the multiplication sign, which includes powers.
- Write the following problem on the board: Simplify $7 + (6 + 5^2) \times 3$.
- Explain the steps on the board to pupils and make sure they understand.

Solution:

$$\begin{aligned}
 7 + (6 + 5^2) \times 3 &= 7 + (6 + 25) \times 3 && \text{Start inside brackets, using "of" first.} \\
 &= 7 + (31) \times 3 && \text{Add inside the brackets.} \\
 &= 7 + 93 && \text{Multiply} \\
 &= 100 && \text{Add}
 \end{aligned}$$

- Write another problem on the board: Evaluate $3 + 6 \times (5 + 4) \div 3 - 7$ using the order of operations.
- Explain the steps on the board to pupils and make sure they understand.

Solution:

$$\begin{aligned}
 3 + 6 \times (5 + 4) \div 3 - 7 &= 3 + 6 \times 9 \div 3 - 7 && \text{Bracket} \\
 &= 3 + 6 \times 3 - 7 && \text{Division} \\
 &= 3 + 18 - 7 && \text{Multiplication} \\
 &= 21 - 7 && \text{Addition} \\
 &= 14 && \text{Subtraction}
 \end{aligned}$$

7. Write the following problem on the board: Evaluate $\left(2\frac{1}{3} + 4\frac{1}{2}\right) \div 8\frac{1}{5}$
8. Ask pupils to solve the problem with seatmates.
9. Invite a volunteer to perform the operation in the bracket first on the board:

$$2\frac{1}{3} + 4\frac{1}{2} = \frac{7}{3} + \frac{9}{2} = \frac{14+27}{6} = \frac{41}{6}$$

10. Invite another volunteer to solve $\frac{41}{6} \div 8\frac{1}{5}$ on the board:

$$\frac{41}{6} \div 8\frac{1}{5} = \frac{41}{6} \div \frac{41}{5} = \frac{41}{6} \times \frac{5}{41} = \frac{5}{6}$$

11. Write the following problem on the board: Evaluate $\left(3\frac{1}{2} + 7\right) \div \left(4\frac{1}{3} - 3\right)$.
12. Ask pupils to solve the problem with seatmates.
13. Invite a volunteer to solve the problem on the board. Allow the other pupils to be involved.

Solution:

$$\begin{aligned} \left(3\frac{1}{2} + 7\right) \div \left(4\frac{1}{3} - 3\right) &= \left(\frac{7}{2} + 7\right) \div \left(\frac{13}{3} - 3\right) && \text{Convert mixed to improper fractions} \\ &= \left(\frac{7+14}{2}\right) \div \left(\frac{13-9}{3}\right) && \text{Solve inside brackets} \\ &= \frac{21}{2} \div \frac{4}{3} && \text{Divide} \\ &= \frac{21}{2} \times \frac{3}{4} \\ &= \frac{63}{8} = 7\frac{7}{8} \end{aligned}$$

Practice (16 minutes)

1. Write the following problems on the board:
 - a. Evaluate $150 \div (6 + 3 \times 8) - 5$
 - b. Evaluate $\frac{1}{2} + \frac{1}{4}$ of $\frac{2}{3} \div \frac{5}{6}$
 - c. Evaluate $6 + 2^3 - 4 \times 8$
2. Ask pupils to solve the problems independently in their exercise books. They may discuss with seatmates if needed.
3. Walk around to check for understanding and clear any misconceptions.
4. Invite volunteers from the class to solve the problems on the board.

Solutions:

a.

$$\begin{aligned} 150 \div (6 + 3 \times 8) - 5 &= 150 \div (6 + 24) - 5 && \text{Multiply inside brackets} \\ &= 150 \div 30 - 5 && \text{Add inside brackets} \\ &= 5 - 5 && \text{Divide} \\ &= 0 && \text{Subtract} \end{aligned}$$

b.



$\frac{1}{2} + \frac{1}{4}$ of $\frac{2}{3} \div \frac{5}{6}$	$= \frac{1}{2} + \left(\frac{1}{4} \times \frac{2}{3}\right) \div \frac{5}{6}$	Multiply ("of")
	$= \frac{1}{2} + \frac{2}{12} \div \frac{5}{6}$	
	$= \frac{1}{2} + \frac{1}{6} \div \frac{5}{6}$	Simplify the fraction
	$= \frac{1}{2} + \frac{1}{6} \times \frac{6}{5}$	Divide
	$= \frac{1}{2} + \frac{1}{5}$	
	$= \frac{5+2}{10} = \frac{7}{10}$	Add

C.

$6 + 2^3 - 4 \times 8$	$= 6 + 8 - 4 \times 8$	Power ("of")
	$= 6 + 8 - 32$	Multiply
	$= 14 - 32$	Add
	$= -18$	Subtract

Closing (1 minute)

1. For homework, have pupils do the practice activity PHM1-L031 in the Pupil Handbook.

Lesson Title: Review of Numbers and Numerations	Theme: Numbers and Numeration	
Lesson Number: M1-L001	Class: SSS 1	Time: 40 minutes
 Learning Outcomes By the end of the lesson, pupils will be able to: <ol style="list-style-type: none"> 1. Identify prime numbers and prime factors. 2. Calculate LCM and HCF. 	 Preparation None	

Opening (3 minutes)

1. Review factors by asking pupils to list the factors of 20 and 45.
2. Have pupils volunteer to give the factors, and list their answers on the board. (Answers: 20: 1, 2, 4, 5, 10, 20; 45: 1, 3, 5, 9, 15, 45)
3. Write on the board: Write down the first 5 multiples of 20.
4. Allow pupils to brainstorm and call out their answers. Write them on the board. (Answers: 20, 40, 60, 80, 100)
5. Explain that in today's lesson pupils will revise how to identify prime numbers and prime factors and also learn how to calculate LCM and HCF. These are topics from JSS.

Teaching and Learning (20 minutes)

1. Explain prime numbers:
 - Prime numbers are numbers that are greater than 1 and cannot be divided evenly by any other number except 1 and itself.
 - Zero and 1 are not considered prime numbers.
 - When talking about prime numbers, we are referring to whole numbers.
2. Write on the board: Identify the prime numbers between 0 and 30.
3. Ask pupils to volunteer to answer. (Answer: 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29)
4. Explain prime factors:
 - A prime factor is a prime number that divides exactly into another given number.
 - Every positive integer has its own unique set of prime factors.
5. Write on the board: What are the prime factors of 12?
6. Ask pupils to volunteer to answer. (Answers: 2 and 3)
7. Write on the board: Write down the prime factors of: a. 40 and b. 50.
8. Ask pupils to work with seatmates to write the prime factors. Invite volunteers to write the answers on the board. (Answers: **a. 2, 5; b. 2, 5**)
9. Explain LCM:
 - Learning to find prime factors of numbers will help you find the least common multiple (LCM) and the highest common factor (HCF) of numbers.
 - We are going to use prime factorisation to find the LCM. The prime factorisation of a number is the list of all prime numbers that result in the given number when multiplied together.
 - To find the LCM:
 - Find the prime factorisation of each number.
 - Find the prime factors that appear in any one of the prime factorisations.

- Find the product of these primes using each prime the most number of times that it appears in any one of the prime factorisations.
10. Write on the board: Find the least common multiple of 24 and 60.
 11. Invite a volunteer to write the prime factorisation of 24. (Answer: $24 = 2 \times 2 \times 2 \times 3$)
 12. Invite a volunteer to write the prime factorisation of 60. (Answer: $60 = 2 \times 2 \times 3 \times 5$)
 13. Have a pupil volunteer to identify the prime factors of 24 and 60. (Answer: **2, 3, 5**)
 14. Explain:
 - There are three 2s (in 24 and two 2s in 60)
 - There is only one 3 (in 24 and in 60)
 - There is only one 5 (in 60)
 15. Ask pupils to multiply these prime factors in their exercise books. Invite a volunteer to write the answer on the board. (Answer: $LCM = 2 \times 2 \times 2 \times 3 \times 5 = 120$)
 16. Explain HCF:
 - We are going to use prime factorisation in finding the highest common factor (HCF) of 2 or more numbers.
 - Find the prime factors that appear in both of the prime factorisations.
 - Multiply the prime factors that occur in both numbers. Multiply each common prime factor the least number of times it appears in any one of the prime factorisations.
 - The HCF is the largest of all the common factors.
 17. Write on the board: Find the HCF of 20 and 30
 18. Remind pupils to use prime factorisation (the same as above in finding the LCM) to find the HCF.
 19. Ask volunteers to list the prime factorisation of 20 and 30. (Answers: $20 = 2 \times 2 \times 5$ and $30 = 2 \times 3 \times 5$)
 20. Ask a volunteer to identify the prime factors that are common with the two numbers. (Answer: 2 and 5)
 21. Ask pupils to multiply the common prime numbers in their exercise books. Invite a volunteer to write the answer on the board. (Answer: $HCF = 2 \times 5 = 10$)

Practice (15 minutes)

1. Write the following three problems on the board:
 - a. Write down the prime factors of 35, 50 and 80.
 - b. Find the least common multiple (LCM) of 18, 30 and 45.
 - c. Find the highest common factor (HCF) of 24, 36 and 48.
2. Ask pupils to solve the problems in their exercise books.
3. After 10 minutes, ask them to discuss their answers with seatmates.
4. Review the answers with them and clear up any misconceptions.

Solutions:

a. **35: 5,7; 50:2,5; 80:2,5**

b.

$$30 = 2 \times 3 \times 5$$

$$45 = 3 \times 3 \times 5$$

c. $24 = 2 \times 2 \times 2 \times 3$

$$36 = 2 \times 2 \times 3 \times 3$$



$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$\mathbf{LCM = 2 \times 3 \times 3 \times 5 = 90}$$

$$\mathbf{HCF = 2 \times 2 \times 3 = 12}$$

Closing (*2 minutes*)

1. Have 2-3 pupils volunteer to explain prime numbers, prime factors and the difference between LCM and HCF in their own words.
2. For homework, have pupils do the practice activity PHM1-L001 in the Pupil Handbook.

Lesson Title: Introduction to probability – Part 1	Theme: Statistics and Probability	
Lesson Number: M3-L097	Class: SSS 3	Time: 40 minutes
 Learning Outcomes By the end of the lesson, pupils will be able to: 1. Define, use, and give examples of terms used in probability. 2. Use the language of probability to describe events in real life.	 Preparation 1. Write the table giving PROBABILITY TERMS AND DEFINITIONS found at the end of this lesson plan on the board. Leave the Examples column blank. 2. Write the questions also found at the end of this lesson plan on the board.	

Opening (4 minutes)

1. Ask pupils to think about something they are likely to do when they go home today.
2. Invite volunteers to answer. (Example answers: do their homework; have a meal; listen to music; watch a film; play football with friends)
3. Tell pupils that after today's lesson, they will be able to define, use, and give examples of terms used in probability. They will also be able to use the language of probability to describe events in real life.

Teaching and Learning (20 minutes)

1. Explain:
 - Probability is used to describe how likely or unlikely it is for something to happen.
 - It is used in many different fields such as statistics, commerce, gambling, insurance, science and technology,
 - Before we can study probability, we need to be able to define and use the terms used in probability.
2. Point to the list of words for question a. on the board and explain: Because probability is based on uncertainty and chance, in everyday life we use words such as those shown.
3. Ask pupils to work with seatmates to answer question a.
4. Invite volunteers to give answers and note some of the answers on the board. The answers will very likely be different depending on the time of year, who is answering the question, and so on.

Solution:

a. Statement

i. It will rain tomorrow

ii. The sun will rise in the east tomorrow

iii. You are late for school tomorrow

iv. You complete all your Maths practice correctly

v. A human being grows to 20 feet tall

vi. Your favourite football team wins its next match.

- ii. is a certainty – the sun always rises in the east.

- v. is impossible. No human being will grow to be 20 feet tall.

- The total opposite of a certain event is an impossible event.

- The only impossible statement is v. All the others have a degree of uncertainty depending

Example answers

likely / unlikely depending on the time of year

certain

likely / unlikely

likely / unlikely

impossible

likely / unlikely

on time of year, the person answering and other similar factors.

5. Explain:

- We have different terms to explain and describe probability.
- Give each term and definition in the table of PROBABILITY TERMS AND DEFINITION.
- Give some of the examples in the example column.
- Ask pupils to discuss with seatmates to come up with more examples.
- Invite volunteers to give their examples.

6. Go through all the terms in the table making sure pupils are clear on each term.

7. Explain: New probability terms will be defined as we encounter them during the topic.

8. Invite a volunteer to fully assess question b. on the board and extract the given information.
(Example answer: give all the possible outcomes of given experiments)

Solution:

b. Given: give all the possible outcomes of given experiments

i. guessing the gender of a new-born child: $S = \{\text{girl, boy}\}$

guessing the prime numbers less than 20: $S = \{2, 3, 5, 7, 11, 13, 17, 19\}$

9. Ask pupils to work with seatmates to answer question b. iii.

10. Invite a volunteer to give their answer. The rest of the class should check and correct their answer.

guessing a vowel $S = \{a, e, i, o, u\}$

11. Ask pupils to work with seatmates to answer question c.

12. Invite volunteers to give their examples of each type of probability outcome.
Examples are given below in the solution. Pupils will give their own examples.

Solution:

c. Given: describe two events for each given probability outcome

certain

It will rain in August

Independence Day will fall on 27 April next year

likely

You eat rice for lunch today

You score 100% in your next Maths test

unlikely

People will be able to live on Mars

You win first prize in the state lottery

Impossible

A human being will live to be 200 years old

The sun will rise in the west tomorrow

Practice (15 minutes)

1. Ask pupils to work independently to answer question d.
2. Walk around, if possible, to check the answers and clear misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

- d. Given: give all the possible outcomes of given experiments



guessing a month ending in "ber" $S = \{\text{September, October, November, December}\}$

guessing a multiple of 10 under 100 $S = \{10, 20, 30, 40, 50, 60, 70, 80, 90\}$

arranging the letters of the word cat $S = \{\text{cat, cta, atc, act, tac, tca}\}$

Closing (1 minute)

1. For homework, have pupils do the practice activity PHM3-L097 in the Pupil Handbook.

Lesson Title: Review of Quadratic Equations		Unit: Review of SSS 1	
Lesson Number: TGM2-L003		Class: SSS 2	Time: 40 minutes
 Learning Outcomes By the end of the lesson, pupils will be able to:	 Preparation None		
<ol style="list-style-type: none"> Solve quadratic equations algebraically. Graph and interpret quadratic functions. 			

Opening (1 minute)

- Tell pupils that they will be revising quadratic equations today.
- Ask pupils to explain what quadratic equations are in their own words and allow time for discussion. (Example answers: they are equations that contain the term x^2 ; when graphed they make parabolas; they can be solved by factorisation)

Teaching and Learning (23 minutes)

- Write the following quadratic equation on the board: $y = x^2 + 4x + 3$
- Remind pupils of some facts about quadratic functions and equations:
 - A "quadratic equation" is given in a way that allows you to solve for a variable and does not have a y -value. (Example: $x^2 + 4x + 3 = 0$)
 - A "quadratic function" has a y -value and can be graphed. (Example: $y = x^2 + 4x + 3$)
 - Quadratic equations and functions both contain the term x^2 .
 - We can find the solutions to quadratic equations in several ways:
 - By graphing the related function
 - Using factorisation
 - Completing the square
 - Using the quadratic formula
 - The graph of a quadratic function is a parabola. The solutions (or roots) of a quadratic equation are the points where the parabola crosses the x -axis.
- Tell pupils that we will first solve the quadratic equation on the board graphically. Draw the table of values shown below and a Cartesian plane on the board:

x	-4	-3	-2	-1	0
y					

- Use the quadratic equation to solve for y and write the answer in the table of values.

Solution:

$$\begin{aligned}
 y &= x^2 + 4x + 3 \\
 &= (-4)^2 + 4(-4) + 3 \\
 &= 16 - 16 + 3 \\
 &= 3
 \end{aligned}$$

x	-4	-3	-2	-1	0
y	3				

- Ask pupils to work with seatmates to find the other y -values and complete the table. As they finish, invite volunteers to write the answers on the board.

Solutions:

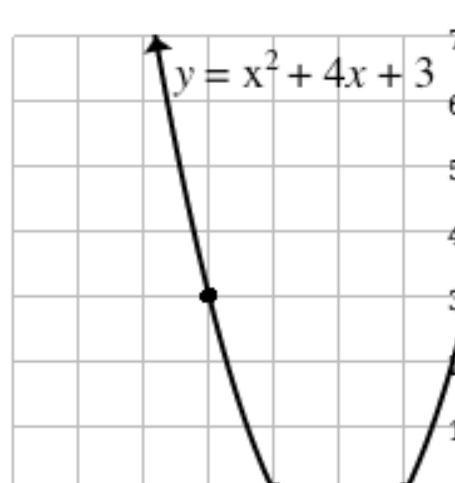
$$\begin{aligned} y &= (-3)^2 + 4(-3) + 3 \\ &= 9 - 12 + 3 \\ &= 0 \end{aligned}$$

x	-4	-3	-2	-1	0
y	3	0	-1	0	3

$$\begin{aligned} y &= (-2)^2 + 4(-2) + 3 \\ &= 4 - 8 + 3 \\ &= -1 \end{aligned}$$

$$\begin{aligned} y &= (0)^2 + 4(0) + 3 \\ &= 0 - 0 + 3 \\ &= 3 \end{aligned}$$

$$\begin{aligned} y &= (-1)^2 + 4(-1) + 3 \\ &= 1 - 4 + 3 \\ &= 0 \end{aligned}$$



6. Invite volunteers to come to the board and plot each of the 5 points in the Cartesian plane (Answer: See graph).
7. Connect the 5 points to make a smooth parabola. (Answer: See graph).
8. Ask pupils to identify the solutions to the quadratic equation. If needed, remind them that the x -intercepts give the solutions. (Answers: $x = -3$; $x = -1$)

9. Tell pupils that we will now solve the same problem by factorisation.

10. Review the process of factorisation:

- Factorisation involves rewriting the quadratic equation in the form $x^2 + 4x + 3 = (x + a)(x + b)$, where a and b are integers.
- a and b should sum to the coefficient of the second term of the equation, and multiply to get the third term. (In this case, $a + b = 4$ and $a \times b = 3$)
- After finding a and b , set each binomial equal to 0 and solve to find the value of x . These values are the roots.

11. Solve the quadratic equation by factorisation, explaining each step:

$$x^2 + 4x + 3 = (x + a)(x + b) \quad \text{Set up the equation}$$

$$a + b = 4$$

$$a \times b = 3$$

Note that the values of a and b must be 3 and 1 to satisfy these equations

$$x^2 + 4x + 3 = (x + 1)(x + 3)$$

Substitute the values of a and b into the equation

$$x + 1 = 0$$

$$x + 3 = 0$$

Set each binomial equal to 0 and solve for x .

$$x = -1$$

$$x = -3$$

These are the roots.

12. Tell pupils that we will now solve the same problem using the quadratic formula.

13. Review the quadratic formula:

- Write on the board: $ax^2 + bx + c = 0$
- Write the quadratic formula on the board: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Tell pupils that to find the roots of the equation, we can simply substitute the values of a , b , and c from a given equation into the formula and solve.

14. Solve the quadratic equation $x^2 + 4x + 3 = 0$ with the quadratic formula, explaining each step:

Solution:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(3)}}{2(1)}$$

Substitute the values of a , b , and c

$$= \frac{-4 \pm \sqrt{4}}{2}$$

Simplify

$$= \frac{-4 \pm 2}{2}$$

Note: The \pm symbol tells us that we can find 2 solutions: one by adding, and the other by subtracting

$$= \frac{-4+2}{2} \text{ and } \frac{-4-2}{2}$$

$$x = -1 \text{ and } -3$$

Identify the 2 roots

5. Remind pupils that we have now found the same solutions using 3 methods: graphing, factorisation, and the quadratic formula.

Practice (15 minutes)

1. Write the problem below on the board.
2. Ask pupils to work individually or with seatmates.
 - a. Using the given table of values, plot the quadratic function $y = x^2 - x - 2$.

x	-2	-1	0	1	2	3
y						

- b. Based on your graph, what are the roots of the equation $x^2 - x - 2 = 0$?
 - c. Choose one algebraic method (factorisation or the quadratic formula) and use it to solve the quadratic equation.
3. Ask pupils to compare answers with seatmates and check their work. If time allows, invite volunteers to write the solutions on the board.

Solutions:

- a. Table of values and graph:

$$y = (-2)^2 - (-2) - 2$$

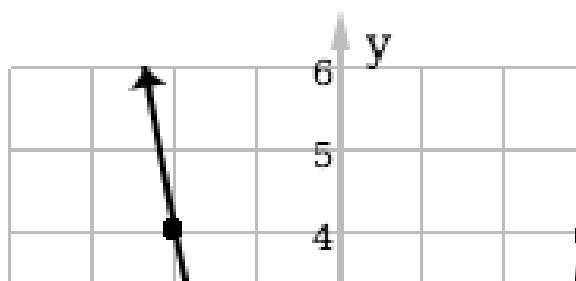
$$= 4 + 2 - 2$$

$$= 4$$

x	-2	-1	0	1	2	3
y	4	0	-2	-2	0	4

$$y = (-1)^2 - (-1) - 2$$

$$= 1 + 1 - 2$$



$$= 0$$

$$\begin{aligned} y &= (0)^2 - (0) - 2 \\ &= 0 - 0 - 2 \\ &= -2 \end{aligned}$$

$$\begin{aligned} y &= (1)^2 - (1) - 2 \\ &= 1 - 1 - 2 \\ &= -2 \end{aligned}$$

$$\begin{aligned} y &= (2)^2 - (2) - 2 \\ &= 4 - 2 - 2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} y &= (3)^2 - (3) - 2 \\ &= 9 - 3 - 2 \\ &= 4 \end{aligned}$$

b. The roots are $x = -1$ and $x = 2$ (the x -intercepts)

c. Pupils may solve by either factorisation or the quadratic formula:

Factorisation

$$\begin{aligned} x^2 - x - 2 &= (x + a)(x + b) \\ &= (x - 2)(x + 1) \end{aligned}$$

$$x - 2 = 0$$

$$x = 2$$

$$x + 1 = 0$$

$$x = -1$$

Quadratic Formula

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{9}}{2}$$



$$= \frac{1 \pm 3}{2}$$

$$= \frac{1+3}{2} \text{ and } \frac{1-3}{2}$$

$$= 2 \text{ and } -1$$

Closing (1 minute)

1. Revise the lesson by asking pupils to list the ways of solving a quadratic equation. (Example answers: graphically, factorisation, quadratic formula, completing the square (not covered in this lesson))
2. For homework, have pupils do the practice activity PHM2-L003 in the Pupil Handbook.

Lesson Title: Simultaneous Linear Equations using Elimination		Theme: Algebraic Processes	
Lesson Number: TGM2-L009		Class: SSS 2	Time: 40 minutes
 Learning Outcome By the end of the lesson, pupils will be able to solve simultaneous linear equations using elimination.	 Preparation Write the problem in the Opening on the board.		

Opening (3 minutes)

- Review solving linear equations. Write the following problem on the board: If $3x + 2y = 18$, find the value of y when $x = 2$.
- Allow time for pupils to answer in their exercise books.
- Invite a volunteer to come to the board to show the solution to the linear equation.

Solution:

$$\begin{array}{rcl}
 3x + 2y & = & 18 \\
 3(2) + 2y & = & 18 \qquad \text{Substitute } x = 2 \\
 6 + 2y & = & 18 \\
 2y & = & 18 - 6 \qquad \text{Transpose} \\
 2y & = & 12 \\
 \frac{2y}{2} & = & \frac{12}{2} \qquad \text{Divide both sides by 2} \\
 y & = & 6
 \end{array}$$

- Explain to pupils that today's lesson is solving simultaneous linear equations using the method of elimination.

Teaching and Learning (20 minutes)

- Write two linear equations on the board:

$$\begin{array}{rcl}
 2x + 3y & = & 10 \qquad (1) \\
 2x + y & = & 2 \qquad (2)
 \end{array}$$

- Explain:
 - To solve for the unknown variable in one linear equation we use the given value as in our example on the board.
 - To solve for 2 linear equations with 2 unknown variables, x and y , we need to solve them together (simultaneously) so that they satisfy both equations.
 - We call that solving "simultaneous linear equations".
 - There are several methods: elimination, substitution and by graphing.
 - We will be using the method of elimination in this lesson.
- Ask pupils to look for the variable with the same co-efficient in both equations.
- Ask pupils to identify the variable. (Answer: x).
- Ask pupils to give the value of the co-efficient. (Answer: 2)

6. Explain the solution step by step.

Solution:

$$2x + 3y = 10 \quad (1)$$

$$-(2x + y = 2) \quad (2)$$

$$\hline 0 + 2y = 8$$

$$2y = 8$$

$$\frac{2y}{2} = \frac{8}{2}$$

$$y = 4$$

Subtract equation (2) from equation (1).

Note: The negative sign will change all the signs across the bracket

Divide both sides of the equation by the coefficient of y (which is 2)

7. Substitute the value of $y = 4$ in either equation 1 or 2 to find the value of x

$$2x + 3y = 10 \quad (1)$$

$$2x + 3(4) = 10$$

$$2x + 12 = 10$$

$$2x = 10 - 12$$

$$2x = -2$$

$$\frac{2x}{2} = \frac{-2}{2}$$

$$x = -1$$

Substitute $y = 4$ in equation (1)

Transpose 12

Divide throughout by 2

8. The solution is therefore $x = -1, y = 4$.

9. Write the solution on the board as an ordered pair: $(-1, 4)$

10. Ask pupils to check that the solution is correct by substituting the values for x and y in equation (2).

Check:

$$2x + y = 2 \quad (2)$$

$$2(-1) + 4 = 2$$

Substitute $x = -1, y = 4$ in equation (2)

$$-2 + 4 = 2$$

$$2 = 2$$

$$\text{LHS} = \text{RHS}$$

11. Write on the board:

$$2x + 5y = 2 \quad (1)$$

$$3x + 2y = 25 \quad (2)$$

12. Ask the pupils what they notice about these two equations. (Answer: None of the variables have the same co-efficient).

13. Explain:

- We eliminate the x -variable first and solve for the y -variable.
- Make the coefficients of x the same by multiplying equation (1) throughout by the coefficient of x in equation (2), and multiplying equation (2) throughout by the coefficient of x in equation (1).

Solution:

$$\begin{array}{rcl}
 6x + 15y & = & 6 \\
 6x + 4y & = & 50 \\
 \hline
 0 + 11y & = & -44 \\
 \frac{11y}{11} & = & \frac{-44}{11} \\
 y & = & -4
 \end{array}$$

(3) Multiply equation (1) $\times 3$

(4) Multiply equation (2) $\times 2$

Subtract equation (4) from equation (3).

Divide throughout by 11

$$\begin{array}{rcl}
 2x + 5y & = & 2 \\
 2x + 5(-4) & = & 2 \\
 2x - 20 & = & 2 \\
 2x & = & 2 + 20 \\
 2x & = & 22 \\
 \frac{2x}{2} & = & \frac{22}{2} \\
 x & = & 11
 \end{array}$$

(1)

Substitute $y = -4$ in equation (1)

Transpose -20

Divide throughout by 5

14. The solution is therefore $x = 11, y = -4$, or the ordered pair $(11, -4)$.

15. Ask pupils to work with seatmates to check the solution, $x = 11, y = -4$. They should substitute these values in equation (2).

16. Invite a volunteer to write their solution on the board.

Solution:

$$\begin{array}{rcl}
 3x + 2y & = & 25 \\
 3(11) + 2(-4) & = & 25 \\
 33 - 8 & = & 25 \\
 25 & = & 25 \\
 \text{LHS} & = & \text{RHS}
 \end{array}$$

(2)

Substitute $x = 11, y = -4$

17. The solution is verified as $x = 11, y = -4$.

18. Explain:

- It does not matter which variable we choose to eliminate first. This same procedure can be used to eliminate the y -variable first, then solve for the x -variable.
- To make the coefficients of y the same, we multiply equation (1) throughout by the coefficient of y in equation (2), and multiply equation (2) throughout by the coefficient of y in equation (1).

Practice (15 minutes)

1. Ask pupils to solve the given pairs of equations using the method of elimination:

a. $3x + 4y = -1$
 $3x + 8y = 4$

b. $5x - 2y = 15$
 $3x + 5y = 9$

- Ask pupils to exchange books and check each other's work.
- Invite volunteers to come to the board to show their solutions.

Solutions:

a.

$$\begin{array}{rcl}
 3x + 4y & = & -1 \\
 3x + 8y & = & 4 \\
 \hline
 0x - 4y & = & -5 \\
 -4y & = & -5 \\
 \frac{-4y}{-4} & = & \frac{-5}{-4} \\
 y & = & \frac{5}{4}
 \end{array}$$

b.

$$\begin{array}{rcl}
 5x - 2y & = & 15 \\
 3x + 5y & = & 9 \\
 \hline
 15x - 6y & = & 45 \\
 15x + 25y & = & 45 \\
 \hline
 0 - 31y & = & 0 \\
 y & = & 0
 \end{array}$$

$$\begin{array}{rcl}
 3x + 8y & = & 4 \\
 3x + 8\left(\frac{5}{4}\right) & = & 4 \\
 3x + 10 & = & 4 \\
 3x & = & 4 - 10 \\
 3x & = & -6 \\
 \frac{3x}{3} & = & \frac{-6}{3} \\
 x & = & -2
 \end{array}$$

$$\begin{array}{rcl}
 5x - 2y & = & 15 \\
 5x - 2(0) & = & 15 \\
 5x - 0 & = & 15 \\
 5x & = & 15 \\
 \frac{5x}{5} & = & \frac{15}{5} \\
 x & = & 3
 \end{array}$$

$x = 3, y = 0$

$x = -2, y = \frac{5}{4}$

Closing (2 minutes)

- Ask pupils to write in their exercise books the basic method to eliminate the x -variable from 2 linear equations. (Example answer: Make the coefficients of x the same by multiplying equation (1) throughout by the coefficient of x in equation (2), and multiplying equation (2) throughout by the coefficient of x in equation (1)).
- Invite a volunteer to read out their answer. Correct any mistakes and confirm that pupils have written the method correctly in their exercise books.
- For homework, have pupils do the practice activity PHM2-L009 in the Pupil Handbook.

Session 4: Number bases, real numbers and BODMAS

120 minutes

Session Objectives

By the end of the session, participants will be able to solve problems on:

- Number bases
- Real, rational and irrational numbers
- Order of operations

Materials

Chart 4.1 Learning outcomes

Mathematics lesson plans SSS 1-3

Chart paper and markers

Handout 4.1 Real Number Cards

Chart 4.2 Real Number Subset Labels

Chart 4.3 Real Number Venn Diagram

Activity outline

Introduction	3 minutes
Activity 1: Number bases	40 minutes
Activity 2: Real, Rational, Irrational numbers	40 minutes
Activity 3: Order of operations	35 minutes
Summary	2 minutes

Session Introduction3 minutes

- Welcome participants.
- Display chart 4.1 (learning outcomes) and go through the learning outcomes with participants.

Activity 1: Number bases30 minutes

In mathematics, a Base is the number of different digits or combination of digits and letters that a system of counting uses to represent numbers. For example, the most common base used today is the decimal system. Because "dec" means 10, it uses the 10 digits from 0 to 9.

- Write the following symbols on the chalkboard: 7 VII •• 13
- Ask: What observations can you make about the four numbers written on the board? *(At this point, some of the participants may not know that VII and •• are numbers.)* Explain to participants that all of these symbols represent quantities in various number systems. In fact, all of the symbols represent the same quantity, 7.
- Remind participants that we use a decimal number system, also called a Base 10 system. Because this is our accepted number system, we do not write or say Base 10 each time we write a number.
- Ask participants to look at the symbol, VII and ask: Does anyone know what number system is being used here? *(Roman Numeral system)* Explain to participants that in the Roman Numeral system, V represents 5 and I represents 1, so 7 is written VII.
- Ask participants to try to identify the number system for ••. *(Mayan Number System is being used. The short horizontal line represents 5 while each dot on top represents a 1.)*
- Ask participants to explain how 13 could represent the number
- Explain to participants that the Roman and Mayan number systems use entirely different symbols to represent their numbers. *(Some number systems use the same digits as our decimal system but in a different way. In this case 7 in Base 10 is the same as 13 in Base 4. The decimal system uses ten unique digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The number after 9 is 10, a combination of two unique digits. In the decimal number system, the order and position of the digits determines the value. For example, 43 and 34 are not the same thing.)*
- Ask participants: What does the number 43 represent? *(4 tens and 3 ones, or 40 + 3)* What does 34 represent? *(3 tens and 4 ones, or 30 + 4)*. How are 52

and 502 different? (*52 represents 5 tens and 2 ones, or $50 + 2$; 502 represents 5 hundreds, 0 tens and 2 ones, or $500 + 2$.*)

- Solve together (complete the expansion of the number 4823)

$$4 \ (1000) + 8 \ (\underline{\quad}) + \underline{\quad}(\underline{\quad}) + \underline{\quad}(\underline{\quad})$$

What do you notice about all the numbers in parentheses? *Going right to left, each number is 10 times greater than the last.*

We can write the expansion of 4,823 another way: $4(1000) + 8(100) + 2(10) + 3(1) = 4(10^3) + 8(10^2) + 2(10^1) + 3(10^0)$

Now, using the notation above, write the following numbers in expanded form: 48, 15, 211

Answers: $48 = 4(10^1) + 8(10^0)$; $15 = 1(10^1) + 5(10^0)$; $211 = 2(10^2) + 1(10^1) + 1(10^0)$

- Remind participants that $10^0 = 1$
- Have participants consider the number 13 in Base 4. (*Point out to participants that 13 in Base 4 should be read as "one three in base four" instead of "thirteen in base four."*) In groups, ask participants to discuss what they think 13 in Base 4 means and how it could be equivalent to 7 in Base 10. (*13 in Base 4 means 1 four and 3 ones, or $4 + 3$*)
- Write 13 in Base 4 in expanded form as a class. ($1(4) + 3(1) = 1(4^1) + 3(4^0)$). *Participants may have to be reminded that $4^0 = 1$*
- Have participants complete the following with a partner:
 - What are the unique digits used in the Base 4 Numeration System? 0, 1, 2, 3
 - How do we count in Base 4? 0, 1, 2, 3, 10, 11, 12, 13, 20, 21, 22, 23...
 - Complete the expansion of the 231 in Base 4. = $2(4^2) + 3(4^1) + 1(4^0)$
 - Write the following numbers in expanded form:
 - $25_6 = 2(6^1) + 5(6^0)$
 - $12_4 = 1(4^1) + 2(4^0)$
 - $33_5 = 3(5^1) + 3(5^0)$
- If time permits, play the game below

Activity 2: Real, Rational, Irrational numbers

30 minutes

- Distribute Handout 4.1 Real Numbers Cards. Discuss various characteristics of the numbers. Then, direct participants to cut the cards apart and sort them any way they like. When they are finished sorting, have them discuss with partners how they did their sorts. Lead a discussion about the different ways they sorted the numbers, asking them to explain the processes they used.
- Show Chart 4.2 Real Number Subset Labels. Have participants write them and cut them apart and arrange them in any order. Have participants assign each number card to a subset. Point out how some numbers belong in more than one subset and discuss the characteristics of each subset.
- Have participants work with partners to sort the number cards into rational and irrational numbers. Then, have them sort the rational numbers into integers, whole numbers, and/or natural numbers. When they have finished sorting, discuss the fact that some numbers can appear in more than one subset, e.g., 4 is a rational number, an integer, a whole number, and a natural number. Explain that the attributes of one subset can be contained in whole or in part in another subset. Explain the process of sorting numbers into the *most specific* subset.
- Show the Real Number System Venn Diagram, Chart 4.3. Have participants copy the diagram and write the names of the subsets in the appropriate areas on the diagram and then write the numbers from the number cards in the *most specific* subsets. Finally, have participants add two more numbers to each subset, explaining why the only number that can be uniquely in the "Whole Numbers" subset area is zero.

Questions (ask participants orally)

- To which subset(s) of the real number system does the number -0.75 belong? Why? (*rational; though it's negative, it can be expressed as a fraction involving only 2 integers*)
- Is the square root of 15 rational or irrational? How do you know? (*irrational because it's never-ending and never repeating. It can't be written as a fraction*)
- Identify whether a number can be both whole and irrational, and explain why or why not. (*impossible for a number to be both rational and irrational*)
- Identify which subset of the real number system contains the most rational numbers, and explain why. (*integers*)
- Identify whether pi (π) is rational or irrational, and explain why. (*irrational; it is an approximation*)

Activity 3: Order of Operations

25 minutes

- Review with participants the process of finding the area of a rectangle by displaying three rectangles with the dimensions (9" x 4", 9" x 5", and 9" x 8").
- Ask participants to explain the most direct way to find the total area of any two of these rectangles. They should recognize that the most direct way is by finding the area of each rectangle and then adding the two areas, e.g., $9 \times 4 = 36$; $9 \times 5 = 45$; $36 + 45 = 81$.
- Ask them to explain the most direct way to find the total area of all three rectangles. (Find the area of each, and then add the three areas.)
- Put the following problem on the board: $4 \times 3 + 2 \times 5 = x$. Have participants solve the problem and then share their answer with a partner. Ask the participants whether anyone got an answer of 70. If so, ask how they got that answer. They will probably say that they multiplied 4 and 3 to get 12, then added 2 to 12 to get 14, and finally multiplied 14 by 5 to get 70. Ask the participants if anyone got an answer of 22. If so, ask how they got that answer. The participants who got this answer should explain that they multiplied 4 by 3 to get 12, then multiplied 2 by 5 to get 10, and finally added 12 and 10 to get 22. Discuss with the participants why using these two different procedures would give such different answers.
- Have participants write an expression to model the process they used in the first question (finding areas of rectangles) to find the total area of two rectangles. Participants should write one of the following expressions: $9 \times 4 + 9 \times 5$, $9 \times 4 + 9 \times 8$, or $9 \times 5 + 9 \times 8$.
- Select one of these expressions and use it to model on the board the two procedures used in the warm-up: (1) doing all operations left to right and (2) doing all multiplications first and then doing addition.
- Have participants work in pairs to construct an explanation of why they must use the second procedure when finding the area of two rectangles. Allow plenty of time for discussion.
- Allow pairs time to present their explanations. At the end, summarize the discussion to explain why the second procedure is correct.
- Have the participants write a rule, based on the discussion, for simplifying expressions that involve multiplication and addition. The rule should state that *all multiplication must be done before any addition*.
- Have the participants write an expression to find the total area of all three rectangles from the warm-up (e.g., $9 \times 4 + 9 \times 5 + 9 \times 8$), and have participants use their rule to simplify the expression.
- Have the participants work in pairs to see if there is another way to solve their original warm-up problem. Since the rectangles have one dimension the same, they could be put next to each other to create one big rectangle.

For example, the 9×4 and 9×5 rectangles together create a 9×9 rectangle with an area of 81.

- Have participants write an expression that models the above approach. One expression could be $9 \times (4 + 5)$. To simplify this expression, explain that 4 and 5 need to be added first and then the sum multiplied by 9 to get 81.
- Have participants write a rule for simplifying expressions that involve parentheses and multiplication. The rule should state that *all expressions inside of parentheses must be simplified before any multiplication*.
- Ask the participants to simplify 32 and then add 5 to the answer. Have the participants write an expression that models the problem. Participants will most likely write $32 + 5$. Explain that the commutative property allows the expression also to be written $5 + 32$.
- Have the participants write a rule for simplifying expressions that involve exponents and addition. The rule should state that *all exponents must be simplified before any addition*.
- Have participants write a new rule that combines the three rules they have written. The rule should state that *when simplifying, expressions inside parentheses must be simplified first, then all exponents must be simplified, then all multiplication is done, and, finally, all addition is done*. Explain that this "order of operations" rule includes division with multiplication and subtraction with addition. *Multiplication and division must be done from left to right first, and then addition and subtraction is done from left to right*.
- Help participants create a complete "order of operations" rule similar to the following:

When simplifying, do all expressions inside parentheses first, then all exponents, then all multiplication and division operations from left to right, and finally all addition and subtraction operations from left to right.

- It might be helpful for participants to see how to organize the order of operations in a numbered list of steps, as shown below:

- *Do any work within parentheses () or other grouping symbols [] first.*
- *Do any work with exponents (powers) or roots.*
- *Do any multiplication and division in order from left to right.*
- *Do any addition and subtraction in order from left to right.*

- Have participants create and write two expressions, each containing parentheses, exponents, and all operations. On a separate sheet of paper, have them simplify their expressions, showing each step in the order of operations and the final answer. Have them exchange their problems with a

partner, simplify each other's expressions, and discuss the problems until agreement is reached on the correct order of operations and final answer.

Summary: _____ 2 minutes

- Display the learning outcomes and go through them

Materials

Handout 4.1 Real Number Cards

Give participants the numbers below cut up.

3.987	-7	19994.5
$\frac{1}{5}$	0.8	-0.35
3.14	80	$\sqrt{2}$
900	-21	5.444
5	-88	212
$\sqrt{7}$	$\sqrt{22}$	-4.5

Chart 4.2 Real Number Subset Labels

Rational numbers

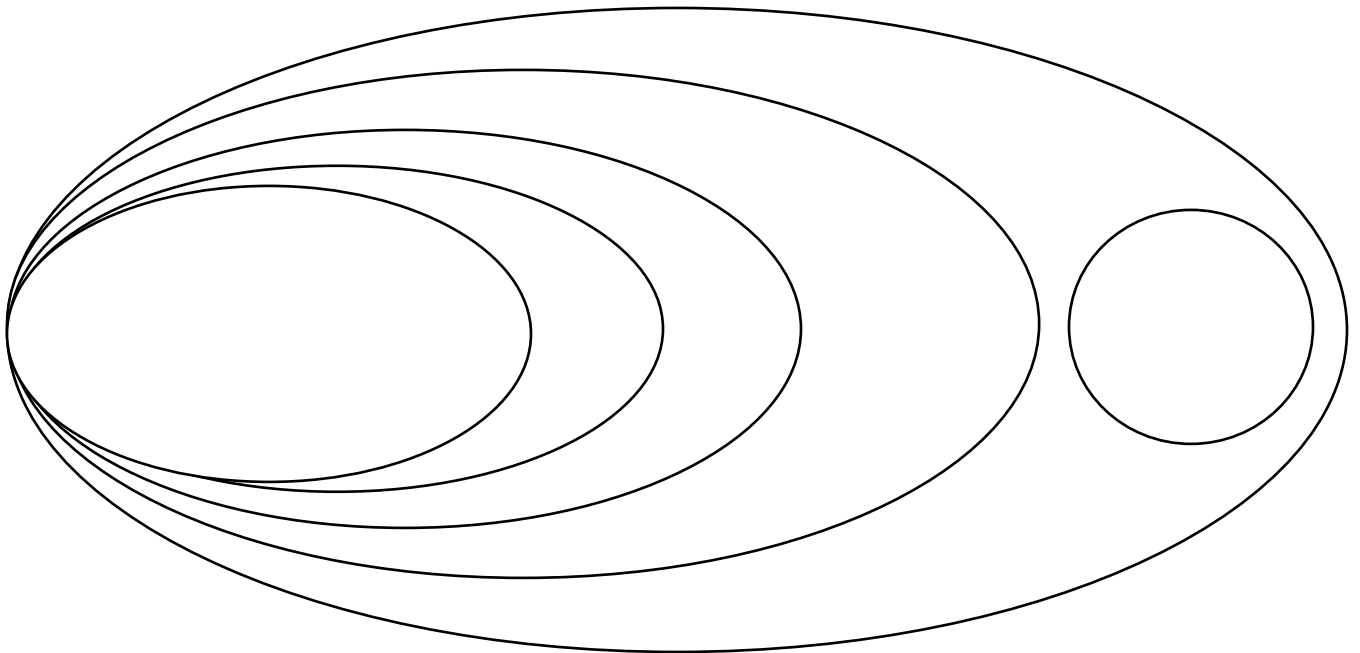
Integers

Whole numbers

Natural numbers

Irrational Numbers

Chart 4.3 Real Number Venn Diagram



Session 5: Indices, Notations, Logarithms/Antilogarithms 120 minutes

Learning outcomes

By the end of the session, participants will be able to:

- Simplify problems with numbers expressed in index notation
- Understand why putting numbers in standard form is useful
- Convert numbers to and from standard form; multiply and divide numbers in standard form.
- Logarithms and Antilogarithms

Materials

Chart 5.1: Learning outcomes

Chart 5.2: Problems on indices

SSS 1-3 Mathematics Lesson Plan Manuals

Handout 5.1: Logarithm Table

Handout 5.2: Anti-logarithm Table

Activity outline

Introduction	3 minutes
Activity 1: Solving problems with Indices/notations expressions	30 minutes
Activity 2: Understanding and using the standard expression	40 minutes
Activity 3: Logarithms and Antilogarithms	45 minutes
Summary	2 minutes

Session Introduction

3 minutes

- Welcome the participants to the session.
- Explain that in the session, you will be considering the following aspects of Number and Numeration that is indices, standard form, logarithms, and antilogarithms
- Go through Chart 5.1: Learning outcomes.

Activity 1: Understanding Indices and notations

30 minutes

- Explain that mathematics deals with numbers that can be unbelievably large and difficult to manipulate. To minimize this problem, mathematicians have come up with ways to express such numbers. Some of these ways are the use of indices and standard forms. In this session, we shall look at both. Understanding this makes problem solving in mathematics a bit easier.
- Get participants attention and write: $2 \times 2 \times 2 =$ on the flip chart or on the chalkboard.
- Ask participants the following questions one at a time.
- What number is multiplied? (2)
- How many times is the number multiplied? (3 times)
Can someone come and express this same multiplication in a shorter way? (Yes, 2^3)
- Explain that the lower number is called the base and the upper number indicates the number of times the number is multiplied and is called the index (plural indices).
- Explain that: $2 \times 2 \times 2$ can be simplified as 2^3 .
- Explain that writing or expressing numbers in this way is called index notation and the number is read as: *the base to the power of the index*. For example, 2^3 is read as: 'two to power three'.
- Explain that: *the power or index tells us how many times the base should be multiplied by itself*.
- Take any concern and explain that you are going to give them an opportunity to apply their understanding.
- Ask participants to individually simplify $7 \times 7 \times 7 \times 7 \times 7$ and to expand 5^6 .
- Ask volunteers to solve the problems on the chalkboard/flip chart paper with explanation.
- Discuss their solutions (7^5 and $5 \times 5 \times 5 \times 5 \times 5 \times 5$) and take any concern.
- Explain that numbers expressed with notations can be added, subtracted, divided and multiplied with ease by applying the laws of indices.

- Write on the board: First law of indices $a^m \times a^n = a^{m+n}$
- Ask: Who can explain to the group what the first law of indices states?

<p><u>First Law of Indices</u></p> $a^m \times a^n = a^{m+n}$ $2^5 \times 2^3 = 2^{5+3}$ $= 2^8$	<p><u>Second Law of Indices</u></p> $\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$ $\frac{6^5}{6^3} = 6^5 \div 6^3 = 6^{5-3}$ $= 6^2$
<p><u>Third Law of Indices</u></p> $a^0 = 1$ $\frac{6^5}{6^5} = 6^5 \div 6^5 = 6^{5-5}$ $= 6^0$ $= 1$	<p><u>Fourth Law of Indices</u></p> $(a^m)^n = a^{mn}$ $(3^3)^2 = 3^{3 \times 2}$ $= 3^6$
<p><u>Fifth Law of Indices</u></p> $(a \times b)^n = a^n \times b^n$ $(2 \times 3)^4 = 2^4 \times 3^4$	<p><u>Sixth Law of Indices</u></p> $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$ $\left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3}$

- Guide a participant to say that: When we multiply expressions (or numbers) with the same base, we add the indices.
- Write on the board: $2^5 \times 2^3$
- Ask the class to use the law to work out the answer.
- Have a participant volunteer to solve the question on the board. (Answer: As shown in the table under 'First law of indices'.)
- Write the heading and the second law of indices on the board.
- Guide a participant to say that: When we divide expressions (or numbers) with the same base, we subtract the indices.
- Write on the board: $\frac{6^5}{6^3}$
- Ask participants to work in pairs to solve the third law of indices.
- Have participants volunteer to call out the answer. Write it on the board. (Answer: As shown in the table under 'Third law of indices').
- Say: Sometimes when we subtract the indices we end up with nothing or zero. The third law of indices explains what to do when this happens.
- Write the information for the third law on the board.
- Say: 6^0 gives 1 because when we divide a number by itself we get 1 except when the base is 0
- Point out on the example where we are dividing 6^5 by itself.
- Say: In the fourth law, we have an index raised to another index. In this case, we multiply the 2 indices as shown on the board. The fifth and sixth laws deal with bases that are different.
- Write the fifth law and ask participants to work out the example: $(2 \times 3)^4$. (Answer: As shown in the table).
- Write the sixth law and ask participants to work out the example: $\left(\frac{3}{5}\right)^3$. (Answer: As shown in the table)
- Display Chart 5.2 and ask the participants to solve them in pairs.
- Take responses from a pair at a time and address concerns.

- Explain that in some instances the numbers are replaced by letters and that the same laws are applied in solving them

Activity 2: Understanding and using the standard expression 40 minutes

The objective of this activity is to appreciate the need to put big and small numbers in a more readable form, how to write numbers in standard form, and calculations involving numbers in standard form.

- Explain that expressing numbers in standard form is another way of overcoming the writing of numbers with large sizes. It also reduces the chances of introducing errors in writing, copying, or transferring figures. Understanding the standard form of expression becomes easier with the understanding of the use of indices.
- What do you notice when you try this on your calculator: 111111111... X 111111111... (until you can't enter any more digits)
- Write $a \times 10^b$ on a chart paper. 'a' is a number between 1 and 10 (excluding 10) while 'b' can be any number (can be negative)
- Tell participants that Standard Form or Scientific Notations is useful for two reasons: It allows us to write really small or really big numbers concisely and it allows us to compare small and big numbers. Example: Which is bigger? (23400000000000 or 23400000000000; 2.34×10^{14} or 2.34×10^{13})
- To convert from standard form, we just consider how many times we're multiplying or dividing by 10. ($3 \times 10^4 = ?$; $1.24 \times 10^5 = ?$; $6 \times 10^{-3} = ?$; 1.01×10^{-1})
- Check for understanding by asking participants to convert from these: 3.012×10^7 ; 7.5×10^1 ; 8.31×10^0 ; 2.4×10^{-6}
- To convert to standard form, we enumerate how many times we need to move the decimal point until we're in the range 1 to 10 (excluding 10) Example $20000 = ?$; $0.0043 = ?$; $931,000,000 = ?$; $0.000001001 = ?$
- Show participants how to correct numbers not in standard form (If the number at the front is not between 1 and 10 (excluding 10), it's not in standard form. We can fix this!) $200 \times 10^4 = 2 \times 10^2 \times 10^4 = 2 \times 10^6$
- Write $12 \times 10^4 = 1.2 \times 10^1 \times 10^4 = 10^5$; $0.003 \times 10^5 = 3 \times 10^{-2} \times 10^5 = 3 \times 10^3$; $210 \times 10^{-3} = 2.1 \times 10^2 \times 10^{-3} = 2.1 \times 10^{-1}$
- Show how to multiply in standard form $(2 \times 10^3) \times (4 \times 10^4) = 8 \times 10^7$; $(7 \times 10^5) \times (6 \times 10^6) = 42 \times 10^{11} = 4.2 \times 10 \times 10^{11} = 4.2 \times 10^{12}$
- Participants try the following: $(7 \times 10^7) \times (2 \times 10^{-3})$; $(9 \times 10^{-2}) \times (9 \times 10^{-3})$; $8 \times 10^8 / 2 \times 10^3$; $4 \times 10^5 / 8 \times 10^{-2}$
- Test for understanding: $9 \times 10^3 / 2 \times 10^6$; $2 \times 10^8 / 8 \times 10^{-5}$
- Share your strategy with the whole group

Activity 3: Logarithms and Antilogarithms45 minutes

- Review simple equations that involve indices. Write the following problem on the board: Find the values of the unknown:
 - a. $3^x = 9$ b. $2^x = 128$
- Ask participants to solve the problems independently.
- Invite volunteers to write the solutions on the board.
- Solutions:

$$\begin{aligned} \text{a. } 3^x &= 9 \\ 3^x &= 3^2 \\ x &= 2 \end{aligned}$$

$$\begin{aligned} \text{b. } 2^x &= 128 \\ 2^x &= 2^7 \\ x &= 7 \end{aligned}$$

- Explain to participants that today's session is on logarithms and their relationship to indices.
- Explain: Logarithms are the "opposite" of exponentials, just as subtraction is the opposite of addition and division is the opposite of multiplication.
- Write on the board: $y = b^x$ is equivalent to $\log_b y = x$.
- Read the statement aloud so that participants understand how to read logarithms: "'y equals b to the power x' is equivalent to 'log base b of y equals x'."
- Label the base and index in each statement so the relationship is clear to participants:

$$\begin{array}{ccc} y = b^x & \longleftarrow & \text{power} \\ \uparrow & & \\ \text{base} & & \end{array} \qquad \begin{array}{ccc} \log_b y = x & \longleftarrow & \text{power} \\ \uparrow & & \\ \text{base} & & \end{array}$$

- Explain:
 - The value of the subscripted "b" is "the base of logarithm", just as b is the base in the exponential expression b^x .
 - We can rewrite any exponential expression in this form as a logarithm.
- Write the following problem on the board: Convert $6^3 = 216$ to the equivalent logarithmic expression.
- Explain to participants that to convert, the base (i.e. the 6) remains the same, but the 3 and the 216 switch sides.
- Write on the board: $\log_6(216) = 3$
- Write on the board: Convert $\log_4(1024) = 5$ to the equivalent exponential expression.

- Ask participants to work with partners to rewrite the logarithm as an exponential expression.
- Take responses from 2 pairs. (Answer: $4^5 = 1024$)
- Display Chart 5.2, the summary table, which shows the relationship between the index and logarithm.

Index	Logarithm
$x = a^y$	$\log_a x = y$
$100 = 10^2$	$\log_{10} 100 = 2$
$64 = 4^3$	$\log_4 64 = 3$
$0.001 = 10^{-3}$	$\log_{10} 0.001 = -3$

- Ask participants to complete the table on chart 5.3 with partners.

Convert numbers in index form to logarithmic form and vice-versa.

No.	Index Form	Logarithmic Form
	$10^4 = 10,000$	
		$\log_p M = k$
	$2^x = y$	
		$\log_3(xy) = 2$
	$7^2 = 49$	

- Take responses from participants to complete the table as below
Answers:

No.	Index Form	Logarithmic Form
a.	$10^4 = 10,000$	$\log_{10} 10,000 = 4$
b.	$p^k = M$	$\log_p M = k$
c.	$2^x = y$	$\log_2 y = x$
d.	$3^2 = xy$	$\log_3(xy) = 2$
e.	$7^2 = 49$	$\log_7 49 = 2$

- Explain:
 - The base of logarithms can be written in any positive number except 1 and the logarithm of a negative number does not exist.
 - When the base of a logarithm is not written it means it is in base 10.
- Write on the board: $\log 100 = 2$

- Explain:
 - Although the base is not written, we can assume it is 10.
 - This is an equivalent statement to $10^2 = 100$

• Write on the board: $\log_{10} 100 = 2$ is equivalent to $10^2 = 100$

• Write on the board:

Write the following as logarithms:

- $10^5 = 100,000$
- $5^{-3} = 0.008$
- $a^b = -43$

Write the following as exponential expressions:

- $\log_5 125 = 3$
- $\log_9 27 = m$.

- Ask participants to solve the problem independently in their exercise books.
- Take quick responses from them (Answers: a. $\log_{10} 100,000 = 5$ or $\log 100,000 = 5$; b. $\log_5 0.008 = -3$; c. $\log_a -43 = b$; d. $5^3 = 125$; e. $9^m = 27$; f. $2^{-3} = 0.125$)

• Write on the board: Simplify $x = \log_3 9$.

• Explain:

- We can solve this logarithm using its relationship to indices.
- The solution to the logarithm $\log_3 9$ is the value of x .

• Discuss: How can we rewrite this logarithm? (Answer: $9 = 3^x$ or $3^x = 9$)

• Write the solution on the board and explain to participants.

Solution:

$$\begin{aligned} x &= \log_3 9 \\ 9 &= 3^x && \text{Change to index form} \\ 3^2 &= 3^x && \text{Substitute } 9 = 3^2 \\ 2 &= x \end{aligned}$$

• Write another problem on the board: Find the value of $\log_3 27$

• Explain:

- This logarithm is not equal to anything, but we can still write it in index form.
- Set it equal to x , then solve for x to find its value.

• Write on the board: $\log_3 27 = x$

- Ask participants to write the index form boldly in their notepad. They raise their solution up when you say 'Show me' (Answer: $27 = 3^x$ or $3^x = 27$)
- Go through same process to find the value of x .
- Solution:

$$\begin{array}{ll} \log_3 27 = x & \\ 27 = 3^x & \text{Change to index form} \\ 3^3 = 3^x & \text{Substitute } 27 = 3^3 \\ 3 = x & \end{array}$$

- Write another problem on the board: Find the value of $\log_4 2$
- Ask a volunteer to express the number in indices form on the board. (Answer: $2 = 4^x$)
- Solve on the board, explaining each step:

$$\begin{array}{ll} \log_4 2 = x & \\ 2 = 4^x & \text{Change to index form} \\ 2 = 2^{2x} & \text{Substitute } 4 = 2^2 \\ 1 = 2x & \text{Set the powers equal} \\ \frac{1}{2} = x & \text{Divide throughout by 2} \end{array}$$

- Explain:
 - Recall and apply the rules for solving simple equations involving indices.
 - Make the bases of the indices the same, then solve for x .
- Write 3 more problems on the board:
 - Solve for x if $\log_3 81 = x$
 - Find the value of $\log_{25} 5$
 - Simplify $\log_8 \frac{1}{64} = y$
- Ask participants to solve the problems with partners.
- Walk around to check for understanding. Provide explanations as needed.
- Invite volunteers to solve the problems on the board.

Solutions:

$$\begin{array}{ll} \text{a. } \log_3 81 = x & \text{b. } \log_{25} 5 = x \\ 3^x = 81 & 25^x = 5 \\ 3^x = 3^4 & 5^{2x} = 5^1 \\ x = 4 & 2x = 1 \end{array}$$

$$x = \frac{1}{2}$$

$$\begin{aligned} \text{c. } \log_8 \frac{1}{64} &= y \\ 8^y &= \frac{1}{64} \\ 8^y &= 64^{-1} \\ 8^y &= 8^{2(-1)} \\ 8^y &= 8^{-2} \\ y &= -2 \end{aligned}$$

- Write on the board: $\log_{10} 76.83 = 1.8856$
- Explain:
 - We can write the logarithm of each number as a decimal number.
 - This can be done using a calculator or logarithm tables. Today we will use logarithm tables.
 - We will only be working with logarithms in base 10.
 - In the result of a logarithm of any number there are two parts, an integer (whole number) before the decimal point and a fraction after the point.
 - The whole number part is called the characteristic and the decimal part is called the mantissa.

- Draw on the board:

Characteristic: \rightarrow 1.8856 \leftarrow Mantissa:
 whole number decimal part

- Explain how to find the characteristic and mantissa:
 - The mantissa is found using a logarithm table.
 - The characteristic can be found by expressing the number you are finding the logarithm of (in this case 76.83) in standard form. The power on the 10 is the characteristic.
- Write on the board: Convert to the form $a \times 10^n$, and n is the characteristic.
- Write it as a problem on the board: Find $\log_{10} 76.83$
- Ask a volunteer to express the number 76.83 in standard form. (Answer: 7.683×10^1)
- Explain: The whole number part (characteristic) is 1, because the power on 10 is 1.

- As a general rule, if the number is greater than 1, then the characteristic is the number of digits before the decimal point minus one. (For example, the characteristic of 104.6 is 2 since the whole number part has 3 digits.)
- Explain: We will now find the mantissa of $\log_{10} 76.83$ from the logarithm table.
- Hold up the logarithm table. Explain the steps for finding the mantissa. As you explain, move your fingers and point at the part of the chart you are referring to:
 - $\log 76.83 = 1.$ "*something*". We are looking for the decimal part.
 - To find the decimal part, go along the row beginning with 76 and under 8, which gives 8854.
 - Now find the number in the differences column headed 3. This number is 2.
 - Add 2 to 8854 to get 8856. Therefore $\log 76.83 = 1.8856$.
- Write the answer on the board: $\log 76.83 = 1.8856$
- Explain: When using tables, always make sure that the number taken from the differences column is in the same row as the rest of the figures.
- Ask participants to open their Participant Handbook to the logarithm table. Ask them to work with partners to find the same mantissa as you have just demonstrated.
- Walk around to check for understanding and clear any misconceptions.
- Write the following problem on the board: Find $\log 37$.
- Solve on the board. Explain the solution to participants:

Step 1. Find the characteristic: Change 37 to standard form. The characteristic is the power on 10, which is 1: $37 = 3.7 \times 10^1$

Step 2. Find the mantissa in the logarithm table: $37 \rightarrow 0.5682$

Step 3. Rewrite $\log 37$ using the characteristic and mantissa: 1.5682
- Write the solution to the same problem on the board in powers of 10, and explain each step to participants:

$$\begin{aligned}
 37 &= 3.7 \times 10^1 && \text{Write in standard form} \\
 &= 10^{0.5682} && \text{From log table; } 3.7 = 10^{0.5682} \\
 &\quad \times 10^1 \\
 &= 10^{0.5682+1} && \text{Apply law of indices} \\
 &= 10^{1.5682}
 \end{aligned}$$

Hence $\log 37 = 1.5682$

- Write the following problem on the board: Find the logarithm of 513.6.
- Solve on the board and using the log table, explaining each step to participants. Ask them to follow along in their own log table.

Solution:

$$\begin{aligned}
 513.6 &= 5.136 \times 10^2 && \text{Write in standard form} \\
 &= 10^{0.7106} && \text{From table; look for 51.36, which gives} \\
 &\quad \times 10^2 && 0.7106 \\
 &= 10^{0.7106+2} && \text{Apply the law of indices} \\
 &= 10^{2.7106}
 \end{aligned}$$

Hence $\log 513.6 = 2.7106$

- Write on the board: Find the logarithms of: a. 4137 b. 8.403
- Ask participants to solve the problem with partners and write the answers in their exercise books.
- Walk around to check for understanding. Explain as needed.
- Invite two volunteers to give their answers on the board and explain how they used the log table. Ask other participants to check their work.

$$\begin{array}{ll}
 \text{a. } 4137 = 4.137 \times 10^3 & \text{b. } 8.403 = 8.403 \times 10^0 \\
 = 10^{0.6167} \times 10^3 & = 10^{0.9245} \times 10^0 \\
 = 10^{0.6167+3} & = 10^{0.9245} \times 1 \\
 = 10^{3.6167} & = 10^{0.9245}
 \end{array}$$

Therefore $\log 4137 = 3.6167$

Therefore $\log 8.403 = 0.9245$

Explain:

- Antilogarithms are the opposite of logarithms. They “undo” logarithms.
- They are called “antilogs” for short.
- Write on the board: If $\log 673.4 = 2.8283$, then $\text{antilog } 2.8283 = 673.4$.
- Explain:
 - We use tables of antilogarithms to solve antilog problems.
 - We look for antilogs in a table the same way as logarithms. However, we must use a different table. Antilogarithms and logarithms each have their own table.
 - When finding the antilog, look up the fractional part only. Using the example on the board, we would drop the 2 and look for 0.8283.
- Write on the board: Find the antilog of 0.5768.

- Hold up the antilog table. Explain the steps for finding the antilog. As you explain, move your fingers and point at the part of the chart you are referring to:
 - Put the finger under .57, run it along to the column headed 6. Here the number is 3767.
 - Run your finger to the number under 8 in the differences (it is 7).
 - Add the number under the columns for 6 and difference 8: $3767 + 7 = 3774$.
 - We know that there is 1 integer digit in the antilog, because the integer in the problem (the characteristic) is 0. We get 3.774.
 - Explain how to determine the decimal's placement:
 - We moved the decimal point based on the characteristic in the problem (the integer digit). Move the decimal point one more space than the value of the characteristic.
 - In the example, the characteristic was 0. Therefore, we moved 1 space from the left side, and changed 3774 to 3.774
 - Recall that when we calculated logs, we gave the characteristic the number of digits before the decimal place minus 1. Therefore, when finding antilogs, we do the opposite. We give the result one more integer digit than the characteristic given in the problem.
 - Write on the board: if $\log n = 2.3572$, find n .
 - Find the antilog of both sides of the equation: $\text{antilog}(\log n) = \text{antilog}(2.3572)$
 - Remind participants that antilog "undoes" log because they are opposites.
 - Write on the board: $n = \text{antilog}(2.3572)$
- Explain the solution to participants. Hold up the antilog tables to show how to find the answer, and write the answer on the board.
- Solution:
- To find n means we have to use the antilog table. As we know, $\log n$ consists of two parts, the characteristic and mantissa.
 - In this case, the characteristic is 2, and the mantissa is 0.3572. Use the antilog table to find the antilog of 0.3572.
 - The antilog of 0.3572 is 2276 (demonstrate this).
 - Since the characteristic is 2, put the decimal 3 places from the left side.
 - Therefore $n = 227.6$

- Write on the board: Find the antilog of 2.7547
- Explain the solution to participants. Hold up the antilog tables to show how to find the answer, and write the answer on the board.

Solution:

- The fractional part of 2.7547 is 0.7547. 0.7547 in the antilog tables gives 5684 (demonstrate this).
- The integer part of 2.7547 is 2. This shows that there are three digits before the decimal point.
- Therefore $\text{antilog } 2.7547 = 568.4$
- Write on the board: Find the number whose logarithm is 5.3914.
- Write this on the board as both a logarithm and antilog: $\log x = 5.3914$, $x = \text{antilog } 5.3914$
- Explain the solution to participants. Hold up the antilog tables to show how to find the answer, and write the answer on the board.

Solution:

- The fractional part of 5.3914 is 0.3914. 0.3914 in the antilog tables gives 2462.
- The integer 5 shows that there are six digits before the decimal point.
- Therefore, the number whose logarithm is 5.3914 is 246,200.
- Write on the board: Find the antilogarithms of the following numbers:
a. 1.5752 b. 4.5724
- Ask participants to work with partners to solve the problems.
- Walk around to check for understanding and clear any misconceptions.
- Invite two volunteers to write their answers on the board and explain using the antilog tables. (Answers: a. $\text{antilog } 1.5752 = 37.60$; b. $\text{antilog } 4.5724 = 37,360$)

Multiplication and division of numbers greater than 1 using logarithms.

- Write on the board: Evaluate 34.83×5.427
- Explain the product rule:
 - We can multiply 2 numbers using logarithms.
 - There are 3 steps:
 - Find the logarithms of the numbers.
 - Add the logarithms.
 - Find the antilogarithm of the result.

- Solve the problem on the board. Explain each step to participants.

Solution:

Step 1. Find the logarithms of the numbers (use the table).

$$\log 34.83 = 1.5420$$

$$\log 5.427 = 0.7346$$

Step 2. Add the logarithms.

$$\begin{array}{r} 1.5420 \\ + 0.7346 \\ \hline 2.2766 \end{array}$$

Step 3. Find the antilog of 2.2766.

From the table, we have 1891. The integer part should be 3 digits, since the characteristic is 2. Therefore, the solution is 189.1.

- Write another problem on the board: Evaluate $85.73 \div 39.63$
- Explain the division rule:
 - We can also divide 2 numbers using logarithms.
 - There are 3 steps:
 - Find the logarithms of the numbers.
 - Subtract the logarithm of the denominator from the numerator.
 - Find the antilogarithm of the result.
- Explain: Note that for multiplication, the second step is to add the logarithms. For division, the second step is to subtract the logarithms.
- Solve the problem on the board. Explain each steps to participants.

Solution:

Step 1. and Step 2. Find the logarithms and subtract. Use the table as shown.

Numbers	Logarithms
85.73	1.9332
39.63	– 1.5980
Subtract the logs for division	0.3352

Step 3. Find the antilog of 0.3352.

From the table, we have 2164. The integer part should be 1 digit, since the characteristic is 0. Therefore, the solution is 2.164.

- Explain:
 - Calculations using logarithms follows the laws of indices.

- Recall that the logarithms we are handling are based on powers of 10, although there is no need to write out the base 10 every time.
- Thus, when we add logarithms for multiplication and subtract for division, this is similar to applying the multiplication and division rules for indices.
- Write the following problem on the board: Evaluate $42.87 \times 23.82 \times 1.127$
- Explain: When you multiply more numbers, follow the same 3 steps.
- Ask volunteers to give the 3 steps needed to solve the problem. (Answer: Find the logarithms, add, and find the antilogarithm.)
- Ask participants to solve the problem with partners.
- Invite a volunteer to write the solution on the board and explain.

Solution:

Numbers	Logarithms
42.87	1.6321
23.82	1.3770
1.127	+ 0.0518
Add the logs for multiplication	3.0609

Antilog of 3.0609 = 1150. The solution is 1150.

- Write another problem on the board: Evaluate $\frac{675.2}{35.81}$
- Ask participants to work with partners to solve the problem.
- Invite a volunteer to write the solution on the board.

Solution:

Number	Logarithm
675.2	2.8294
35.81	– 1.5540
Subtract the logs for division	1.2754

The antilog of 1.2754 = 1,886. The solution is 18.86.

Session summary

2 minutes

- Go over the learning outcomes and emphasize the important points.
- Thank the participants for their participation and remind them of the need to be punctual for the next session.

Materials (1 per participant)
Handout 5.1: Logarithm Table

COMMON LOGARITHM TABLE

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345</																		

Handout 5.2: Anti-Logarithm Table

FRITHLOGARTAIM

ANTI-LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
-00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0 0 1	1 1 1	2 2 2
-01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 1	1 1 1	2 2 2
-02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
-03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
-04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
-05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
-06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1	1 1 1	2 2 2
-07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
-08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0 1 1	1 1 2	2 2 3
-09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
-10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
-11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
-12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1	1 2 2	2 2 3
-13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0 1 1	1 2 2	2 3 3
-14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0 1 1	1 2 2	2 3 3
-15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0 1 1	1 2 2	2 3 3
-16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0 1 1	1 2 2	2 3 3
-17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0 1 1	1 2 2	2 3 3
-18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0 1 1	1 2 2	2 3 3
-19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0 1 1	1 2 2	3 3 3
-20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0 1 1	1 2 2	3 3 3
-21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0 1 1	2 2 2	3 3 3
-22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0 1 1	2 2 2	3 3 3
-23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0 1 1	2 2 2	3 3 4
-24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
-25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1	2 2 2	3 3 4
-26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1	2 2 3	3 3 4
-27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1	2 2 3	3 3 4
-28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0 1 1	2 2 3	3 4 4
-29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1	2 2 3	3 4 4
-30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0 1 1	2 2 3	3 4 4
-31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1	2 2 3	3 4 4
-32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
-33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4
-34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1 1 2	2 3 3	4 4 5
-35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1 1 2	2 3 3	4 4 5
-36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1 1 2	2 3 3	4 4 5
-37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2	2 3 3	4 4 5
-38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1 1 2	2 3 3	4 4 5
-39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2	2 3 3	4 5 5
-40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2	2 3 4	4 5 5
-41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
-42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
-43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
-44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
-45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
-46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
-47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
-48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 3 4	5 6 6
-49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 4 4	5 6 6

FRITHLOGARTAIM

ANTI-LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
-50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1 1 2	3 4 4	5 6 7
-51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1 2 2	3 4 5	5 6 7
-52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1 2 2	3 4 5	5 6 7
-53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1 2 2	3 4 5	6 6 7
-54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1 2 2	3 4 5	6 6 7
-55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1 2 2	3 4 5	6 7 7
-56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1 2 3	3 4 5	6 7 8
-57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1 2 3	3 4 5	6 7 8
-58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1 2 3	4 4 5	6 7 8
-59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1 2 3	4 5 5	6 7 8
-60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1 2 3	4 5 6	6 7 8
-61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1 2 3	4 5 6	7 8 9
-62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1 2 3	4 5 6	7 8 9
-63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1 2 3	4 5 6	7 8 9
-64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1 2 3	4 5 6	7 8 9
-65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1 2 3	4 5 6	7 8 9
-66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1 2 3	4 5 6	7 9 10
-67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1 2 3	4 5 7	8 9 10
-68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1 2 3	4 6 7	8 9 10
-69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1 2 3	5 6 7	8 9 10
-70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1 2 4	5 6 7	8 9 11
-71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1 2 4	5 6 7	8 10 11
-72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1 2 4	5 6 7	9 10 11
-73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1 3 4	5 6 8	9 10 11
-74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1 3 4	5 6 8	9 10 12
-75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1 3 4	5 7 8	9 10 12
-76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1 3 4	5 7 8	9 11 12
-77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1 3 4	5 7 8	10 11 12
-78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1 3 4	6 7 8	10 11 13
-79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1 3 4	6 7 9	10 11 13
-80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1 3 4	6 7 9	10 12 13
-81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2 3 5	6 8 9	11 12 14
-82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2 3 5	6 8 9	11 12 14
-83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2 3 5	6 8 9	11 13 14
-84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2 3 5	6 8 10	11 13 15
-85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2 3 5	7 8 10	12 13 15
-86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2 3 5	7 8 10	12 13 15
-87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2 3 5	7 9 10	12 14 16
-88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2 4 5	7 9 11	12 14 16
-89													

Chart 5.2: The summary table

Index	Logarithm
$x = a^y$	$\log_a x = y$
$100 = 10^2$	$\log_{10} 100 = 2$
$64 = 4^3$	$\log_4 64 = 3$
$0.001 = 10^{-3}$	$\log_{10} 0.001 = -3$

Chart 5.3: Pair exercise

Convert numbers in index form to logarithmic form and vice-versa.

No.	Index Form	Logarithmic Form
	$10^4 = 10,000$	
		$\log_p M = k$
	$2^x = y$	
		$\log_3(xy) = 2$
	$7^2 = 49$	

Session 6 Model Teaching (continuation) 150 minutes

Session objectives

By the end of the session, participants will be able to:

- Practice identifying 2 stars and 1 wish from the model lessons of others
- Become familiar with the LPMs

Session outline

Session	Introduction	3 minutes
Activity	Model lessons / feedback	115 minutes
Session	Review	2 minutes

Session introduction 3 minutes

- Remind participants that they will continue the model teaching from yesterday.

Activity 1: Model lessons 115 minutes

- Call participants up one at a time (in sequential order with regards to the LPM) to deliver their lesson. Each participant should teach only for ~5-10 minutes (depending on the number of participants in the group).
- The participants who are not teaching at the time should take observation notes in their notebooks and actively participate as “students” in the lesson.
- Ask some teachers to start at the very beginning of the lesson (Opening) and ask others to start in teaching and learning, practice and closing. This will enable the group to see different parts of the lesson structure.
- After each participant is done delivering his/her lesson, de-brief as a group / in pairs to come to an agreement on the 2 stars and 1 wish.

- At least a few times, ensure that participants are being specific enough in giving examples of the 2 stars and actionable enough when it comes to the 1 wish.

Session review

5 minutes

- Thank participants for preparing well for their model lessons
- Ask for general reflections on how it felt to deliver the lessons as well as do the observations and any “key takeaways” from this activity
- Review the session objectives
- Thank participants for their active learning over the last two days. Remind them that this is just the start and they will receive more support in school from SSOs.

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