Free Quality School Education

Ministry of Basic and Senior Secondary Education

Pupils' Handbook for

Senior Secondary *Mathematics*

SSS II

Term III

Foreword

These Lesson Plans and the accompanying Pupils' Handbooks are essential educational resources for the promotion of quality education in senior secondary schools in Sierra Leone. As Minister of Basic and Senior Secondary Education, I am pleased with the professional competencies demonstrated by the writers of these educational materials in English Language and Mathematics.

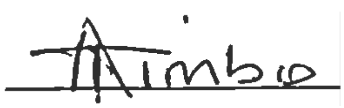
The Lesson Plans give teachers the support they need to cover each element of the national curriculum, as well as prepare pupils for the West African Examinations Council's (WAEC) examinations. The practice activities in the Pupils' Handbooks are designed to support self-study by pupils, and to give them additional opportunities to learn independently. In total, we have produced 516 lesson plans and 516 practice activities – one for each lesson, in each term, in each year, for each class. The production of these materials in a matter of months is a remarkable achievement.

These plans have been written by experienced Sierra Leoneans together with international educators. They have been reviewed by officials of my Ministry to ensure that they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognized techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make the best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new lesson plans. It is really important that the Lesson Plans and Pupils' Handbooks are used, together with any other materials they may have.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank our partners for their continued support. Finally, I also thank the teachers of our country for their hard work in securing our future.



Mr. Alpha Osman Timbo

Minister of Basic and Senior Secondary Education

The policy of the Ministry of Basic and Senior Secondary Education, Sierra Leone, on textbooks stipulates that every printed book should have a lifespan of three years.

To achieve thus, DO NOT WRITE IN THE BOOKS.

Table of Contents

Introduction

to the Pupils' Handbook

These practice activities are aligned to the Lesson Plans, and are based on the National Curriculum and the West Africa Examination Council syllabus guidelines. They meet the requirements established by the Ministry of Basic and Senior Secondary Education.

1. The practice activities will not take the whole term, so use any extra time to revise material or re-do activities where you made mistakes.
2. Use other textbooks or resources to help you learn better and practise what you have learned in the lessons.
3. Read the questions carefully before answering them. After completing the practice activities, check your answers using the answer key at the end of the book.
4. Make sure you understand the learning outcomes for the practice activities and check to see that you have achieved them. Each lesson plan shows these using the symbol to the right.

   Learning Outcomes

5. Organise yourself so that you have enough time to complete all of the practice activities. If there is time, quickly revise what you learned in the lesson before starting the practice activities. If it is taking you too long to complete the activities, you may need more practice on that particular topic.
6. Seek help from your teacher or your peers if you are having trouble completing the practice activities independently.
7. Make sure you write the answers in your exercise book in a clear and systematic way so that your teacher can check your work and you can refer back to it when you prepare for examinations.
8. Congratulate yourself when you get questions right! Do not worry if you do not get the right answer – ask for help and continue practising!

KEY TAKEAWAYS FROM SIERRA LEONE'S PERFORMANCE IN WEST AFRICAN SENIOR SCHOOL CERTIFICATE EXAMINATION – GENERAL MATHEMATICS[1]

This section, seeks to outline key takeaways from assessing Sierra Leonean pupils' responses on the West African Senior School Certificate Examination. The common errors pupils make are highlighted below with the intention of giving teachers an insight into areas to focus on, to improve pupil performance on the examination. Suggestions are provided for addressing these issues.

Common errors

1. Errors in applying principles of BODMAS
2. Mistakes in simplifying fractions
3. Errors in application of Maths learned in class to real-life situations, and vis-a-versa.
4. Errors in solving geometric constructions.
5. Mistakes in solving problems on circle theorems.
6. Proofs are often left out from solutions, derivations are often missing from quadratic equations.

Suggested solutions

1. Practice answering questions to the detail requested
2. Practice re-reading questions to make sure all the components are answered.
3. If possible, procure as many geometry sets to practice geometry construction.
4. Check that depth and level of the lesson taught is appropriate for the grade level.

[1] This information is derived from an evaluation of WAEC Examiner Reports, as well as input from WAEC examiners and Sierra Leonean teachers.

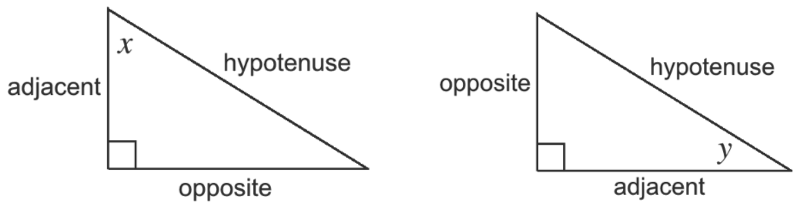
Lesson Title: Review of sine, cosine and tangent	**Theme:** Trigonometry
Practice Activity: PHM2-L097	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to identify the trigonometric ratios (SOHCAHTOA).

Overview

Trigonometric ratios can be applied to the angle and sides of right-angled triangles. We use 3 types of sides (adjacent, opposite and hypotenuse) in trigonometric ratios. The sides "adjacent" and "opposite" are determined by their relationship to the angle in question.

See examples:



The 3 trigonometric ratios in this lesson are:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{O}{H}$$
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{A}{H}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{O}{A}$$

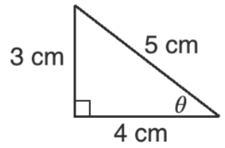
Sin, cos, and tan are the abbreviations that we use for sine, cosine, and tangent. Trigonometric functions are functions of angles. The theta symbol (θ) is shown here, and it is often used to represent angles.

We use the term SOHCAHTOA as a way of remembering the ratios:

a. SOH stands for "sine equals opposite over hypotenuse".
b. CAH stands for "cosine equals adjacent over hypotenuse".
c. TOA stands for "tangent equals opposite over adjacent".

Solved Examples

1. Apply the trigonometric ratios to θ:



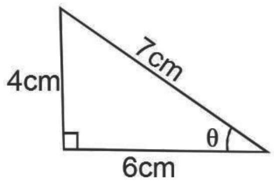
Solution:

$$\sin\theta = \frac{O}{H} = \frac{3}{5}$$

$$\cos\theta = \frac{A}{H} = \frac{4}{5}$$

$$\tan\theta = \frac{O}{A} = \frac{3}{4}$$

2. Apply the trigonometric ratios to angle θ on the triangle below:



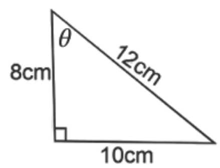
Solution:

$$\sin\theta = \frac{O}{H} = \frac{4}{7}$$

$$\cos\theta = \frac{A}{H} = \frac{6}{7}$$

$$\tan\theta = \frac{O}{A} = \frac{4}{6} = \frac{2}{3}$$

3. Apply the trigonometric ratios to θ:



Solution:

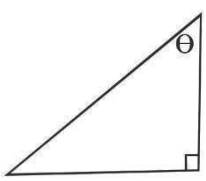
$$\sin\theta = \frac{O}{H} = \frac{10}{12} = \frac{5}{6}$$

$$\cos\theta = \frac{A}{H} = \frac{8}{12} = \frac{2}{3}$$

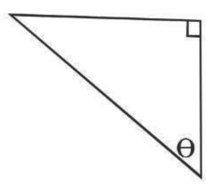
$$\tan\theta = \frac{O}{A} = \frac{10}{8} = \frac{5}{4} = 1\frac{1}{4}$$

4. Identify and label the sides as "opposite", "adjacent" or "hypotenuse" based on their relative position to θ.

a.

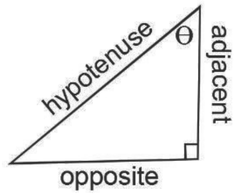


b.

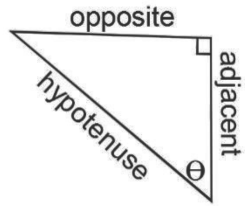


Solutions:

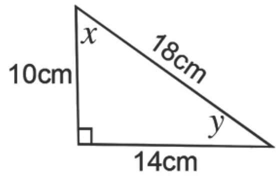
a.



b.



5. For the triangle below, apply the trigonometric ratios to both angles x and y.



Solution:

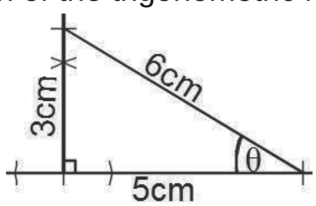
$\sin x = \frac{O}{H} = \frac{14}{18} = \frac{7}{9}$ $\sin y = \frac{O}{H} = \frac{10}{18} = \frac{5}{9}$

$\cos x = \frac{A}{H} = \frac{10}{18} = \frac{5}{9}$ $\cos y = \frac{A}{H} = \frac{14}{18} = \frac{7}{9}$

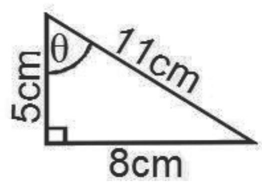
$\tan x = \frac{O}{A} = \frac{14}{10} = \frac{7}{5} = 1\frac{2}{5}$ $\tan y = \frac{O}{A} = \frac{10}{14} = \frac{5}{7}$

Practice

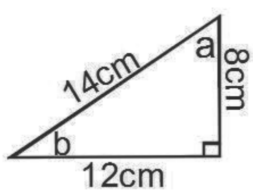
1. For the triangle below, apply each of the trigonometric ratios to angles θ.



2. Apply the trigonometric ratios to angle θ, for the triangle below:



3. For the triangle below, apply the trigonometric ratios to both angles a and b. Simplify your answer.



Lesson Title: Application of sine, cosine and tangent	**Theme:** Trigonometry
Practice Activity: PHM2-L098	**Class:** SSS 2

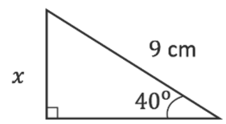
Learning Outcome
By the end of the lesson, you will be able to apply the trigonometric ratios of tangent, sine and cosine to solve right-angled triangles, using trigonometric tables if available.

Overview

This lesson uses trigonometric ratios introduced in the previous lesson to solve for missing sides in a triangle. Trigonometric tables should be used to find the trigonometric function of the given angles to 4 decimal places. If you do not have access to trigonometric tables, a calculator may be used.

Solved Examples

1. Find the measure of missing side x:

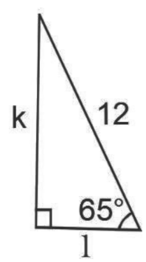


Solution:

Substitute the known angle and side into the formula $\sin\theta = \frac{O}{H}$. Use $\sin 40° = 0.6428$ (from the sine table or a calculator).

$$\begin{aligned} \sin 40° &= \frac{x}{9} \\ 9 \times \sin 40° &= x \\ 9 \times 0.6428 &= x \\ x &= 5.7852 \\ x &= 5.8 \text{ cm to 1 d.p.} \end{aligned}$$

2. Find the lengths of sides k and l in the diagram below. Give your answers to the nearest whole number.



Solution:
Apply the sine ratio to find k:

$$\sin 65^o = \frac{opposite}{hypotenuse}$$

$$\sin 65^o = \frac{k}{12}$$

$$\therefore k = 12 \times \sin 65° \qquad \text{Find } \sin 65° \text{ in the sine table}$$

$$k = 12 \times 0.9063$$

$$k = 10.88 = 11$$

Apply the cosine ratio to find l:

$$\cos 65^o = \frac{adjacent}{hypotenuse}$$

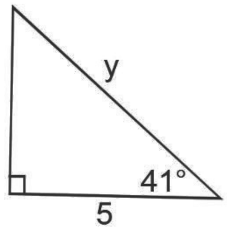
$$\cos 65^o = \frac{l}{12}$$

$$l = 12 \times \cos 65°$$

$$l = 12 \times 0.4226$$

$$l = 5.0712 = 5$$

3. Find the length of the side marked y in the diagram below, correct to 1 decimal place.



Solution:
Apply the cosine ratio:

$$\cos 41^o = \frac{adjacent}{hypotenuse}$$

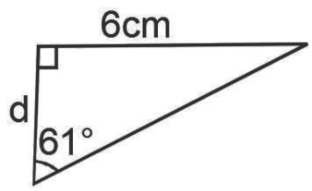
$$\cos 41^o = \frac{5}{y}$$

$$0.7547 \times y = 5$$

$$y = \frac{5}{0.7547}$$

$$y = 6.6$$

4. Find the length of d in the diagram below, correct to the nearest whole number.



Solution:

Apply the tangent ratio:

$$\tan 61^{o} = \frac{opposite}{adjacent}$$

$$\tan 61^{o} = \frac{6}{d}$$

$$d \tan 61° = 6$$

$$d \times 1.804 = 6$$

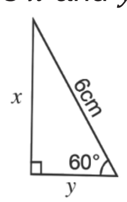
$$d = \frac{8}{1.804}$$

$$d = 4.435$$

$$d = 4 \text{ cm}$$

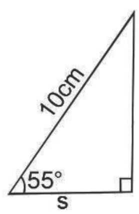
Practice

1. Find the measures of the missing sides x and y:

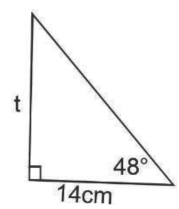


2. Find the measure of the lettered sides in the lettered sides in the diagrams below. Give your answers to 1 decimal place.

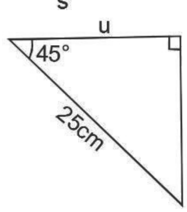
a.



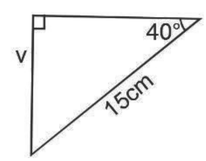
b.



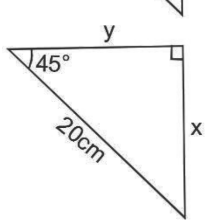
c.



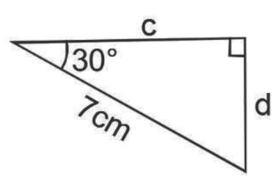
d.



e.



f.



Lesson Title: Special angles (30°, 45°, 60°)	Theme: Trigonometry
Practice Activity: PHM2-L099	**Class:** SSS 2

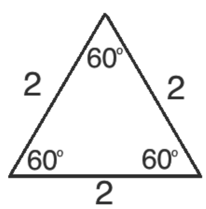
Learning Outcomes
By the end of the lesson, you will be able to:
1. Derive and identify the trigonometric ratios of special angles 30°, 45° and 60°.
2. Identify that $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

Overview

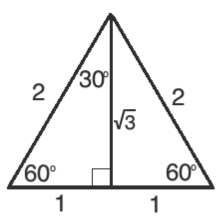
The trigonometric ratios of special angles 30°, 45°, and 60° can be found from equilateral and isosceles triangles.

To find the trigonometric ratios of special angles 30° and 60°:

- Draw an equilateral triangle with sides of length 2 units.



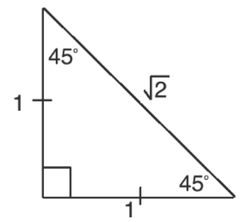
- Bisect one angle so that you have a right-angled triangle with 30° and 60° angles, as shown:



- The angle bisector also bisects the opposite side, so that each segment has length 1.
- The height of the triangle can be found by applying Pythagoras' theorem.
- Use one of the right-angled triangles, and apply the 3 trigonometric ratios to both the 30° angle and the 60° angle.

To find the trigonometric ratios of special angle 45°:

- Draw an isosceles triangle where the 2 sides that form a right angle have length 1.



- The hypotenuse of the triangle can be found by applying Pythagoras' theorem.
- Choose either 45° angle, and apply the 3 trigonometric ratios.

Using the ratios for special angles 30°, 45° and 60°, you can observe that $\tan\theta = \frac{\sin\theta}{\cos\theta}$. This relationship is true for any angle θ.

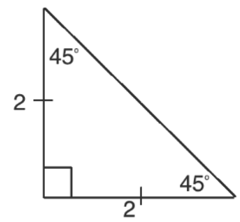
Solved Examples

1. Use the equilateral triangle shown in the Overview to find the trigonometric ratios of 30°.
   Solution:
$$\sin 30° = \frac{O}{H} = \frac{1}{2}$$
$$\cos 30° = \frac{A}{H} = \frac{\sqrt{3}}{2}$$
$$\tan 30° = \frac{O}{A} = \frac{1}{\sqrt{3}}$$

2. Use the equilateral triangle shown in the Overview to find the trigonometric ratios of 60°.
   Solution:
$$\sin 60° = \frac{O}{H} = \frac{\sqrt{3}}{2}$$
$$\cos 60° = \frac{A}{H} = \frac{1}{2}$$
$$\tan 60° = \frac{O}{A} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

3. Use the isosceles triangle shown in the Overview to find the trigonometric ratios of 45°.
   Solution:
$$\sin 45° = \frac{O}{H} = \frac{1}{\sqrt{2}}$$
$$\cos 45° = \frac{A}{H} = \frac{1}{\sqrt{2}}$$
$$\tan 45° = \frac{O}{A} = \frac{1}{1} = 1$$

4. Using the isosceles triangle below, find the trigonometric ratios of 45°.



Solution:

Step 1. Find the length of the hypotenuse:

$$\begin{aligned} 2^2 + 2^2 &= c^2 && \text{Substitute 1 and 1 into the formula} \\ 4 + 4 &= c^2 && \text{Simplify} \\ 8 &= c^2 \\ \sqrt{8} &= \sqrt{c^2} && \text{Take the square root of both sides} \\ 2\sqrt{2} &= c \end{aligned}$$

Step 2. Find the trigonometric ratios of 45 using the lengths of this triangle:

$$\sin 45° = \frac{O}{H} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\cos 45° = \frac{A}{H} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\tan 45° = \frac{O}{A} = \frac{2}{2} = 1$$

Note that these are the same as the ratios found using the triangle in problem 3. We will find the same result for any similar triangle. That is, any isosceles triangle with 45-degree angles.

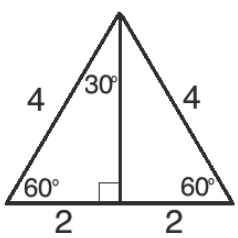
5. Use the formula $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to find $\tan 30°$ using the ratios for $\sin 30°$ and $\cos 30°$.

Solution:

$$\tan 30° = \frac{\sin 30°}{\cos 30°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \div \frac{\sqrt{3}}{2} = \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Practice

1. Using the equilateral triangle shown below, find the trigonometric ratios of 30° and 60°.



2. Use the formula $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to find $\tan 45°$ using the ratios for $\sin 45°$ and $\cos 45°$.
3. If $\sin\theta = \frac{\sqrt{2}}{2}$ and $\cos\theta = -\frac{\sqrt{2}}{2}$, what is $\tan\theta$?
4. If $\sin 0° = 0$ and $\cos 0° = 1$, find $\tan 0°$.

Lesson Title: Applying special angles	Theme: Trigonometry
Practice Activity: PHM2-L100	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to use the special angles 30°, 45°, and 60° to solve problems.

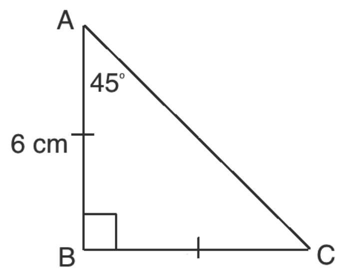
Overview

The trigonometric ratios of special angles 30°, 45°, and 60° can be used to find the measures of the sides of a right-angled triangle.

To solve, use the ratio for the special angles that were found in the previous lesson. Set them equal to ratios you observe in the triangle given in the problem, and solve for the unknown side. Remember that the ratio problem should only have 1 unknown side. You must use the known side, and choose the correct trigonometric ratio.

Solved Examples

1. Find the lengths of sides BC and AC:



Solution:
To find $|BC|$:
Set $\tan 45°$ equal to the ratio from the triangle in the problem, and the ratio $\tan 45° = 1$ we found in the previous lesson:

$$\tan 45° = \frac{|BC|}{6} = 1$$

Multiply both sides by 6 to solve for $|BC|$:

$$|BC| = 6$$

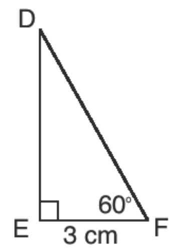
To find $|AC|$:
Set $\cos 45°$ equal to the ratio from the triangle in the problem, and the ratio of $\cos 45°$ we found in the previous lesson:

$$\cos 45° = \frac{6}{|AC|} = \frac{1}{\sqrt{2}}$$

Cross-multiply to solve for $|AC|$:

$$|AC| = 6\sqrt{2}$$

2. Find the measures of DE and DF:



Solution:

Find $|DE|$:

$$\tan 60° = \frac{|DE|}{3} = \sqrt{3}$$

$$|DE| = 3\sqrt{3}$$

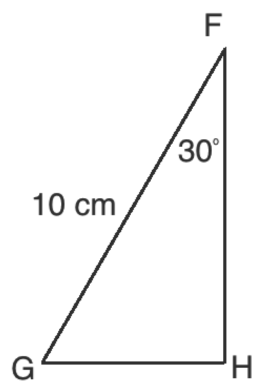
Find $|DF|$:

$$\cos 60° = \frac{3}{|DF|} = \frac{1}{2}$$

$$|DF| = 3 \times 2$$

$$|DF| = 6$$

3. Find the measures of FH and GH:



Solution:

Find $|FH|$:

$$\cos 30° = \frac{|FH|}{10} = \frac{\sqrt{3}}{2}$$

$$2|FH| = 10\sqrt{3}$$

$$|FH| = \frac{10\sqrt{3}}{2}$$

$$|FH| = 5\sqrt{3}$$

Find $|GH|$:

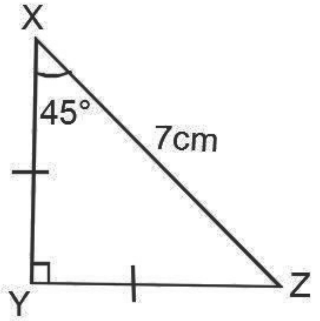
$$\sin 30° = \frac{|GH|}{10} = \frac{1}{2}$$

$$|GH| = \frac{10}{2}$$

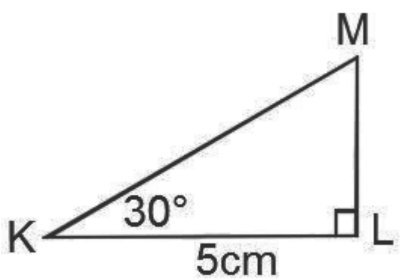
$$|GH| = 5$$

Practice

1. Find |XY| and |YZ|

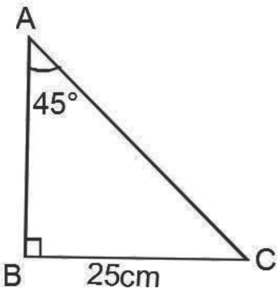


2. Find $|KM|$ and $|LM|$

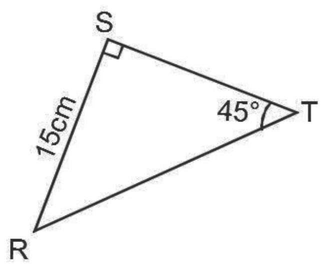


3. Find the length of the missing sides of the following triangles.

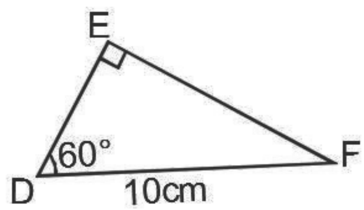
a.



b.

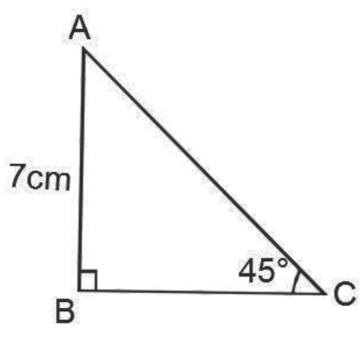


c.

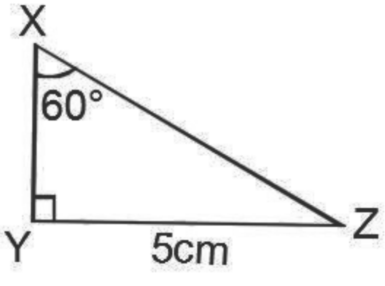


4. Find the lengths of the unknown sides.

a.



b.



Lesson Title: Inverse trigonometry	**Theme:** Trigonometry
Practice Activity: PHM2-L101	**Class:** SSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify that inverse trigonometric functions "undo" the corresponding trigonometric functions.
2. Apply inverse trigonometric functions to find unknown angles.

Overview

The inverse of a function is its opposite. It's another function that can undo the given function. Inverse functions are shown with a power of -1. For example, inverse sine is $\sin^{-1} x$. Inverse sine "undoes" sine: $\sin^{-1}(\sin\theta) = \theta$.

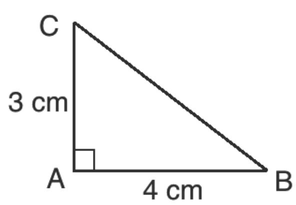
The other inverse trigonometric functions are $\cos^{-1} x$ and $\tan^{-1} x$. The inverse functions are also sometimes called "arcsine", "arccosine", and "arctangent".

You can use inverse trigonometric functions to find the degree measure of an angle. You can use the trigonometric tables ("log books") or calculators. Using trigonometric tables, you will work backwards. Find the decimal number in the chart, and identify the angle that it corresponds to.

At times, you can solve inverse trigonometry problems without a trigonometry table or calculator. This is generally only true for special angles. For example, if you see $x = \sin^{-1}(\frac{1}{2})$, you may identify that $\frac{1}{2}$ is the sine ratio of the special angle 30°. Therefore, the answer would be $x = 30°$.

Solved Examples

1. Find the measure of angle B:



Solution:

Step 1. Identify which function to use. The opposite and adjacent sides are known, so we will use $\tan^{-1}$.

Step 2. Find the tangent ratio. This is the ratio that you will "undo" with $\tan^{-1}$ to find the angle:

$$\tan B = \frac{3}{4} = 0.75$$

Step 3. Find $\tan^{-1}$ of both sides to find the angle measure:

$$\begin{aligned} \tan B &= 0.75 \\ \tan^{-1}(\tan B) &= \tan^{-1}(0.75) \\ B &= \tan^{-1}(0.75) \end{aligned}$$

Calculate $\tan^{-1}(0.75)$ using either a calculator or trigonometry table.

To calculate $\tan^{-1}(0.75)$ using the tangent table: Look for 0.75 in the table. It is not there, but 0.7481 is there. If we add 0.0018 to 0.7481, it will give us 0.75. Find 18 in the "add differences" table, and it corresponds to 7. Therefore, the angle is 36.87.

Answer: $B = 36.87°$

2. Find the following using trigonometry tables ("log books"):
    a. $\sin^{-1}(0.5015)$
    b. $\cos^{-1}(0.7891)$

    Solutions:

    a. Solution using a **sine table**:
    - Find 0.5015 in the trigonometric table for sine.
    - It is in row 31, under the first column (.0). This means that the angle has measure 31.0°.

    b. Solution using a **cosine table**:
    - Look for 0.7891 in the cosine table. It is in row 37, under the column for .9.
    - This gives us the angle 37.9°.

3. Find x if $x = \sin^{-1}(\frac{\sqrt{3}}{2})$.

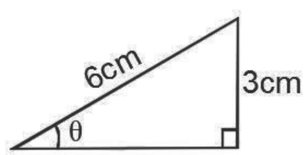
    Solution:

    Since the inverse sine is the opposite of sine, we can take the sine of both sides to eliminate it.

    $$\begin{aligned} \sin x &= \sin(\sin^{-1}(\frac{\sqrt{3}}{2})) \\ \sin x &= \frac{\sqrt{3}}{2} \end{aligned}$$

    We know from the lesson on special angles that $\sin 60° = \frac{\sqrt{3}}{2}$. Therefore, the answer is $x = 60°$.

4. Use the diagram below to calculate the value of θ.



    Solution:

    Apply the sine ratio, then take the inverse sine of both sides.

$$\begin{aligned} \sin\theta &= \frac{O}{H} = \frac{3}{6} = \frac{1}{2} \\ \sin^{-1}(\sin\theta) &= \sin^{-1}(0.5) \\ \theta &= 30° \end{aligned}$$ Sine ratio

We know from the lesson on special angles that $\sin 30° = \frac{1}{2}$. Therefore, the inverse sine of $\frac{1}{2}$ is $30°$.

5. Find x if $x = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

   Solution:

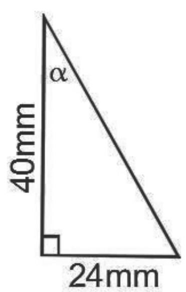
   Since the inverse cosine is the opposite of cosine, we can take the cosine of both sides to eliminate it.

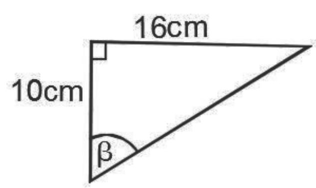
$$\begin{aligned} \cos x &= \cos\left(cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) \\ \cos x &= \frac{1}{\sqrt{2}} \end{aligned}$$

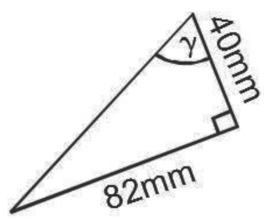
   We know from the lesson on special angles that $\cos 45° = \frac{1}{\sqrt{2}}$. Therefore, the answer is $x = 45°$.

Practice

1. Use trigonometry tables to find the angles whose tangents are:
   a. 0.4452 b. 3.2709 c. 0.0768 d. 0.3977
2. Use trigonometry tables to find the angles whose sines are:
   a. 0.3420 b. 0.8746 c. 0.9344 d. 0.6250
3. Use trigonometry tables to find the angles whose cosines are:
   a. 0.3582 b. 0.9265 c. 0.0163
4. Calculate the sizes of the angles marked α, β and γ to the nearest degree.

a.


b.


c.


5. Find:
   a. x if $x = \sin^{-1}\left(\frac{1}{2}\right)$
   b. x if $x = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$
   c. x if $x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Lesson Title: Trigonometry and Pythagoras' Theorem	**Theme:** Trigonometry
Practice Activity: PHM2-L102	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve right-angled triangles using trigonometric ratios and Pythagoras' Theorem.

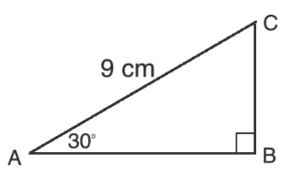
Overview

To "solve" a triangle means to find any missing side or angle measures. You are familiar with several methods for solving triangles, including: trigonometric and inverse trigonometric functions, Pythagoras' theorem, and finding angle measures by subtracting from 180°.

When you have a triangle with missing sides and angles, you need to decide how to solve for them. In some cases, you could solve a problem using different methods. For example, in some cases the side of a right-angled triangle could be solved with Pythagoras' theorem or trigonometry. Choose the method you prefer, or the one that is best for the given problem.

Solved Examples

1. Find the missing sides and angles:



Solution:

Calculate $|AB|$:

$\cos 30^{o} = \frac{\lvert AB\rvert}{8}$	Apply the cosine ratio
$8 \times \cos 30^{o} = \lvert AB\rvert$	Multiply throughout by 8
$8 \times \frac{\sqrt{3}}{2} = \lvert AB\rvert$	Use the special angle ratio
$\lvert AB\rvert = 4\sqrt{3}$ cm	

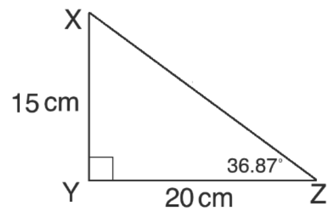
Calculate $|BC|$:

$\sin 30^{o} = \frac{\lvert BC\rvert}{8}$	Apply the sine ratio
$8 \times \sin 30^{o} = \lvert BC\rvert$	Multiply throughout by 8
$8 \times \frac{1}{2} = \lvert BC\rvert$	Use the special angle ratio

$|BC| = 4$ cm

Calculate $\angle C$: $180° - 90° - 30° = 60°$

2. Find the missing sides and angles:



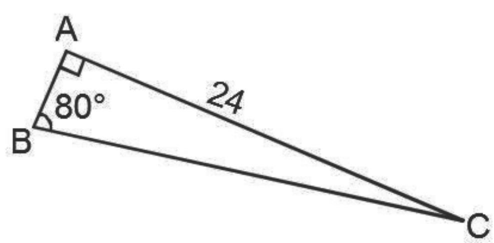
Solutions:

Calculate $|XZ|$:

$15^2 + 20^2$	$=$	$|XZ|^2$	Substitute the sides into the formula
$225 + 400$	$=$	$|XZ|^2$	Simplify
625	$=$	$|XZ|^2$	
$\sqrt{625}$	$=$	$\sqrt{|XZ|^2}$	Take the square root of both sides
25 cm	$=$	$|XZ|$	

Calculate $\angle X$: $180° - 90° - 36.87° = 53.13°$

3. Calculate the missing sides and angle. Give your answers to 1 decimal place.



Solutions:

Calculate |AB|

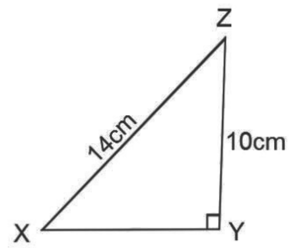
$\tan 80°$	$=$	$\frac{24}{\lvert AB \rvert}$	Apply the tangent ratio
$\lvert AB \rvert$	$=$	$\frac{24}{\tan 80°}$	Change the subject
	$=$	$\frac{24}{5.671}$	
	$=$	4.2	

Calculate |BC|

$\lvert BC \rvert^2$	$=$	$\lvert AB \rvert^2 + \lvert AC \rvert^2$
$\lvert BC \rvert^2$	$=$	$(4.2)^2 + (24)^2$
$\lvert BC \rvert^2$	$=$	$17.64 + 576$
$\lvert BC \rvert^2$	$=$	593.64
$\sqrt{\lvert BC \rvert^2}$	$=$	$\sqrt{593.64}$
$\lvert BC \rvert$	$=$	24.4 cm

Calculate $\angle C$: $180° - 90° - 80° = 10°$

4. Find the missing side and both angles of the triangle. Give your answers to 1 decimal place.



Solution:

First find angle X:

$$\begin{aligned} \sin X &= \tfrac{10}{14} = 0.7143 \\ \sin^{-1}(\sin x) &= \sin^{-1}(0.7143) \quad \text{Use the sine table} \\ x &= 45.6° \end{aligned}$$

Calculate $\angle Z$: $180° - 90° - 45.6° = 44.4°$

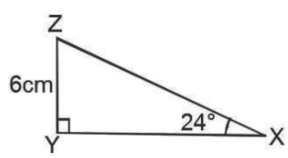
Calculate $|XY|$

$$\begin{aligned} |XY|^2 + 10^2 &= 14^2 \\ |XY|^2 + 100 &= 196 \\ |XY|^2 &= 196 - 100 \\ |XY| &= \sqrt{96} = 9.8 \text{ cm} \end{aligned}$$

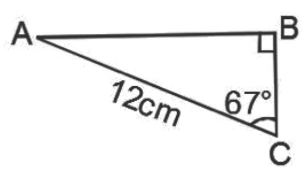
Practice

1. Find the unknown sides in the triangles below. Give your answers to 1 decimal place.

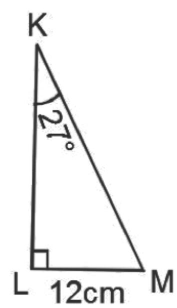
a.



b.

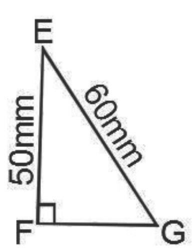


c.

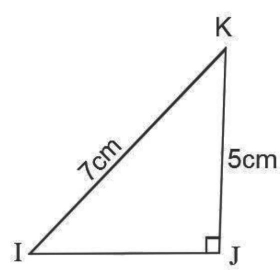


2. Find the missing sides and angles of the following triangles. Give your answers to 1 decimal place.

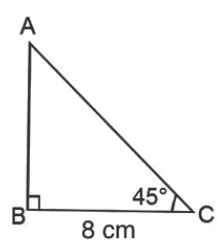
a.



b.



c.



Lesson Title: Angles of elevation	Theme: Trigonometry
Practice Activity: PHM2-L103	**Class:** SSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Calculate angles of elevation.
2. Calculate height and distance associated with an angle of elevation.

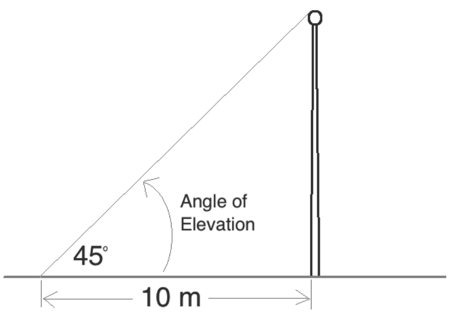
Overview

Elevation is related to height. Problems on angles of elevation handle the angle that is associated with the height of an object. An angle of elevation is measured a certain distance away from an object.

Angle of elevation problems generally deal with 3 measures: the angle, the distance from the object, and the height of the object. You may be asked to solve for any of these measures. These can be solved for using trigonometry to find distances, and inverse trigonometry to find angles.

Solved Examples

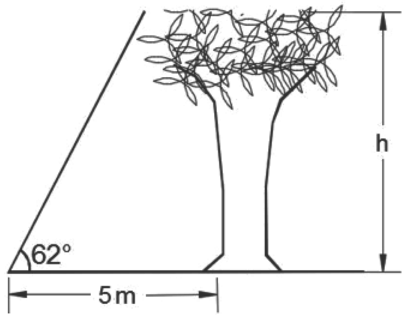
1. At a point 10 metres away from a flag pole, the angle of elevation of the top of the pole is 45°. What is the height of the pole?
   Solution:
   First, draw a diagram:



   Solve using the tangent ratio, because we are concerned with the sides opposite and adjacent to the angle.

$\tan 45°$	$=$	$\frac{h}{10}$	Set up the equation
1	$=$	$\frac{h}{10}$	Substitute $\tan 45° = 1$
10 m	$=$	h	

2. From a point 5 m away from a tree, the angle of elevation of the top of the tree was $62°$. How tall was the tree? Give your answer to 1 decimal place.
   Solution:



$\tan 62° = \frac{h}{5}$ — Apply the tangent rule

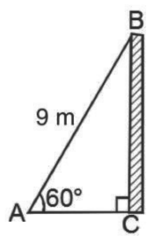
$5 \tan 62° = h$ — Multiply throughout by 5

$5(1.881) = h$

$9.405 = h$

$h = 9.4$ m

3. A ladder 9m long rests against a vertical wall. If the ladder makes an angle of $60°$ with the ground, find the distance between the foot of the ladder and the wall.
   Solution:
   First, draw a diagram:



$\cos 60° = \frac{\text{AC}}{\text{AB}} = \frac{\text{AC}}{9\text{ m}}$ — Apply the cosine rule

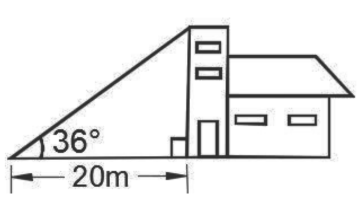
$9 \cos 60° = \text{AC}$ — Multiply throughout by 90

$9 \times 0.5 = \text{AC}$

$4.5\text{ m} = \text{AC}$

The foot of the ladder is 4.5 metres from the wall.

4. A ladder learning on the roof of a house vertically built forms an angle of $36°$ with the ground. If the foot of the ladder is 20 m from the wall, what is the height of the house? Give your answer to 2 decimal places.
   Solution:
   First, draw a picture:



$$\tan 36° = \frac{x}{20\text{ m}}$$ Set up the equation

$$0.7265 = \frac{x}{20\text{ m}}$$

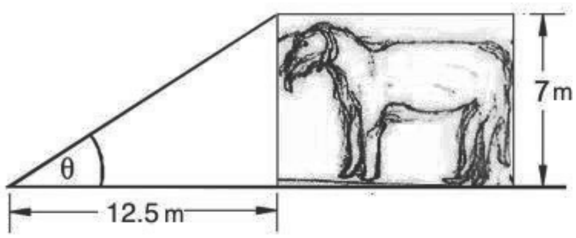
$$20 \times 0.7265 = x$$

$$14.53 = x$$

The house is 14.53 metres tall.

5. There is a giant dog with a height of 7 metres. What is the angle of elevation from a point 12.5 metres away from a dog, to the top of his head? Give your answer to the nearest degree.

Solution:



$$\tan\theta = \frac{7}{12.5}$$

$$\tan\theta = 0.56$$

$$\theta = \tan^{-1}(0.56)$$

$$\theta = 29.25$$

$\theta = 29°$ to the nearest degree

Practice

Give your answers to 1 decimal place.

1. A communication pole is 6 m tall. At a distance F metres away from the pole, the angle of elevation is $63°$. Find F.
2. A ladder learning against a vertical wall makes an angle of $24°$ with the wall. The foot of the ladder is 5 m from the wall. Find the length of the ladder.
3. A vertical stick is 8 m high, and the length of its shadow is 6 m. What is the angle of elevation of the sun.
4. The shadow of a vertical pole 15 m high is 20 m long. What is the angle of elevation of the sun?
5. A ladder leaning against a vertical wall makes an angle of $21°$ with the wall. The foot of the ladder is 5 m from the wall. How high up the wall does the ladder reach?
6. What is the angle of elevation of the top of a spire 240 m high from a point on the ground 200 m from the foot of it?

Lesson Title: Angles of depression	**Theme:** Trigonometry
Practice Activity: PHM2-L104	**Class:** SSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Calculate angles of depression.
2. Calculate depth and distance associated with an angle of depression.

Overview

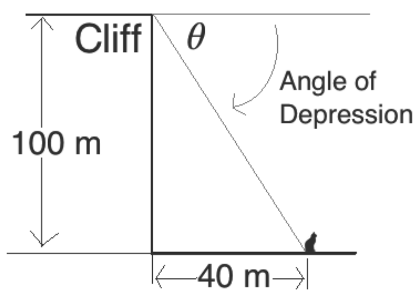
"Depression" is the opposite of elevation. "Depressed" means downward. So if there is an angle of depression, it is an angle in the downward direction.

The angle of depression is the angle made with the **horizontal** line. See the diagram in Solved Example 1. The horizontal line is at the height of the cliff.

Angle of depression problems generally deal with 3 measures: the angle, the horizontal distance, and the depth of the object. Depth is the opposite of height. It is the distance downward. You may be asked to solve for any of these 3 measures.

Solved Examples

1. A cliff is 100 metres tall. At a distance of 40 metres from the base of the cliff, there is a cat sitting on the ground. What is the angle of depression of the cat from the cliff?
   Solution:
   First, draw a diagram:

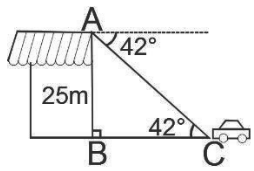


   Solve using the tangent ratio, because we are concerned with the sides opposite and adjacent to the angle.

$\tan\theta$	$=$	$\frac{100}{40} = 2.5$	Set up the equation
$\tan^{-1}(\tan\theta)$	$=$	$\tan^{-1}(2.5)$	Take the inverse tangent
θ	$=$	68.2	Use the tangent tables

2. From the top of a building with height 25 m, the angle of depression of a taxi is $42°$. Find the distance between the car and the top of the building.

Solution:
First, draw a diagram:



The distance required is AC.

$$\sin 42° = \frac{25}{AC} \quad \text{Apply the sine ratio}$$

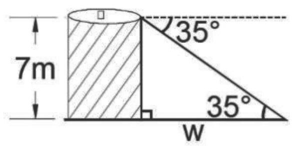
$$AC \sin 42° = 25 \quad \text{Multiply throughout by AC}$$

$$AC = \frac{25}{\sin 42°} \quad \text{Divide throughout by } \sin 42°$$

$$AC = \frac{25}{0.6691}$$

$$AC = 37.4 \text{ m}$$

3. A water tank is 7 metres high. A certain point is w metres away from the tank, at an angle of depression of $35°$ from the top of the tank. Find w to the nearest whole number.
Solution:



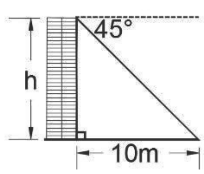
$$\tan 35° = \frac{7}{x} \quad \text{Set up the equation}$$

$$0.7002 = \frac{7}{x} \quad \text{Substitute } \tan 35° = 0.7002$$

$$x = \frac{7}{0.7002} \quad \text{Change subject}$$

$$x = 10 \text{ m}$$

4. A cat is sitting 10 metres from the base of a pillar wall. The angle of depression of the cat from the top of the pillar wall is $45°$. What is the height of the pillar wall?
Solution:



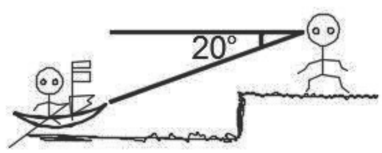
$$\tan 45° = \frac{h}{10}$$

$$1 = \frac{h}{10} \quad \text{Substitute } \tan 45° = 1$$

$$10 = h \quad \text{Multiply throughout by } 10$$

The height of the pillar wall is 10 metres.

5. Amara is standing on the top of a cliff. He has to lower his eyes through an angle of $20°$ in order to look at a fishing boat. His eyes are 1.5 metres from the ground. If the boat is 80 metres from the cliff, find the height of the cliff.
   Solution:
   First, draw a diagram:



Find the height of Amara's eyes from the fishing boat using the tangent ratio:

$$\tan 20° = \frac{h}{80}$$

$$0.3640 = \frac{h}{80}$$ Substitute $\tan 20° = 0.3639$

$$0.3640 \times 80 = h$$ Multiply throughout by 80

$$29.12 = h$$

$$\mathrm{h} = 29.1 \text{ m}$$

Subtract Amara's height from the total height to find the height of the cliff:
Cliff height $= 29.1 - 1.5 = 27.6$ metres

Practice

1. A boy is standing on top of a cliff. His eyes are 21 metres above the sea. He sees a boat out to sea. The boat is at sea level and the angle of depression of the boat from the boy's eye is $4°$. How far is the boat from the bottom of the cliff? Give your answer to the nearest metre.
2. The angle of the depression from the top of a building of height 30 m of a stationary vehicle is $41°$. Find the distance between the car and the top of the building, correct to 1 decimal place.
3. A man is standing on the top of a cliff. His eye is 100 metres above the water, and he observes that the angle of depression of a boat at sea is $18°$. How far is the boat from the cliff? Give your answer to the nearest metre.
4. A point k is on the same horizontal level as the base of a flagpole. If the distance from k to the pole is 16 metres and the height of the pole is 27 metres, calculate the angle of depression of k from the top of the pole. Give your answer to the nearest degree.
5. Boima throws a stone off the top of a tower, from a height of 6 metres. The stone landed on the ground. The angle of depression of the stone from the top of the tower is $26.10°$. How far is the stone from the tower? Give your answer to the nearest metre.

Lesson Title: Applications of angles of elevation and depression – Part 1	**Theme:** Trigonometry
Practice Activity: PHM2-L105	**Class:** SSS 2

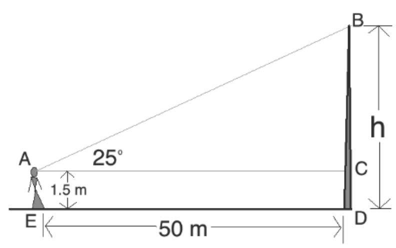
Learning Outcome
By the end of the lesson, you will be able to solve practical problems related to angles of elevation and depression.

Overview

This lesson is on practical problems related to angles of elevation and depression. You already have all of the information you need to solve these problems. However, they may have extra steps.

Solved Examples

1. A woman standing 50 metres from a flag pole observes that the angle of elevation of the top of the pole is 25°. Assuming her eye is 1.5 metres above the ground, calculate the height of the pole to the nearest metre.
   Solution:
   First, draw a diagram:



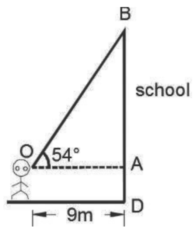
To find the height of the flag pole, we must find the length of BC, then add it to the height of the woman's eye (CD or AE), which is 1.5 metres.

Step 1. Find $|BC|$:

$\tan 25°$	$=$	$\frac{BC}{50}$	Set up the equation
0.4663	$=$	$\frac{BC}{50}$	Substitute $\tan 25° = 0.4663$ (from table)
50×0.4663	$=$	BC	Multiply throughout by 50
BC	$=$	23.315	metres

Step 2. Add: $h = BC + CD = 23.315 + 1.5 = 24.815$
Rounded to the nearest metre, the height of the pole is 25 m.

2. George wants to measure the height of his school building. He is standing about 9 metres from the building and observes that the angle of elevation of the top of the building is $54°$. Assuming his eye is 1.57 metres above the ground, calculate the height of the building.
   Solution:
   First, draw a diagram:



Step 1. Find $|AB|$:

$$\tan 54° = \frac{\text{AB}}{\text{OA}} \quad \text{Set up the equation}$$

$$\tan 54° = \frac{\text{AB}}{\text{9m}}$$

$$1.376 = \frac{\text{AB}}{9} \quad \text{Substitute } \tan 54° = 1.376 \text{ (from table)}$$

$$9 \times 1.376 = \text{AB} \quad \text{Multiply throughout by } 9$$

$$\text{AB} = 12.4 \text{ metres}$$

Step 2. Add the height of his eye:

$h = \text{DB} = \text{DA} + \text{AB}$

$= \text{DB} = 1.57 + 12.4$

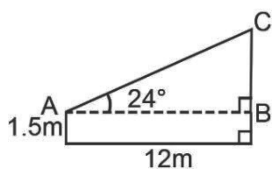
$= 13.97$ metres

Therefore, George concluded that the school building was about 14 m high.

3. A man walks 12m directly away from a tree and from this position, the angle of elevation of the top of the tree is $24°$. If the measurement is taken from a point 1.5 metres above ground level, find the height of the tree. Give your answer correct to 3 significant figures.

   Solution:

   First, draw a diagram:



Step 1. Find $|BC|$. From the figure,

$$\tan 24° = \frac{\text{BC}}{12} \quad \text{Set up the equation}$$

$$0.4452 = \frac{\text{BC}}{12} \quad \text{Substitute } \tan 24° = 0.4452 \text{ (from table)}$$

$$12 \times 0.4452 = \text{BC} \quad \text{Multiply throughout by } 12$$

$$\text{BC} = 5.34 \text{ m} \quad \text{(to 3 s.f)}$$

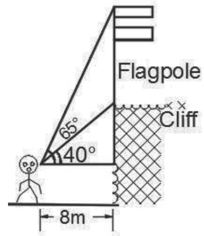
Step 2. Add: $h = 5.34\text{m} + 1.5\text{m}$

$= 6.84$ m (to 3 s.f)

4. A military post guard is standing 8 metres from a cliff, which has a pole mounted on top. He is looking at the country's national flag on top of the cliff. He observed that the angle of elevation of the top and bottom of the pole are $40°$ and $65°$ respectively. Find the height of the flag pole.

Solution:

First, draw a diagram:



Step 1. Find the height of the portion with an angle of elevation of $65°$. Call it x

$\tan 65°$	$=$	$\frac{x}{8}$	Set up the equation
2.145	$=$	$\frac{x}{8}$	Substitute $\tan 65° = 2.145$ (from table)
2.145×8	$=$	x	Multiply throughout by 8
x	$=$	17.16 m	

Step 2. Find the height of the portion with an angle of elevation of $40°$, call it y.

$\tan 40°$	$=$	$\frac{y}{8}$	Set up the equation
0.8391	$=$	$\frac{y}{8}$	Substitute $\tan 40° = 0.8391$ (from table)
0.8391×8	$=$	y	Multiply throughout by 8
y	$=$	6.71 m	

Step 3. Subtract: $h = x - y = 17.16 - 6.71 = 10.45$ m

Answer: The flagpole is 10.5 metres tall.

Practice

1. A man stands at a distance of 16.6 metres away from the base of a tower. He discovers that the angle of elevation of the top of the tower is $64°$. If his eyes are 2.81 metres from the ground, calculate the height of the tower. Give your answer correct to 3 significant figures.
2. A girl looks through a window of a building and sees an orange fruit on the ground 50 metres away from the foot of the building. If the window is 9 metres from the ground, calculate, correct to the nearest degree, the angle of depression of the orange from the window.
3. A flagpole is flying on the corner of a roof of a tall building. Tomagbandi is standing on the same level as the bottom of the building, 10 metres away. Tomagbandi measures the angle of elevation of the top of the building, and finds that it is $69°$. He then measures the angle of elevation of the top of the flagpole and finds that it is $73°$. Calculate the height of the flagpole.
4. A hawk on top of a tree, 20 metres high, views a chick on the ground at an angle of depression of $39°$. Find correct to two significant figures the distance of the chick from the bottom of the tree.

Lesson Title: Applications of angles of elevation and depression – Part 2	Theme: Trigonometry
Practice Activity: PHM2-L106	**Class:** SSS 2

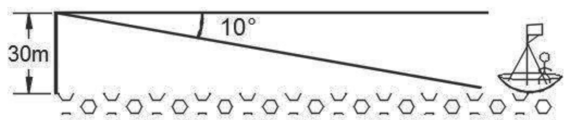
Learning Outcome
By the end of the lesson, you will be able to solve practical problems related to angles of elevation and depression.

Overview

This lesson is on practical problems related to angles of elevation and depression. You already have all of the information you need to solve these problems. However, they may have extra steps.

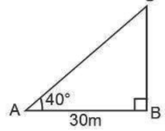
Solved Examples

1. From the top of a light-tower 30 metres above sea level, a boat is observed at an angle of depression of $10°$. Calculate the distance of the boat from the foot of the light-tower, correct to 2 significant figures.
   Solution:
   First, draw a diagram:



$\tan 10°$	$=$	$\frac{30}{d}$	Set up the equation
0.1763	$=$	$\frac{30}{d}$	Substitute $\tan 10° = 0.1763$ (from table)
$0.1763 \times d$	$=$	30	Multiply throughout by d
d	$=$	$\frac{30}{0.1763}$	
d	$=$	170 metres	

2. The angle of elevation of the top of a tower from a point of the horizontal ground, 30 metres away from the foot of the tower is $40°$. Calculate the height of the tower to 2 significant figures.
   Solution:
   First, draw a diagram:



$$\tan 40° = \frac{BC}{AB}$$

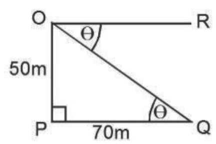
$$\tan 40° = \frac{BC}{30m} \quad \text{Set up the equation}$$

$$0.8391 = \frac{BC}{30m} \quad \text{Substitute } \tan 40° = 0.8391$$

$$0.8391 \times 30 = BC \quad \text{Multiply throughout by } 30$$

$$BC = 25 \text{ m} \quad \text{Simplify}$$

3. A point Q is on the same horizontal level as the foot of tower. If the distance of Q from the foot of the tower is 70 m and the height of the tower is 50 m, find the angle of depression of Q from the top of the tower. Give your answer to the nearest degree.
   Solution:



$$\tan\theta = \frac{OP}{PQ} = \frac{50}{70} \quad \text{Set up the equation}$$

$$\tan\theta = \frac{50}{70}$$

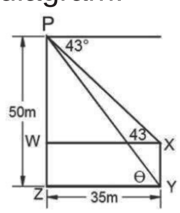
$$\tan\theta = 0.7143$$

$$\theta = \tan^{-1}(0.7143) \quad \text{Read from the tangent table}$$

$$\theta = 36°$$

4. Philip looked out from the window of a building at a height of 50 metres, and observed that the angle of depression of the top of a flagpole was $43°$. If the foot of the pole is 35 m from the foot of the building and on the same horizontal ground, find, correct to the nearest whole number, the:
   a. Angle of depression of the foot of the pole from Philip.
   b. The height of the flag pole.

   Solutions:
   First, draw a diagram:



   a. In Δ PZY of the diagram, θ is equal to the angle of depression of the foot of the pole from Philip.

$$\tan\theta = \frac{50}{35}$$

$$\tan\theta = 1.4286$$

$$\theta = \tan^{-1}(1.4286)$$

$$\theta = 55°$$

Hence, the angle of depression of the foot of the pole from Philip is $55°$.

b. Find the height associated with the angle of $43°$. Subtract this from the height of Philip (50 m) to find the height of the flag pole.

$\tan 43°$	$=$	$\frac{\text{PW}}{\text{WX}}$	Set up the equation
$\tan 43°$	$=$	$\frac{\text{PW}}{35}$	
0.9325	$=$	$\frac{\text{PW}}{35}$	Substitute $\tan 43° = 0.9325$
35×0.9325	$=$	PW	Multiply throughout by 35
PW	$=$	32.64 m	
h	$=$	$50 - \text{PW}$	
h	$=$	$50 - 32.64$	
h	$=$	17.36 m	

Practice

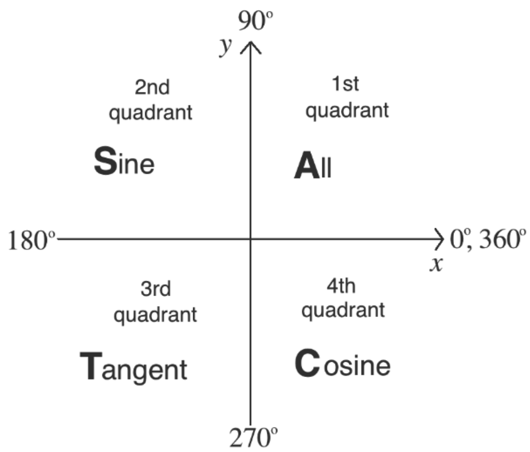
1. The angle of depression of a boat from the mid-point of a vertical cliff is $35°$. If the boat is 120 m from the foot of the cliff, calculate the height of the cliff to the nearest metre.
2. From the top of a cliff, the angle of depression of a boat on the sea is $32°$. If the height of the cliff above sea level is 30 metres, calculate correct to 2 significant figures the horizontal distance of the boat from the bottom of the cliff.
3. Joe wants to calculate the angle of elevation of the top of his father's house. He is standing about 28 metres from the building and the building is 12 metres tall. Assuming that his eye is 1.5 metres above the ground, calculate the angle of elevation of the top of the building from his eye. Give your answer to the nearest degree.
4. A point v is 45 metres away from the foot of a building on the same horizontal ground. From the point v, the angle of elevation to point R on the side of the building is $38°$. Find the height of R from the ground. Give your answer to two decimal places.
5. Mariam looked out from the window of a building at a height of 45 metres, and observed that the angle of the depression of the top of a flag pole was $39°$. If the foot of the pole is 20 metres from the foot of the building and on the same horizontal ground, find correct to the nearest whole number, the:
   a. Angle of depression of the foot of the pole from Mariam, correct to the nearest degree.
   b. The height of the flag pole, correct to 1 decimal place.

Lesson Title: The general angle – Part 1	Theme: Trigonometry
Practice Activity: PHM2-L107	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to extend sine, cosine, and tangent ratios of acute angles to obtuse and reflex angles.

Overview

This lesson is on finding the trigonometric ratios of obtuse and reflex angles. Previous lessons only handled acute angles. Notice that right triangles cannot have obtuse or reflex angles. However, we can still find trigonometric ratios for them.



Angles are centred at the origin of this chart, which is the point where the x- and y-axes cross. Positive angles open in the counter-clockwise direction.

There are 4 quadrants that an angle could lie in. An angle in the first quadrant is acute, and an angle in the second quadrant is obtuse. An angle in the third or fourth quadrant is a reflex angle. The quadrant that an angle lies in tells you whether the result of the trigonometric ratio will be positive or negative.

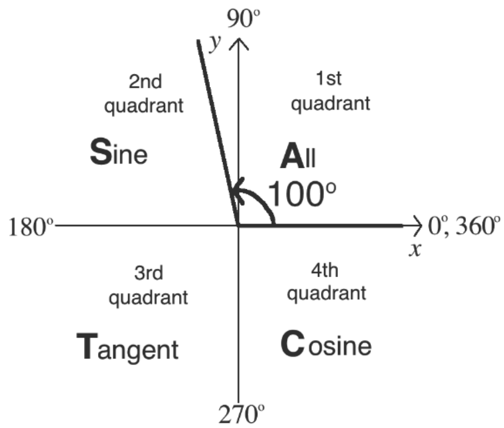
In the diagram above, the words "All, Cosine, Tangent, and Sine" tell you which ratios are **positive** in the given quadrants. We use the word "ACTS" to remember which ratios are positive. The word ACTS starts in the **first** quadrant and goes in a **clockwise** direction.

Remember that A stands for "all", which means that all of the trigonometric ratios are positive in the first quadrant. The letters C, T, and S in other quadrants stand for trigonometric ratios, and tell us that they will be positive. All other ratios will be negative. For example, the second quadrant has an S. This means that sine will be positive, while cosine and tangent will be negative.

Each obtuse or reflex angle has an **associated acute angle.** This is the acute angle that it forms with the x-axis when it is laid on the 4 quadrants in the diagram above.

To find the ratio of an obtuse or reflex angle, find the ratio of the associated acute angle. Then, apply the correct sign for that quadrant.

For example, consider the angle 100°:



It forms an 80° angle with the x-axis in the 2nd quadrant. Therefore, to find a trigonometric ratio of 100°, you would find the ratio of 80° and apply the appropriate sign (positive or negative).

Solved Examples

1. Find $\sin 100°$
   Solution:
   Step 1. Find the sine of the associated acute angle: $\sin 80° = 0.9848$
   Step 2. Keep the positive sign, because sine is positive in the 2nd quadrant: $\sin 100° = 0.9848$

2. Find $\cos 165°$
   Solution:
   Step 1. Find the cosine of the associated acute angle: $\cos 15° = 0.9659$
   Step 2. Change the sign to negative, because cosine is negative in the 2nd quadrant: $\cos 165° = -0.9659$

3. Find $\tan 230°$
   Solution:
   Step 1. Find the tangent of the associated acute angle: $\tan 50° = 1.192$
   Step 2. Keep the positive sign, because tangent is positive in the 3rd quadrant: $\tan 50° = 1.192$

4. Without using tables, find the values of the following in simplified form, using surds where necessary.
   a. $\tan 330°$ b. $\cos 300°$ c. $\sin 315°$
   a. Find $\tan 330°$

Solution:

$330°$ is in the 4th quadrant, therefore its tangent ratio is negative. The associated acute angle with the x-axis is $30°$

$$\tan 330° = -\tan 30 = \frac{-\sqrt{3}}{3}.$$

b. Find $\cos 300°$

Solution:

$300°$ is in the 4th quadrant, there its cosine ratio is positive. The associated acute angle with the x-axis is $60°$.

$$\cos 300° = (+)\cos 60° = \frac{1}{2}.$$

c. Find $\sin 315°$

Solution:

$315°$ is in the 4th quadrant, there its ratio is negative. The associated acute angle with the x-axis is $45°$.

$$\sin 315° = -\sin 45° = \frac{-\sqrt{2}}{2}.$$

Practice

1. Find the following to 4 decimal places using tables:
   a. $\tan 201°$ b. $\cos 112°$ c. $\sin 130°$ d. $\cos 334°$
2. Find the following without using tables. Write them in their simplified form, using surds where necessary:
   a. $\sin 330°$ b. $\sin 135°$ c. $\tan 120°$ d. $\cos 210°$

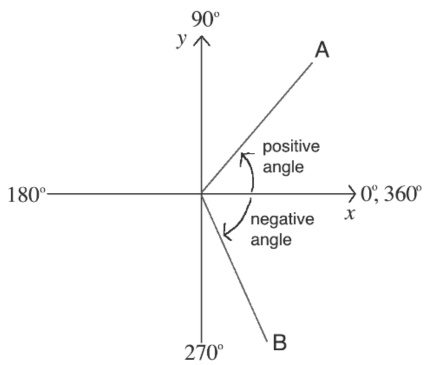
Lesson Title: The general angle – Part 2	**Theme:** Trigonometry
Practice Activity: PHM2-L108	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to express a positive or negative angle of any size in terms of an equivalent positive angle between 0° and 360°, and find the trigonometric ratios.

Overview

This lesson is a continuation of the previous lesson. It covers negative angles, and angles larger than 360°.

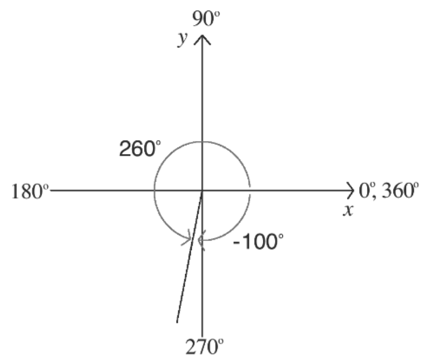
Consider the diagram below:



Angles that open in the **counter-clockwise** direction have **positive** values. These are the angles we have worked with so far. The angle formed by A is positive. Angles that open in the **clockwise** direction have **negative** values. The angle formed by B is negative.

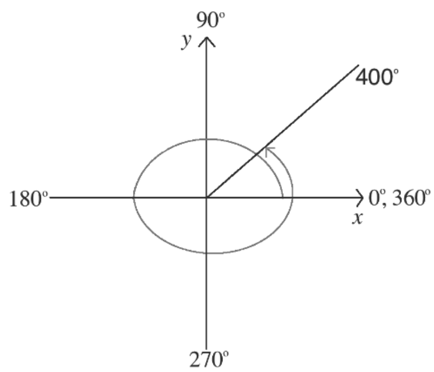
For each negative angle, there is a corresponding positive angle. The positive angle is the remainder of the full revolution (360°) that is not covered by the negative angle. The corresponding positive angle is found by subtracting the absolute value of the negative angle from 360°.

For example, consider $-100°$. The corresponding positive angle is $360° - 100° = 260°$. This is shown below:



The trigonometric ratio of a negative angle is equal to the same trigonometric ratio of the corresponding positive angle. For example, $\sin(-100°) = \sin 260°$.

Consider an angle larger than 360°. For example, 400°. It is more than 1 full rotation. This is shown below:



To find a trigonometric ratio of an angle that is more than 360°, divide by 360° and find the remainder. The remainder will be a number less than 360°. Find the trigonometric ratio of the remainder. For example, to calculate $\cos 400°$ you would calculate $\cos 40°$ because $400° \div 360 = 1 \text{ remainder } 40°$.

Solved Examples

1. Find $\sin(-100°)$
   Solution:
   Step 1. Find the associated positive angle: $360° - 100° = 260°$
   Step 2. Find $\sin 260°$ following the steps from the previous lesson:
   - The corresponding acute angle is 80°.
   - $\sin 80° = 0.9848$
   - Because the angle ($260°$ or $-100°$) lies in the third quadrant, the sine ratio is negative.
   - $\sin 260° = -0.9848$

   The answer is -0.9848, because $\sin(-100°) = \sin 260°$.

2. Find $\cos 400°$

Solution:
Step 1. Divide by 360 and find the remainder: $400° \div 360° = 1 \text{ remainder } 40°$
Step 2. Find the cosine of the remainder: $\cos 40° = 0.7660$
The answer is 0.7660, because $\cos 400° = \cos 40°$.

3. Find $\sin(600°)$
Solution:
Step 1. Divide by $360°$ and find the remainder:
$$600° \div 360° = 1 \text{ remainder } 240°$$
Step 2. Find the sine of the remainder, $\sin 240°$.
This requires finding the corresponding acute angle that $240°$ forms with the x-axis (from the previous lesson)

Corresponding acute angle: $240° - 180° = 60°$

Note that sine is negative in the third quadrant, where $240°$ lies.

Therefore, $\sin(600°) = \sin(240°) = -\sin(60°) = -\frac{\sqrt{3}}{2}$ or -0.8660

4. Find $\cos(-390°)$
Solution:
This is a negative number greater than $360°$, which requires additional steps.

Step 1. Divide by $360°$ and find the remainder:
$$-390° \div 360° = -1 \text{ remainder}(-30°)$$
Step 2. Find the cosine of the remainder, $\cos(-30°)$.
The positive angle that corresponds to $-30°$ is $330°$. Also note that the angle formed by $330°$ and the x-axis is $30°$, and cosine is positive in the 4th quadrant.
Therefore, we have:
$$\cos(-390°) = \cos(-30°) = \cos(330°) = \cos(30°)$$

Using the tables or special angle $30°$, we have $\cos(-390°) = 0.8660 = \frac{\sqrt{3}}{2}$

5. Find $\cos(-300°)$
Solution:
Step 1. Find the associated positive angle: $360° - 300° = 60°$
Step 2. Find $\cos(60°)$ using either the tables or special angle $60°$:
$$\cos(-300°) = \cos(60°) = \frac{1}{2}$$

Practice

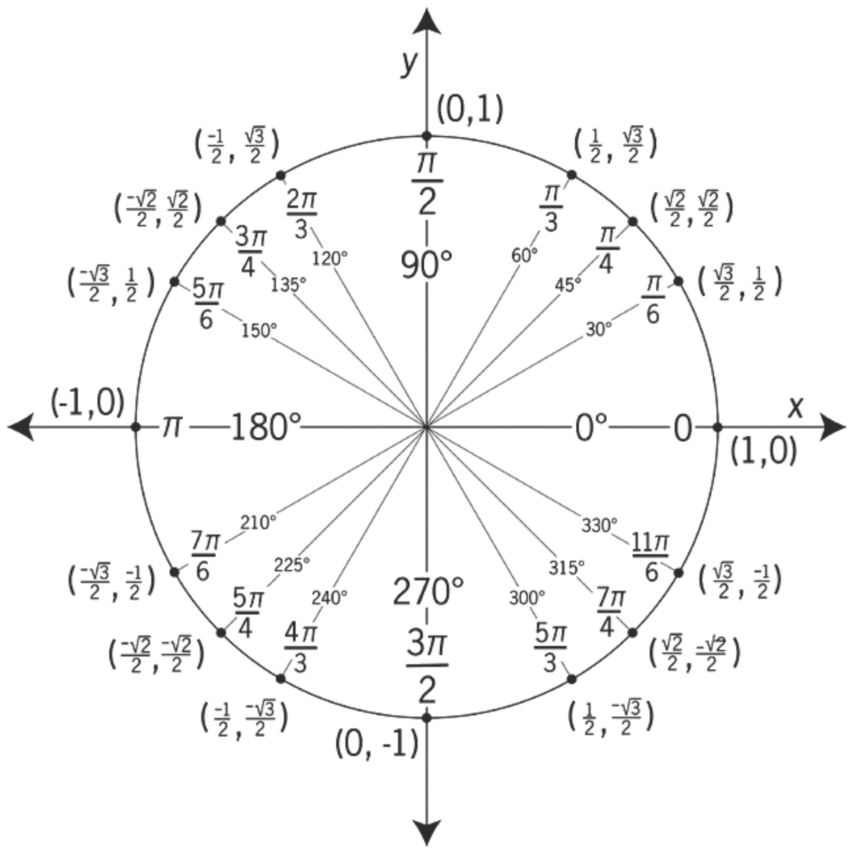
1. Find the values of the following using tables. Give each answer to 4 decimal places:
    a. $\cos(-241°)$
    b. $\tan(-315°)$
    c. $\cos(-380°)$
    d. $\sin(403°)$
2. Find the values of the following using their relationships to special angles. Write them in their simplified form, using surds where necessary:
    a. $\sin(480)$
    b. $\cos(-390)$
    c. $\cos(-210)$
    d. $\sin(-120)$

Lesson Title: The unit circle	Theme: Trigonometry
Practice Activity: PHM2-L109	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to define $\sin\theta$ and $\cos\theta$ as ratios within a unit circle.

Overview

This lesson is on identifying the sine and cosine ratios of angles between 0° and 360° using the unit circle, which is shown below.



Unit Circle[2]

The unit circle is drawn on the Cartesian plane so that the length of its radius is 1 unit. Any point P on the circle forms an angle where each side of the angle is a radius of the circle. Each point P on the circle has coordinates that are an ordered pair. The

[2] Licensed under a Creative Commons Attribution 4.0 International License. OpenStax College, Precalculus. OpenStax CNX. http://cnx.org/contents/fd53eae1-fa23-47c7-bb1b-972349835c3c@.

x-value of the ordered pair is the cosine of the angle formed by P, and the y-value of the ordered pair is the sine of the angle formed by P.

That is, $x = \cos\theta$ and $y = \sin\theta$.

The angles in quadrant 1 of the unit circle are special angles. The angles in other quadrants correspond to the special angles. For example, look at angle 150°. Its corresponding acute angle (the acute angle it forms with the x-axis) is 30°, which is a special angle.

Remember ACTS, the rule for deciding whether of trigonometric ratios are positive or negative. You can also observe that this rule is true for all of the angles shown in the unit circle.

Solved Examples

1. Find: $\sin 60°$
   Solution:
   Identify 60° on the unit circle. Identify the y-coordinate of the point, $\frac{\sqrt{3}}{2}$.
   $\sin 60° = \frac{\sqrt{3}}{2}$

2. Find: $\cos 240°$
   Solution:
   Identify 240° on the unit circle. Identify the x-coordinate of the point, $\frac{-1}{2}$.
   $\cos 240° = \frac{-1}{2}$.

3. Find the value of $\tan 240°$ using the unit circle.
   Solution:
   Since only cosine and sine ratios are in the unit circle, use the relationship $\tan\theta = \frac{\sin\theta}{\cos\theta}$. Substitute and simplify:

$$\begin{aligned}\tan 240° &= \frac{\sin 240°}{\cos 240°} \\ &= \left(-\frac{\sqrt{3}}{2}\right) \div \left(-\frac{1}{2}\right) \\ &= \frac{\sqrt{3}}{2} \times \frac{2}{1} \\ &= \sqrt{3}\end{aligned}$$

4. Evaluate: $\frac{1+\cos 240°}{1-\cos 240°}$
   Solution:
   Substitute values from the unit circle and simplify:

$$\frac{1+\cos 240°}{1-\cos 240°} = \frac{1+\left(-\frac{1}{2}\right)}{1-\left(-\frac{1}{2}\right)}$$

$$= \frac{1-\frac{1}{2}}{1+\frac{1}{2}}$$

$$= \frac{\frac{1}{2}}{\frac{3}{2}}$$

$$= \frac{1}{2} \div \frac{3}{2}$$

$$= \frac{1}{2} \times \frac{2}{3}$$

$$= \frac{1}{3}$$

Practice

1. Find the following using the unit circle:
   a. $\cos 225°$
   b. $\sin 330\ °$
   c. $\tan 300°$
2. Evaluate the following using the unit circle:
   a. $2 \sin 30° \cos 120°$
   b. $\frac{\tan 240°}{\cos 210°}$
   c. $\cos^2 225° + \sin^2 225°$; note that this is the same as $(\cos 225°)^2 + (\sin 225°)^2$.

Lesson Title: Problem solving with trigonometric ratios	**Theme:** Trigonometry
Practice Activity: PHM2-L110	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to solve various problems using the sine, cosine, and tangent ratios of any angle between 0° and 360°.

Overview

This lesson is on problem solving using the trigonometric ratios. You will use information that you have learned in previous lessons.

Solved Examples

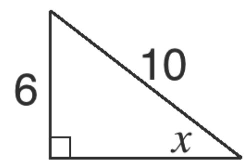
1. Find the value of $\tan 315°$ without using a calculator.
   Solution:
   Use the fact that $\tan x = \frac{\sin x}{\cos x}$, and the values for $\sin 315°$ and $\cos 315°$ from the unit circle.
   $$\tan 315° = \frac{\sin 315°}{\cos 315°} = \frac{-\sqrt{2}}{2} \div \frac{\sqrt{2}}{2} = \frac{-\sqrt{2}}{2} \times \frac{2}{\sqrt{2}} = -1$$

2. Given that $2\sin(x + 4) - 1 = 0$ where $0° \leq x \leq 90°$, find the value of x.
   Solution:
   Step 1. Solve for the trigonometric function:
   $$\begin{aligned} 2\sin(x+4) - 1 &= 0 \\ 2\sin(x+4) &= 1 \\ \sin(x+4) &= \tfrac{1}{2} \end{aligned}$$

   Step 2. Identify that $\sin 30° = \frac{1}{2}$, which is a special angle. Therefore:
   $$\begin{aligned} \sin(x+4) &= \tfrac{1}{2} \\ \sin(x+4) &= \sin 30° \\ x + 4 &= 30° \\ x &= 30° - 4 \\ x &= 26° \end{aligned}$$

3. Given that $\sin x = 0.6$ and $0° \leq x \leq 90°$, evaluate $4\cos x + \sin x$. Give your answer in the form $\frac{m}{n}$, where m and n are integers.

Solution:



Draw the triangle where $\sin x = 0.6 = \frac{6}{10}$. →

Use Pythagoras' theorem to find the unknown side:

$$\begin{aligned} 6^2 + b^2 &= 10^2 \\ 36 + b^2 &= 100 \\ b^2 &= 100 - 36 \\ b^2 &= 64 \\ \sqrt{b^2} &= \sqrt{64} \\ b &= 8 \end{aligned}$$

Find $\cos x$: $\cos x = \frac{A}{H} = \frac{8}{10}$

Therefore, $4\cos x + \sin x = 4\left(\frac{8}{10}\right) + \frac{6}{10} = \frac{32}{10} + \frac{6}{10} = \frac{38}{10} = \frac{19}{5}$

4. If $\sin(x - 20)° = \cos 45°$ and $0° \leq x \leq 90°$, find the value of x.

Solution:

We know that $\cos 45° = \sin 45° = \frac{\sqrt{2}}{2}$ because these are special angles.

Therefore:

$$\begin{aligned} \sin(x - 20)° &= \cos 45° \\ x - 20 &= 45° \\ x &= 65° \end{aligned}$$

Practice

Evaluate the following without tables.

1. $\frac{\sin 60°}{\cos 60°}$
2. $\frac{\sin 300° + \tan 300°}{\cos 300°}$
3. $\frac{\cos 60° \sin 60°}{\tan 30°}$
4. If $\cos(x - 30°) = \sin 60°$ and $0° \leq x \leq 90°$, find the value of x.
5. $\cos^2 330°$

Lesson Title: Graph of $\sin\theta$	**Theme:** Trigonometry
Practice Activity: PHM2-L111	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to use the unit circle to draw the graphs of $\sin\theta$ for $0 \leq \theta \leq 360°$ and solve related trigonometric problems.

Overview

This lesson is on graphing the sine function. The process is similar to graphing other types of functions. Use a table of values, with degrees as the x-values, and the results of the sine function as the y-values. To graph trigonometric functions, we usually use intervals such as 30° or 45° between x-values. Often, we are asked to graph a trigonometric function over the interval $0° \leq x \leq 360°$ or $0° \leq x \leq 180°$.

You may find the values for your table of values using any method, unless a specific method is given in the problem. You may find the values in the unit circle and leave them in surd form. Alternatively, you may use a calculator or sine table, and give decimals in the table instead of surds.

Solved Examples

1. Draw the graph of $y = \sin x$ for values of x from 0° to 360°, using intervals of 45°.
    a. Use the graph to solve $\sin x = 0$.
    b. Find the truth set of the equation $\sin x = \frac{1}{2}$.

Solutions:
First, make a table of values. The x-values in our table of values will be degrees between 0° and 360°. We want intervals of 45°, so add 45° to each x-value to get the next value for the table. Find the sine of each value in the table using the unit circle.

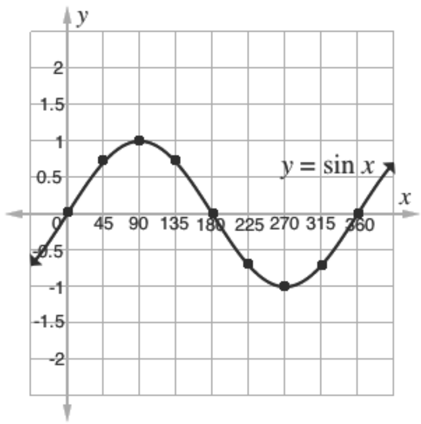
x	$0°$	$45°$	$90°$	$135°$	$180°$	$225°$	$270°$	$315°$	$360°$
$\sin x$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0

Alternatively, you may give $\sin x$ as decimal numbrs:

x	$0°$	$45°$	$90°$	$135°$	$180°$	$225°$	$270°$	$315°$	$360°$
$\sin x$	0	0.7071	1	0.7071	0	-0.7071	-1	-0.7071	0

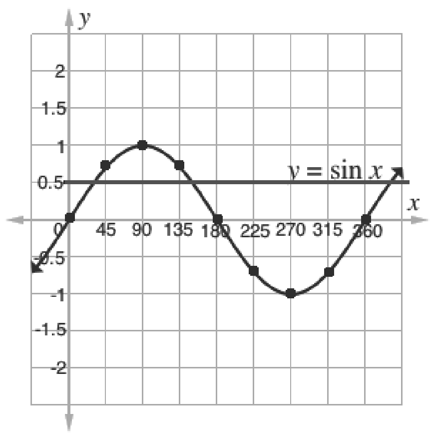
Each column is an ordered pair, which can be plotted on the Cartesian plane. For example: $(0°, 0), (45°, \frac{\sqrt{2}}{2}), (90°, 1)$

Plot each of the points and connect them in a curve:



a. We want to find the places on the graph of sine where $y = 0$ on the interval of concern, $0° \leq x \leq 360°$. This is similar to solving a quadratic equation. We have graphed the function, and we want to find where it crosses the x-axis.
Answers: $x = 0°, 180°, 360°$

b. To find the truth set, we find all points in the given interval where this equation is true. This equation tells us that $y = \frac{1}{2}$. Draw a horizontal like at $y = \frac{1}{2}$, and find all the points at which the line intersects the curve of $y = \sin x$.



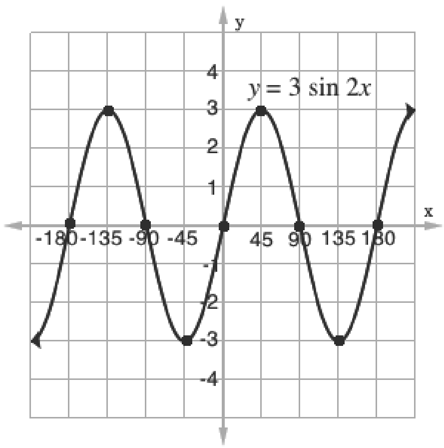
Identify the approximate x-values (within $0° \leq x \leq 360°$) at which the line and curve intersect.
Answers: 30°, 150°

2. Complete the following for the function $y = 3 \sin 2x$:
   a. Draw the graph of $y = 3 \sin 2x$ for values of x from $-180°$ to $180°$. Use intervals of 45°.
   b. State the period of the graph. The period is the distance in the x-direction that it takes the sine curve to complete one cycle. The cycles repeat.
   c. State the maximum and minimum value of $3 \sin 2x$ and the values of x at which these occur on the interval $-180° \leq x \leq 180°$.
   Solution:

a. Complete a table of values to find $y = 3\sin 2x$ for each x-value:

x	$2x$	$\sin 2x$	$3\sin 2x$
$-180°$	$-360°$	0	0
$-135°$	$-270°$	1	3
$-90°$	$-180°$	0	0
$-45°$	$-90°$	-1	-3
$0°$	$0°$	0	0
$45°$	$90°$	1	3
$90°$	$180°$	0	0
$135°$	$270°$	-1	-3
$180°$	$360°$	0	0

Graph:



b. From the drawing we can see that the graph repeats itself every $180°$; so the period of the graph is 180.

c. The maximum value of $3\sin 2x$ is 3. This occurs when $x = -135°$ and $x = 45°$

The minimum value of $3\sin 2x$ is -3. This occurs when $x = -45°$ and $x = 135°$.

Practice

1. Create a table of values and draw the graph of $y = -\sin x$ from $0°$ to $360°$ using intervals of $45°$.
    a. Use the graph to solve $-\sin x = 1$.
    b. Find the truth set of the equation $-\sin x = -\frac{1}{2}$.
2. Draw the graph of $y = -2\sin x$ for values of x from $0°$ to $180°$. Using intervals of $30°$.
    a. From the graph, find y when $x = 170°$.
    b. Find the truth set of the equation $-2\sin x + 1 = 0$.

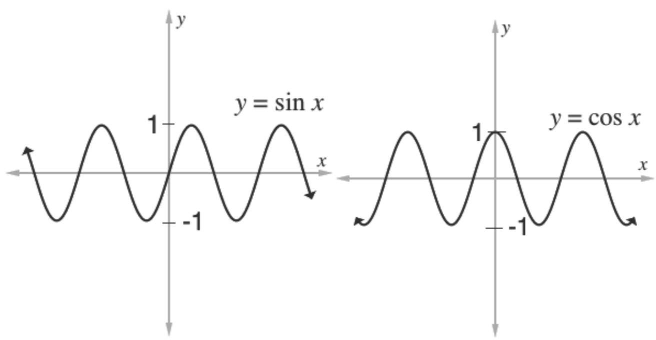
Lesson Title: Graph of $\cos\theta$	**Theme:** Trigonometry
Practice Activity: PHM2-L112	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to use the unit circle to draw the graphs of $\cos\theta$ for $0 \leq \theta \leq 360°$ and solve related trigonometric problems.

Overview

This lesson is on graphing the cosine function. The process is similar to graphing the sine function. Use a table of values, with degrees as the x-values, and the results of the cosine function as the y-values. As with sine, we usually use intervals such as 30° or 45° between x-values. Often, we are asked to graph a trigonometric function over the interval $0° \leq x \leq 360°$ or $0° \leq x \leq 180°$.

Notice that the sine and cosine curve have the same shape. They both go on forever in both x-directions, and remain between $y = -1$ and $y = 1$. However, they have different starting points. $y = \sin x$ intersects the origin. $y = \cos x$ intersects the y-axis at $y = 1$.



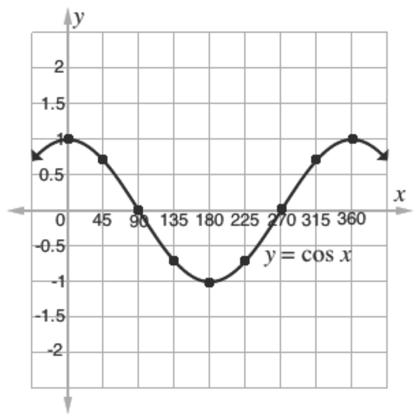
Solved Examples

1. Draw the graph of $y = \cos x$ for values of x from 0° to 360°, using intervals of 45°.
   a. Use the graph to solve $\cos x = 0$.
   b. Find the truth set of the equation $\cos x = 1$.

   Solutions:
   First, make a table of values. The x-values in our table of values will be degrees between 0° and 360°. We want intervals of 45°, so add 45° to each x-value to get the next value for the table. Find the cosine of each value in the table using the unit circle.

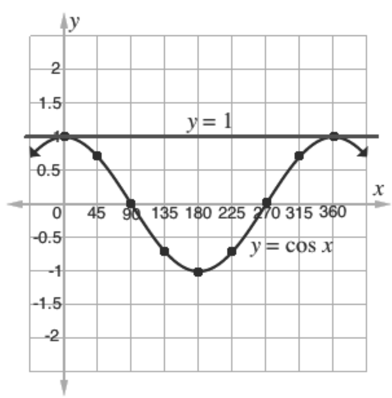
x	$0°$	$45°$	$90°$	$135°$	$180°$	$225°$	$270°$	$315°$	$360°$
$\cos x$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1

Plot each of the points and connect them in a curve:



c. We want to find the places on the graph of the cosine where $y = 0$ on the interval of concern, $0° \leq x \leq 360°$.
Answers: $x = 90°, 270°$

b. To find the truth set, we find all points in the given interval where this equation is true. This equation tells us that $y = 1$. Draw a horizontal like at $y = 1$, and find all the points at which the line intersects the curve of $y = \cos x$.

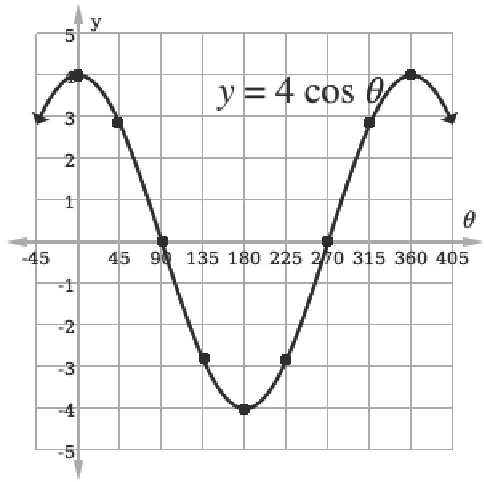


Answers: $x = 0°, 360°$

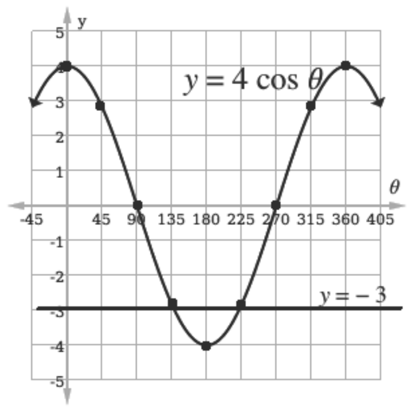
2. Draw the graph of $y = 4\cos\theta$ for value of θ between $0°$ to $360°$, using intervals of $45°$. Find the truth set of the equation $4\cos\theta = -2$.
Solution:
First make a table of values. The x-values in our table of values will be degrees between $0°$ to $360°$ with intervals of $45°$. Find the cosine of each value in the table. Decimals are given in the example below, which can be found using a calculator or log book.

θ	0°	45°	90°	135°	180°	225°	270°	315°	360°
cos θ	0	0.71	0	−0.71°	−1	−0.71°	0	0.71	1
4 cos θ	0	2.84	0	−2.84°	−4	−2.84°	0	2.84°	4

Draw the graph:



To find the truth set, draw a horizontal line at $y = -3$ (see below). Note that it intersects the curve at slightly more than 135, and slightly less than 225. Therefore, the truth set is approximately $\theta = 140°, 220°$.



Practice

1. Draw the graph of $y = 2\cos x$ for values of θ from $0°$ to $360°$, using intervals of $45°$. From your graph, find y when $x = 200°$.
2. Draw the graph of $y = \cos^2\theta$ for values of θ from $0°$ to $180°$, using intervals of $30°$. From your graph:
   a. Find y when $\theta = 45°$.
   b. Find the truth set of $\cos^2\theta = 1$.

Lesson Title: Graphs of $\sin\theta$ and $\cos\theta$	**Theme:** Trigonometry
Practice Activity: PHM2-L113	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to use the unit circle to draw the graphs of functions of the form $y = a\sin\theta + b\cos\theta$ for $0° \leq \theta \leq 360°$ and solve related trigonometric problems.

Overview

This lesson is on graphing functions that contain both sine and cosine, such as of $y = 2\sin x + \cos x$. The process is similar to graphing the sine or cosine function alone. Use a table of values, with degrees as the x-values, and the results of the complete function as the y-values. It is useful to have extra rows in the table to perform calculations. For example, for the function above, you would have a row for calculating $2\sin x$, and a row for calculating $\cos x$. In the last row, you would add them together. The last row are y-values that are used for plotting points.

Because we will be doing operations on values in different rows of the table, it is easiest to give the values as decimals, instead of using their values from the unit circle.

After graphing, the shape of the curve will be similar to what we have seen before with $y = \sin x$ and $y = \cos x$.

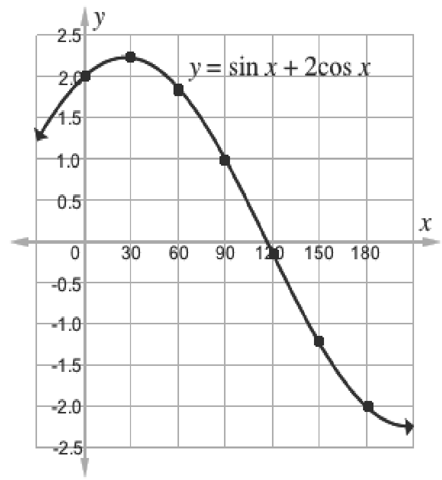
Solved Examples

1. Draw the graph of $y = 2\sin x + \cos x$ for values of x from 0° to 180°, using intervals of 30°. Use the graph to approximate the solution of $\sin x + 2\cos x = 0$.
   Solution:
   First, make a table of values. The x-values in our table of values will be degrees between 0° and 360°. We want intervals of 30°, so add 30° to each x-value to get the next value for the table.

   Find the sine and cosine of each x-value using the trigonometric tables. Remember to multiply each cosine value by 2. Add the sine and cosine rows to find the values for the last row.

x	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$
$\sin x$	0	0.5	0.9	1	0.9	0.5	0
$2\cos x$	2.0	1.7	1.0	0	−1.0	−1.7	−2.0
$\sin x + 2\cos x$	2.0	2.2	1.9	1.0	−0.1	−1.2	−2.0

   Plot each of the points and connect them in a curve:



To solve $\sin x + 2\cos x = 0$, we need to find the places on the graph where $y = 0$ on the interval of concern, $0° \leq x \leq 180°$. We can observe that the curve intersects the x-axis just before 120°. A good estimate for the solution is 115°.

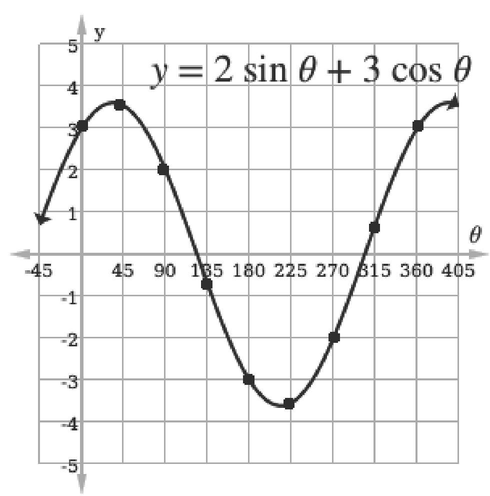
2. Draw the graph of $y = 2\sin\theta + 3\cos\theta$ for $0° \leq \theta \leq 360°$, using intervals of $45°$. From the graph, find:
   a. The solutions of the equation $2\sin\theta + 3\cos\theta = 2$
   b. Solve the equation $2\sin\theta + 3\cos\theta = 0$.

   Solutions:

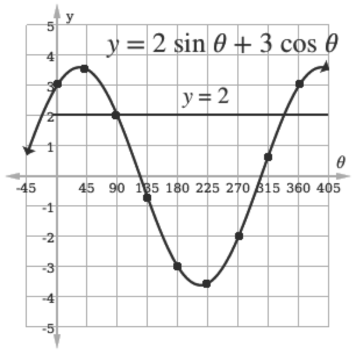
   Create a table of values:

θ	$0°$	$45°$	$90°$	$135°$	$180°$	$225°$	$270°$	$315°$	$360°$
$2\sin\theta$	0	1.41	2	1.41	0	-1.41	-2	-1.41	0
$3\cos\theta$	3	2.12	0	-2.12	-3	-2.12	0	2.12	3
$2\sin\theta + 3\cos\theta$	3	3.53	2	-0.71	-3	-3.53	-2	0.71	3

Draw the graph using the first and last rows of the table:



a. Draw a horizontal line at $y = 2$, and find the points at which it intersects the curve. The answers are $\theta = 90°, 340°$. The second value is approximate.



b. To solve $2\sin\theta + 3\cos\theta = 0$, find the values of θ when $y = 0$. These are the points where the curve intersects the horizontal axis. These are approximately $\theta = 125°, 300°$.

Practice

1. Draw the graph of $y = \sin x + \cos x$ for values of x from $0°$ to $360°$, using intervals of $30°$. Use your graph to find the approximate values of x for which $\sin x + \cos x = 0.75$.
2. Draw the graph of $y = 3\sin x - 2\cos x$ for $0° \leq x \leq 180°$, using intervals of $30°$. From the graph, find:
   a. The truth set of $y = 2$.
   b. The solution of $3\sin x - 2\cos x = 0$.

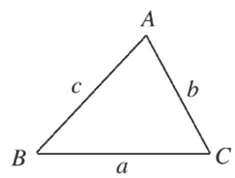
Lesson Title: The sine rule	**Theme:** Trigonometry
Practice Activity: PHM2-L114	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to derive the sine rule and use it to calculate lengths and angles in triangles.

Overview

We have previously solved for side lengths of right-angled triangles using Pythagoras' theorem and trigonometry. We also used similar triangles and theorems to solve for the sides of certain types of triangles.

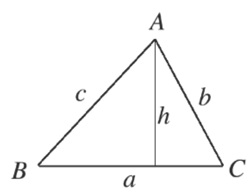
The sine rule allows us to solve for missing side of any triangle, as long as we have enough information. The sine rule is given by the ratios $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ for a triangle:



For triangle ABC, the angles are usually labeled with capital letters, while the sides are labeled with lower case letters. Using the sine rule, we can solve a triangle if we are given **any 2 angles and 1 side** in the problem, or if we are given **2 sides and the angle opposite 1 of them**.

The sine rule can be derived using information we already know about triangles, as follows:

Step 1. Draw the perpendicular (height) from A to BC:



Step 2. Write the sine ratio for angles B and C:

$$\sin B = \frac{h}{c} \text{ and } \sin C = \frac{h}{b}$$

Step 3. Solve each equation for h:

$$h = c \sin B \text{ and } h = b \sin C$$

Step 4. Set the 2 formulae for h equal to one another:

$$c \sin B = b \sin C$$

Step 5. Divide throughout by $\sin B$ and $\sin C$:

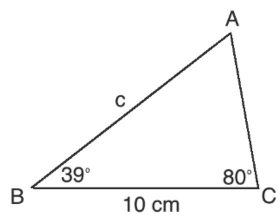
$$c = \frac{b \sin C}{\sin B}$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

This shows part of the sine rule. We have shown that it is true for c and b. We could draw an additional perpendicular from C to AB to show that the same is true for a and b. Thus, we have the sine rule, which says that 3 fractions are equal.

Solved Examples

1. Find the length of missing side c:



Solution:

Use two fractions from the sine rule: $\frac{a}{\sin A} = \frac{c}{\sin C}$

Substitute known values (a and C) into the formula:

$$\frac{10}{\sin A} = \frac{c}{\sin 80°}$$

There are 2 unknowns. Find A by subtracting the known angles of the triangle from 180: $A = 180° - (39° + 80°) = 61°$

Substitute $A = 61°$ into the formula, and solve for c:

$$\frac{10}{\sin 61°} = \frac{c}{\sin 80°}$$

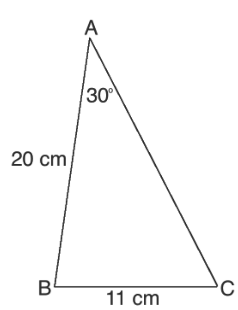
$$10 \times \sin 80° = c \times \sin 61°$$

$$c = \frac{10 \times \sin 80°}{\sin 61°}$$

$$c = \frac{10 \times 0.9848}{0.8746}$$ Substitute values from the sine table

$c = 11.26$ cm Simplify

2. Find angles B and C in the triangle below:



Solution:

We have enough info to find C with the formula: $\frac{a}{\sin A} = \frac{c}{\sin C}$.

Substitute the values and solve:

$$\frac{11}{\sin 30°} = \frac{20}{\sin C} \quad \text{Substitute in the formula}$$

$$11 \times \sin C = 20 \times \sin 30° \quad \text{Cross multiply}$$

$$\sin C = \frac{20 \times \sin 30°}{11} \quad \text{Solve for } C$$

$$\sin C = \frac{20 \times 0.5}{11} = \frac{10}{11}$$

$$\sin C = 0.9091$$

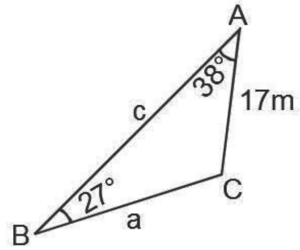
$$C = \sin^{-1} 0.9091 \quad \text{Take the inverse sine of both sides}$$

$$C = 65.38° \quad \text{Use the sine table}$$

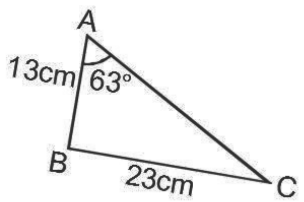
Subtract A and C from 180 to find B: $B = 180° - (30° + 65.38°) = 84.82°$

Practice

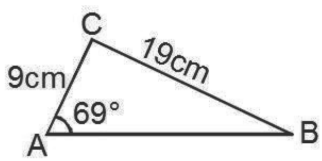
1. Find the lengths of the missing sides, a and c, to the nearest metre:



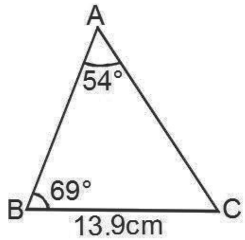
2. Find the measures of angles B and C, correct to 1 decimal place:



3. Find angles B and C, and side AB, correct to the nearest whole number:



4. Find angle C and sides b and c in the triangle, correct to the nearest whole number:



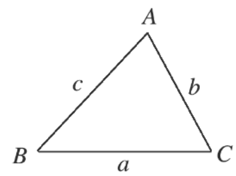
Lesson Title: The cosine rule	**Theme:** Trigonometry
Practice Activity: PHM2-L115	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to derive the cosine rule and use it to calculate lengths and angles in triangles.

Overview

In some cases, we do not have enough information to use the sine rule to solve a problem. The cosine rule allows us to solve triangles where the sine rule cannot be used. We can use the cosine rule if **two sides and the angle between them** are given.

The cosine rule says that for a triangle:



The following are true:

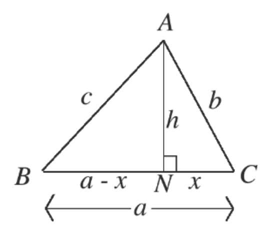
$$c^2 = a^2 + b^2 - 2ab\ \cos C$$
$$b^2 = a^2 + c^2 - 2ac\ \cos B$$
$$a^2 = b^2 + c^2 - 2bc\ \cos A$$

The formulae above are used to solve for the sides. The subject of each formula can be changed to solve for the angles:

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$
$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$
$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

The cosine rule can be derived using information we already know about triangles, as follows:

Step 1. Draw the perpendicular (height) from A to BC. Label the triangle as shown:



Step 2. Apply Pythagoras' theorem to the right-angled triangle ABN on the left-hand side of triangle ABC, then solve as follows:

c^2	$=$	$(a-x)^2 + h^2$	Pythagoras' theorem
	$=$	$a^2 - 2ax + x^2 + h^2$	Expand the binomial

In the triangle, $x^2 + h^2 = b^2$ (Pythagoras' theorem on triangle ACN). Substitute this in the equation above:

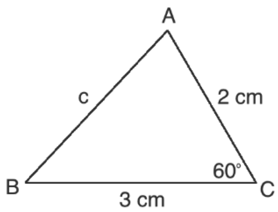
$$c^2 = a^2 - 2ax + b^2$$

In triangle ACN, we can find the cosine ratio of C as: $\cos C = \frac{x}{b}$. Solving for x, we have $x = b\cos C$. Substitute this in the formula:

$$\begin{aligned} c^2 &= a^2 - 2ab\cos C + b^2 \\ &= a^2 + b^2 - 2ab\cos C \end{aligned}$$

Solved Examples

1. Find the length of missing side c:

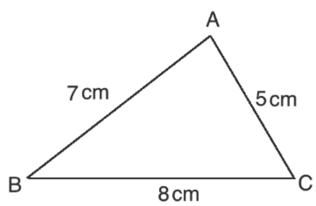


Solution:

Apply the formula $c^2 = a^2 + b^2 - 2ab\ \cos C$:

c^2	=	$a^2 + b^2 - 2ab\ \cos C$	Formula
	=	$3^2 + 2^2 - 2(3)(2)\ \cos 60°$	Substitute values from triangle
	=	$3^2 + 2^2 - 2(3)(2)(0.5)$	Substitute $\cos 60° = 0.5$
	=	$9 + 4 - 12\ (0.5)$	Simplify
	=	$13 - 6$	
c^2	=	7	
c	=	$\sqrt{7}$ = 2.65 cm to 2 d.p.	Take the square root of both sides

2. Find the measures of angles A, B, and C using the cosine rule:



Solution:

Use each formula for finding the measure of each angle. Substitute the appropriate values and simplify:

$$\begin{aligned} \cos A &= \frac{5^2+7^2-8^2}{2(5)(7)} = \frac{25+49-64}{70} = \frac{10}{70} = 0.1429 \\ \cos A &= 0.1429 \\ A &= \cos^{-1} 0.1429 \\ A &= 81.8° \text{ to 1 d.p.} \end{aligned}$$

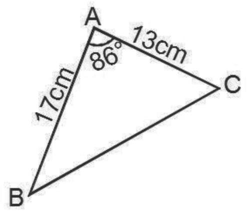
$$\begin{aligned} \cos B &= \frac{7^2+8^2-5^2}{2(7)(8)} = \frac{49+64-25}{112} = \frac{88}{112} = 0.7857 \\ \cos B &= 0.7857 \\ B &= \cos^{-1} 0.7857 \\ B &= 38.2° \text{ to 1 d.p.} \end{aligned}$$

$$\cos C = \frac{5^2+8^2-7^2}{2(5)(8)} = \frac{25+64-49}{80} = \frac{40}{80} = 0.5$$
$$\cos C = 0.5$$
$$C = \cos^{-1} 0.5$$
$$C = 60°$$

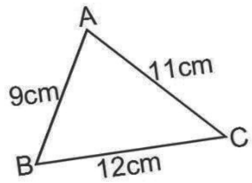
Check your work by adding the angles together: $A + B + C = 81.8° + 38.2° + 60° = 180°$.

Practice

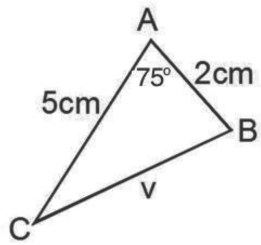
1. Find the length of the missing side a, correct to the nearest centimetre.



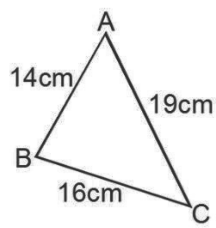
2. Find the angles A, B and C in the triangle below, to the nearest degree.



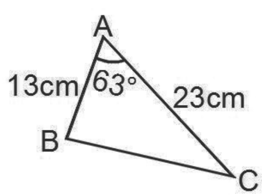
3. Find the length of v, correct to 1 decimal place.



4. Find the measure of angle A, correct to the nearest degree:



5. Find the angles B, C and length a in the triangle below. Give your answers correct to the nearest whole number.



Lesson Title: Application of sine and cosine rules	Theme: Trigonometry
Practice Activity: PHM2-L116	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to use the sine and cosine rules to solve triangles.

Overview

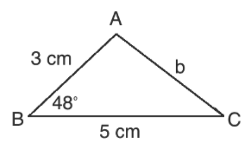
This lesson is on using the sine and cosine rules to solve problems. You will need to decide which rule to use for a given problem. In some cases, there could be more than 1 different way of solving a problem.

Recall the cases where you can use each rule:

- **Sine rule:** use when given 2 angles and any side, or 2 sides and the angle opposite 1 of them.
- **Cosine rule:** use when given two sides and the included angle.

Solved Examples

1. In the triangle below, find b, A, and C:



Solution:
Consider the steps before solving. We can use the cosine rule to find the missing side, b. After finding b, we can use either the sine rule or the cosine rule for finding angles to find A and C. It is easier to use the sine rule to find a missing angle. Find the value of the smaller angle (C) first. Finding the sine of an acute angle is less ambiguous than finding the sine of an obtuse angle. Once we have 2 angles, we can solve for the 3rd by subtracting from 180°.

Step 1. Use the cosine rule to find b:

b^2	$=$	$a^2 + c^2 - 2ac\ \cos B$	Formula
	$=$	$3^2 + 5^2 - 2(3)(5)\ \cos 48°$	Substitute values from triangle
	$=$	$3^2 + 5^2 - 2(3)(5)(0.6691)$	Substitute $\cos 48° = 0.6691$
	$=$	$9 + 25 - 30\ (0.6691)$	Simplify
	$=$	$34 - 20.073$	
b^2	$=$	13.927	
b	$=$	$\sqrt{13.927} = 3.73$ cm to 2 d.p.	Take the square root of both sides

Step 2. Use the sine rule to find C:

$$\frac{3.73}{\sin 48°} = \frac{3}{\sin C}$$

$$3.73 \times \sin C = 3 \times \sin 48° \quad \text{Cross multiply}$$

$$\sin C = \frac{3 \times \sin 48°}{3.73}$$

$$\sin C = \frac{3 \times 0.7431}{3.73} \quad \text{Substitute values from the sine table}$$

$$\sin C = 0.5977 \quad \text{Simplify}$$

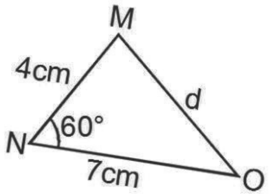
$$C = \sin^{-1} 0.5977$$

$$C = 36.71°$$

Step 3. Subtract to find A:

$$A = 180° - (48° + 36.71°) = 95.29°$$

2. In the triangle MNO, find d, M and O, correct to 1 decimal place:



Solution:

Step 1. Use the cosine rule to find d:

$$d^2 = m^2 + o^2 - 2mo\cos 60° \quad \text{Formula}$$

$$= 7^2 + 4^2 - 2(7)(4)\cos 60° \quad \text{Substitute values from triangle}$$

$$= 7^2 + 4^2 - 2(7)(4)(0.5) \quad \text{Substitute } \cos 60° = 0.5$$

$$= 49 + 16 - 56(0.5) \quad \text{Simplify}$$

$$= 65 - 28$$

$$d^2 = 37 \quad \text{Take the square root of both sides}$$

$$d = \sqrt{37} = 6.1 \text{ cm}$$

Step 2. Use the sine rule to find O:

$$\frac{6.1}{\sin 60°} = \frac{4}{\sin O}$$

$$6.1 \times \sin O = 4 \times \sin 60° \quad \text{Cross multiply}$$

$$\sin O = \frac{4 \times \sin 60°}{6.1}$$

$$\sin O = \frac{4 \times 0.8660}{6.1}$$

$$\sin O = \frac{3.464}{6.1}$$

$$\sin O = 0.5679$$

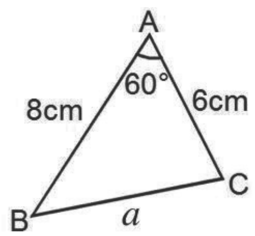
$$O = \sin^{-1}(0.5679)$$

$$O = 34.6°$$

Step 3. Subtract to find M:

$M = 180° - (60° + 34.6°) = 85.4°$

3. Find the missing sides and angles in the triangle below:



Solution:

Step 1. Use the cosine rule to find a:

a^2	$=$	$b^2 + c^2 - 2bc\cos 60°$	Formula
	$=$	$6^2 + 8^2 - 2(6)(8)\cos 60°$	Substitute values from triangle
	$=$	$36 + 64 - 2(6)(8)(0.5)$	Substitute $\cos 60° = 0.5$
	$=$	$100 - 96(0.5)$	Simplify
	$=$	$100 - 48$	
a^2	$=$	52	Take the square root of both sides
a	$=$	$\sqrt{52} = 7.2$ cm	

Step 2. Use the sine rule to find B:

$$\frac{7.2}{\sin 60°} = \frac{6}{\sin B}$$

$$7.2 \times \sin B = 6 \times \sin 60° \quad \text{Cross multiply}$$

$$\sin B = \frac{6 \times \sin 60°}{7.2}$$

$$\sin B = \frac{6 \times 0.8660}{7.2}$$

$$\sin B = \frac{5.196}{7.2}$$

$$\sin B = 0.7217$$

$$\theta = \sin^{-1}(0.7217)$$

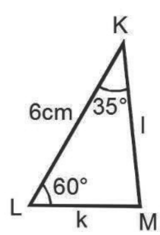
$$\theta = 46.2°$$

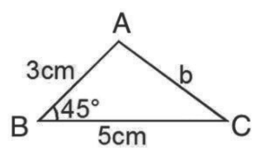
Step 3. Subtract to find C:

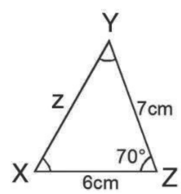
$C = 180° - (60° + 46.2°) = 73.8°$

Practice

1. Find the missing sides and angles in the triangles below. Give your answers correct to 1 decimal place.

a.


b.


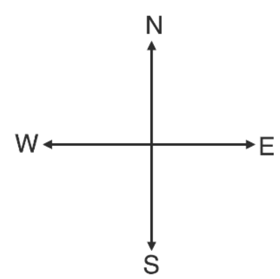
c.


Lesson Title: Compass bearings	**Theme:** Bearings
Practice Activity: PHM2-L117	**Class:** SSS 2

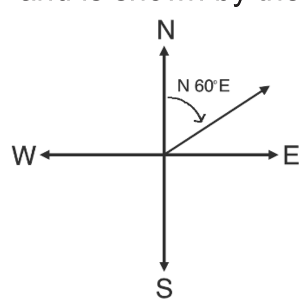
Learning Outcomes
By the end of the lesson, you will be able to:
1. Interpret bearings in terms of compass directions.
2. Draw diagram representations of bearing statements.

Overview

This lesson is on compass bearings, which are related to the compasses that are used to navigate. Navigational compasses are used by ships or on land to determine the correct direction or route to travel. Information from compasses can also be used to determine distances. A compass has 4 directions that form right angles with each other, as shown:

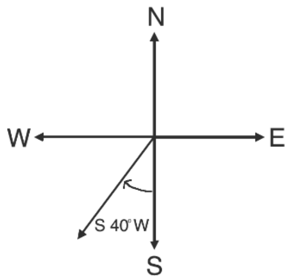


Compass bearings are measured from north or south. A number of degrees in either the east or west direction is given in degrees. For example, the compass bearing N 60° E means "60° east of north" and is shown by the arrow in the diagram below:



Solved Examples

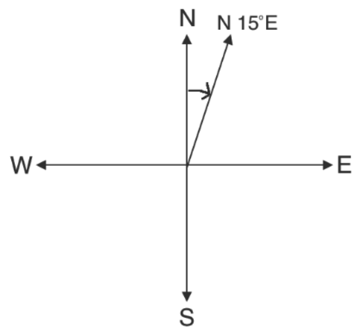
1. Draw a diagram to show S 40° W.
   Solution:
   Use a protractor to measure 40° from south, toward west. Draw a line with an arrow in this direction:



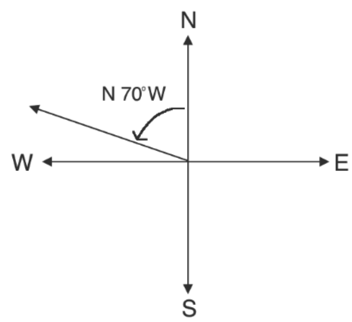
2. Draw diagrams to show the following bearings:
    a. N 15° E
    b. N 70° W
    c. S 28° E
    d. S 75° W

    Solutions:

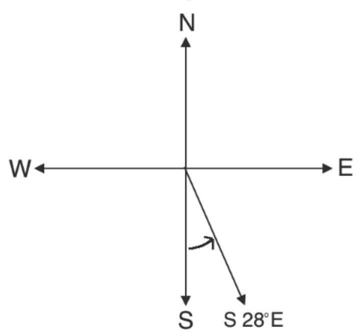
a.



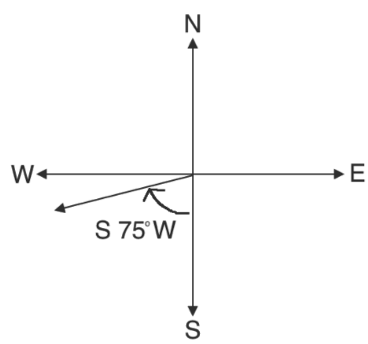
b.



c.



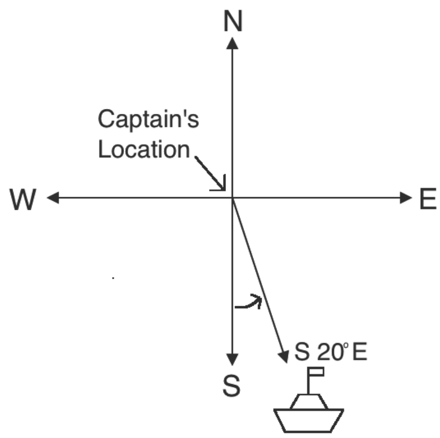
d.



3. A ship captain sees another ship at a bearing of S 20° E from his location. Draw this in a diagram.

    Solution:

    Draw a compass, then draw a ship at a bearing of S 20° E. Label the captain's location.



Practice

1. For each of the following, draw a compass showing East, West, North, and South. Draw an arrow to show each bearing.
    a. $\text{N } 30° \text{ E}$
    b. $\text{S } 25° \text{ E}$
    c. $\text{N } 85° \text{ W}$
    d. $\text{S } 60° \text{ W}$
2. A ship sailed at a bearing of $\text{N } 45° \text{ E}$. Draw this in a diagram.

Lesson Title: Three figure bearings	**Theme:** Bearings
Practice Activity: PHM2-L118	**Class:** SSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify angles measured clockwise from the geographic north.
2. Represent bearings as angles in three digits.
3. Solve simple problems involving three figure bearings.

Overview

Three-figure bearings are bearings given in 3 digits. These 3 digits give the angle of the bearing in the clockwise direction from the geographic north. The angles range from 000° to 360°. They must always have 3 digits, even when they're actually less than 100 degrees.

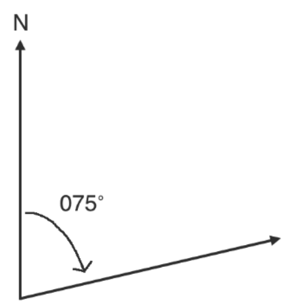
Three-figure bearing diagrams have one arrow pointing in the north direction (shown below) and another arrow showing the bearing.

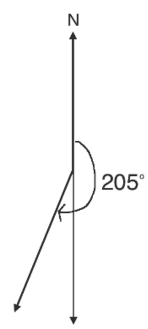
Solved Examples

1. Draw diagrams to show the following bearings:
    a. 009°
    b. 075°
    c. 205°

    Solutions:

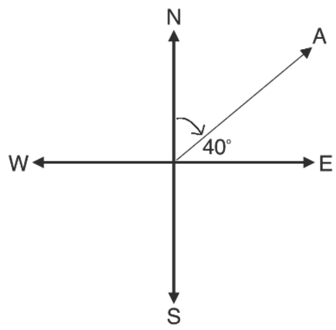
    a.


    b.


    c.


    Note that for bearings greater than 180, the vertical line can be extended down. For part c., the vertical line is extended down and used to measure 25° clockwise from south (because $205° - 180° = 25°$).

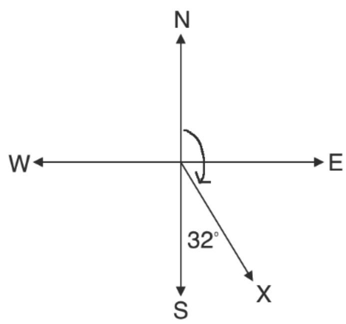
2. Find the three-figure bearing of A:



Solution:
We need to find the angle that the bearing A forms with the north. The 4 directions are given, and we know east is 90° from north. Subtract the given angle from 90°: $A = 90° - 40° = 50°$

The three-figure bearing is 050°.

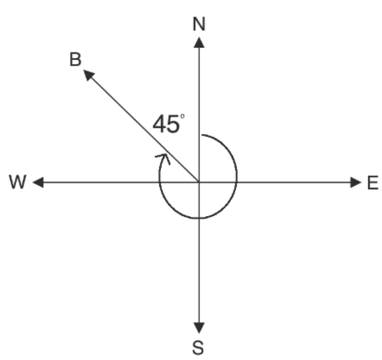
3. Find the three-figure bearing of X:



Solution:
We know that south is 180° from north. Subtract the given angle from 180° to find the bearing of X from the north direction. $X = 180° - 32° = 148°$

The three-figure bearing is 148°.

4. Find the three-figure bearing of B:



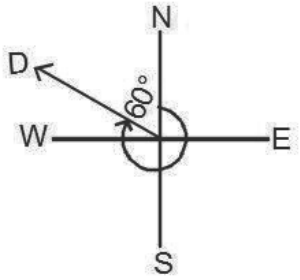
Solution:
We know that a full revolution is 360°. Subtract the given angle from 360° to find the bearing of B from north. $B = 360° - 45° = 315°$

The three-figure bearing is 315°.

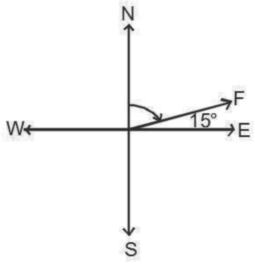
Practice

1. Write the 3-figure bearing for each of the following diagrams:

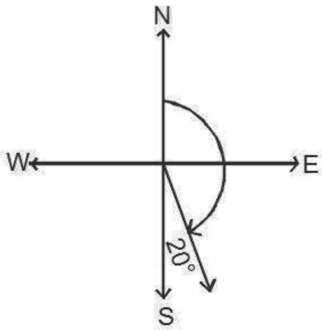
a.



b.



c.



2. A boat at sea is on a bearing of $205°$ from your current location. Draw a diagram for this.

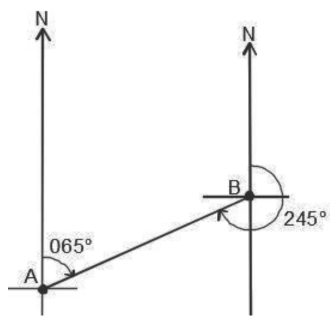
Lesson Title: Reverse bearings	**Theme:** Bearings
Practice Activity: PHM2-L119	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to find the reverse bearing of a given bearing.

Overview

When we talk about “reverse” bearings, we must have 2 points. Consider 2 points A and B. We have the bearing from A to B, and we have the bearing from B to A. These are different, because bearings are about direction. A to B is a different direction than B to A. They are reverse. Reverse bearings are sometimes called “back bearings”.

Consider the following example:



The bearing from A to B is 065°, and the bearing from B to A is 245°. For both bearings, we use the line that joins them and the north direction. We find the bearing of the line joining them from the north.

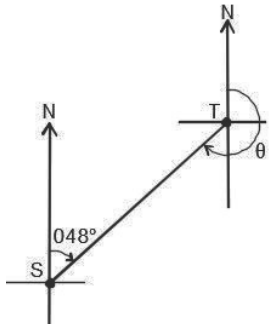
Depending on the size of the first bearing, you will add or subtract 180° to find the reverse bearing:

- Reverse bearing $= \theta + 180°$ if θ is less than $180°$
- Reverse bearing $= \theta - 180°$ if θ is more than $180°$

Note that there are 2 ways that bearings can be described in problems. “The bearing of T from S” is the same as “the bearing from S to T”.

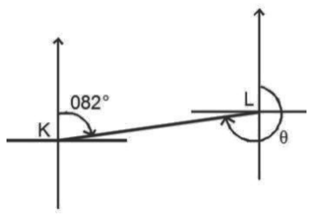
Solved Examples

1. If the bearing of T from S is 048°, find the bearing of S from T.
   Solutions:
   First draw a diagram to visualise the problem:



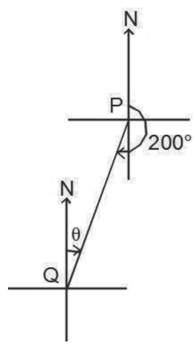
Add 180° to find the reverse bearing, since the bearing from S to T is less than 180°: $\theta = 48° + 180° = 228°$

2. If the bearing of L from K is $082°$, find the bearing of K from L:
Solution:
Diagram**:**



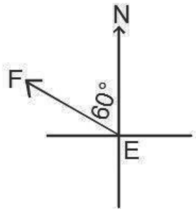
Add: $\theta = 082° + 180 = 262°$

3. If the bearing of P from Q is $250°$, find the bearing of Q to P:
Solution:
Diagram:



Subtract, because the bearing is greater than $180°$:
$\theta = 250° - 180° = 070°$

4. In the diagram below, find:
   a. The bearing of F from E
   b. The bearing of E from F



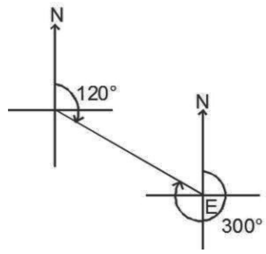
Solutions:

a. Find the bearing from the north. Subtract the given angle $(60°)$ from $360°$.

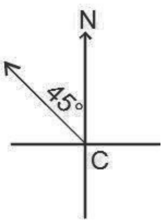
$360° - 60° = 300°$

b. Find the reverse bearing using the result from a.

$300° - 180° = 120°$



5. In the diagram below, find:
   a. The bearing of D from C
   b. The bearing of C from D



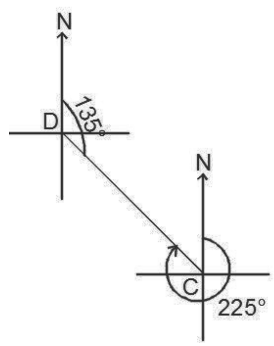
Solutions:

a. Find the bearing from north. Subtract the given angle $(45°)$ from $360°$.

$360° - 45° = 315°$

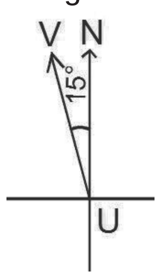
b. Find the reverse bearing using the result from a.

$315° - 180° = 135°$



Practice

1. If the bearing of B from A is $038°$, find the bearing of A from B. Draw a diagram showing both bearings.
2. If the bearing from X to Y is $210°$, find the bearing from Y to X. Draw a diagram.
3. The bearing from J to K is $085°$, draw a diagram and find the bearing from K to J.
4. In the diagram below, find:
   a. The bearing of V from U
   b. The bearing of U from V



Lesson Title: Bearing problem solving – Part 1	**Theme:** Bearings
Practice Activity: PHM2-L120	**Class:** SSS 2

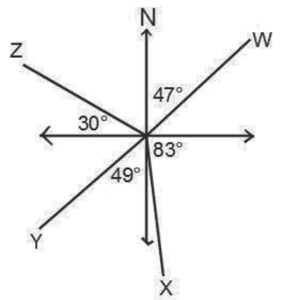
Learning Outcome
By the end of the lesson, you will be able to draw diagrams and solve bearings problems that do not involve distances.

Overview

This lesson uses information from the previous lessons on bearings to solve problems. When you encounter a bearings problem, it is best to draw a diagram first.

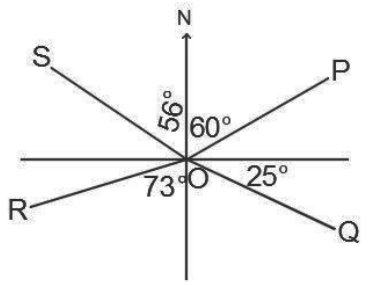
Solved Examples

1. Find the three-figure bearings of each of the points W, X, Y and Z from point O.



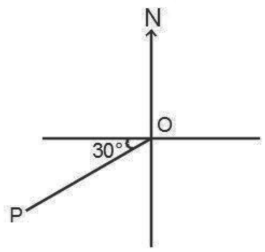
Solution:
Point W: $047°$
Point X: $90° + 83° = 173°$
Point Y: $180° + 49° = 229°$
Point Z: $180° + 90° + 30° = 300°$ or $270° + 30° = 300°$

2. Find three figure bearings of each of the points P, Q, R and S from point O.



Solution:
Point P: $060°$
Point Q: $90° + 25° = 115°$
Point R: $180° + 73° = 253°$
Point S: $360° - 56° = 304°$

3. In the diagram below, find:
   a. The bearing of P from O
   b. The bearing of O from P



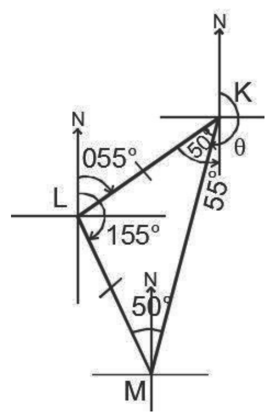
Solutions:

a. The angle formed by OP and the south direction: $90° - 30° = 60°$
   Therefore, the bearing of P from O: $180° + 60° = 240°$

b. The bearing of O from P is the reverse bearing: $240° - 180° = 060°$

4. If the bearing of B from A is $040°$, find the bearing of A from B.
   Solution:
   Calculate the reverse bearing: $40° + 180° = 200$

5. The bearing of K from L is $055°$ and the bearing of M from L is $155°$. If L is equidistant from K and M, find the bearing of M from K.
   Solution:
   Draw a diagram for the problem:



Note that K, L, and M form an isosceles triangle. We seek to find the angle marked θ in the diagram.

Find the interior angle of the triangle at L: $155° - 55° = 100°$

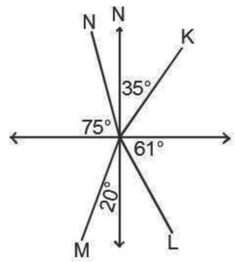
Since the other 2 interior angles of the triangle are equal, subtract the known angle $(100°)$ from $180°$ and then divide by 2: $K = M = \frac{180° - 100°}{2} = \frac{80°}{2} = 40°$

Use the alternate angle to the bearing of K from L. This is a $55°$ angle at K, as marked in the diagram. This gives the angle formed by KM and the south line, which is $55° - 50° = 5°$

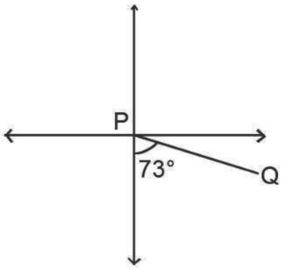
Therefore, the bearing of M from K: $180° + 5° = 185°$.

Practice

1. Find the three-figure bearings of the points K, L, M and N from point O.



2. In the diagram, find:
    a. The bearing of Q from P
    b. The bearing of P from Q



3. The bearing of X from Y is $062°$ and the bearing of Z from Y is $174°$, where XY and Z are three points on the plane. If Y is equidistant from X and Z, find the bearing of Z from X.

Lesson Title: Distance-bearing form and diagrams	**Theme:** Bearings
Practice Activity: PHM2-L121	**Class:** SSS 2

Learning Outcomes

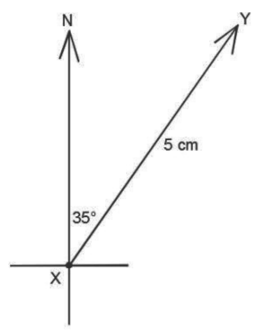
By the end of the lesson, you will be able to:

1. Write the distance and bearing of one point from another as (r, θ).
2. Interpret a distance-bearing problem and draw a corresponding diagram.

Overview

This lesson is on distance-bearing form. This is another way to describe bearings that use the distance between two points.

Consider the example:



The bearing from X to Y can be written as $\overrightarrow{XY} = (5 \text{ cm}, 035°)$. The distance and three-point bearing are given in brackets.

In general, the position of point Q from another point P can be represented by $\overrightarrow{PQ} = (r, \theta)$, where r is the distance between the 2 points, and θ is the three-point bearing from P to Q.

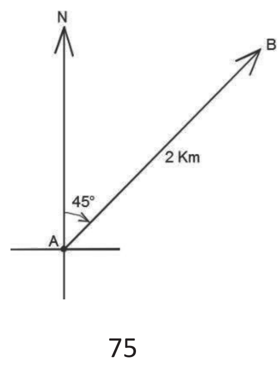
Solved Examples

1. A hunter starts at point A and travels through the bush 2 km in the direction 045° to point B. Give the bearing and draw a diagram.

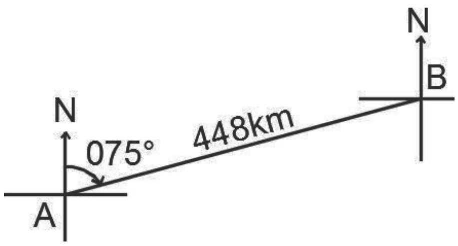
   Solution:

   Bearing: $\overrightarrow{AB} = (2 \text{ km}, 045°)$

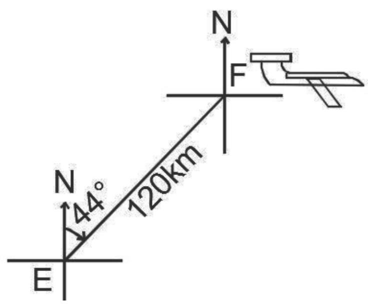
   Diagram:



2. A ship sailed 448 km from port A to port B on a bearing of $075°$. Write the ship's bearing and a diagram.
Solution: $\overline{AB} = (448 \text{ km}, 075°)$



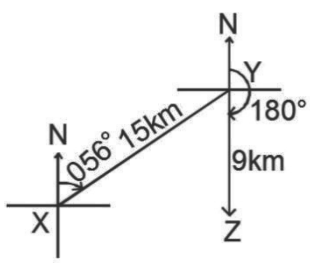
3. An airplane flies from a town E on a bearing of $44°$ to another town F at a distance of 120 km. Write the bearing of the airplane and draw a diagram.
Solution: $\overline{EF} = (120 \text{ km}, 044°)$



4. A trader that is moving in a city with his goods starts at point X and travels 15 km in the direction $056°$ to point Y. He then travels 9 km south to point Z.
   - Write the bearing from X to Y.
   - Write the bearing from Y to Z.
   - Draw a diagram to show the trader's movement.

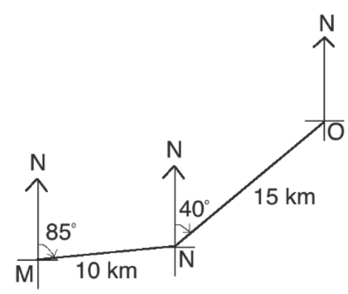
Solutions:

a. $\overrightarrow{XY} = (15 \text{ km}, 056°)$
b. $\overrightarrow{YZ} = (9 \text{ km}, 180°)$
c. Diagram →



5. A train traveled 10 km at a bearing of $085°$ from station M to station N. It then traveled 15 km at a bearing of $040°$ from the second station to the last station, O.
   a. Give the bearing from M to N.
   b. Give the bearing from N to O.
   c. Draw the diagram.

   Solutions:
   a. $\overrightarrow{MN} = (10 \text{ km}, 085°)$
   b. $\overrightarrow{NO} = (15 \text{ km}, 040°)$
   c. Diagram →

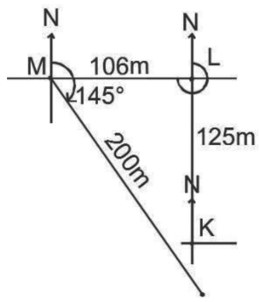


6. Ali ran at a distance of 125 metres due north, then 106 metres due west. He then ran 200 metres on a bearing of $145°$
   - Write the bearing for each of his 3 run
   - Draw a diagram of his movement.

   Solutions:

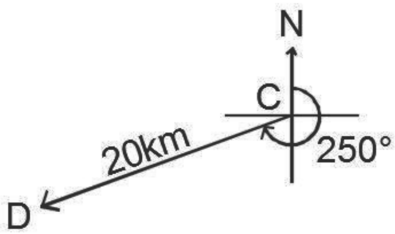
   a. $(125\text{ m}, 000°)$; $(106\text{ m}, 270°)$; $(200\text{ m}, 145°)$

   b.



Practice

1. Write the bearing from point C to D as shown in the diagram below:



2. A pilot starts at Airport S and flies 20 km in the direction $056°$ to Airport T. He then flies south to airport U at a distance of 15 km.
   a. Write the bearing from S to T.
   b. Write the bearing from T to U.
   c. Draw a diagram.
3. A boat travels 70 km from point W at a bearing of $280°$ to point X. It then travels 65 km from point X at a bearing of $120°$ to point Y.
   a. Write the bearing from W to X.
   b. Write the bearing from X to Y.
   c. Draw a diagram.
4. An ice-cream seller walks a distance of 120 metres due north, then 100 metres due west. He then walks 80 metres on a bearing of $140°$.
   a. Write the bearing for each of his 3 walks.
   b. Draw a diagram of his movement.

Lesson Title: Bearing problem solving – Part 2	Theme: Bearings
Practice Activity: PHM2-L122	**Class:** SSS 2

Learning Outcomes

By the end of the lesson, you will be able to:

1. Solve bearings problems with right triangles.
2. Apply Pythagoras' theorem and trigonometric ratios to calculate distance and direction.

Overview

This lesson is on solving bearing problems. You will use Pythagoras' theorem and trigonometric ratios to solve for distance and direction.

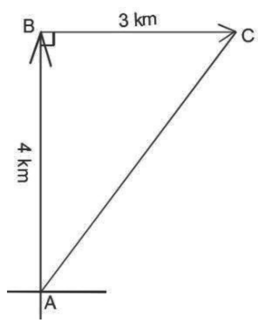
Recall that when you encounter a bearings problem, the first step is to draw a diagram. At times, this diagram forms a right-angled triangle. You can apply Pythagoras' theorem and trigonometry to find missing sides and angles in such triangles.

Solved Examples

1. Hawa walked 4 km from point A to B in the north direction, then 3 km from point B to C in the east direction.
   a. How far is she from her original position?
   b. What is the bearing from A to C?

   Solutions:

   First, draw a diagram:



   Draw her movement in the north direction and the east direction. These two lengths can be connected to form a triangle, as shown above.

   a. Use Pythagoras' theorem to find the distance from C to A:

$\|AB\|^2 + \|BC\|^2$	$=$	$\|AC\|^2$	Apply Pythagoras' theorem
$4^2 + 3^2$	$=$	$\|AC\|^2$	Substitute known lengths
$16 + 9$	$=$	$\|AC\|^2$	Simplify
25	$=$	$\|AC\|^2$	
$\sqrt{25}$	$=$	$\sqrt{\|AC\|^2}$	Take the square root of both sides

$$5\text{ km} = |AC|$$

She is 5 km from her original position.

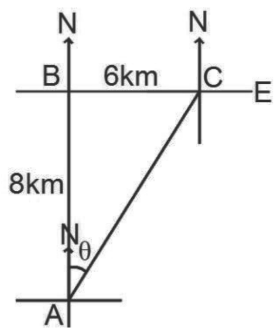
b. Use trigonometry to find the angle of the bearing from A to C. We can choose any trigonometric function, because we know the lengths of all 3 sides. Let's use tangent:

$\tan A$	$=$	$\frac{3}{4} = 0.75$	Apply the tangent ratio
$\tan^{-1}(\tan A)$	$=$	$\tan^{-1}(0.75)$	Take the inverse tangent of both sides
A	$=$	$\tan^{-1}(0.75)$	
A	$=$	$36.87°$	From the tangent table

The bearing from A to C is $\overrightarrow{AC} = (5\text{ km}, 037°)$.

2. Suppose a car leaves a point A and moves northwards for 8 kilometers to point B before turning right. It then moves eastward for 6 kilometres to point C. Represent the bearing of the car's displacement in distance bearing form.

Solution:



Use Pythagoras theorem to find the distance C to A.

$\|AB\|^2 + \|BC\|^2$	$=$	$\|AC\|^2$	Apply Pythagoras' theorem
$8^2 + 6^2$	$=$	$\|AC\|^2$	Substitute the known lengths
$64 + 36$	$=$	$\|AC\|^2$	Simplify
100	$=$	$\|AC\|^2$	
$\sqrt{100}$	$=$	$\sqrt{\|AC\|^2}$	Take the square root of both sides
10 km	$=$	$\|AC\|$	

Use the tangent ratio to find θ:

$\tan\theta$	$=$	$\frac{6}{8}$	Apply the tangent ratio
$\tan\theta$	$=$	0.75	
$\tan^{-1}(\tan A)$	$=$	$\tan^{-1}(0.75)$	Take the inverse tangent of both sides
A	$=$	$\tan^{-1}(0.75)$	
A	$=$	$36.87°$	

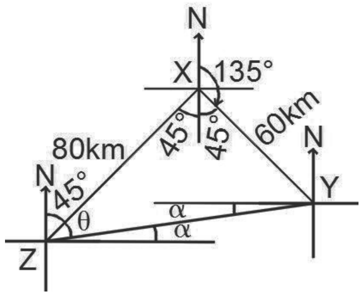
The bearing from A to C is $\overrightarrow{AC} = (10\text{ km}, 037°)$.

3. y is 60 km away from x on a bearing of $135°$, and z is 80 km away from x on a bearing of $225°$. Find the:

   a. Distance of z from y

   b. Bearing of z from y

Solutions:



a. Note that the points X, Y and Z form a right-angled triangle, where X is a right angle. Use Pythagoras' theorem:

$\lvert ZY\rvert^2$	$=$	$\lvert XZ\rvert^2 + \lvert XY\rvert^2$	Apply Pythagoras' theorem
$\lvert ZY\rvert^2$	$=$	$80^2 + 60^2$	Substitute the known lengths
$\lvert ZY\rvert^2$	$=$	$6400 + 3600$	Simplify
$\lvert ZY\rvert^2$	$=$	$10{,}000$	
$\sqrt{\lvert ZY\rvert^2}$	$=$	$\sqrt{10{,}000}$	Take the square root of both sides
ZY	$=$	100 km	

b. From the bearing diagram, the bearing of z from y is $270°$ – α. First find the angle labeled θ, and use it to find α and the bearing.

Use the tangent ratio to find θ:

$\tan\theta$	$=$	$\frac{60}{80} = \frac{3}{4}$	Apply the tangent ratio
$\tan\theta$	$=$	0.75	
θ	$=$	$\tan^{-1}(0.75)$	Take the inverse tangent of both sides
θ	$=$	$37°$	From the tangent table

Find α using the right angle at z: $\alpha = 90° - 45° + 37° = 8°$

Therefore, the bearing of z from y is: $270° - \alpha = 270° - 8° = 262°$.

Practice

1. A ship leaves port P and sails 15 km on a bearing of $045°$ to port Q. It then sails 20 km on a bearing of $135°$ to port R.
   a. Represent the information in a diagram.
   b. Calculate correct to the nearest whole number:
      i. The distance from P to R.
      ii. The bearing of R from P.
2. A village P is 10 km from a village Q, on a bearing $065°$. Another village R is 8 km from Q on a bearing of $155°$. Calculate:
   a. The distance of R from P to the nearest kilometre.
   b. The bearing of R from P, to the nearest degree.
3. Town P is 15 km due east of Q. Another town O is 8 km due south of Q.
   a. Draw a diagram for the problem
   b. Find the distance from O to P.
   c. Find the bearing of P from O.

Lesson Title: Bearing problem solving – Part 3	**Theme:** Bearings
Practice Activity: PHM2-L123	**Class:** SSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Solve bearings problems with acute and obtuse triangles.
2. Apply the sine and cosine rules to calculate distance and direction.

Overview

This lesson is on solving bearing problems. You will use the sine and cosine rules to solve for distance and direction.

Recall that the sine and cosine rules can be used to find missing sides and angles in acute and obtuse triangles. These may be applied to bearings problems that feature such triangles in their diagrams. Recall the information that is needed to apply either the sine rule or cosine rule:

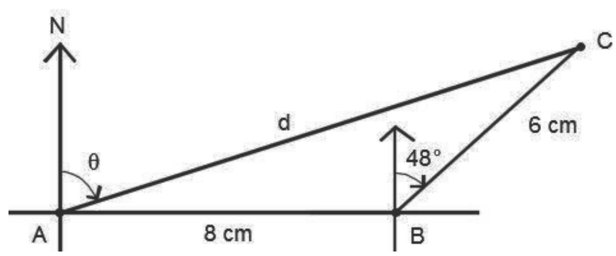
- **Sine Rule:** Use if given any 2 angles and 1 side, or 2 sides and the angle opposite 1 of them.
- **Cosine Rule:** Use if given two sides and the angle between them.

If needed, revise the sine and cosine rules. The sine rule is in lesson 114 of this book (PHM2-T3-W29-L114), and the cosine rule is in lesson 115 (PHM2-T3-W29-L115).

Solved Examples

1. A woman walks due east from point A to point B, a distance of 8 kilometres. She then changes direction and walks 6 km to point C on a bearing of 048°.
   a. What is the distance from A to C?
   b. What is the bearing of C from A?

   Solution:
   First, draw a diagram for the problem:



   a. Use the cosine rule to find $|AC|$, because we know 2 sides and the angle between them.

$\vert AC\vert^2$	$=$	$\vert AB\vert^2 + \vert BC\vert^2 - 2\vert AB\vert\vert BC\vert \cos B$	Formula
	$=$	$8^2 + 6^2 - 2(8)(6)\ \cos(90+48)°$	Substitute values from triangle

$$
\begin{aligned}
&= 64 + 36 - 96\ \cos 138° \\
&= 100 - 96\,(-0.7431) && \text{Substitute } \cos 138° = -0.7431 \\
&= 100 + 71.3376 \\
|AC|^2 &= 171.3376 \\
|AC| &= \sqrt{171.3376} = 13.09 \text{ km to 2 d.p.} && \text{Take the square root of both sides}
\end{aligned}
$$

b. We now have enough information to use the sine rule to find the angle of the bearing from C to A. Calculate the angle inside the triangle at A, and subtract it from 90° to find the bearing.

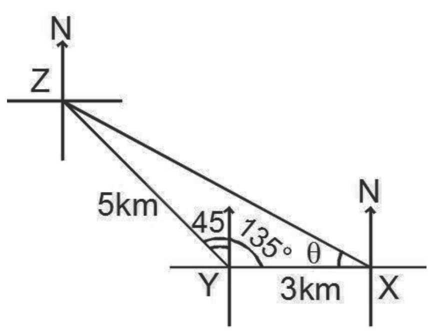
$$
\begin{aligned}
\frac{6}{\sin A} &= \frac{13.09}{\sin 138°} && \text{Substitute in the formula} \\
\sin A &= \frac{6 \sin 138°}{13.09} && \text{Solve for } A \\
\sin A &= \frac{6 \times 0.6691}{13.09} \\
\sin A &= 0.3067 \\
A &= \sin^{-1} 0.3067 && \text{Take the inverse sine of both sides} \\
A &= 17.86° && \text{Use the sine table}
\end{aligned}
$$

Round to $18°$, and subtract from $90°$ to find the bearing: $90° - 18° = 72°$

The bearing of C from A is $\overrightarrow{AC} = (13.09 \text{ km}, 72°)$.

2. A cyclist starts from a point X and rides 3 km due west to a point Y. He changes direction and rides 5 km at a bearing of $315°$ to a point Z.
   a. How far is he from the starting point, correct to the nearest km?
   b. Find the bearing of Z from X, to the nearest degree.

   Solutions:



a. Using the cosine formula. Let y be the distance from X to Z.

$$
\begin{aligned}
y^2 &= 5^2 + 3^2 - 2(5)(3)\cos 135° && \text{Substitute from the triangle} \\
y^2 &= 25 + 9 - 30(-0.7071) && \text{Substitute } \cos 135 = -0.7071 \\
y^2 &= 25 + 9 + 21.213 \\
y^2 &= 55.213 \\
y &= \sqrt{55.213} && \text{Take the square from both sides} \\
y &= 7.4305 \\
y &= |XZ| = 7 \text{ km}
\end{aligned}
$$

b. Use the sine rule to find the angle θ in the diagram. This can be added to $270°$ to find the bearing.

$$\frac{5}{\sin\theta} = \frac{7}{\sin 135}$$

$$\sin\theta = \frac{5\sin 135}{7}$$

$$\sin\theta = \frac{5(0.7071)}{7}$$ Substitute $\sin 135 = 0.7071$

$$\sin\theta = 0.5051$$

$$\theta = \sin^{-1}(0.5051)$$

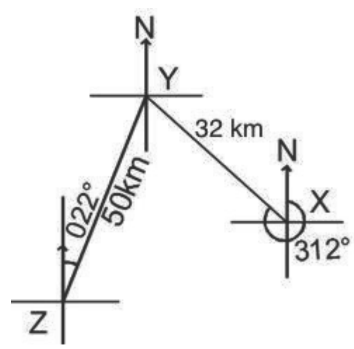
$$\theta = 30.34°$$

Bearing of Z from X: $270° + \theta = 270 + 30° = 300°$

Practice

1. Y is 60 km away from X on a bearing of $135°$. Z is 80 km away from X on a bearing of $225°$. Find the:
   a. Distance of Z from Y
   b. Bearing of Z from Y
2. The diagram below shows the positions of three points X, Y and Z on a plane. The bearing of Y from X is $312°$ and that of Y from Z is $022°$. If $|XY| = 32$ km and $|ZY| = 50$ km, calculate, correct to one decimal place:

   a. $|XZ|$
   b. The bearing of Z from X



Lesson Title: Bearing problem solving – Part 4	**Theme:** Bearings
Practice Activity: PHM2-L124	**Class:** SSS 2

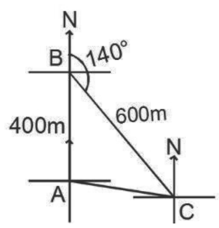
Learning Outcome
By the end of the lesson, you will be able to solve bearing problems using the appropriate rules or theorems.

Overview

This lesson is on solving bearing problems. You will use information from the previous lessons to solve various bearings problems, including: triangle properties, Pythagoras' theorem, trigonometric ratios, and sine and cosine rules.

Solved Examples

1. A motorist rode 400 metres due north, then 600 metres at a bearing of $140°$. How far is he from his original location?
   Solution:
   First, draw a diagram:



   Step 1. Find the angle inside the triangle at B: $B - 180° - 140° = 40°$
   Step 2. Apply the cosine rule.

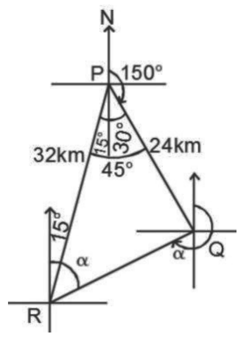
$\lvert AC\rvert^2$	$=$	$\lvert AB\rvert^2 + \lvert BC\rvert^2 - 2\lvert AB\rvert\lvert BC\rvert \cos B$	Formula
	$=$	$400^2 + 600^2 - 2(400)(600)\cos 40$	Substitute values from the triangle
	$=$	$160{,}000 + 360{,}000 - 480{,}000 \cos 40°$	Substitute $\cos 40° = 0.7660$
	$=$	$520{,}000 - 367{,}680$	
$\lvert AC\rvert^2$	$=$	$152{,}320$	
AC	$=$	$\sqrt{152{,}320} = 390.3$	Take the square root of both sides

   Answer: The motorist is 390.3 metres from his original location.

2. The bearing of Q from P is $150°$ and the bearing of P from R is $015°$. Q and R are 24 km and 32 km, respectively, from P.
   a. Represent this information in a diagram.
   b. Calculate the distance between Q and R, correct to two decimal places.
   c. Find the bearing of R from Q, correct to the nearest degree.

Solution:

a. Diagram:



b. By sine rule:

$$QR^2 = 24^2 + 32^2 - 2(24)(32)\cos 45°$$ Substitute values from the triangle

$$= 576 + 1024 - 1536\cos 45°$$ Simplify

$$= 576 + 1024 - 1536(0.7071)$$ Substitute $\cos 45° = 0.7071$

$$QR^2 = 1600 - 1086.1056$$

$$QR^2 = 513.8944$$

$$QR = \sqrt{513.8944} = 22.67 \text{ km}$$ Take the square root of both sides

c. Let the bearing of R from Q be β. But $\beta = \alpha + 180°$ and $\alpha = \theta + 15°$, where θ is the interior angle of the triangle at R.

Find θ using the sine rule:

$$\frac{24}{\sin\theta} = \frac{22.67}{\sin 45}$$

$$\sin\theta = \frac{24\sin 45}{22.67}$$

$$\sin\theta = \frac{24\times 0.7071}{22.67}$$

$$\sin\theta = \frac{16.9704}{22.67}$$

$$\sin\theta = 0.7486$$

$$\theta = \sin^{-1}(0.7486)$$

$$\theta = 48.47$$

$$\therefore \alpha = 48.47° + 15° = 63.47°$$

Hence $\beta = 63.47° + 180°$

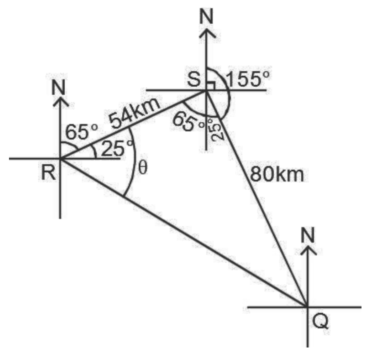
$$\beta = 243.47°$$

The bearing of R from Q is $243°$.

3. A ship sails from point R on bearing $065°$ to port S, a distance of 54 km. It then sails on a bearing of $155°$ to port Q, a distance of 80 km. Find, correct to one decimal place:
   a. The distance between R and Q.
   b. The bearing of Q from R.

   Solutions:

   First, draw a diagram:



a. Note that the interior angle of the triangle at S is a right angle. Therefore, Pythagoras' theorem can be used to find $|RQ|$.

By Pythagoras' theorem,

$$
\begin{aligned}
|RQ|^2 &= 54^2 + 80^2 \\
&= 2916 + 6400 \\
&= 9316 \\
|RQ| &= \sqrt{9316} \\
|RQ| &= 96.5 \text{ km}
\end{aligned}
$$

b. Use the tangent ratio to find the angle labeled θ in the diagram. This can be added to $65°$ to find the bearing of Q from R.

$$
\begin{aligned}
\tan\theta &= \frac{80}{54} = \\
\theta &= \tan^{-1}(1.4815) \\
\theta &= 55.98 \\
\theta &= 56°
\end{aligned}
$$

The bearing of Q from R is $65° + \theta = 65° + 56° = 121°$.

Practice

1. A ship travels 3 km from a port P on a bearing of $080°$ and then 4 km on a bearing $047°$. Find its distance and bearing from P.
2. A boat sails 4 km due west and then 8 km due south. Find it bearings from its original position to the nearest degree.
3. A man starts at A and walks 2 km on a bearing of $017°$ to point B. He then walks 3 km on a bearing of $107°$ to point C. What is the bearing of C from A?
4. A bus moves from Moyamba on a bearing of $75°$ to Massiaka, a distance of 130 km. It arrives in Massiaka, but changes course and moves to Kono, a distance of 90 km, on a bearing of $160°$.
    a. Find the distance from Moyamba to Kono, to the nearest kilometre.
    b. Find the bearing of Kono from Moyamba, to the nearest degree.
5. John's house is 20 km from his school on a bearing $80°$. Kadie's house is 12 km from the same school on a bearing of $155°$. Calculate the distance of John's house from Kadie's house to the nearest kilometre.

Lesson Title: Drawing pie charts	Theme: Statistics and Probability
Practice Activity: PHM2-L125	**Class:** SSS 2

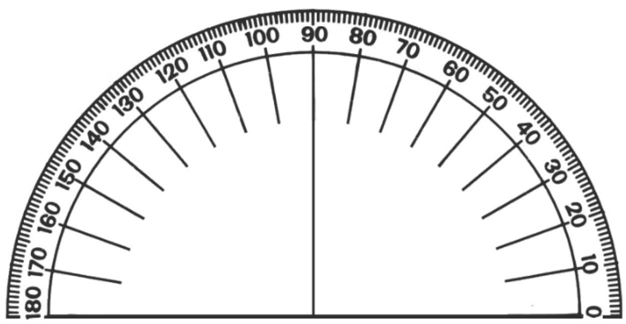
Learning Outcome
By the end of the lesson, you will be able to draw pie charts from given data.

Overview

A pie chart is a type of graph in which a circle is divided into sectors that each represent a portion of the whole. Each sector of the pie chart is a certain percentage of the whole, and the percentages in the chart add up to 100%.

To draw a pie chart accurately, we must use a protractor. The entire circle is 360°, and each segment is one part of the whole. We must find what part of the whole each segment is, and assign a degree to it. Then, we use a protractor to draw an angle inside the pie chart with that degree.

If you do not have a protractor, you can make one with paper. Trace this protractor with a pen onto another piece of paper.



Solved Examples

1. A group of pupils was surveyed to find their favourite fruits. The data is in the table below. Create a pie chart for this data.

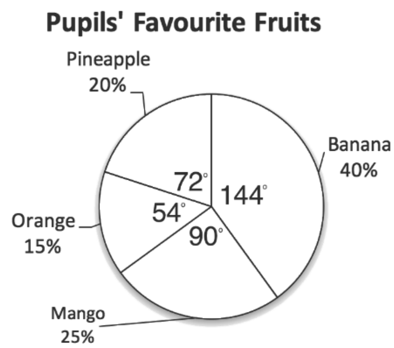
Favourite Fruit	Frequency	Percentage
Banana	16	40%
Mango	10	25%
Orange	6	15%
Pineapple	8	20%
TOTAL	40	100%

Solution:
First, calculate the degree measure for each fruit. Give each frequency as a fraction of the whole (40), and multiply that fraction by 360°.

$$\text{Banana} = \frac{16}{40} \times 360° = 144°$$
$$\text{Mango} = \frac{10}{40} \times 360° = 90°$$
$$\text{Orange} = \frac{6}{40} \times 360° = 54°$$
$$\text{Pineapple} = \frac{8}{40} \times 360° = 72°$$

Now, draw the pie chart using the degrees you found:

- Draw the empty circle and heading, "Pupils' Favourite Fruits".
- Draw each segment using a protractor. For example, here are the steps for drawing the "Banana" segment:
  - Place the centre of the protractor on the centre of the pie chart and place the bottom of the protractor exactly along one radius of the circle.
  - Find the angle measurement for banana, 144°.
  - Use a straight edge to draw another radius from the centre at 144°.
- Label each sector as shown.



2. The table below shows how a family spends their money in one day.

Items	**Amount Spent**
Food	Le 15,000.00
House rent	Le 9,000.00
Electricity	Le 10,000.00
Transportation	Le 24,000.00
Other	Le 2,000.00

a. Represent the information on a pie chart.
b. What percentage does the family spend on transportation?

Solutions:

a. **Step 1.** Find each quantity as a fraction of the whole, and multiply it by 360 to find the degree of its segment.
Calculate the total: $15{,}000 + 8{,}000 + 10{,}000 + 24{,}000 + 3{,}000 = 60{,}000$

Calculate the measures of the segments:

$$\text{Food} = \frac{15{,}000}{60{,}000} \times 360° = 90°$$

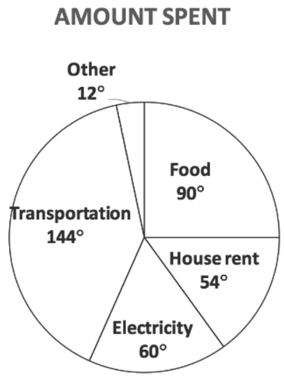
$$\text{House rent} = \frac{9{,}000}{60{,}000} \times 360° = 54°$$

$$\text{Electricity} = \frac{10{,}000}{60{,}000} \times 360° = 60°$$

$$\text{Transportation} = \frac{24{,}000}{60{,}000} \times 360° = 144°$$

$$\text{Other} = \frac{2{,}000}{60{,}000} \times 360° = 12°$$

Step 2. Use a protractor to draw the pie chart:



b. Find transportation as a percentage of the total amount spent:

$$\frac{24{,}000}{60{,}000} \times 100\% = 0.4 \times 100\% = 40\%$$

Practice

1. In a high school, a census was taken regarding the favourite subjects of pupils. It gave the following results:

Subjects	Number of pupils
Mathematics	200
English Language	50
Literature	90
Biology	100
Government	60
History	80
Economics	140

   a. Represent the above information on a pie chart.
   b. What percentage of pupils are enrolled in biology?
2. The cost incurred on infrastructural development by a mining company is as follows: Wages: 45%, Taxes: 15%; Material: 30%; Transport: 10%. Show this information in a pie chart.
3. In a secondary school in Sierra Leone, a census taken regarding the various ethnic groups gives the following results.

Ethnic group	Number of pupils
Temne	80
Mende	100
Susu	60
Krio	200
Fullah	50
Limba	90
Others	140

a. Represent the above data on a pie chart.
b. What percentage of pupils fall into other groups?
c. What percentage of pupils are either Temne or Mende?

Lesson Title: Interpretation of pie charts	**Theme:** Statistics and Probability
Practice Activity: PHM2-L126	**Class:** SSS 2

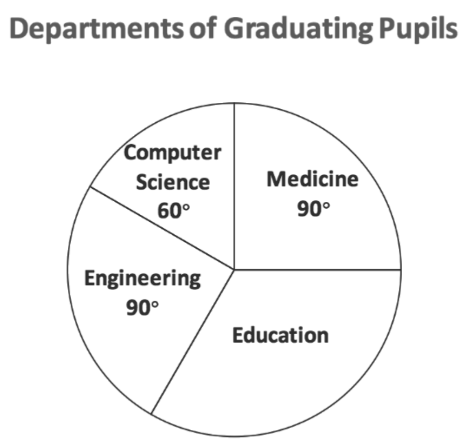
Learning Outcome
By the end of the lesson, you will be able to interpret and solve pie chart problems.

Overview

This lesson is on interpreting pie charts and solving problems related to pie charts. Many of these problems involve calculating the number of items that fall into a given sector of a pie chart. This is often done by calculating the size of the segment as a proportion of the whole. Proportions can be made using degrees ($\frac{x}{360°}$ where x is the degree measure of the proportion of concern), or they may be percentages. Multiply the proportion by the total number represented in the chart to find the number in a given segment.

Solved Examples

1. This year, 1,000 pupils graduated from a certain university. The pie chart below shows the departments they graduated from. Use it to answer the questions.



a. How many pupils graduated from the education department?
b. What percentage of the total graduated from the medicine department?

Solutions:

a. This solution involves multiply steps:

Step 1. Find the degree measure of Education:

Education measure $= 360° - (90° + 60° + 90°) = 120°$

Step 2. Multiply the proportion by the total number of pupils to calculate those studying Education:

Number in Education $= \frac{120}{360} \times 1{,}000$

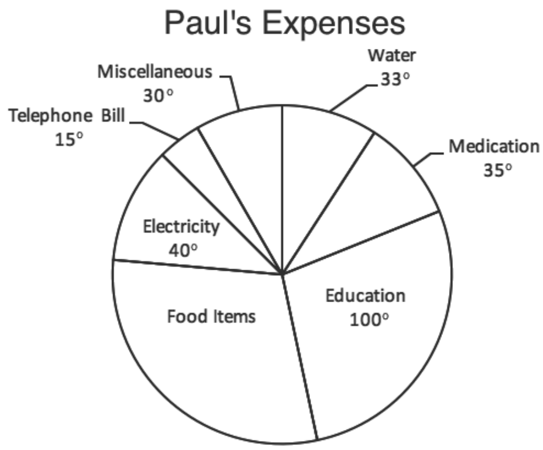
$= \frac{1}{3} \times 1{,}000$

$$= 333.3$$
$$= 333 \quad \text{Round to a whole number}$$

Answer: 333 pupils are graduating from education.

b. Write medicine as a percentage of the whole using its degree measure:
Pupils graduating from medicine $= \frac{90}{360} \times 100\% = 25\%$

2. In the year 2016, Paul's monthly salary was Le $24{,}000.00$. He spent his money on medication, education of his children, food items, electricity charges, telephone bills, water and miscellaneous expenses. His monthly expenses are shown in the pie chart below.



a. How much did he spend each month on food items?
b. What percentage of his total was allocated to Education?

Solutions:

a. This solution involves multiple steps:
Step 1. Find the degree measure of food items:
Food items measure $= 360° - (100° + 35° + 33° + 30° + 15° + 40°)$
$= 360° - 253°$
$= 107°$
Step 2. Multiply the proportion by the total amount to calculate for food items.
Spent on food $= \frac{107}{360} \times 24{,}000.00$
$= \text{Le}7{,}133.33$

b. Write education as a percentage of the whole using its degree measurement.
Education$= \frac{100}{360} \times 100\% = \frac{10}{36} \times 100\% = 27.8\%$

3. Using the pie chart in Solved Example 2, find:
a. The percentage of money spent on medication.
b. The amount spent on miscellaneous items in one year.

Solutions:

a. Calculate the percentage using the degree measurement on medication:

Medication $= \frac{35}{360} \times 100\% = 9.7\%$

b. **Step 1.** Find the percentage spent on miscellaneous using the degree:

Percentage spent on miscellaneous $= \frac{30}{360} \times 24{,}000.00 = 8.3\%$

Step 2. Use the percentage to calculate for the amount spent on miscellaneous items per month:

Monthly spending on miscellaneous $= \frac{8.3}{100} \times 24{,}000 = \text{Le } 1{,}992.00$

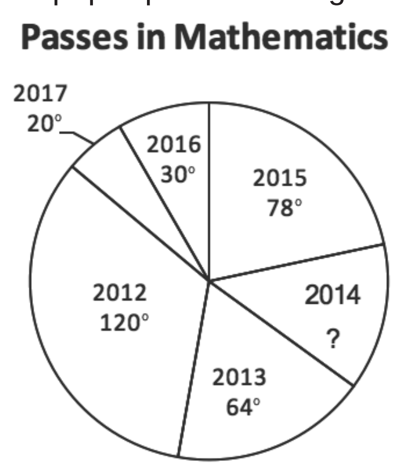
Step 3. Multiply by 12 to find the spending in one year:

Spending on miscellaneous in 1 year $= \text{Le}1{,}992.00 \times 12 = \text{Le } 23{,}904.00$

Practice

1. The pie chart represents the number of items of food stuffs for sale in a shop. If there are 80 packs of biscuits, how many packs of chocolate are there?

2. The pie chart below shows the passes in Mathematics in a secondary school from 2012 to 2017. In total, 900 pupils passed during the 6-year period.



   a. How many pupils passed in the year 2014?
   b. What percentage of passes occurred in either 2012 or 2013?

Lesson Title: Drawing and interpretation of bar charts	Theme: Statistics and Probability
Practice Activity: PHM2-L127	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to draw and interpret bar charts.

Overview

This lesson is on drawing and interpreting bar charts based on given data. Bar charts are used to compare different quantities. Therefore, data is given in a table showing different quantities or frequencies, and you are asked to draw a bar chart to represent the data. You will then answer questions based on the information in the chart.

Solved Examples

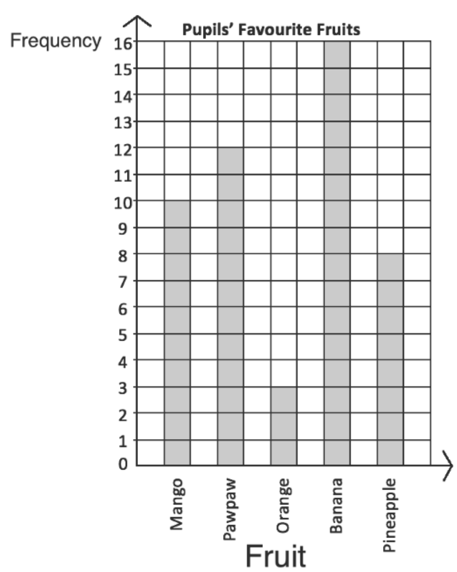
1. A teacher surveyed a class to learn the favourite fruits of the class members. The result of the survey is shown in the following frequency table:

Favourite Fruits	
Mango	10
Pawpaw	12
Orange	3
Banana	16
Pineapple	8

   a. Draw a bar chart to represent the data.
   b. Which fruit is most popular?
   c. Which fruit is least popular?
   d. What percentage of pupils prefer banana?
   e. How many pupils prefer either mango or pawpaw?

Solutions:

a. Bar chart:



b. Banana is most popular. It has the highest bar, which indicates the greatest frequency.
c. Orange is the least popular. It has the lowest bar, which indicates the lowest frequency.
d. Find the total number of pupils: $10 + 12 + 3 + 16 + 8 = 49$ pupils
Find the number of pupils who prefer banana (16) as a percentage of the whole: $\frac{16}{49} \times 100\% = 32.7\%$.
e. The word "or" tells us to include both categories of pupils: those who prefer mango, and those who prefer pawpaw. Add the frequencies: $10 + 12 = 22$ pupils.

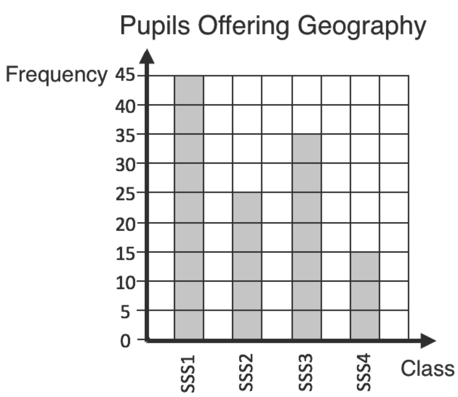
2. The table below shows the number of pupils that are offering geography in a senior secondary school Freetown.

Level	No. of Pupils
SSS1	45
SSS2	25
SSS3	35
SSS4	15

a. Draw a bar chart to present the data.
b. Which class has the greatest number of pupils that offer geography?
c. Which class has the fewest pupils that offer geography?
d. What is the difference between the number of pupils that offer geography in SSS 1 and business SSS 2?

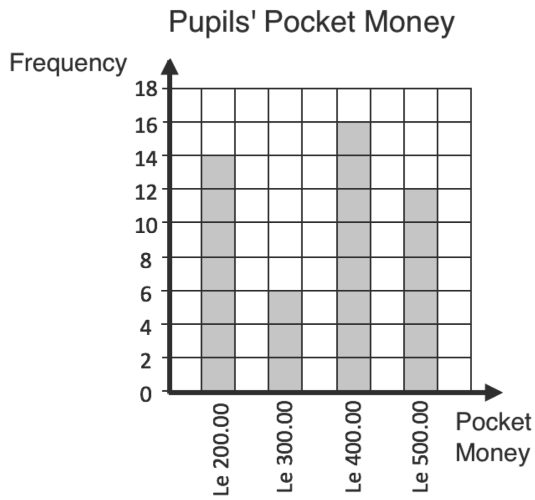
Solutions:

a. Draw the graph as shown:



b. SSS 1
c. SSS 4
d. 20 pupils

3. The bar chart below shows the pocket money received by pupils in a class in a week. Use the bar chart to answer the questions below.



a. Find the total number of pupils in the class.
b. How many pupils received either Le 300.00 or Le 500.00 for a week?
c. What percentage of pupils received Le 400.00 per week?

Solutions:

a. Add the heights of the bars: $14 + 6 + 16 + 12 = 48$ pupils
b. Add the heights of the bars for Le 300.00 and Le 500.00:
   $6 + 12 = 18$ pupils
c. The heights of the bar for Le 400.00 is 16 pupils. Find 16 as a percentage of 48, the total number of pupils in the class:
   $\frac{16}{48} \times 100 = 33.3\%$

Practice

1. The following is the result a survey conducted by an NGO on the occupations of adults in a community in Kailahun district. Draw a bar chart to show the information.

Occupation	**No.**
Farmers	40
Traders	10
Health workers	15
Teachers	25
Other	20

2. The table below shows the marks of pupils on a test. No pupil scored lower than 40% or higher than 85%. Draw a bar chart for the information and use it to answer the questions below.

Marks	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%
No. of Pupils	1	2	4	3	0	2	5	8	1	2

a. How many pupils took the test?
b. If 55% is the passing mark, how many pupils passed the test?
c. What percentage of pupils passed the test?
d. How many pupils scored 65% or 70%?

Lesson Title: Mean, Median, and Mode	**Theme:** Statistics and Probability
Practice Activity: PHM2-L128	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate the mean, median, and mode of a list of ungrouped data.

Overview

This lesson is on calculating the mean, median, and mode of a list of ungrouped data. Ungrouped data is information that is listed individually. We will work with grouped data in future lessons.

The **mean** is a number that can tell us where the middle of the data is. It is also commonly known as the "average". To find the mean of a set of data, add the numbers together and divide the total by the number of items. The quotient is the mean.

The number in the middle when the numbers are listed in ascending or descending order is called the **median**. When there is an even number of items in the list, there are 2 numbers in the middle. The median is found by calculating the mean of these 2 numbers.

The **mode** is the number that appears most often in a list. It can often be easily observed. If no number appears more than once, there is no mode. If multiple numbers appear more than once, there are multiple modes.

Solved Examples

1. Ten pupils received the following scores on their Maths exam: 87, 100, 76, 92, 90, 95, 85, 67, 99 and 95. Calculate the mean, median, and mode of the scores.
   Solution:
   Mean:
   Add the numbers: $87 + 100 + 76 + 92 + 90 + 95 + 85 + 67 + 99 + 95 = 886$
   Divide by the number of pupils: $886 \div 10 = 88.6$
   The mean score is 88.6.
   Median:
   List the numbers in ascending order: 67, 76, 85, 87, 90, 92, 95, 95, 99, 100
   Identify the middle of the list: 90, 92
   Since there is not one number in the middle, find the mean of the 2 numbers in the middle. Add them together and divide by 2: median $= \frac{90+92}{2} = 91$
   Mode:
   Note that 95 is the only number that occurs more than once, so it is the mode.

2. On her exams, Fatu scored x% in Mathematics, 90% in English, 95% in Biology, and 80% in Chemistry. If her mean score for all subjects was 88%, what is the value of x?
Solution:
This problem must be solved backwards. We are given the mean, and we must solve for one of the numbers in the list. The first step is to set up an equation. Then solve for x using algebra.

88	$=$	$\frac{x+90+95+80}{4}$	Set up the equation
88	$=$	$\frac{x+265}{4}$	Simplify
4×88	$=$	$x + 265$	Multiply throughout by 4
352	$=$	$x + 265$	
$352 - 265$	$=$	x	Subtract 265 from both sides
87	$=$	x	

3. Eleven pupils estimated the length of a classroom in metres as follows: $6, 9, 11, 11, 12, 12, 12, 12, 13, 15, 19$. Find the following for their estimates:
   a. Mode b. Median c. Mean

Solutions:
a. The mode is 12 m, since 12 is the most frequent piece of data
b. The median is 12, which is in the middle.
c. The mean is $\frac{6+9+11+11+12+12+12+12+13+15+19}{11} = \frac{132}{11} = 12$

4. Find x if the mean of five numbers $1, 3, 6, 8$ and x is 6.
Solution:
Apply the formula for mean, and solve for x:

$$\text{Mean} = 6 = \frac{1+3+6+8+x}{5}$$
$$5 \times 6 = 18 + x$$
$$30 = 18 + x$$
$$30 - 18 = x$$
$$12 = x$$

5. The mean weight of 8 buckets of water is 6.1 kg. The weight of 7 buckets are recorded as follows (in kilogrammes): 9.8, 6.4, 7.2, 5.4, 3.2, 4.2, and 2.9. what is the weight of the eighth bucket?
Solution:

Mean $= 6.1$	$=$	$\frac{9.8+6.4+7.2+5.4+3.2+4.2+2.9+w}{8}$	Mean formula
6.1	$=$	$\frac{39.1+w}{8}$	Simplify
6.1×8	$=$	$39.1 + w$	Multiply both sides by 8
48.8	$=$	$39.1 + w$	
$48.8 - 39.1$	$=$	w	Subtract 39.1 from both sides

$$9.7 = w$$

The weight of the 8th bucket is 9.7 kg.

Practice

1. Agnes received the following scores in examinations in 8 subjects: 68, 75, 80, 56, 77, 68, 98, and 87. Calculate the mean, median and mode of her scores, correct to the nearest whole number.
2. The shoe sizes of five pupils are $10, 9, 10, 11$ and 8.
    a. Find the median shoe size.
    b. Find the modal shoe size.
    c. Calculate the mean shoe size, correct to 1 decimal place.
3. The number of goals scored by a team in 9 football matches are as follows:
    $3, 5, 7, 7, 8, 8, 8, 11, 15$. Which of the following statements are true of these scores:
    a. The mean is greater than the mode.
    b. The mode and the median are equal.
    c. The mean, median and mode are equal.
4. A group of pupils measured their pepper seedlings in the school garden. They obtained the following results: 10.8 cm, 10.9 cm, 10.7 cm, 10.8 cm, 10.8 cm, 11.8 cm, 10.7 cm, 10.9 cm and 10.8 cm. Calculate the following, correct to 1 decimal place:
    a. Median
    b. Mode
    c. Mean

Lesson Title: Mean, median, and mode from a table or chart	Theme: Statistics and Probability
Practice Activity: PHM2-L129	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate mean, median, and mode from a frequency table or a bar chart.

Overview

In the previous lesson, you calculated mean, median, and mode from a list. In this lesson, you will do calculations on data from a table or chart.

To calculate **mean**, use multiplication to find the sum of numbers that are the same. In other words, multiply each value by its frequency. Add all of the results together.

Finally, divide by the number of items as usual.

To find the **median**, identify where the item in the middle falls (in which bar of the chart, or row/column of the frequency table). Give the value that this bar or column/row represents.

Recall that mode is the value with the highest frequency. Therefore, it would have the tallest bar, or the greatest value for frequency in a frequency table.

Solved Examples

1. The table below shows the distribution of marks on an assignment that a class completed. No one scored below 6 marks. Find the mean, median, and mode of the scores.

Marks	6	7	8	9	10
Frequency	3	9	4	3	1

Solution:

Mean:

Find the total marks:

$$6(3)+7(9)+8(4)+9(3)+10(2)=18+63+32+27+10=150$$

Find the number of pupils in the class by adding the frequencies:

$$3+9+4+3+1=20$$

Divide the total marks by the number of pupils: $150 \div 20 = 7.5$

Mean = 7.5 marks

Median:

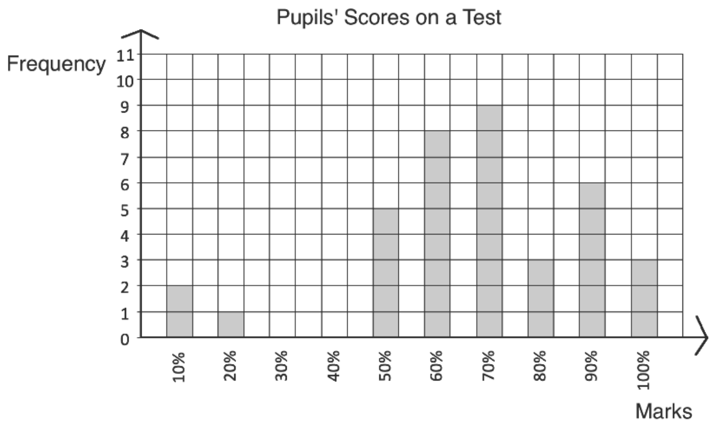
There are 20 pupils in the class. The median is the mean score of the 2 pupils in the middle, which are the 10^{th} and 11^{th} pupils. Find the scores of the 10^{th} and 11^{th} pupils by counting in the frequency table. The first 3 pupils scored 6,

then the next 9 pupils scored 7 marks. That makes 12 pupils in total. Therefore, the 10th and 11th pupils scored 7 marks.
Median = 7 marks

Mode:

Mode is the mark that appears most often. In other words, it has the highest frequency.
Mode = 7

2. The bar chart below shows marks that pupils achieved on a test, as percentages. Find the mean, median, and mode.



Solution:

Mean:

Find the sum of the marks:

$$2(10) + 20 + 5(50) + 8(60) + 9(70) + 3(80) + 6(90) + 3(100)$$
$$= 20 + 20 + 250 + 480 + 630 + 240 + 540 + 300$$
$$= 2{,}480$$

Find the number of pupils: $2 + 1 + 5 + 8 + 9 + 3 + 6 + 3 = 37$
Divide: $2480 \div 37 = 67.02\%$

Median:

Since there are 37 pupils in the class, the 19th pupil is in the middle. There are 18 pupils with a lower score, and 18 pupils with a higher score. Locate the 19th pupil in the bar chart by counting the bars, from least to greatest. The 19th pupil is within 70%. Therefore, median = 70%

Mode:

The highest bar is at 70%; therefore, the mode is 70%.

3. The table below shows the weight distribution of 24 wrestlers. Calculate in kilogrammes the mean, median and mode of the weight of the wrestlers.

Weight in kg	150	160	170
No. of wrestlers	6	10	4

Solution:

Mean:

Find the total weight:

$$6(150) + 10(160) + 4(170) = 900 + 1{,}600 + 680 = 3{,}180$$

Find the number of wrestlers:

$$6 + 10 + 4 = 20$$

Divide the total weight by the number of wrestlers:

$$3{,}180 \div 20 = 159 \text{ kg}$$

Therefore, Mean $= 159$ kg.

Median:

Since there are 20 wrestlers, the median is the mean of the weights of the 10^{th} and 11^{th} wrestlers. Wrestlers 10 and 11 both fall into the column for 160 kg. Therefore, median $= 160$ kg.

Mode:

The weight which appears most frequently is the mode: 160 kg.

Practice

1. 20 candidates sat an exam. The table below shows their scores at the time their final results were released. Find the mean, median and mode of their scores.

Scores	2	3	4	5	6	7
No. of Pupils	2	4	7	2	3	2

2. The table below shows the weights, to the nearest kilogramme, of twelve pupils in a Mathematics class.

Weight in kg	55	57	59	61	63
No. of Pupils	2	1	2	4	3

   a. Draw a bar chart to illustrate the above information.
   b. Find the following: i. The mode; ii. The median of the distribution.
   c. Calculate the mean weight correct to the nearest kilogramme.

Lesson Title: Grouped frequency tables	**Theme:** Statistics and Probability
Practice Activity: PHM2-L130	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to present and interpret grouped data in frequency distribution tables.

Overview

Previous statistics lessons on lists and bar charts have handled ungrouped data. This is the first lesson on grouped data. Grouped data is divided into class intervals that are equal in size, and each cover a range of values.

Grouped data can be arranged in a frequency table. Each class interval is represented on one row.

For example, consider the grouped frequency table:

Pupils' Heights	
Height	**Frequency**
150 – 159 cm	8
160 – 169 cm	7
170 – 179 cm	5
Total	20

There are 3 class intervals represented in the table: 150-159 cm, 160-169 cm, and 170-179 cm. The table tells us how many pupils fall into each class interval. For example, there are 8 pupils in the class interval 150-159 cm. We do not know their exact heights.

Solved Examples

1. Use the frequency table above (Pupils' Heights) to answer the questions below.
   a. Which class interval did the greatest number of pupils fall into?
   b. How many pupils are 160 cm or taller?
   c. How many pupils are shorter than 170 cm?

   Solutions:
   a. The greatest number of pupils fall into 150-159 cm.
   b. Add the frequencies for heights 160 cm and taller:
   $7 + 5 = 12$ pupils
   c. Add the frequencies for all heights under 170 cm:
   $8 + 7 = 15$ pupils

2. The frequency table below shows the scores that one class of pupils received on an exam. Use the table to answer the questions.

Pupils' Scores on an Exam	
Marks (%)	**Frequency**
41-50	3
51-60	2
61-70	4
71-80	11
81-90	10
91-100	5

a. How many pupils are in the class?
b. If pupils need to score more than 60% to pass, how many pupils passed?
c. How many pupils failed?
d. How many pupils scored more than 80%?

Solutions:

a. Add the frequencies to find the total number of pupils: $3 + 2 + 4 + 11 + 10 + 5 = 35$ pupils
b. Add the frequencies for class intervals greater than 60%: $4 + 11 + 10 + 5 = 30$ pupils
c. Add the frequencies for class intervals 60% or lower: $3 + 2 = 5$ pupils
d. Add the frequencies for class intervals greater than 80%: $10 + 5 = 15$ pupils

3. The following are the shoe sizes of 25 adults: 32, 35, 38, 41, 43, 42, 45, 38, 39, 39, 36, 34, 37, 42, 36, 41, 34, 33, 42, 36, 35, 40, 44, 37, 35.
   a. Draw a frequency table using class intervals 32-34, 35-37, 38-40, 41-43, 44-46.
   b. Which interval do the greatest number of pupils fall into?
   c. A certain type of shoe is only available in size 40 or smaller. How many people could wear that shoe?

   Solutions:

   a. First, write the shoe sizes in ascending order: 32, 33, 34, 34, 35, 35, 35, 36, 36, 36, 37, 37, 38, 38, 39, 39, 40, 41, 41, 42, 42, 42, 43, 44, 45.
   Next, draw and fill the frequency table:

Adults' Shoe Sizes	
Size	**Frequency**
32-34	4
35-37	8
38-40	5
41-43	6
44-46	2
Total	25

   b. The greatest number of pupils fall into size 35-37.
   c. Add the frequencies in sizes 40 and smaller: $4 + 8 + 5 = 17$ people

Practice

1. The table gives the distribution of the ages in years of all persons in a town who were under the ages of 40 years on 30th June 1990. The frequency represents thousands of people.

Age	Frequency (thousands)
0 – 4	2
5 – 9	3
10 – 14	6
15 – 19	15
20 – 24	12
25 – 29	7
30 – 34	4
35 – 39	1

   Using the table, calculate:
   a. The total number of persons between 15 and 29 years.
   b. The total number of persons under 20 years.

2. The following gives the weights of 50 children in a sports camp.

64	14	17	48	58	60	43	44	15	32
47	21	23	37	51	26	18	36	22	24
43	45	29	33	46	38	19	38	36	30
31	72	57	41	41	44	54	24	26	41
22	25	35	35	37	36	51	52	19	62

   Using class intervals $10 - 19, 20 - 29,$ ... prepare a frequency table and use it to calculate the:
   a. Total number of children with weights less than 30.
   b. Total number of children weighing between 50 and 79.

3. The table below shows the distribution of times spent by motorists awaiting purchases of petrol at a filling station.

Waiting Time (Min)	No. of Motorists
15 – 19	3
20 – 24	7
25 – 29	10
30 – 34	25
35 – 39	18
40 – 44	2

   Calculate:
   a. The total number of motorists who purchased petrol.
   b. The total number of motorists awaiting purchase of petrol at the filling station for 30 to 44 minutes.

Lesson Title: Drawing histograms	Theme: Statistics and Probability
Practice Activity: PHM2-L131	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to present and interpret grouped data in histograms.

Overview

Grouped data cannot be presented in a bar chart. It must be presented in a histogram. Histograms look like bar charts, but they are actually a different tool for representing data.

Like a bar chart, a histogram consists of vertical bars. However, in histograms, the bar does not represent only 1 piece of data, but a range of data. Each bar represents a **class interval**. Recall that a class interval is a group, represented in 1 row of a grouped frequency table.

In histograms, each bar is centred on a **class mid-point** on the x-axis. Class mid-points are the points that lie exactly in the middle of class intervals. Sometimes the class mid-point is labelled on the x-axis, and sometimes the high and low values of each class interval (called "class boundaries") are labelled. The vertical axis is frequency, which is the same as with bar charts. The bars of a histogram touch each other, unlike bar charts. Histogram bars touch each other because they represent continuous intervals.

Solved Examples

1. Create a histogram for the data shown in the frequency table. →

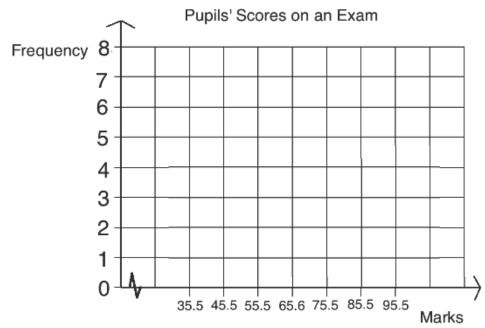
Pupils' Scores on an Exam	
Marks	**Frequency**
31-40	1
41-50	2
51-60	3
61-70	1
71-80	3
81-90	7
91-100	3

Solution:

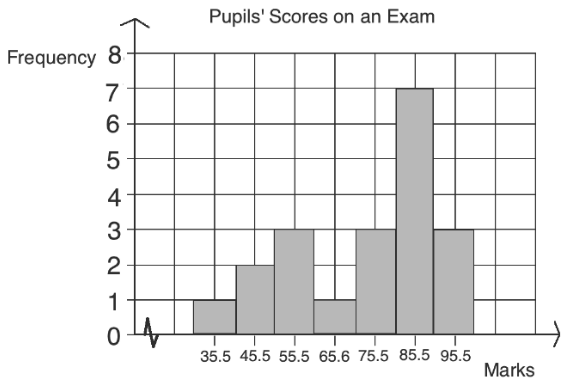
First, draw the axes for the histogram. Label each mid-point, because the bars will be centred on these.

Mid-points can be observed or calculated.

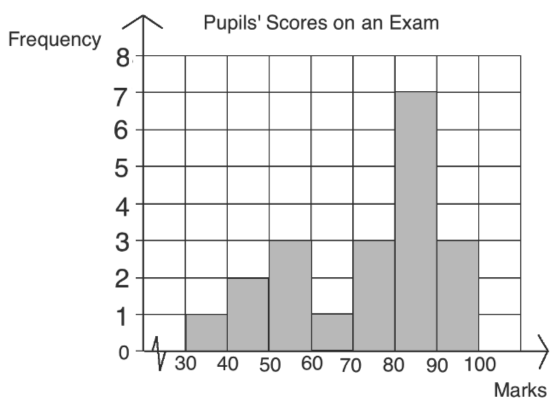
For example, the mid-point of interval 31-40 is $\frac{40+31}{2} = \frac{71}{2} = 35.5$.



Next, draw each bar, making them as tall as the frequencies in the table. Remember to make the bars touch each other:



Any histogram can also be drawn with the class boundaries marked on the x-axis. For example, this is the same histogram:



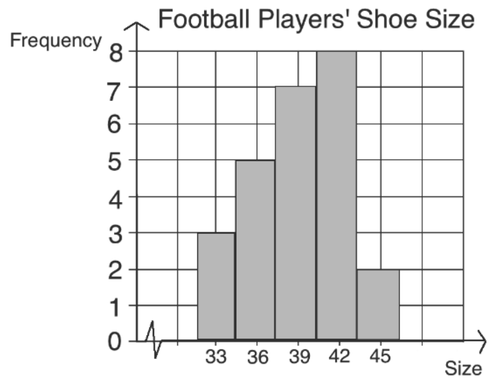
2. A school wants to buy shoes for its football team. They measured the shoe sizes of 25 football players and displayed them in the table below.

Shoe Size	32-34	35-37	38-40	41-43	44-46
Frequency	3	5	7	8	2

   a. Draw a histogram to display the data.
   b. Which class do the greatest number of football players fall into?
   c. The market only has football shoes in size 41 and larger this week. How many football players have to wait to receive their shoes?
   d. How many football players wear shoe sizes 35-40?

Solutions:

a. Identify the class mid-points of the intervals (33, 36, 39, 42, 45). Draw the axes and label the x-axis with the midpoints. Draw the bars using the frequencies given in the table. →



b. Class 41-43, which has the tallest bar.

c. Add the frequencies of classes less than 41: $3 + 5 + 7 = 15$ players.

d. Add the frequencies for classes 35-37 and 38-40: $5 + 7 = 12$ players.

Practice

1. The frequency distribution below shows the weight distribution of 100 patients in a hospital.

Weights (kg)	No. of patients
50 – 59	30
60 – 69	17
70 – 79	16
80 – 89	22
90 – 99	9
100 – 109	4
110 – 119	2

a. Draw a histogram to represent the data.
b. Which class interval contains the most patients?
c. How many patients weigh 70 kg or more?
d. How many patients weigh less than 80 kg?

2. The table below shows the frequency distribution of the marks of 700 candidates in an entrance examination into a model university.

Marks (%)	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89
Frequency	10	50	70	130	120	100	90	70	60

a. Draw a histogram to represent the data.
b. If candidates need to score at least 40% to pass, how many candidates passed?
c. How many candidates failed?
d. How many candidates scored at least 50%?

3. The frequency distribution of the weights of participants in football tournament held in Jupiter is as shown below.

Weights (kg)	40-49	50-59	60-69	70-79	80-89	90-99	100-109
No of participants	9	2	22	25	17	4	16

a. Draw a histogram for the data.
b. How many participants were in the tournament?
c. How many participants weigh less than 70 kg?
d. How many participants weigh at least 60 kg?

Lesson Title: Interpreting histograms	**Theme:** Statistics and Probability
Practice Activity: PHM2-L132	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to interpret information in a histogram, including estimating mode.

Overview

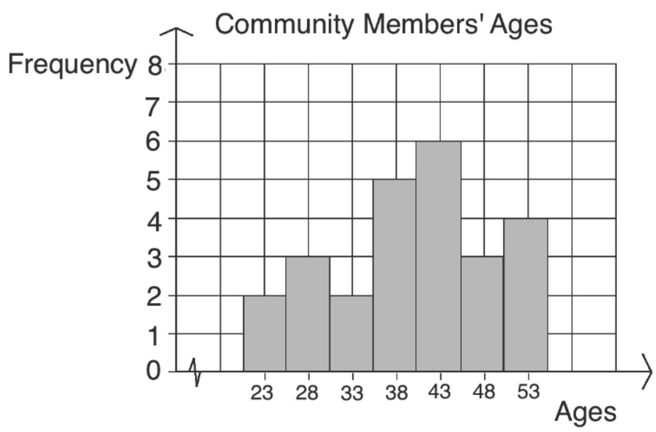
This lesson is on interpreting histograms. Histograms display grouped data, which means that we cannot find the exact median or mode. However, we can use them to estimate the median and mode.

The median class is the class interval that the median value falls into. The modal class is the class interval that the mode is likely to fall into, because it has the greatest frequency. It is the tallest bar in the histogram.

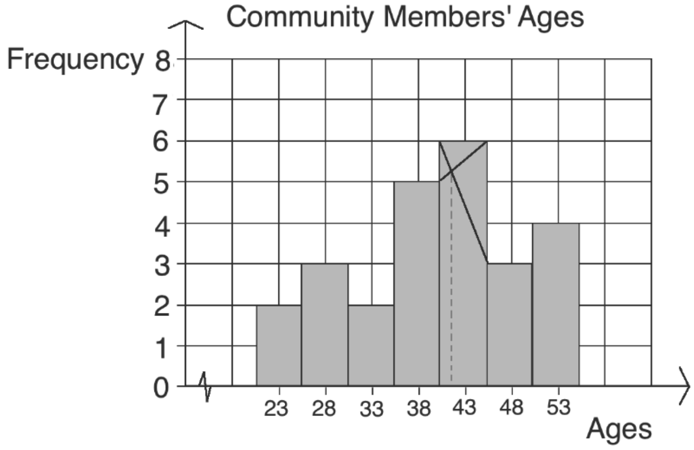
To estimate the mode, draw intersecting lines using vertices of the tallest bar, and draw a vertical dotted line from their intersection to the x-axis. The point on the x-axis where the lines intersect is the estimated mode.

Solved Examples

1. The histogram below displays the ages of 25 community members. Use the histogram to estimate the mode.



Solution:
Draw intersecting lines using vertices of the tallest bar, and draw a vertical dotted line from their intersection to the x-axis:



The point where they intersect on the x-axis is the estimated mode. The estimated mode is approximately 42 years old.

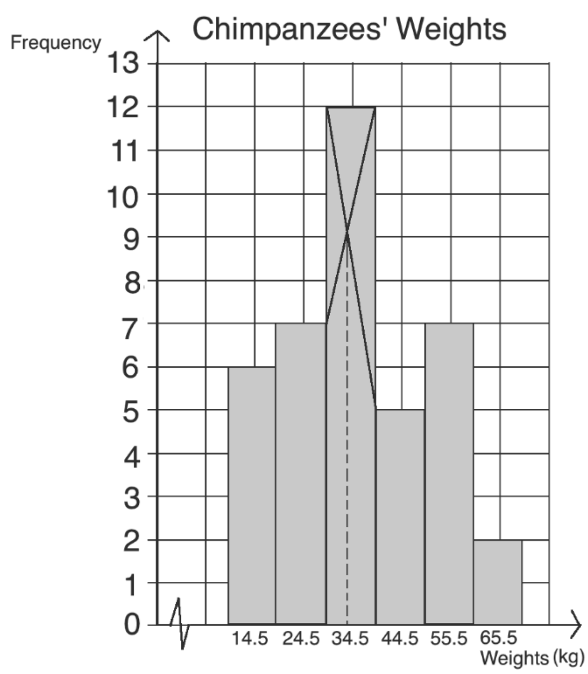
2. A chimpanzee reserve houses 39 chimpanzees that have been rescued from hunters and people who kept them as pets. The weights of the chimpanzees are displayed in the frequency table:

Weight (kg)	10-19	20-29	30-39	40-49	50-59	60-69
Frequency	6	7	12	5	7	2

a. Draw a histogram for the data.
b. What is the median class?
c. What is the modal class?
d. Estimate the mode.

Solutions:

a. Histogram →
b. The median class is the class where the chimpanzee with the median weight falls. The 20th chimpanzee has the median weight. This is in class interval 30-39, which is the median class.
c. The modal class is the tallest bar, which is 30-39.
d. To estimate the mode, draw lines as shown on the histogram. The mode is approximately 34 kg.

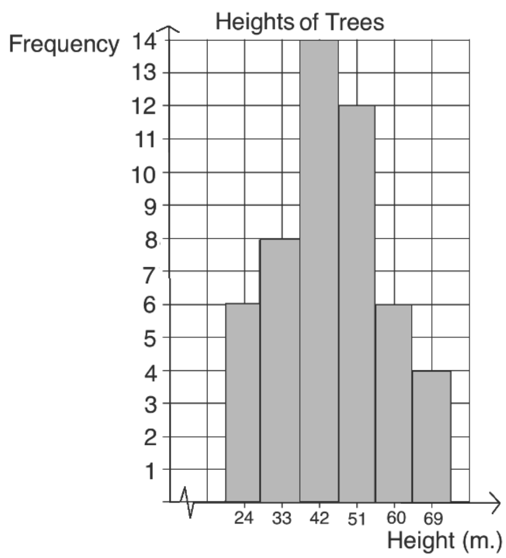


Practice

1. The table below shows the distribution of ages of a group of 50 men in a village.

Age (years)	Frequency
21 – 30	2
31 – 40	7
41 – 50	12
51 – 60	13
61 – 70	10
71 – 80	6

    a. Represent the information in a histogram.
    b. Use your histogram to estimate:
        i. The median class
        ii. The modal class
        iii. The mode
2. The histogram shows the heights (in metres) of trees in a garden. Estimate the mode, correct to two decimal places.



Lesson Title: Frequency polygons	**Theme:** Statistics and Probability
Practice Activity: PHM2-L133	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to present and interpret grouped data in frequency polygons.

Overview

Recall line graphs, which are used to show ungrouped data. We create line graphs by plotting and connecting points. Frequency polygons are similar to line graphs, in the same way that histograms are similar to bar charts.

Frequency polygons are used to display grouped data, which means that we plot class intervals. We must find each class mid-point on the x-axis, and plot the frequency for the corresponding class interval. Recall that for histograms we can use either the class mid-points or class boundaries to draw the bars. For frequency polygons, we must use the mid-points.

The class interval that contains the mode is referred to as the "modal class", and the class interval that contains the median is referred to as the "median class". When data is presented as grouped data in a histogram or frequency polygon, we cannot tell exactly where the median or mode is. We can identify the modal class and median class.

For frequency polygons, the tallest point gives the modal class. The median class contains the median quantity of the data set. For example, in a set of 7 numbers, the 4^{th} number is the median. We can count up in the frequency polygon to find the interval the 4^{th} number falls into.

Solved Examples

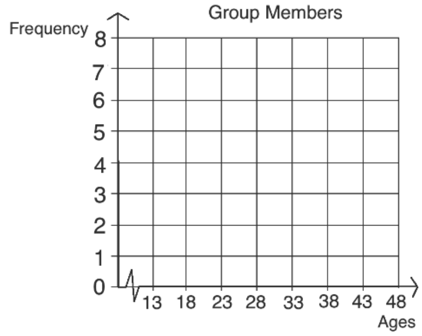
1. A certain women's group has 25 members. Their ages are 21, 42, 35, 26, 32, 19, 23, 27, 29, 38, 41, 42, 27, 35, 18, 30, 31, 26, 24, 41, 22, 35, 37, 23, 20.
    a. Draw a frequency table using class intervals 16-20, 21-25, 26-30, 31-35, 36-40, 41-45.
    b. Draw a frequency polygon.
    c. Identify the modal class and the median class.

Solutions:

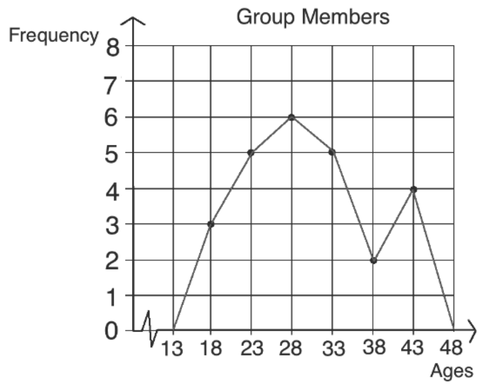
a. Write the numbers in order: 18, 19, 20, 21, 22, 23, 23, 24, 26, 26, 27, 27, 29, 30, 31, 32, 35, 35, 35, 37, 38, 41, 41, 42, 42.
Draw the table:

Group Members	
Ages	**Frequency**
16-20	3
21-25	5
26-30	6
31-35	5
36-40	2
41-45	4

b. Draw the axes for the frequency polygon, using the class mid-points on the x-axis:



Plot the points and connect them. Normally we extend the line of the frequency polygon to the mid-point of what would be the next interval, if that interval existed in the data. In our data set, we don't have any women in the class intervals that contain 13 and 48. Therefore, we can extend the line down to zero.



c. **Modal class:** The highest point gives the modal class. Since the highest point is at 28, the modal class is 26-30. We can also observe this from the frequency table, because it has the greatest frequency.
Median class: The median class contains the age that is in the middle. In a set of 25 women, the 13^{th} woman has the median age. We can count up in the frequency polygon in the same way that we did for histogram. The 13^{th} woman is in the interval 26-30, so this is the median class.

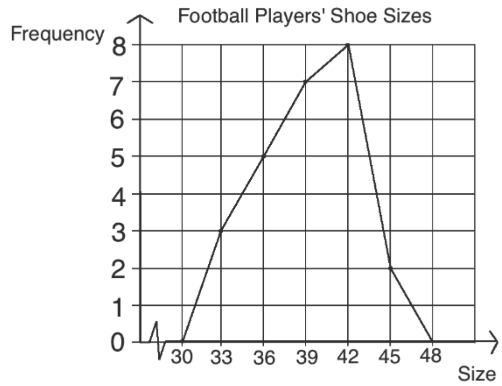
2. A school wants to buy shoes for its football team. They measured the shoe sizes of 25 football players and displayed them in the table below.

Shoe Size	32-34	35-37	38-40	41-43	44-46
Frequency	3	5	7	8	2

a. Draw a frequency polygon to display the data.
b. What is the modal class?
c. What is the median class?
d. The market only has football shoes in size 41 and larger this week. How many football players have to wait to receive their shoes?

Solutions:

a. Frequency polygon (using class mid-points 33, 36, 39, 42, 45) →
b. The modal class is 41-43.
c. The median class is where the 13^{th} pupil falls, which is 38-40.
d. Add the frequencies of classes less than 41: $3 + 5 + 7 = 15$ pupils.



Practice

1. The table below shows the distribution of marks achieved by 100 pupils in a chemistry test.

Marks	21-30	31-40	41-50	51-60	61-70	71-80	81-90
No. of Pupils	10	13	16	18	22	12	9

a. Draw a frequency polygon to display the data.
b. What is the modal class?
c. What is the median class?
d. If the passing mark is 51%, how many pupils passed the test?

2. Mr. North measured the time it took for his pupils to run around a track. Here are the results:

Time in Seconds	Frequency
41 – 45	1
46 – 50	5
51 – 55	8
56 – 60	4
61 – 65	7
66 – 70	4

a. Draw a frequency polygon to show the data.
b. How many pupils ran around the track?
c. What is the modal class?
d. What is the median class?

Lesson Title: Mean of grouped data	**Theme:** Statistics and Probability
Practice Activity: PHM2-L134	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate and interpret the estimated mean of grouped data.

Overview

When data is divided into groups, we cannot determine the value of each piece of data in the set. Therefore, we cannot determine the exact mean. We can **estimate** the mean of grouped data using a formula.

The formula for estimated mean is $\bar{x} = \frac{\Sigma fx}{\Sigma f}$, where f is frequency, and x is the corresponding class mid-point.

Recall that the sigma symbol (Σ) tells us to find the sum. The numerator tells us to find the sum of each frequency multiplied by each corresponding class mid-point. The denominator tells us to find the sum of the frequencies.

Solved Examples

1. In one village, 15 farmers have just harvested their pepper. The table below shows the amount of pepper they harvested in kilogrammes. Estimate how much pepper each farmer harvested on average.

Farmers' Harvests	
Pepper (kg)	**Frequency**
0 – 4	2
5 – 9	5
10 – 14	4
15 – 19	3
20 – 24	1
Total	15

Solution:
First, find each class mid-point. These can be added to the table:

Farmers' Harvests		
Pepper (kg)	**Frequency**	**Mid-point**
0 – 4	2	2
5 – 9	5	7
10 – 14	4	12
15 – 19	3	17
20 – 24	1	22
Total	15	

Then, use the frequencies and mid-points to calculate the estimated mean:

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{(2\times2)+(5\times7)+(4\times12)+(3\times17)+(1\times22)}{2+5+4+3+1}$$
$$= \frac{4+35+48+51+22}{15}$$
$$= \frac{160}{15}$$
$$= 10.67 \text{ to 2 d.p.}$$

2. The following are the weights in kilogrammes of 50 children in a sports camp.

64	14	17	48	58	60	43	44	15	32
47	21	23	37	51	26	10	36	22	24
43	45	29	33	46	38	19	38	36	30
31	72	57	41	41	44	54	24	26	41
22	25	35	35	37	36	51	52	19	62

Using class intervals $10 - 19$, $20 - 29$, ... prepare a frequency table and use it to calculate the mean weight.

Solution:

Frequency table:

Weight (kg)	Mid-point (x)	Frequency	fx
10 – 19	14.5	6	87
20 – 29	24.5	10	245
30 – 39	34.5	13	448.5
40 – 49	44.5	11	489.5
50 – 59	54.5	6	327
60 – 69	64.5	3	193.5
70 – 79	74.5	1	74.5
		$\sum f = 50$	$\sum fx = 1865$

Mean: $\bar{x} = \frac{\sum fx}{\sum f} = \frac{1865}{50} = 37.3$

Practice

1. The marks obtained in a test by 40 pupils are as follows:

78	42	30	33	66	39	72	60
54	44	42	24	45	60	72	30
39	36	66	33	36	84	18	45
27	78	33	42	67	63	30	63
60	27	30	81	27	33	72	18

   a. Construct a frequency table, using class intervals of $10 - 19$, $20 - 29, 30 - 39$, and so forth.
   b. Calculate the mean.

2. The table below shows the distribution of children per house hold in a survey. Using the table, find the mean, correct to the nearest whole number.

No. of Children	0	1	2	3	4	5
Frequency	7	3	5	11	10	14

3. The data below shows the distribution of marks obtained by 50 pupils in a test. Create a frequency table and calculate the mean to the nearest whole number.

10	73	19	78	24	78	34	34	35	35
37	59	41	63	41	65	45	55	55	58
48	49	49	53	53	55	45	55	65	67
58	38	59	43	61	44	48	68	79	83
70	14	74	29	76	31	48	68	85	90

Lesson Title: Median of grouped data	**Theme:** Statistics and Probability
Practice Activity: PHM2-L135	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to calculate and interpret the estimated median of grouped data.

Overview

This lesson is on estimating median from grouped data. We have found the median class for grouped data in previous lessons. When we know the position of the median, we can find which class it falls in. However, if we do not know each piece of data, we cannot determine the exact median. We can **estimate** the median of grouped data using a formula.

The estimated median is given by the formula: $L + \left[\frac{\frac{n}{2} - (\sum f)_L}{f_m}\right] \times c$, where:

- L is the lower class boundary of the group containing the median.
- n is the total frequency of the data.
- $(\sum f)_L$ is the total frequency for the groups **before** the median group.
- f_m is the frequency of the median group.
- c is the group width.

Solved Examples

1. In one village, 17 farmers have just harvested cassava. The table below shows the amount of cassava they harvested in kilogrammes. Estimate the median amount of cassava that was harvested.

Farmers' Harvests	
Cassava (kg)	**Frequency**
10 – 14	1
15 – 19	3
20 – 24	6
25 – 29	5
30 – 34	2
Total	17

Solution:
First, identify that the median falls into class interval 20-24. Since there are 17 farmers, the 9^{th} farmer has the median harvest. Eight farmers harvested more, and 8 farmers harvested less. The 9^{th} farmer is in interval 20-24.

Identify the value for each variable in the formula, substitute them, and evaluate.

$$\text{Median} = L + \left[\frac{\frac{n}{2} - (\Sigma f)_L}{f_m}\right] \times c$$

$= 20 + \left[\frac{\frac{17}{2} - (1+3)}{6}\right] \times 5$ Substitute the values

$= 20 + \left[\frac{8.5-4}{6}\right] \times 5$ Simplify

$= 20 + \left[\frac{4.5}{6}\right] \times 5$

$= 20 + 0.75 \times 5$

$= 20 + 3.75$

$= 23.75$

2. The table below shows the height distribution of 40 children in a village. Estimate the median.

Height (cm)	110-118	119-127	128-136	137-145	146-154	155-163	164-172
Frequency	9	3	4	5	2	5	12

Solution:

First, identify that the median falls into class interval 137-145. Since there are 40 children, the median height is the average of the 20th and 21st children's height. The 20th and 21st children are in interval 137-145.

Identify the value for each variable in the formula, substitute them, and evaluate.

$$\text{Median} = L + \left[\frac{\frac{n}{2} - (\Sigma f)_L}{f_m}\right] \times c$$

$= 137 + \left[\frac{\frac{40}{2} - (9+3+4)}{5}\right] \times 9$ Substitute values

$= 137 + \left[\frac{20-16}{5}\right] \times 9$ Simplify

$= 137 + \left[\frac{4}{5}\right] \times 9$

$= 137 + 0.8 \times 9$

$= 137 + 7.2$

$=$ 144.2 cm

3. The data below shows the age distribution of dwellers in a village.

Age	**Frequency**
20 – 29	12
30 – 39	13
40 – 49	39
50 – 59	28
60 – 69	11
70 – 19	7
80 – 89	10

Calculate the median age of dwellers in the village.

Solution:

Identify the values of variables that will be used in calculating the median. Add the frequencies to find the total number of residents: $12 + 13 + 39 + 28 + 11 + 7 + 10 = 120$. The median is the mean of the 60th and 61st positions. These both lie in the interval 40-49.

$$\text{Median} = L + \left[\frac{\frac{n}{2} - (\Sigma f)_L}{f_m}\right] \times c = 40 + \left[\frac{\frac{120}{2} - (12+13)}{39}\right] \times 10$$ Substitute values

$$= 40 + \left[\frac{60-25}{39}\right] \times 10$$ Simplify

$$= 40 + \left[\frac{35}{39}\right] \times 10$$

$$= 40 + 0.9 \times 10$$

$$= 40 + 9$$

= 49 years old

Practice

1. The table below shows the ages of teachers from a certain school. Estimate the median age, correct to the nearest whole number.

Age (years)	25-29	30-34	35-39	40-44	45-49	50-54	55-59
Frequency	1	2	4	5	3	3	2

2. The marks scored by 20 pupils in an examination are given in the frequency table below. Estimate the median, correct to 1 decimal place.

Marks	$0-4$	$5-9$	$10-14$	$15-19$	$20-24$	$25-29$	$30-34$
No. of pupils	2	5	2	7	2	1	1

3. The table below shows the height, in millimetres, of a sample of 250 seedlings on an experimental farm. Estimate the median height to the nearest whole number.

Seedling height (mm)	**Frequency**
0 – 4	40
5 – 9	45
10 – 14	60
15 – 19	40
20 – 24	30
24 – 29	20
30 – 34	10
35 – 39	5

Lesson Title: Practice with mean, median, and mode of grouped data	Theme: Statistics and Probability
Practice Activity: PHM2-L136	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to estimate the mean, median, and mode of grouped data.

Overview

This lesson is on practice estimating the mean, median, and mode of grouped data. Recall that the mean and median are estimated using the formula:

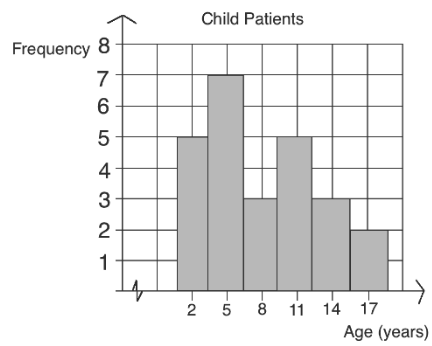
Estimated mean $= \bar{x} = \frac{\sum fx}{\sum f}$

Estimated median $= L + \left[\frac{\frac{n}{2} - (\sum f)_L}{f_m}\right] \times c$

The mode of grouped data is estimated by drawing intersecting lines in the modal class on a histogram, and identifying the estimated mode on the x-axis.

Solved Examples

1. The histogram at right shows the ages of 25 patients in a hospital.
   a. Use the histogram to estimate the mean age of the patients.
   b. Estimate the median age.
   c. Estimate the mode of the patients' ages.



Solutions:

a. **Mean:**

$\bar{x} = \frac{\sum fx}{\sum f} = \frac{(5\times2)+(7\times5)+(3\times8)+(5\times11)+(3\times14)+(2\times17)}{5+7+3+5+3+2}$ Substitute the frequencies and mid-points

$= \frac{10+35+24+55+42+34}{25}$ Simplify

$= \frac{200}{25}$

$=$ 8 years old

b. **Median:**

Median $= L + \left[\frac{\frac{n}{2} - (\sum f)_L}{f_m}\right] \times c = 7 + \left[\frac{\frac{25}{2} - (5+7)}{3}\right] \times 3$ Substitute the values

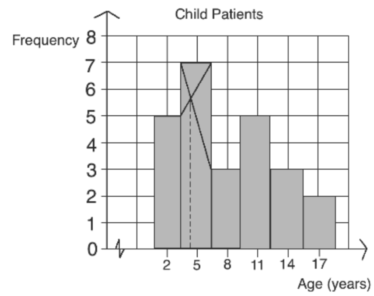
$= 7 + \left[\frac{12.5-12}{3}\right] \times 3$ Simplify

$= 7 + \left[\frac{0.5}{3}\right] \times 3$

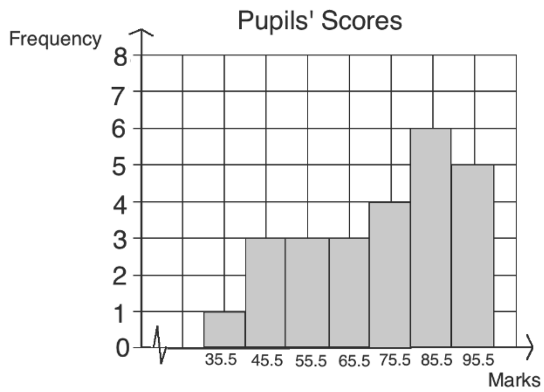
$= \quad 7 + 0.5$

$= \quad 7.5$ years old

c. **Mode:** Draw on the histogram as shown. The estimated mode is 4 years old.



2. Use the histogram showing 25 pupils' scores on an examination to estimate the mean, median, and mode scores.



Solution:

Mean:

$$\bar{x} = \frac{(1\times35.5)+(3\times45.5)+(3\times55.5)+(3\times65.5)+(4\times75.5)+(6\times85.5)+(5\times95.5)}{1+3+3+3+4+6+5}$$

$$= \frac{35.5+136.5+166.5+196.5+302+513+477.5}{25}$$

$$= \frac{1{,}827.5}{25} = 73.1 \text{ marks}$$

Median:

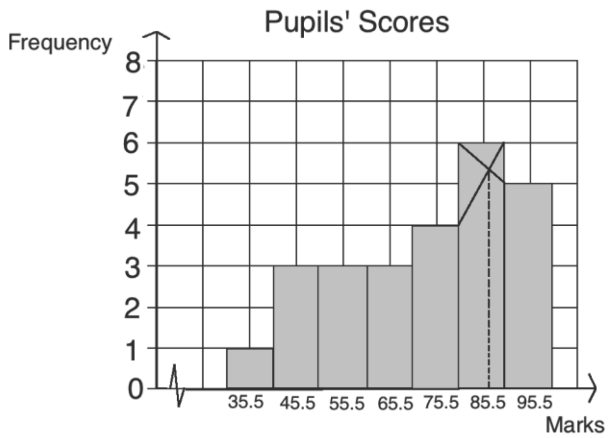
$$\text{Median} = L + \left[\frac{\frac{n}{2} - (\sum f)_L}{f_m}\right] \times c \quad = \quad 71 + \left[\frac{\frac{25}{2} - (2+3+3+3)}{4}\right] \times 10$$

$$= \quad 71 + \left[\frac{13.5-11}{4}\right] \times 10$$

$$= \quad 71 + 3.125$$

$$= \quad 74.13 \text{ marks}$$

Mode: Approximately 87 marks (see histogram below).



Practice

1. The table below shows the ages of teachers from a certain school. Estimate, correct to the nearest whole number:
    a. The mean
    b. The median

Age (Years)	Frequency
25 – 29	1
30 – 34	2
35 – 39	4
40 – 44	5
45 – 49	3
50 – 54	3
55 – 59	2

2. The table below shows the frequency distribution of marks scored by some pupils in an examination. Estimate, correct to 1 decimal place:
    a. The mean
    b. The median
    c. The mode

Marks (%)	21 – 30	31 – 40	41 – 50	51 – 60	61 - 70	71 – 80
Frequency	2	5	12	15	10	6

3. The distribution below gives the weights (in kg) of bags in a store. Estimate the mean and median of the distribution to 2 decimal places.

Weight (kg)	$10 - 21$	$22 - 33$	$34 - 45$	$46 - 57$	$58 - 69$
No. of bags	15	5	21	10	29

Lesson Title: Cumulative frequency tables	**Theme:** Statistics and Probability
Practice Activity: PHM2-L137	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to construct cumulative frequency tables.

Overview

This lesson is on cumulative frequency tables. When something is "cumulative", it increases in quantity by successive addition. The quantity grows.

Cumulative frequency tables look similar to the frequency tables you have seen in previous lessons, but they have a column for cumulative frequency. To find the cumulative frequency for a row, add the frequency for that row to the cumulative frequency of the rows above it.

The cumulative frequency for the last class interval should be equal to the total frequency in the data set.

Solved Examples

1. The table below shows the pepper harvested by 20 farmers in kilogrammes.

Farmers' Harvests	
Pepper (kg)	**Frequency**
0 – 4	2
5 – 9	6
10 – 14	7
15 – 19	4
20 – 24	1
Total	20

   a. Construct a cumulative frequency table of the data.
   b. How many farmers harvested 9 kg or less?
   c. How many farmers harvested less than 20 kg of pepper?

   Solutions:

   a. Add a column to the table for cumulative frequency:

Farmers' Harvests		
Pepper (kg)	**Frequency**	**Cumulative Frequency**
0 – 4	2	2
5 – 9	6	6 + 2 = 8
10 – 14	7	7 + 8 = 15
15 – 19	4	4 + 15 = 19
20 – 24	1	1 + 19 = 20
Total	20	

b. Find the cumulative frequency for class intervals up to 9 kg. This is in the "cumulative frequency" column of the row for class interval 5-9.
Answer: 8 farmers.
c. Find the cumulative frequency for all class intervals less than 20. This is in the "cumulative frequency" column of the row for class interval 15-19.
Answer: 19 farmers

2. The scores of 20 pupils on a Maths test are: 91, 85, 72, 71, 69, 90, 85, 74, 60, 59, 74, 55, 52, 64, 89, 94, 85, 87, 55, 61.
   a. Draw a cumulative frequency table for the data, using class intervals 51-60, 61-70, 71-80, 81-90, 91-100.
   b. How many pupils scored 80 or fewer marks?
   c. If 71 marks is passing, how many pupils failed?

Solutions:

a. Write the numbers in ascending order: 52, 55, 55, 59, 60, 61, 64, 69, 71, 72, 74, 74, 85, 85, 85, 87, 89, 90, 91, 94.
Draw the cumulative frequency table:

Pupils' Scores		
Marks	**Frequency**	**Cumulative Frequency**
51 – 60	5	5
61 – 70	3	5 + 3 = 8
71 – 80	4	8 + 4 = 12
81 – 90	6	12 + 6 = 18
91 – 100	2	18 + 2 = 20
Total	20	

b. Find the cumulative frequency in the row for class interval 71-80. Answer: 12
c. Find the cumulative frequency in the row for class interval 61-70. Answer: 8

Practice

1. The tables below gives the results of a survey done by a chicken farmer. It shows how many eggs his chickens laid each day for 200 days.

Eggs	Frequency
45 – 49	13
50 – 54	24
55 – 59	48
60 – 64	50
65 – 69	39
70 – 74	18
75 – 79	8

   a. Construct a cumulative frequency table of the data
   b. On how many days did his chickens lay 59 eggs or less?
   c. On how many days did his chickens lay at least 70 eggs?

2. The scores of 50 pupils in Mathematics at WASSCE are as follows:
   20, 25, 29, 26, 20, 21, 24, 23, 20, 28, 31, 36, 33, 39, 37, 34, 38, 45, 49, 46, 50, 52, 53, 52, 59, 57, 56, 54, 55, 52, 51, 56, 63, 64, 62, 67, 69, 66, 73, 76, 74, 70, 71, 77, 79, 74, 75, 86, 84, 83.
   a. Draw a cumulative frequency table for the data using class intervals $20-29$ $30-39$, $40-49$, …..
   b. How many pupils scored 59 or fewer marks?
   c. If 70 marks is passing, how many pupils failed?
3. The table below shows the frequency distribution of the marks of 800 candidates in an examination.

Marks (%)	Frequency
0 – 9	10
10 – 19	40
20 – 29	80
30 – 39	140
40 – 49	170
50 – 59	130
60 – 69	100
70 – 79	70
80 – 89	40
90 – 99	20

   a. Construct a cumulative frequency table.
   b. How many candidates scored 69 or less?
   c. If 60 marks is passing, how many candidates passed?

Lesson Title: Cumulative frequency curves	**Theme:** Statistics and Probability
Practice Activity: PHM2-L138	**Class:** SSS 2

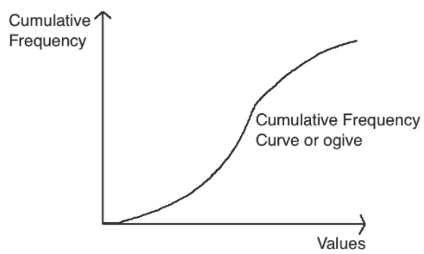
Learning Outcome
By the end of the lesson, you will be able to construct cumulative frequency curves and estimate the median.

Overview

This lesson is on cumulative frequency curves, which are graphs that are constructed based on cumulative frequency tables.

A **cumulative frequency (c.f.) curve** can be graphed in a similar way to line graphs and frequency polygons. Cumulative frequency curves can also be called "**ogive**".

This is the basic shape of a cumulative frequency curve:



A c.f. curve increases as you move in the positive direction along the x-axis. For the x-values, we will plot the upper-class boundary of each class interval. This is the highest data point in each class interval. Usually there is a space of 1 unit between each class interval. For the purpose of graphing, we will take the point in the middle of the class intervals. For example, if one class interval ends at 60 and the next starts at 61, we will plot the value 60.5. For the y-value, we will plot the cumulative frequency from the table.

Once the points are plotted, connect them with a smooth curve.

We can estimate the median using the cumulative frequency curve. Remember that we cannot find the exact median from grouped data, so our result will only be an estimate. To estimate median, identify the location of the median value on the y-axis, and find the corresponding x-value on the c.f. curve.

Solved Examples

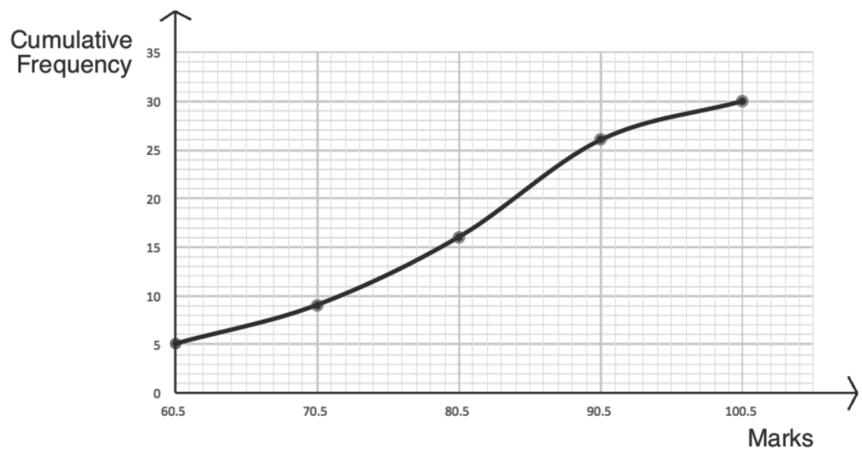
1. The table below gives pupils' scores on a Maths tests.
   a. Complete the cumulative frequency table, and use it to:
   b. Draw a cumulative frequency curve.
   c. Use your curve to estimate the median.

Solutions:

a. Fill the columns:

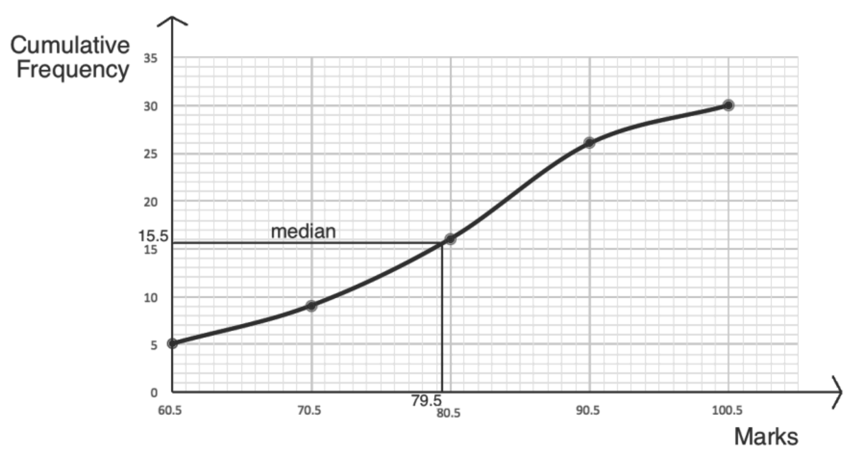
Pupils' Scores on a Maths Test			
Marks	**Frequency**	**Cumulative Frequency**	**Upper Class Interval**
51 – 60	5	5	60.5
61 – 70	4	5 + 4 = 9	70.5
71 – 80	7	7 + 9 = 16	80.5
81 – 90	10	10 + 16 = 26	90.5
91 – 100	4	4 + 26 = 30	100.5
Total	30		

b. Plot the points from the table. The last column (upper class interval) is used for the x-axis, and cumulative frequency is used for the y-axis. Connect the points with a smooth curve.



c. Recall that the median mark is scored by the pupil in the middle. There are 30 pupils in the data set, so the median is the mean of the scores of the 15^{th} and 16^{th} pupils. To find the estimated median using the curve:
   - Find the mark that corresponds to the 15.5^{th} pupil.
   - Draw a horizontal line on the board 15.5 on the y-axis (see below).
   - Draw a vertical line connecting this point on the curve to the x-axis. Identify the number of marks given at this point.

The median can be estimated to be 79.5 marks.



Practice

1. The table below shows the frequency distribution of marks scored by 600 candidates in a Mathematics test.

Marks	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69	70 - 79	80 – 89	90 – 99
Frequency	85	15	130	125	120	45	35	45

   a. Construct a cumulative frequency table.
   b. Construct the cumulative frequency curve.
   c. Use the curve to estimate the median.

2. The table below gives the weights in kilogrammes of people living in a village.

Weight (kg)	Frequency	Upper class interval	Cumulative Frequency
40 – 49	8		
50 – 59	15		
60 – 69	23		
70 – 79	12		
80 – 89	17		
90 – 99	25		
Total			

   a. Fill the empty columns
   b. Draw the cumulative frequency curve.
   c. Use the curve to estimate the median.

3. The following is the distribution of the heights of 80 pupils participating in a sport festival measured in centimetres.

Height (cm)	120 – 124	125 – 129	130 – 134	135 – 139	140 – 144
Frequency	9	5	13	8	5

   a. Construct a cumulative frequency table.
   b. Draw an ogive from the data.
   c. From your ogive, estimate the median

Lesson Title: Quartiles	**Theme:** Statistics and Probability
Practice Activity: PHM2-L139	**Class:** SSS 2

Learning Outcomes
By the end of the lesson, you will be able to:
1. Estimate quartiles using a cumulative frequency curve.
2. Calculate the interquartile range.
3. Calculate the semi-interquartile range.

Overview

Quartiles divide a data set into 4 equal parts. The word “quartile” is related to “quarter”, which means fourths. The lower quartile (Q_1) is one-quarter of the way from the bottom of the data. The upper quartile (Q_3) is one-quarter of the way from the top of the data set. The second quartile (Q_2) is the median, or the middle quartile. When we find quartiles from grouped data, the results are only estimates.

We estimate Q_1 and Q_3 using formulae to find their placement. The upper and lower quartiles are located at $Q_1: \frac{1}{4}(n+1)$ and $Q_3: \frac{3}{4}(n+1)$, where n is the total frequency. After finding the placement of the quartiles in the dataset, we can use the cumulative frequency curve to find the value of each quartile. This process is very similar to the one we used in the previous lesson to estimate median.

Just as we can calculate the range of a data set, we can calculate the interquartile range. The interquartile range can be found by subtracting the lower quartile from the upper quartile ($Q_3 - Q_1$). The interquartile range represents how spread out the middle half of the data is.

The semi-interquartile range tells us about one quarter of the data set (“semi” means half, so it is half of the interquartile range). The semi-interquartile range is given by the formula: $Q = \frac{Q_3 - Q_1}{2}$. Half of the entire dataset lies within the semi-interquartile range from the median.

Solved Examples

1. The table below gives the cassava harvests of 17 farmers.

Farmers’ Harvests			
Cassava (kg)	**Frequency**	**Upper Class Interval**	**Cumulative Frequency**
10 – 14	1	14.5	1
15 – 19	3	19.5	3+1=4
20 – 24	6	24.5	6+4=10
25 – 29	5	29.5	5+10=15
30 – 34	2	34.5	2+15=17
Total	17		

a. Draw the cumulative frequency curve.
b. Use the curve to estimate the quartiles of the distribution.
c. Calculate the interquartile range.
d. Calculate the semi-interquartile range.

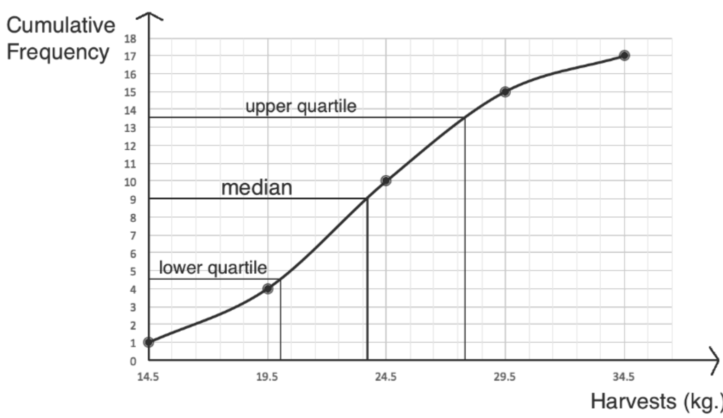
Solutions:

a. Plot the points from the table, using the 3rd column on the x-axis and 4th column on the y-axis. Connect the points with a smooth curve (see curve below).
b. For Q_2, recall that the median mark is scored by the pupil in the middle. There are 17 farmers in the data set, so the median is the harvest of the 9th farmer. Use the formulae to find the place of each quartile on the board:

$$Q_1: \frac{1}{4}(n+1) = \frac{1}{4}(17+1) = \frac{1}{4}(18) = \frac{18}{4} = 4\frac{1}{2}$$
$$Q_3: \frac{3}{4}(n+1) = \frac{3}{4}(17+1) = \frac{3}{4}(18) = \frac{54}{4} = 13\frac{1}{2}$$

To find the lower quartile, we need to identify the $4\frac{1}{2}$th farmer on the y-axis. To find the upper quartile, we need to identify the $13\frac{1}{2}$th farmer.

Draw lines to identify each of the 3 quartiles:



The estimated quartiles are $Q_1 = 20.1$ kg, $Q_2 = 23.5$ kg, and $Q_3 = 27.7$ kg

c. Interquartile range: $Q_3 - Q_1 = 27.7 - 20.1 = 6.6$ kg
   a. Note that the interquartile range tells us how spread out the harvests are. They are spread out over 6.6 kg.
d. Semi-interquartile range: $Q = \frac{Q_3 - Q_1}{2} = \frac{27.7-20.1}{2} = \frac{6.6}{2} = 3.3$ kg
   - Note that the semi-interquartile range tells us that about half of the farmers grew a harvest within 3.3 kg of the median.

Practice

1. The table below shows the frequency distribution of marks scored by 400 pupils in a Mathematics test.

Marks	Frequency	Upper class interval	Cumulative Frequency
10 – 19	25	19.5	25
20 – 29	45	29.5	70
30 – 39	10	39.5	80
40 – 49	80	49.5	160
50 – 59	60	59.5	220
60 – 69	70	69.5	290
70 – 79	40	79.5	330
80 – 89	20	89.5	350
90 – 99	50	99.5	400
Total	400		

a. Draw the cumulative frequency curve.
b. Use the curve to estimate the quartiles of the distribution.
c. Calculate the interquartile range.
d. Calculate the semi-interquartile range.

2. The table below shows the frequency distribution of the weights of children living in a village, according to a household survey that was conducted.

Weight (kg)	Frequency	Upper class interval	Cumulative Frequency
0 – 9	7		
10 – 19	3		
20 – 29	5		
30 – 39	11		
40 – 49	10		
50 – 59	14		
Total	50		

a. Fill the empty columns of the table.
b. Draw the cumulative frequency curve.
c. Use the curve to estimate the median weight.
d. Use the curve to estimate the upper and lower quartiles.
e. Calculate the interquartile range.
f. Calculate the semi-interquartile range.

Lesson Title: Practice with cumulative frequency	**Theme:** Statistics and Probability
Practice Activity: PHM2-L140	**Class:** SSS 2

Learning Outcome
By the end of the lesson, you will be able to construct cumulative frequency tables and curves, and use them to estimate the median, quartiles, interquartile range, and semi-interquartile range**.**

Overview

This lesson is practice of the information from the previous 2 lessons. You should be able to construct a cumulative frequency table and curve, then use it to estimate the median, quartiles, interquartile and semi-interquartile range.

Solved Examples:

1. The table below shows the weights of 50 children in a primary school.

Weight (kg)	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34	35 – 39	40 – 44
Frequency	3	7	9	5	11	6	9

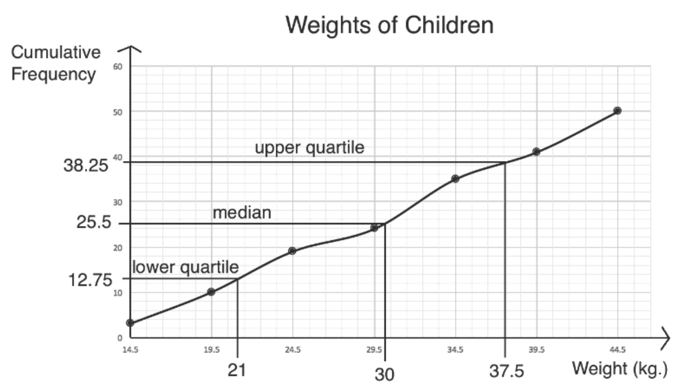
   a. Prepare a cumulative frequency table for the data
   b. Draw the ogive from the cumulative frequency table.
   c. Use your ogive to find: i. The median; ii. The upper and lower quartiles.
   d. Calculate the interquartile range.
   e. Calculate the semi-interquartile range.

   Solutions:

   a. Cumulative frequency table

Masses (kg)	Frequency	Upper class interval	Cumulative
10 – 14	3	14.5	3
15 – 19	7	19.5	$3 + 7 = 10$
20 – 24	9	24.5	$10 + 9 = 19$
25 – 29	5	29.5	$19 + 5 = 24$
30 – 34	11	34.5	$24 + 11 = 35$
35 – 39	6	39.5	$35 + 6 = 41$
40 – 44	9	44.5	$41 + 9 = 50$

   b. Cumulative frequency curve:



a. i. Position of the median: $\frac{1}{2}(n+1) = \frac{1}{2}(50+1) = 25.5$

Estimated median: 30 kg

ii. Q_1: $\frac{1}{4}(n+1) = \frac{1}{4}(50+1) = \frac{1}{4}(51) = \frac{51}{4} = 12.75$

Estimated lower quartile: 21 kg

Q_3: $\frac{3}{4}(n+1) = \frac{3}{4}(50+1) = \frac{3}{4}(51) = \frac{153}{4} = 38.25$

Estimated upper quartile: 37.5 kg

b. Interquartile range: $Q_3 - Q_1 = 37.5 - 21 = 16.5$ kg

c. Semi-interquartile range: $\frac{Q_3 - Q_1}{2} = \frac{16.5}{2} = 8.25$ kg

2. The frequency distribution shows the marks of 100 pupils in a Mathematics test.
   a. Prepare a cumulative frequency table.
   b. Draw the cumulative frequency curve for the distribution.
   c. Use your curve to estimate:
      i. The median.
      ii. The lower and upper quartile.
   d. Calculate the interquartile range.
   e. Calculate the semi-interquartile range.

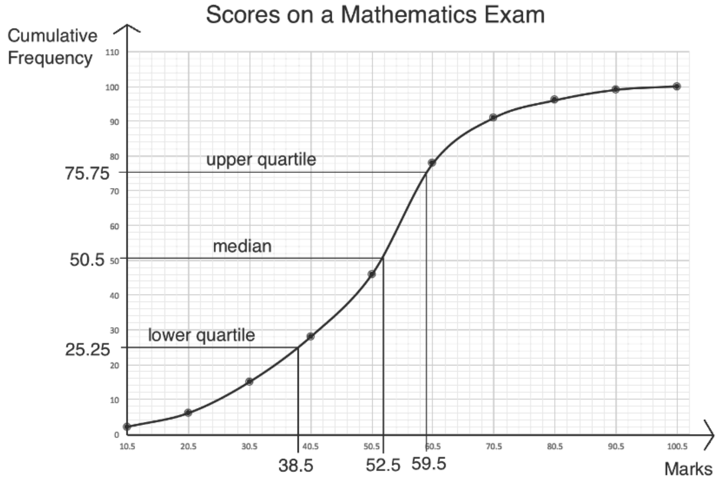
Marks	No. of Pupils
1 – 10	2
11 – 20	4
21 – 30	9
31 – 40	13
41 – 50	18
51 – 60	32
61 – 70	13
71 – 80	5
81 – 90	3
91 – 100	1

Solutions:

a. Cumulative frequency table:

Marks	Frequency	Upper class interval	Cumulative Frequency
1 – 10	2	10.5	2
11 – 20	4	20.5	$4 + 2 = 6$
21 – 30	9	30.5	$6 + 9 = 15$
31 – 40	13	40.5	$15 + 13 = 28$
41 – 50	18	50.5	$28 + 18 = 46$
51 – 60	32	60.5	$46 + 32 = 78$
61 – 70	13	70.5	$78 + 13 = 91$
71 – 80	5	80.5	$91 + 5 = 96$
81 – 90	3	90.5	$96 + 3 = 99$
91 – 100	1	100.5	$99 + 1 = 100$

b. Cumulative frequency curve:



c. i. Position of Median: $\frac{1}{2}(n+1) = \frac{1}{2}(100+1) = 50.5$

Estimated median: 52.5 marks

ii. $Q1: \frac{1}{4}(n+1) = \frac{1}{4}(100+1) = \frac{1}{4}(101) = \frac{101}{4} = 25.25$

Estimated lower quartile: 38.5 marks

$Q3: \frac{3}{4}(n+1) = \frac{3}{4}(100+1) = \frac{3}{4}(101) = \frac{303}{4} = 75.75$

Estimated upper quartile: 59.5 marks

d. To calculate the interquartile: $Q_3 - Q_1 = 59.5 - 38.5 = 21$ marks

e. To calculate the semi-interquartile range: $\frac{Q_3 - Q_1}{2} = \frac{21}{2} = 10.5$ marks

Practice

1. The table below gives the frequency distribution of marks scored by some pupils in Mathematics. No pupil scored below 40 marks.

Marks	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	8	14	28	46	25	19

   a. Create a cumulative frequency table.
   b. Draw the cumulative frequency curve.
   c. Use the curve to estimate the median mark.
   d. Use the curve to estimate the lower and the upper quartiles.
   e. Calculate the interquartile range.
   f. Calculate the semi-interquartile range.

2. The frequency distribution of the weights of 100 participants in a high jump competition is shown below.

Weight (kg)	20-29	30-39	40-49	50-59	60-69	70-79
Frequency	10	18	22	25	16	9

   a. Prepare a cumulative frequency table for the data.
   b. Draw the cumulative frequency curve.
   c. From the curve, estimate:
      i. The median.
      ii. The semi-interquartile range.

Answer Key – Term 3

Lesson Title: Review of sine, cosine and tangent
Practice Activity: PHM2-L097

1. $\sin\theta = \frac{1}{2}; \cos\theta = \frac{5}{6};\ \tan\theta = \frac{3}{5}$
2. $\sin\theta = \frac{8}{11}; \cos\theta = \frac{5}{11}; \tan\theta = 1\frac{3}{5}$
3. $\sin a = \frac{6}{7}; \cos a = \frac{4}{7}; \tan a = 1\frac{1}{2}; \sin b = \frac{4}{7}; \cos b = \frac{6}{7}; \tan b = \frac{2}{3}$

Lesson Title: Application of sine, cosine and tangent
Practice Activity: PHM2-L098

1. $x = 5.2$ cm; $y = 3.0$ cm
2. a. $s = 5.7$ cm; b. $t = 15.6$ cm; c. $u = 17.7$ cm; d. $v = 9.6$ cm; e. $x = 14.1$ cm and $y = 14.1$ cm; f. $c = 6.1$ cm and $d = 3.5$ cm

Lesson Title: Special angles (30°, 45°, 60°)
Practice Activity: PHM2-L099

1. $\sin 30° = \frac{1}{2}; \cos 30° = \frac{\sqrt{3}}{2}$.
2. $\tan 45° = \frac{\sin 45°}{\cos 45°} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}} \div \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{1} = \frac{\sqrt{2}}{\sqrt{2}} = 1$
3. $\tan\theta = -1$
4. $\tan 0° = 0$

Lesson Title: Applying special angles
Practice Activity: PHM2-L100

1. $|XY| = \frac{7\sqrt{2}}{2}$ cm; $|YZ| = \frac{7\sqrt{2}}{2}$ cm
2. $|KM| = \frac{10\sqrt{3}}{3}$ cm; $|LM| = \frac{5\sqrt{3}}{3}$ cm
3. a. $|AB| = 25$cm; $|AC| = 25\sqrt{2}$cm

   b. $|RT| = 15\sqrt{2}$cm; $|ST| = 15$cm

   c. $|EF| = 5\sqrt{3}$cm; $|DE| = 5$cm
4. a. $|AC| = 7\sqrt{2}$cm; $|BC| = 7$cm

   b. $|XZ| = \frac{10\sqrt{3}}{3}$ cm; $|XY| = \frac{5\sqrt{3}}{3}$ cm

Lesson Title:	Inverse trigonometry
Practice Activity:	PHM2-L101

1. a. $24°$ b. $73°$ c. $4.39°$ d. $21.69°$
2. a. $20°$ b. $61°$ c. $69.13°$ d. $38.68°$
3. a. $69.01°$ b. $22.1°$ c. $89.07°$
4. a. $\alpha = 31°$ b. $\beta = 58°$ c. $\gamma = 64°$
5. a. $30°$ b $60°$ c. $30°$

Lesson Title:	Trigonometry and Pythagoras' Theorem
Practice Activity:	PHM2-L102

1. a. $|XY| = 13.5\text{cm}$; $|XZ| = 14.8$ cm
   b. $|BC| = 4.8$ cm; $|AB| = 11.0$ cm
   c. $|KL| = 23.6$ cm; $|KM| = 26.5$ cm
2. a. $\angle G = 56.4°$; $\angle E = 33.6°$; $|FG| = 33.2$ mm
   b. $\angle I = 45.6°$; $\angle K = 44.4°$; $|IJ| = 4.9$ cm
   c. $\angle A = 45°$; $|BC| = 8$; $|AC| = 8\sqrt{2} = 11.3$ cm

Lesson Title:	Angles of elevation
Practice Activity:	PHM2-L103

1. 3.1 metres
2. 12.3 metres
3. $53.1°$
4. $36.9°$
5. 13.0 metres
6. $50.2°$

Lesson Title:	Angles of depression
Practice Activity:	PHM2-L104

1. 300 metres
2. 34.5 metres
3. 308 metres
4. $\theta = 59°$
5. 12 metres

Lesson Title:	Applications of angles of elevation and depression – Part 1
Practice Activity:	PHM2-L105

1. $h = 36.8$ metres
2. $\theta = 10°$
3. $h = 6.7$ metres
4. 25 metres

Lesson Title:	Applications of angles of elevation and depression – Part 2
Practice Activity:	PHM2-L106

1. 168 m
2. 48 m
3. $21°$
4. 35.16 m
5. a. $66°$; b. 28.8 m

Lesson Title:	The general angle – Part 1
Practice Activity:	PHM2-L107

1. a. 0.3839; b. -0.3746; c. 0.7660; d. 0.8988
2. a. $-\frac{1}{2}$; b. $\frac{\sqrt{2}}{2}$; c. $-\sqrt{3}$; d. $-\frac{\sqrt{3}}{2}$

Lesson Title:	The general angle – Part 2
Practice Activity:	PHM2-L108

1. a. -0.4848; b. 1.000; c. 0.9397; d. 0.6820
2. a. $\frac{\sqrt{3}}{2}$; b. $\frac{\sqrt{3}}{2}$; c. $-\frac{\sqrt{3}}{2}$; d. $-\frac{\sqrt{3}}{2}$

Lesson Title:	The unit circle
Practice Activity:	PHM2-L109

1. a. $-\frac{\sqrt{2}}{2}$; b. $-\frac{1}{2}$; c. $-\sqrt{3}$
2. a. $-\frac{1}{2}$; b. -2; c. 1

Lesson Title: Problem solving with trigonometric ratios	
Practice Activity: PHM2-L110	

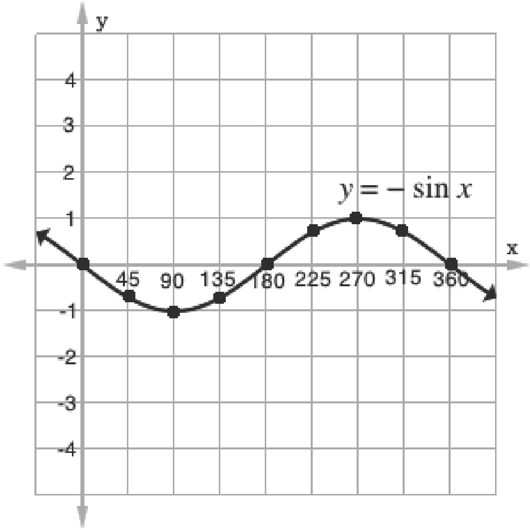
1. $\sqrt{3}$
2. $-3\sqrt{3}$
3. $\frac{3}{4}$
4. $x = 60°$
5. $\frac{3}{4}$

Lesson Title: Graph of $\sin\theta$	
Practice Activity: PHM2-L111	

1. Table of values:

x	$0°$	$45°$	$90°$	$135°$	$180°$	$225°$	$270°$	$315°$	$360°$
$-\sin x$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0

Graph:

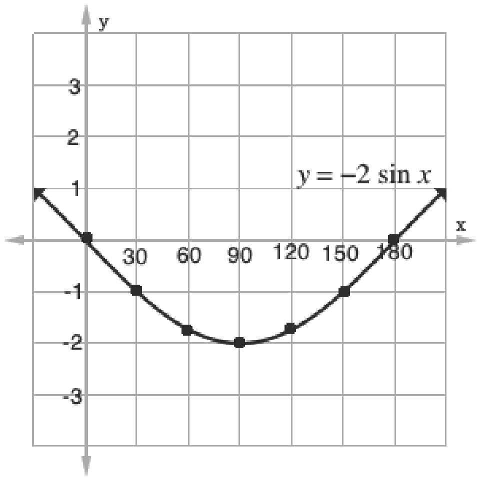


a. $x = 270°$
b. $30°$ and $150°$; where the line $y = -\frac{1}{2}$ intersects the curve.

2. Table of values:

x	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$
$-\sin x$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0
$-2\sin x$	0	-1	$-\sqrt{3}$	-2	$-\sqrt{3}$	-1	0

Graph:



d. Estimate: $y = -0.3$

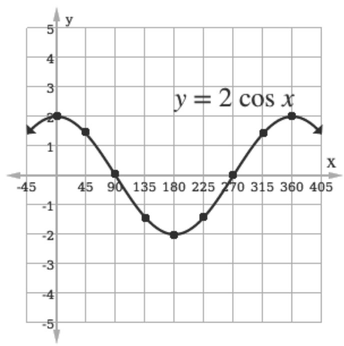
e. $x = 30°, 150°$

Lesson Title:	Graph of $\cos\theta$
Practice Activity:	PHM2-L112

1. Table of values:

x	$0°$	$45°$	$90°$	$135°$	$180°$	$225°$	$270°$	$315°$	$360°$
$\cos x$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1
$2\cos x$	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	$-\sqrt{2}$	0	$\sqrt{2}$	2

Graph:

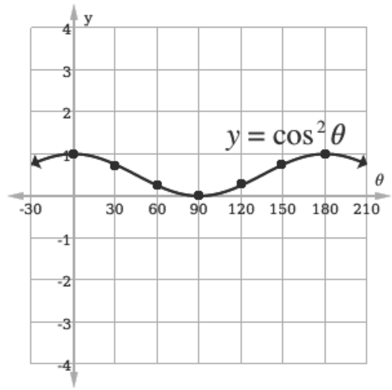


When $x = 200°$, y is approximately -1.8 or -1.9

2. Table of values:

θ	$0°$	$30°$	$60°$	$90°$	$120°$	$150°$	$180°$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\cos^2\theta$	1	$\frac{3}{4}$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{3}{4}$	1

Graph:



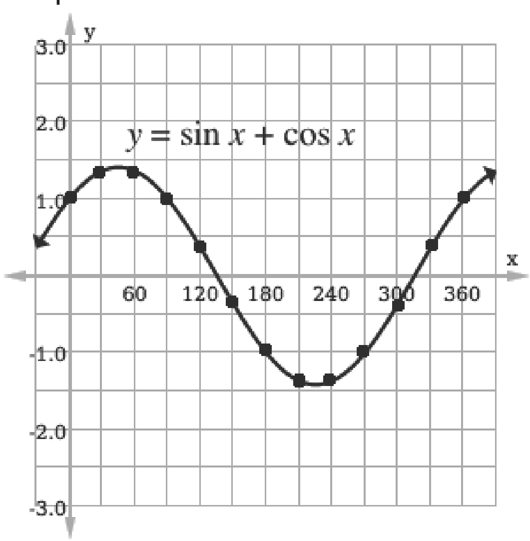
a. approximately 0.5; b. $\theta = 0°, 180°$

Lesson Title:	Graphs of $\sin\theta$ and $\cos\theta$
Practice Activity:	PHM2-L113

1. Table of values:

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\sin x$	0	0.5	0.87	1	0.87	0.5	0	−0.5	−0.87	−1	−0.87	−0.5	0
$\cos x$	1	0.87	0.5	0	−0.5	−0.87	−1	−0.87	−0.5	0	0.5	0.87	1
$\sin x + \cos x$	1	1.37	1.37	1	0.37	−0.37	−1	−1.37	−1.37	−1	−0.37	0.37	1

Graph:

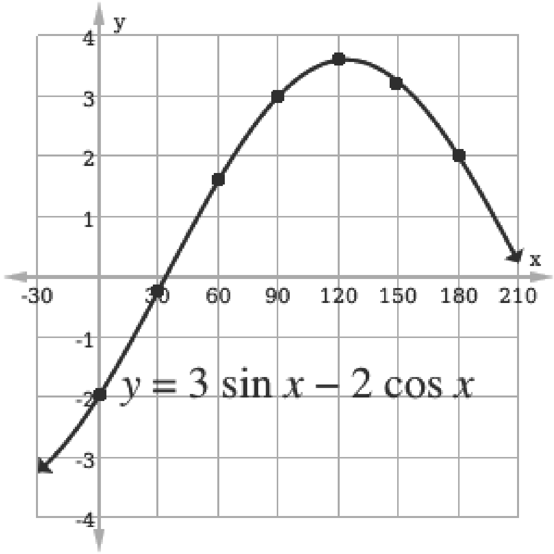


Answer: approximately $x = 105°, 345°$

2. Table of values:

x	0°	30°	60°	90°	120°	150°	180°
$3\sin x$	0	1.5	2.6	3	2.6	1.5	0
$2\cos x$	2	1.7	1	0	−1	−1.7	−2
$y = 3\sin x - 2\cos x$	−2	−0.2	1.6	3	3.6	3.2	2

Graph:



a. $x = 70°, 180°$; b. approximately $x = 35°$

Lesson Title:	The sine rule
Practice Activity:	PHM2-L114

1. $a = 23$ m; $c = 34$ m
2. $B = 86.8°$, $C = 30.2°$
3. $B = 26°$, $C = 85°$ and $AB = 20$ cm
4. $C = 57°$, $b = 16$ cm and $c = 14$ cm

Lesson Title:	The cosine rule
Practice Activity:	PHM2-L115

1. $a = 21$ cm
2. $A = 73°$, $B = 61°$ and $C = 46°$
3. $v = 4.9$ cm
4. $A = 56°$
5. $B = 81°$, $C = 36°$ and $a = 21$ cm

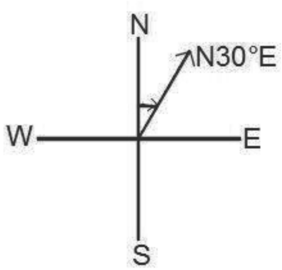
Lesson Title:	Application of sine and cosine rules
Practice Activity:	PHM2-L116

1. $M = 85.0°$, $k = 3.5$ cm, and $l = 5.2$ cm
2. $b = 3.6$ cm, $C = 36.1°$ and $A = 98.9°$
3. $z = 7.5$, $X = 61.3°$ and $Y = 48.7°$

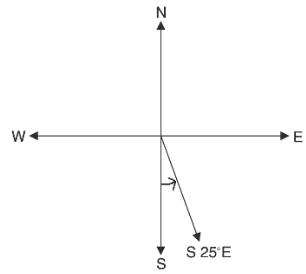
Lesson Title:	Compass bearings
Practice Activity:	PHM2-L117

1.

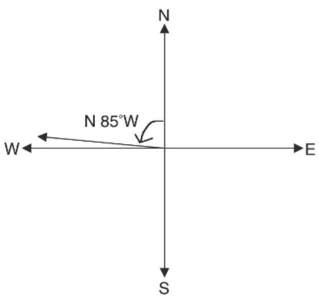
a.



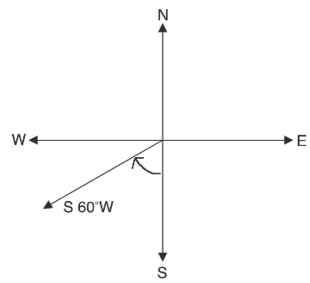
b.



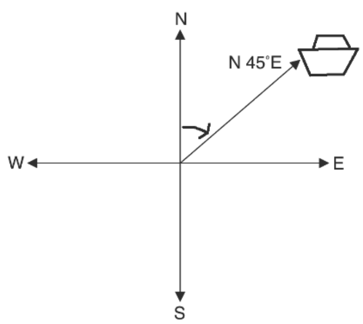
c.



d.

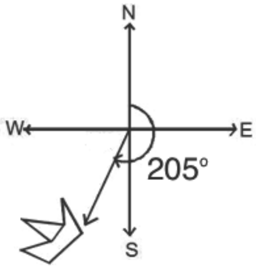


2. Diagram:



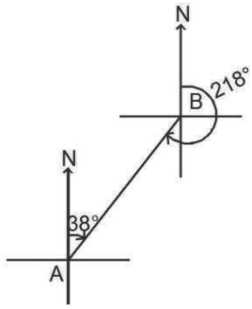
Lesson Title:	Three figure bearings
Practice Activity:	PHM2-L118

1. a. $300°$; b. $075°$; c. $160°$.
2. Diagram:

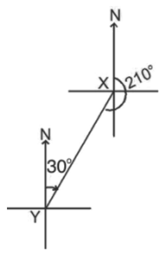


Lesson Title:	Reverse bearings
Practice Activity:	PHM2-L119

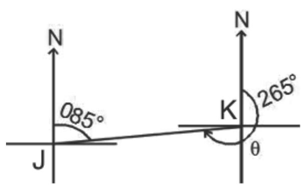
1. $\overrightarrow{BA} = 218°$; diagram:



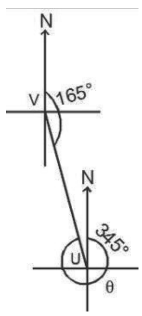
2. $\overrightarrow{YX} = 30°$



3. $\overrightarrow{KJ} = 265°$



4. a. $\overrightarrow{UV} = 345°$; b. $\overrightarrow{VU} = 165°$; diagram:

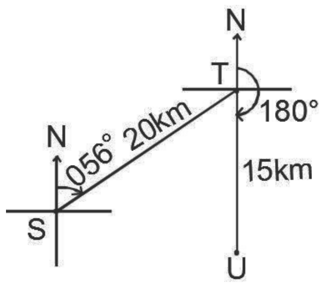


Lesson Title:	Bearing problem solving – Part 1
Practice Activity:	PHM2-L120

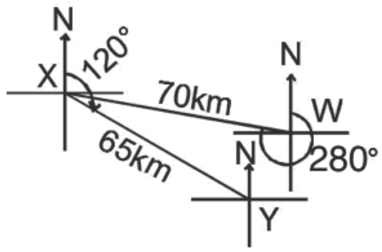
1. Point K: $035°$; Point L: $151°$; Point M: $200°$; Point N: $345°$
2. a. The bearing of Q from $P = 107°$; b. The bearing of P from $Q = 287°$
3. The bearing of Z from $X = 208°$

Lesson Title:	Distance-bearing form and diagrams
Practice Activity:	PHM2-L121

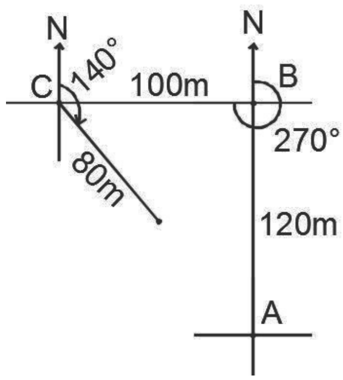
1. $\overrightarrow{CD} = (20 \text{ km}, 250°)$
2. a. $\overrightarrow{ST} = (20 \text{ km}, 056°)$; b. $\overrightarrow{TU} = (15 \text{ km}, 180°)$; c. Diagram:



3. a. $\overrightarrow{WX} = (70 \text{ km}, 280°)$; b. $\overrightarrow{XY} = (65 \text{ km}, 120°)$; c. Diagram:

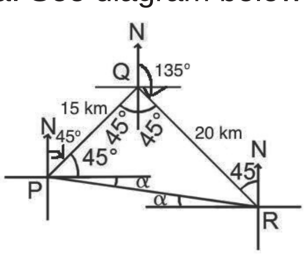


4. a. $(120 \text{ m}, 000°)$; $(100 \text{ m}, 270°)$; $(80 \text{ m}, 140°)$; b. Diagram:

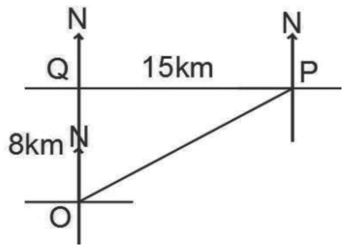


Lesson Title:	Bearing problem solving – Part 2
Practice Activity:	PHM2-L122

1. a. See diagram below; b. i. $\overrightarrow{PR} = 25 \text{ km}$; ii. $278°$



2. a. 13 km; b. $206°$
3. a. See diagram below; b. 17 km; c. $61.9°$



Lesson Title:	Bearing problem solving – Part 3
Practice Activity:	PHM2-L123

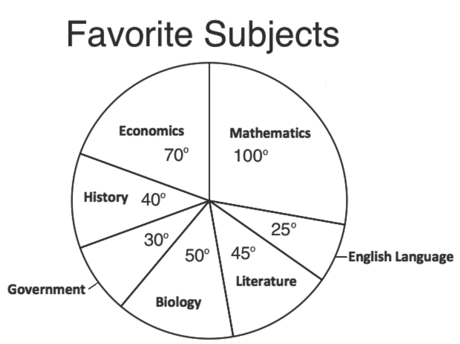
1. a. 100 km; b. $262°$
2. a. 49.3 cm; b. $37.6°$

Lesson Title:	Bearing problem solving – Part 4
Practice Activity:	PHM2-L124

1. The bearing and distance from $P = (6.5 \text{ km}, 061°)$
2. $207°$
3. The bearing of C from A is $73.3°$
4. a. 164 km; b. $108°$
5. a. 19 km

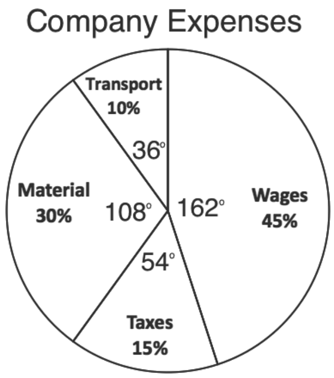
Lesson Title:	Drawing pie charts
Practice Activity:	PHM2-L125

1. A. Pie chart:

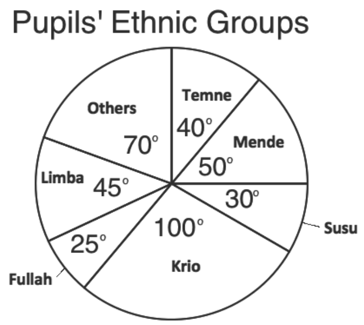


   b. Biology: 14%

2. Pie chart:



3. a. Pie chart:



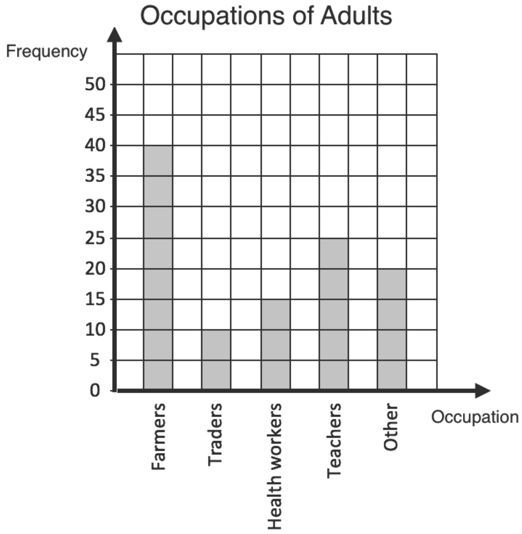
   b. Other ethnic groups: 19%
   c. Temne or Mende: 25%

Lesson Title:	Interpretation of pie charts
Practice Activity:	PHM2-L126

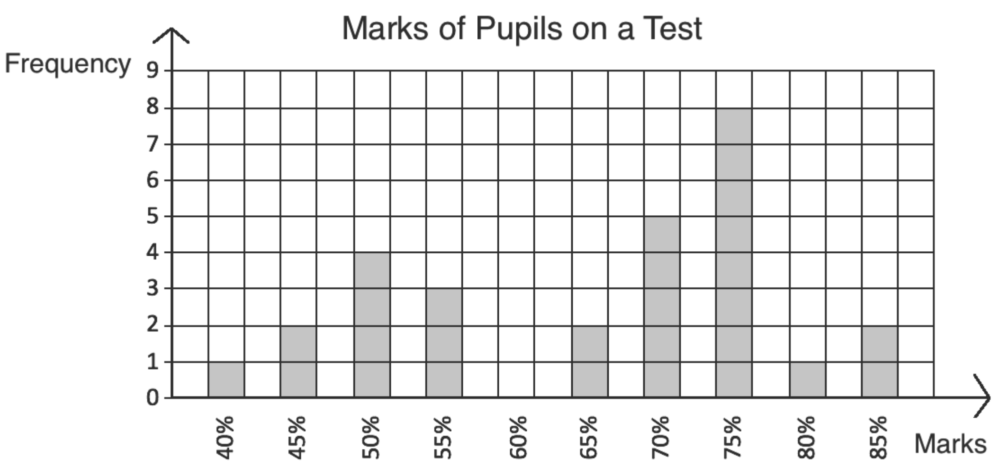
1. 62 packs of chocolate
2. a. 120 pupils; 51.1%

Lesson Title:	Drawing and interpretation of bar charts
Practice Activity:	PHM2-L127

1. Bar chart:



2. Bar chart:



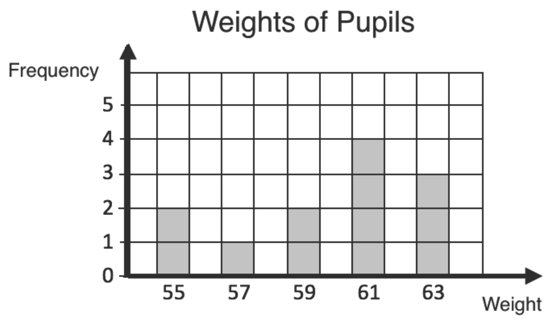
   a. 28 pupils
   b. 21 pupils
   c. 75%
   d. 7 pupils

Lesson Title:	Mean, Median, and Mode
Practice Activity:	PHM2-L128

1. Mean: 76; Median: 76; Mode: 68
2. a. 10; b. 10; c. 9.6
3. Statements (b) and (c) are true
4. a.10.8 cm; b. 10.8 cm; c. 10.9 cm

Lesson Title:	Mean, median, and mode from a table or chart
Practice Activity:	PHM2-L129

1. a. Mean $= 4.3$; b. Median $= 4$; c. Mode $= 4$
2. a. Bar chart:



c. i. Mode $= 61$ kg; ii. Median $= 61$ kg; c. Mean $= 59.8$ kg

Lesson Title:	Grouped frequency tables
Practice Activity:	PHM2-L130

1. a. $34{,}000$ people; b. $26{,}000$ people
2. Frequency table:

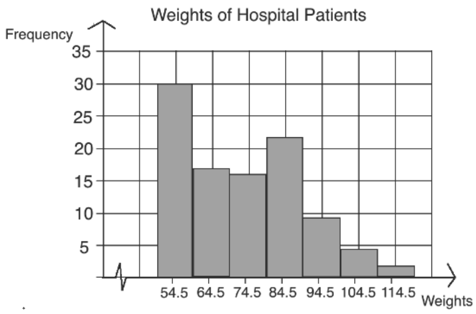
Weight (kg)	**Frequency**
10 – 19	6
20 – 29	10
30 – 39	13
40 – 49	11
50 – 59	6
60 – 69	3
70 – 79	1

a. 16; b. 10

3. a. 65 motorists; b. 45 motorists

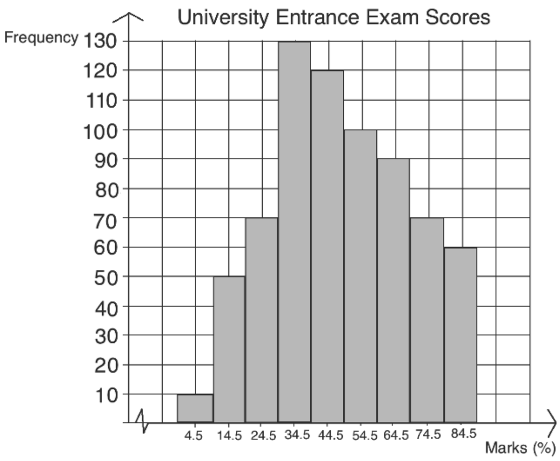
Lesson Title:	Drawing Histograms
Practice Activity:	PHM2-L131

1. a. Histogram:



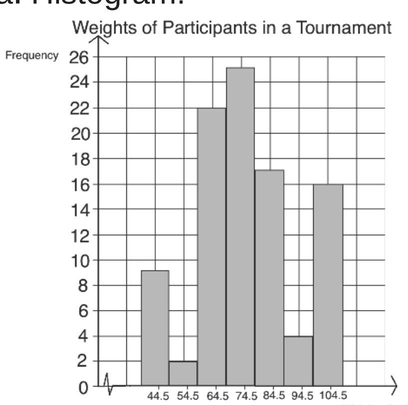
b. $50-59$ kg; c. 53 patients; d. 63 patients

2. a. Histogram:



b. 440 candidates; c. 260 candidates; d. 320 candidates

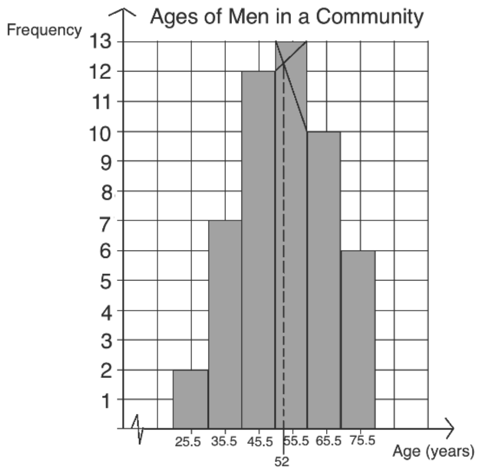
3. a. Histogram:



b. 95 participants; c. 33 participants; d. 84 participants

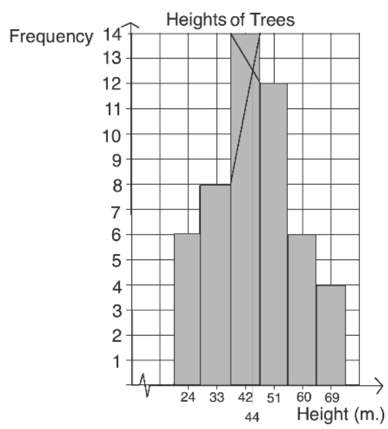
Lesson Title:	Interpreting Histograms
Practice Activity:	PHM2-L132

1. a. Histogram:



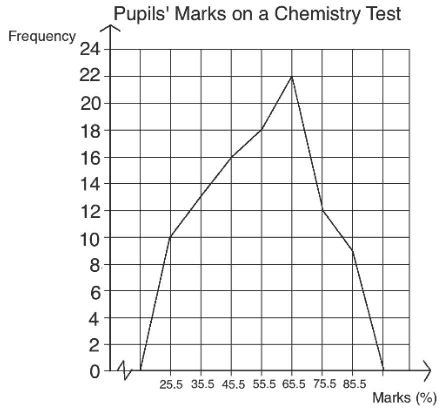
b. Median class: $51 - 60$; c. Modal class: $51 - 60$; d. Estimated mode (see histogram): 54.5

2. Note that the class intervals are: 20-28, 29-37, 38-46, etc. Mode: 44 m. (see histogram below)



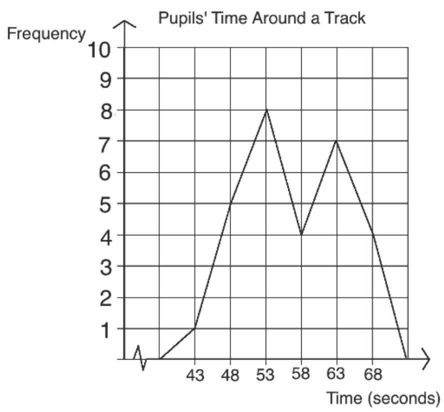
Lesson Title:	Frequency polygons
Practice Activity:	PHM2-L133

1. a. Frequency polygon:



   a. Modal class: $61-70$ marks; c. Median class: $51-60$ marks; d. 61 pupils

2. a. Frequency polygon:



   b. 29 pupils; c. modal class: $51-55$ seconds; median class: $56-60$ seconds

Lesson Title:	Mean of grouped data
Practice Activity:	PHM2-L134

1. a. See frequency table below; b. Mean: $\bar{x} = 48$

Class Interval	Mid-point (x)	Frequency	fx
10 – 19	14.5	2	29
20 – 29	24.5	4	98
30 – 39	34.5	12	414
40 – 49	44.5	6	267
50 – 59	54.5	1	54.5
60 – 69	64.5	8	516
70 – 79	74.5	5	372.5
80 – 89	84.5	2	169
Total		$\sum f$ =40	$\sum fx = 1920$

2. Mean: $\bar{x} = 3$
3. Frequency table: see below; Mean: $\bar{x} = 54$

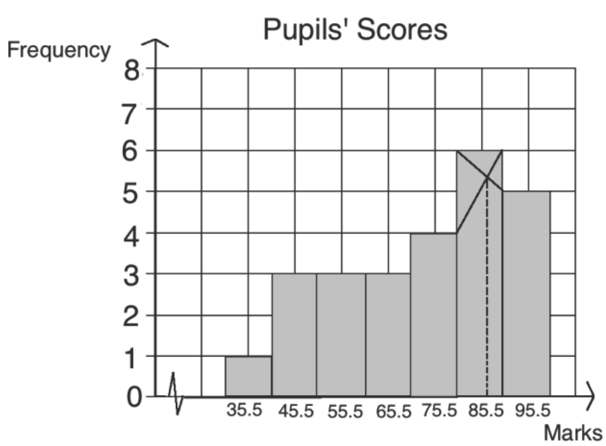
Class Interval	**Mid-point (x)**	**Frequency**	$\boldsymbol{fx}$
10 – 19	14.5	3	43.5
20 – 29	24.5	2	49
30 – 39	34.5	7	241.5
40 – 49	44.5	11	489.5
50 – 59	54.5	10	545
60 – 69	64.5	7	451.5
70 – 79	74.5	7	521.5
80 – 89	84.5	2	169
90 – 99	94.5	2	189
Total		$\sum f$ =50	$\sum fx = 2699.5$

Lesson Title:	Median of grouped data
Practice Activity:	PHM2-L135

1. 43 years
2. 15.7 marks
3. 13 mm

Lesson Title:	Practice with mean, median, and mode of grouped data
Practice Activity:	PHM2-L136

1. a. 43 years; b. 43 years
2. Mean: 73 marks; Median: 74 marks; Mode: 87 marks (see histogram below)



3. Mean: 44.45 kg; Median: 45.43 kg

Lesson Title:	Cumulative frequency tables
Practice Activity:	PHM2-L137

1. a. See table below; b. 85 days; c. 26 days

Eggs	Frequency	Cumulative Frequency
45 – 49	13	13
50 – 54	24	37
55 – 59	48	85
60 – 64	50	135
65 – 69	39	174
70 – 74	18	192
75 – 79	8	200

2. a. See table below; b. 32 pupils; c. 38 pupils

Marks (%)	Frequency	Cumulative Frequency
20 – 29	10	10
30 – 39	7	17
40 – 49	3	20
50 – 59	12	32
60 – 69	6	38
70 – 79	9	47
80 – 89	3	50
Total	50	

3. a. See table below; b. 670 candidates; c. 230 candidates

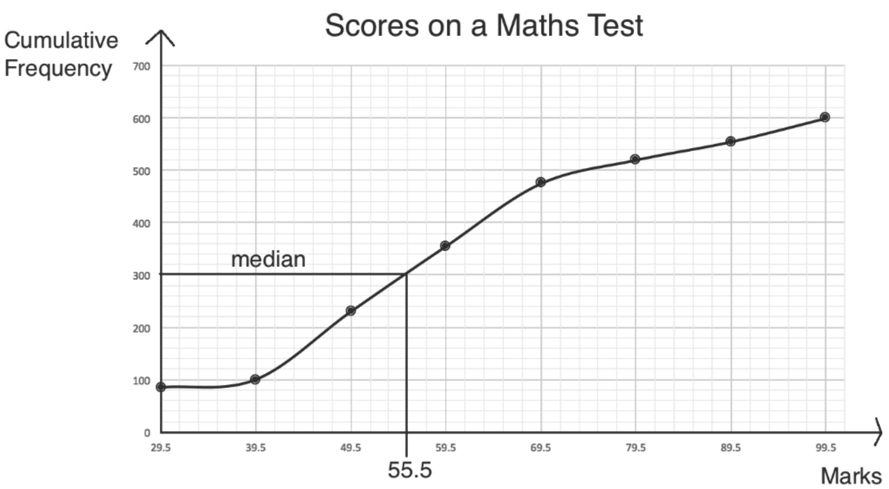
Marks (kg)	Frequency	Cumulative Frequency
0 – 9	10	10
10 – 19	40	50
20 – 29	80	130
30 – 39	140	270
40 – 49	170	440
50 – 59	130	570
60 – 69	100	670
70 – 79	70	740
80 – 89	40	780
90 – 99	20	800

Lesson Title:	Cumulative frequency curves
Practice Activity:	PHM2-L138

1. a. Cumulative frequency table:

Marks	Frequency	Upper class interval	Cumulative Frequency
20 – 29	85	29.5	85
30 – 39	15	39.5	100
40 – 49	130	49.5	230
50 – 59	125	59.5	355
60 – 69	120	695	475
70 – 79	45	79.5	520
80 – 89	35	89.5	555
90 – 99	45	99.5	600
Total	60		

b. Cumulative frequency curve:

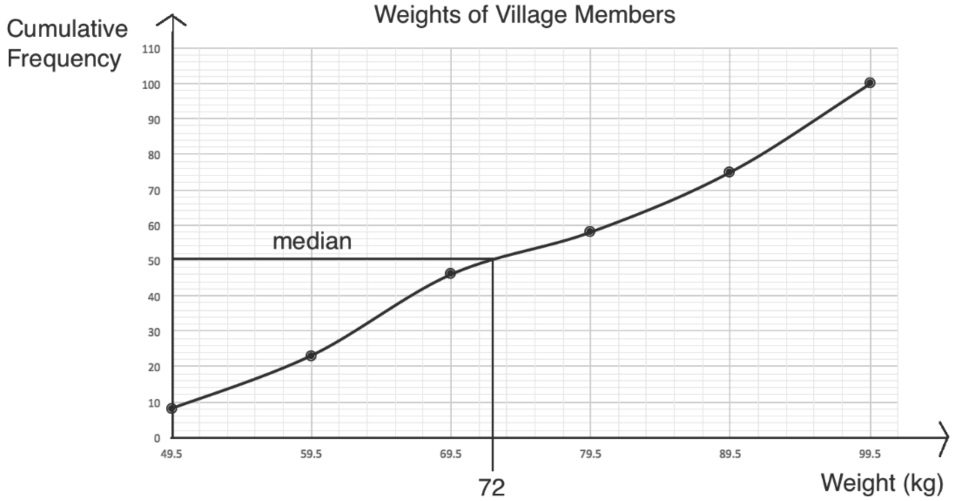


c. Median is approximately 55.5 marks

2. a. Cumulative frequency table:

Weight (kg)	Frequency	Upper class interval	Cumulative Frequency
40 – 49	8	49.5	8
50 – 59	15	59.5	23
60 – 69	23	695	46
70 – 79	12	79.5	58
80 – 89	17	89.5	75
90 – 99	25	99.5	100
Total	100		

b. Cumulative frequency curve or ogive:

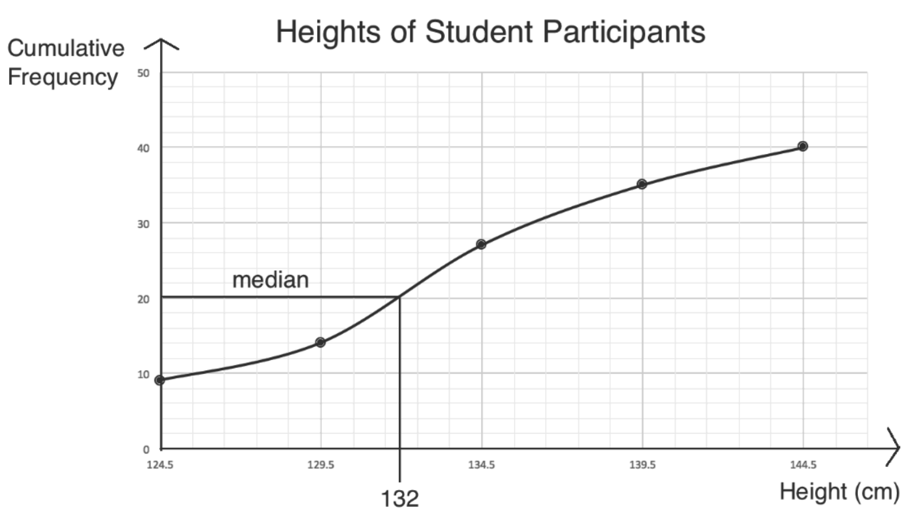


d. Median: approximately 72 kg

3. a. Cumulative frequency table:

Height (cm)	Frequency	Upper class interval	Cumulative Frequency
120 – 124	9	124.5	9
125 – 129	5	129.5	14
130 – 134	13	134.5	27
135 – 139	8	139.5	35
140 – 144	5	144.5	40
Total	40		

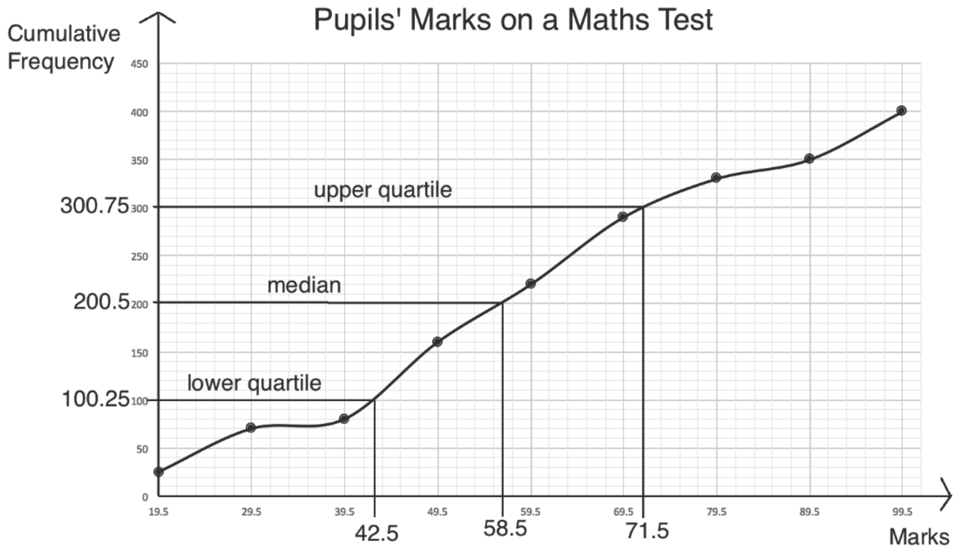
b. Cumulative frequency curve or ogive:



c. Median: approximately 132 cm

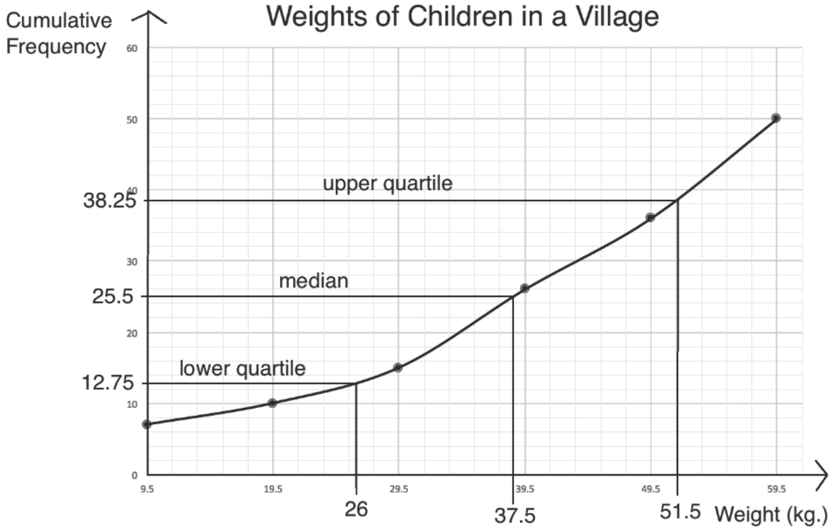
Lesson Title:	Quartiles
Practice Activity:	PHM2-L139

1. a. See cumulative frequency curve below; b. Quartile estimates: $Q_1 = 42.5$ marks, $Q_2 = 58.5$ marks, $Q_3 = 71.5$ marks; c. Interquartile range: 29 marks; d. semi-interquartile range: 14.5 marks.



2. a. See completed table below; b. See cumulative frequency curve below; c. Median: 37.5 kg; d. Upper quartile: 51.5 kg, Lower quartile: 26 kg; e. Interquartile range: 25.5; f. Semi-interquartile range: 12.75.

Masses (kg)	Frequency	Upper class interval	Cumulative Frequency
0 – 9	7	9.5	7
10 – 19	3	19.5	10
20 – 29	5	29.5	15
30 – 39	11	39.5	26
40 – 49	10	49.5	36
50 – 59	14	59.5	50
Total	50		

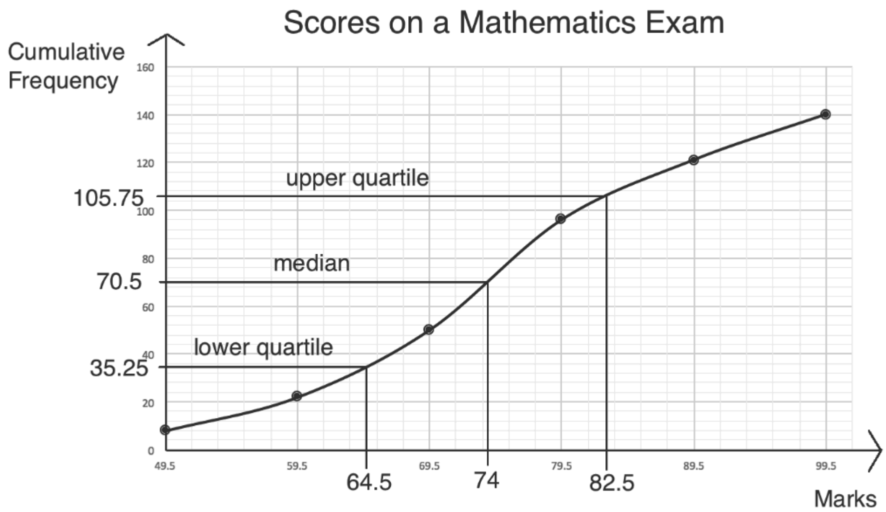


Lesson Title:	Practice with cumulative frequency
Practice Activity:	PHM2-L140

1. a. Cumulative frequency table

Masses (kg)	Frequency	Upper class interval	Cumulative Frequency
40 – 49	8	49.5	8
50 – 59	14	59.5	$8 + 14 = 22$
60 – 69	28	695	$22 + 28 = 50$
70 – 79	46	79.5	$50 + 46 = 96$
80 – 89	25	89.5	$96 + 25 = 121$
90 – 99	19	99.5	$121 + 19 = 140$
Total	140		

b. Cumulative frequency curve:

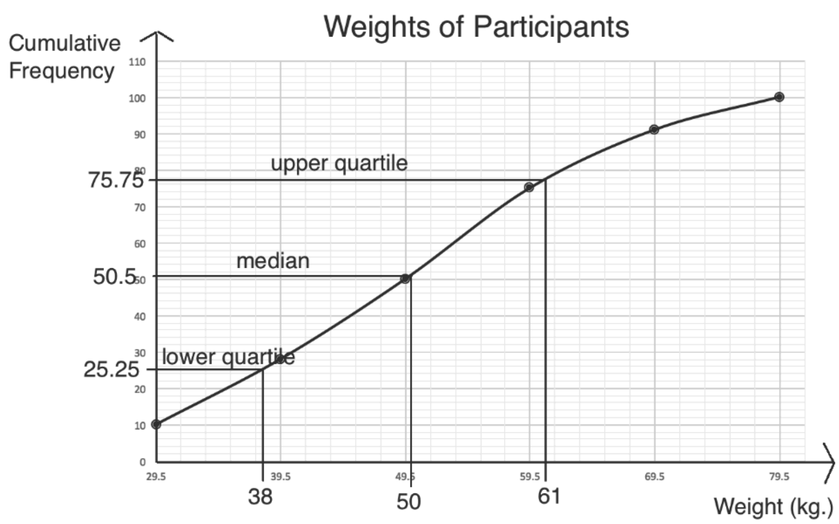


c. Estimated median: 74 marks; d. Estimated lower quartile: 64.5 marks; estimated upper quartile: 82.5 marks; e. Interquartile range: 18 marks; f. Semi-interquartile range: 9 marks.

2. a. Cumulative frequency table

Weight (kg)	No. of Participants	Upper class interval	Cumulative Frequency
20 – 29	10	29.5	10
30 – 39	18	39.5	$10 + 18 = 28$
40 – 49	22	49.5	$28 + 22 = 50$
50 – 59	25	59.5	$50 + 25 = 75$
60 – 69	16	695	$75 + 16 = 91$
70 – 79	9	79.5	$91 + 9 = 100$
Total	100		

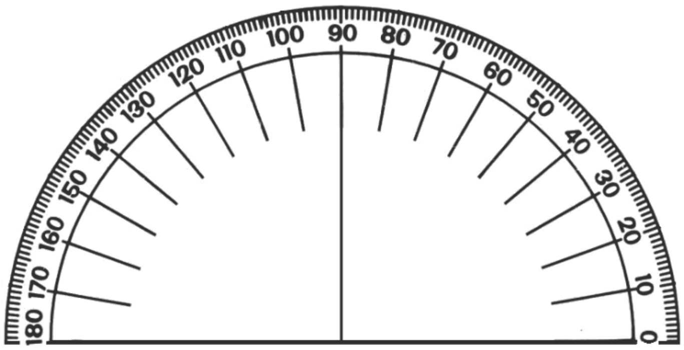
b. Cumulative frequency curve:



c. i. Estimated median: 50 kg; ii. Semi-interquartile range: 11.5 kg

Appendix I: Protractor

You can use a protractor to measure angles. If you do not have a protractor, you can make one with paper. Trace this protractor with a pen onto another piece of paper. Then, cut out the semi-circle using scissors.



Appendix II: Sines of Angles

Sines of Angles (x in degrees)

$x \rightarrow \sin x$

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
0	0·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	2	3	5	7	9	10	12	14	16
1	0·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	2	3	5	7	9	10	12	14	16
2	0·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	2	3	5	7	9	10	12	14	16
3	0·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	2	3	5	7	9	10	12	14	16
4	0·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	2	3	5	7	9	10	12	14	16
5	0·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	2	3	5	7	9	10	12	14	16
6	0·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	2	3	5	7	9	10	12	14	16
7	0·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	2	3	5	7	9	10	12	14	16
8	0·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	2	3	5	7	9	10	12	14	16
9	0·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	2	3	5	7	9	10	12	14	15
10	0·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	2	3	5	7	9	10	12	14	15
11	0·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	2	3	5	7	9	10	12	14	15
12	0·2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	2	3	5	7	9	10	12	14	15
13	0·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	2	3	5	7	8	10	12	14	15
14	0·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	2	3	5	7	8	10	12	14	15
15	0·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	2	3	5	7	8	10	12	13	15
16	0·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	2	3	5	7	8	10	12	13	15
17	0·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	2	3	5	7	8	10	12	13	15
18	0·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	2	3	5	7	8	10	12	13	15
19	0·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	2	3	5	7	8	10	12	13	15
20	0·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	2	3	5	7	8	10	11	13	15
21	0·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	2	3	5	6	8	10	11	13	15
22	0·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	2	3	5	6	8	10	11	13	15
23	0·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	2	3	5	6	8	10	11	13	14
24	0·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	2	3	5	6	8	10	11	13	14
25	0·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	2	3	5	6	8	9	11	13	14
26	0·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	2	3	5	5	8	9	11	12	14
27	0·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	2	3	5	6	8	9	11	12	14
28	0·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	2	3	5	6	8	9	11	12	14
29	0·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	2	3	5	6	8	9	11	12	14
30	0·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	2	3	5	6	8	9	11	12	14
31	0·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	1	3	4	6	7	9	10	12	13
32	0·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	1	3	4	6	7	9	10	12	13
33	0·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	1	3	4	6	7	9	10	12	13
34	0·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	1	3	4	6	7	9	10	12	13
35	0·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	1	3	4	6	7	9	10	11	13
36	0·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	1	3	4	6	7	8	10	11	13
37	0·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	1	3	4	6	7	8	10	11	12
38	0·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	1	3	4	5	7	8	10	11	12
39	0·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	1	3	4	5	7	8	9	11	12
40	0·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	1	3	4	5	7	8	9	11	12
41	0·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	1	3	4	5	7	8	9	10	12
42	0·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	1	3	4	5	6	8	9	10	12
43	0·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	1	3	4	5	6	8	9	10	11
44	0·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	1	2	4	5	6	7	9	10	11
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
45	0·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	1	2	4	5	6	7	9	10	11
46	0·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	1	2	4	5	6	7	8	10	11
47	0·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	1	2	4	5	6	7	8	9	11
48	0·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	1	2	3	5	6	7	8	9	10
49	0·7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	1	2	3	5	6	7	8	9	10
50	0·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	1	2	3	4	6	7	8	9	10
51	0·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	1	2	3	4	5	7	8	9	10
52	0·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	1	2	3	4	5	6	7	8	10
53	0·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	1	2	3	4	5	6	7	8	9
54	0·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	1	2	3	4	5	6	7	8	9
55	0·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	1	2	3	4	5	6	7	8	9
56	0·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	1	2	3	4	5	6	7	8	9
57	0·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	1	2	3	4	5	6	7	8	8
58	0·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	1	2	3	4	5	5	6	7	8
59	0·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	2	3	4	4	5	6	7	8
60	0·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	2	3	3	4	5	6	7	8
61	0·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	2	2	3	4	5	6	7	7
62	0·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	2	2	3	4	5	6	6	7
63	0·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	2	2	3	4	5	5	6	7
64	0·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	2	2	3	4	5	5	6	7
65	0·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	1	2	3	4	4	5	6	7
66	0·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	1	2	3	3	4	5	6	6
67	0·9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	1	2	3	3	4	5	5	6
68	0·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	1	2	3	3	4	4	5	6
69	0·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	1	2	2	3	4	4	5	6
70	0·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	1	2	2	3	3	4	5	5
71	0·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	1	2	2	3	3	4	4	5
72	0·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	1	2	2	3	3	4	4	5
73	0·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	0	1	1	2	2	3	3	4	4
74	0·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	0	1	1	2	2	3	3	4	4
75	0·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	0	1	1	2	2	3	3	3	4
76	0·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	0	1	1	2	2	2	3	3	4
77	0·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	0	1	1	2	2	2	3	3	3
78	0·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	0	1	1	1	2	2	2	3	3
79	0·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	0	1	1	1	2	2	2	3	3
80	0·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	1	1	2	2	2	3
81	0·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	1	1	2	2	2	2
82	0·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	0	1	1	1	1	2	2	2
83	0·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	0	1	1	1	1	1	2	2
84	0·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	0	1	1	1	1	1	1	2
85	0·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	0	1	1	1	1	1	1
86	0·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	0	0	1	1	1	1	1
87	0·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	0	0	0	1	1	1
88	0·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998									
89	0·9998	9999	9999	9999	9999	1·0000	1·0000	1·0000	1·0000	1·0000									
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix III: Cosines of Angles

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
											SUBTRACT Differences								
0	1·0000	1·000	1·000	1·000	1·000	1·000	·9999	·9999	·9999	·9999									
1	0·9998	·9998	·9998	·9997	·9997	·9997	·9996	·9996	·9995	·9995									
2	0·9994	·9993	·9993	·9992	·9991	·9990	·9990	·9989	·9988	·9987	0	0	0	0	0	0	1	1	1
3	0·9986	·9985	·9984	·9983	·9982	·9981	·9980	·9979	·9978	·9977	0	0	0	0	1	1	1	1	1
4	0·9976	·9974	·9973	·9972	·9971	·9969	·9968	·9966	·9965	·9963	0	0	0	1	1	1	1	1	1
5	0·9962	·9960	·9959	·9957	·9956	·9954	·9952	·9951	·9949	·9947	0	0	1	1	1	1	1	1	2
6	0·9945	·9943	·9942	·9940	·9938	·9936	·9934	·9932	·9930	·9928	0	0	1	1	1	1	1	2	2
7	0·9925	·9923	·9921	·9919	·9917	·9914	·9912	·9910	·9907	·9905	0	0	1	1	1	1	2	2	2
8	0·9903	·9900	·9898	·9895	·9893	·9890	·9888	·9885	·9882	·9880	0	1	1	1	1	2	2	2	2
9	0·9877	·9874	·9871	·9869	·9866	·9863	·9860	·9857	·9854	·9851	0	1	1	1	1	2	2	2	3
10	0·9848	·9845	·9842	·9839	·9836	·9833	·9829	·9826	·9823	·9820	0	1	1	1	2	2	2	3	3
11	0·9816	·9813	·9810	·9806	·9803	·9799	·9796	·9792	·9789	·9785	0	1	1	1	2	2	2	3	3
12	0·9781	·9778	·9774	·9770	·9767	·9763	·9759	·9755	·9751	·9748	0	1	1	2	2	2	3	3	3
13	0·9744	·9740	·9736	·9732	·9728	·9724	·9720	·9715	·9711	·9707	0	1	1	2	2	2	3	3	4
14	0·9703	·9699	·9694	·9690	·9686	·9681	·9677	·9673	·9668	·9664	0	1	1	2	2	3	3	3	4
15	0·9659	·9655	·9650	·9646	·9641	·9636	·9632	·9627	·9622	·9617	0	1	1	2	2	3	3	4	4
16	0·9613	·9608	·9603	·9598	·9593	·9588	·9583	·9578	·9573	·9568	0	1	1	2	2	3	3	4	4
17	0·9563	·9558	·9553	·9548	·9542	·9537	·9532	·9527	·9521	·9516	1	1	2	2	3	3	4	4	5
18	0·9511	·9505	·9500	·9494	·9489	·9483	·9478	·9472	·9466	·9461	1	1	2	2	3	3	4	4	5
19	0·9455	·9449	·9444	·9438	·9432	·9426	·9421	·9415	·9409	·9403	1	1	2	2	3	3	4	5	5
20	0·9397	·9391	·9385	·9379	·9373	·9367	·9361	·9354	·9348	·9342	1	1	2	2	3	4	4	5	6
21	0·9336	·9330	·9323	·9317	·9311	·9304	·9298	·9291	·9285	·9278	1	1	2	3	3	4	4	5	6
22	0·9272	·9265	·9259	·9252	·9245	·9239	·9232	·9225	·9219	·9212	1	1	2	3	3	4	5	5	6
23	0·9205	·9198	·9191	·9184	·9178	·9171	·9164	·9157	·9150	·9143	1	1	2	3	3	4	5	6	6
24	0·9135	·9128	·9121	·9114	·9107	·9100	·9092	·9085	·9078	·9070	1	1	2	3	4	4	5	6	7
25	0·9063	·9056	·9048	·9041	·9033	·9026	·9018	·9011	·9003	·8996	1	2	2	3	4	5	5	6	7
26	0·8988	·8980	·8973	·8965	·8957	·8949	·8942	·8934	·8926	·8918	1	2	2	3	4	5	5	6	7
27	0·8910	·8902	·8894	·8886	·8878	·8870	·8862	·8854	·8846	·8838	1	2	2	3	4	5	6	6	7
28	0·8829	·8821	·8813	·8805	·8796	·8788	·8780	·8771	·8763	·8755	1	2	2	3	4	5	6	7	7
29	0·8746	·8738	·8729	·8721	·8712	·8704	·8695	·8686	·8678	·8669	1	2	3	3	4	5	6	7	8
30	0·8660	·8652	·8643	·8634	·8625	·8616	·8607	·8599	·8590	·8581	1	2	3	4	4	5	6	7	8
31	0·8572	·8563	·8554	·8545	·8536	·8526	·8517	·8508	·8499	·8490	1	2	3	4	5	5	6	7	8
32	0·8480	·8471	·8462	·8453	·8443	·8434	·8425	·8415	·8406	·8396	1	2	3	4	5	6	7	8	8
33	0·8387	·8377	·8368	·8358	·8348	·8339	·8329	·8320	·8310	·8300	1	2	3	4	5	6	7	8	9
34	0·8290	·8281	·8271	·8261	·8251	·8241	·8231	·8221	·8211	·8202	1	2	3	4	5	6	7	8	9
35	0·8192	·8181	·8171	·8161	·8151	·8141	·8131	·8121	·8111	·8100	1	2	3	4	5	6	7	8	9
36	0·8090	·8080	·8070	·8059	·8049	·8039	·8028	·8018	·8007	·7997	1	2	3	4	5	6	7	8	9
37	0·7986	·7976	·7965	·7955	·7944	·7934	·7923	·7912	·7902	·7891	1	2	3	4	5	6	7	8	10
38	0·7880	·7869	·7859	·7848	·7837	·7826	·7815	·7804	·7793	·7782	1	2	3	4	5	6	7	9	10
39	0·7771	·7760	·7749	·7738	·7727	·7716	·7705	·7694	·7683	·7672	1	2	3	4	6	7	8	9	10
40	0·7660	·7649	·7638	·7627	·7615	·7604	·7593	·7581	·7570	·7559	1	2	3	5	6	7	8	9	10
41	0·7547	·7536	·7524	·7513	·7501	·7490	·7478	·7466	·7455	·7443	1	2	3	5	6	7	8	9	10
42	0·7431	·7420	·7408	·7396	·7385	·7373	·7361	·7349	·7337	·7325	1	2	4	5	6	7	8	9	11
43	0·7314	·7302	·7290	·7278	·7266	·7254	·7242	·7230	·7218	·7206	1	2	4	5	6	7	8	10	11
44	0·7193	·7181	·7169	·7157	·7145	·7133	·7120	·7108	·7096	·7083	1	2	4	5	6	7	9	10	11
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
											SUBTRACT Differences								
45	0·7071	·7059	·7046	·7034	·7022	·7009	·6997	·6984	·6972	·6959	1	2	4	5	6	7	9	10	11
46	0·6947	·6934	·6921	·6909	·6896	·6884	·6871	·6858	·6845	·6833	1	3	4	5	6	8	9	10	11
47	0·6820	·6807	·6794	·6782	·6769	·6756	·6743	·6730	·6717	·6704	1	3	4	5	6	8	9	10	12
48	0·6691	·6678	·6665	·6652	·6639	·6626	·6613	·6600	·6587	·6574	1	3	4	5	7	8	9	10	12
49	0·6561	·6547	·6534	·6521	·6508	·6494	·6481	·6468	·6455	·6441	1	3	4	5	7	8	9	11	12
50	0·6428	·6414	·6401	·6388	·6374	·6361	·6347	·6334	·6320	·6307	1	3	4	5	7	8	9	11	12
51	0·6293	·6280	·6266	·6252	·6239	·6225	·6211	·6198	·6184	·6170	1	3	4	5	7	8	10	11	12
52	0·6157	·6143	·6129	·6115	·6101	·6088	·6074	·6060	·6046	·6032	1	3	4	5	7	8	10	11	12
53	0·6018	·6004	·5990	·5976	·5962	·5948	·5934	·5920	·5906	·5892	1	3	4	6	7	8	10	11	13
54	0·5878	·5864	·5850	·5835	·5821	·5807	·5793	·5779	·5764	·5750	1	3	4	6	7	9	10	11	13
55	0·5736	·5721	·5707	·5693	·5678	·5664	·5650	·5635	·5621	·5606	1	3	4	6	7	9	10	12	13
56	0·5592	·5577	·5563	·5548	·5534	·5519	·5505	·5490	·5476	·5461	1	3	4	6	7	9	10	12	13
57	0·5446	·5432	·5417	·5402	·5388	·5373	·5358	·5344	·5329	·5314	1	3	4	6	7	9	10	12	13
58	0·5299	·5284	·5270	·5255	·5240	·5225	·5210	·5195	·5180	·5165	1	3	4	6	7	9	10	12	13
59	0·5150	·5135	·5120	·5105	·5090	·5075	·5060	·5045	·5030	·5015	2	3	5	6	8	9	11	12	14
60	0·5000	·4985	·4970	·4955	·4939	·4924	·4909	·4894	·4879	·4863	2	3	5	6	8	9	11	12	14
61	0·4848	·4833	·4818	·4802	·4787	·4772	·4756	·4741	·4726	·4710	2	3	5	6	8	9	11	12	14
62	0·4695	·4679	·4664	·4648	·4633	·4617	·4602	·4586	·4571	·4555	2	3	5	6	8	9	11	12	14
63	0·4540	·4524	·4509	·4493	·4478	·4462	·4446	·4431	·4415	·4399	2	3	5	6	8	9	11	12	14
64	0·4384	·4368	·4352	·4337	·4321	·4305	·4289	·4274	·4258	·4242	2	3	5	6	8	9	11	13	14
65	0·4226	·4210	·4195	·4179	·4163	·4147	·4131	·4115	·4099	·4083	2	3	5	6	8	10	11	13	14
66	0·4067	·4051	·4035	·4019	·4003	·3987	·3971	·3955	·3939	·3923	2	3	5	6	8	10	11	13	14
67	0·3907	·3891	·3875	·3859	·3843	·3827	·3811	·3795	·3778	·3762	2	3	5	6	8	10	11	13	15
68	0·3746	·3730	·3714	·3697	·3681	·3665	·3649	·3633	·3616	·3600	2	3	5	6	8	10	11	13	15
69	0·3584	·3567	·3551	·3535	·3518	·3502	·3486	·3469	·3453	·3437	2	3	5	6	8	10	11	13	15
70	0·3420	·3404	·3387	·3371	·3355	·3338	·3322	·3305	·3289	·3272	2	3	5	7	8	10	12	13	15
71	0·3256	·3239	·3223	·3206	·3190	·3173	·3156	·3140	·3123	·3107	2	3	5	7	8	10	12	13	15
72	0·3090	·3074	·3057	·3040	·3024	·3007	·2990	·2974	·2957	·2940	2	3	5	7	8	10	12	13	15
73	0·2924	·2907	·2890	·2874	·2857	·2840	·2823	·2807	·2790	·2773	2	3	5	7	8	10	12	13	15
74	0·2756	·2740	·2723	·2706	·2689	·2672	·2656	·2639	·2622	·2605	2	3	5	7	8	10	12	13	15
75	0·2588	·2571	·2554	·2538	·2521	·2504	·2487	·2470	·2453	·2436	2	3	5	7	8	10	12	14	15
76	0·2419	·2402	·2385	·2368	·2351	·2334	·2317	·2300	·2284	·2267	2	3	5	7	8	10	12	14	15
77	0·2250	·2233	·2215	·2198	·2181	·2164	·2147	·2130	·2113	·2096	2	3	5	7	9	10	12	13	15
78	0·2079	·2062	·2045	·2028	·2011	·1994	·1977	·1959	·1942	·1925	2	3	5	7	9	10	12	14	15
79	0·1908	·1891	·1874	·1857	·1840	·1822	·1805	·1788	·1771	·1754	2	3	5	7	9	10	12	14	15
80	0·1736	·1719	·1702	·1685	·1668	·1650	·1633	·1616	·1599	·1582	2	3	5	7	9	10	12	14	15
81	0·1564	·1547	·1530	·1513	·1495	·1478	·1461	·1444	·1426	·1409	2	3	5	7	9	10	12	14	16
82	0·1392	·1374	·1357	·1340	·1323	·1305	·1288	·1271	·1253	·1236	2	3	5	7	9	10	12	14	16
83	0·1219	·1201	·1184	·1167	·1149	·1132	·1115	·1097	·1080	·1063	2	3	5	7	9	10	12	14	16
84	0·1045	·1028	·1011	·0993	·0976	·0958	·0941	·0924	·0906	·0889	2	3	5	7	9	10	12	14	16
85	0·0872	·0854	·0837	·0819	·0802	·0785	·0767	·0750	·0732	·0715	2	3	5	7	9	10	12	14	16
86	0·0698	·0680	·0663	·0645	·0628	·0610	·0593	·0576	·0558	·0541	2	3	5	7	9	10	12	14	16
87	0·0523	·0506	·0488	·0471	·0454	·0436	·0419	·0401	·0384	·0366	2	3	5	7	9	10	12	14	16
88	0·0349	·0332	·0314	·0297	·0279	·0262	·0244	·0227	·0209	·0192	2	3	5	7	9	10	12	14	16
89	0·0175	·0157	·0140	·0122	·0105	·0087	·0070	·0052	·0035	·0017	2	3	5	7	9	10	12	14	16
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

Appendix IV: Tangents of Angles

Tangents of Angles (*x* in degrees)

$x \rightarrow \tan x$

											ADD Differences								
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9
0	0·0000	·0017	·0035	·0052	·0070	·0087	·0105	·0122	·0140	·0157	2	3	5	7	9	10	12	14	16
1	0·0175	·0192	·0209	·0227	·0244	·0262	·0279	·0297	·0314	·0332	2	3	5	7	9	10	12	14	16
2	0·0349	·0367	·0384	·0402	·0419	·0437	·0454	·0472	·0489	·0507	2	3	5	7	9	10	12	14	16
3	0·0524	·0542	·0559	·0577	·0594	·0612	·0629	·0647	·0664	·0682	2	4	5	7	9	11	12	14	16
4	0·0699	·0717	·0734	·0752	·0769	·0787	·0805	·0822	·0840	·0857	2	4	5	7	9	11	12	14	16
5	0·0875	·0892	·0910	·0928	·0945	·0963	·0981	·0998	·1016	·1033	2	4	5	7	9	11	12	14	16
6	0·1051	·1069	·1086	·1104	·1122	·1139	·1157	·1175	·1192	·1210	2	4	5	7	9	11	12	14	16
7	0·1228	·1246	·1263	·1281	·1299	·1317	·1334	·1352	·1370	·1388	2	4	5	7	9	11	12	14	16
8	0·1405	·1423	·1441	·1459	·1477	·1495	·1512	·1530	·1548	·1566	2	4	5	7	9	11	12	14	16
9	0·1584	·1602	·1620	·1638	·1655	·1673	·1691	·1709	·1727	·1745	2	4	5	7	9	11	13	14	16
10	0·1763	·1781	·1799	·1817	·1835	·1853	·1871	·1890	·1908	·1926	2	4	5	7	9	11	13	14	16
11	0·1944	·1962	·1980	·1998	·2016	·2035	·2053	·2071	·2089	·2107	2	4	5	7	9	11	13	15	16
12	0·2126	·2144	·2162	·2180	·2199	·2217	·2235	·2254	·2272	·2290	2	4	5	7	9	11	13	15	16
13	0·2309	·2327	·2345	·2364	·2382	·2401	·2419	·2438	·2456	·2475	2	4	6	7	9	11	13	15	17
14	0·2493	·2512	·2530	·2549	·2568	·2586	·2605	·2623	·2642	·2661	2	4	6	7	9	11	13	15	17
15	0·2679	·2698	·2717	·2736	·2754	·2773	·2792	·2811	·2830	·2849	2	4	6	8	9	11	13	15	17
16	0·2867	·2886	·2905	·2924	·2943	·2962	·2981	·3000	·3019	·3038	2	4	6	8	9	11	13	15	17
17	0·3057	·3076	·3096	·3115	·3134	·3153	·3172	·3191	·3211	·3230	2	4	6	8	10	12	13	15	17
18	0·3249	·3269	·3288	·3307	·3327	·3346	·3365	·3385	·3404	·3424	2	4	6	8	10	12	14	16	17
19	0·3443	·3463	·3482	·3502	·3522	·3541	·3561	·3581	·3600	·3620	2	4	6	8	10	12	14	16	18
20	0·3640	·3659	·3679	·3699	·3719	·3739	·3759	·3779	·3799	·3819	2	4	6	8	10	12	14	16	18
21	0·3839	·3859	·3879	·3899	·3919	·3939	·3959	·3979	·4000	·4020	2	4	6	8	10	12	14	16	18
22	0·4040	·4061	·4081	·4101	·4122	·4142	·4163	·4183	·4204	·4224	2	4	6	8	10	12	14	16	18
23	0·4245	·4265	·4286	·4307	·4327	·4348	·4369	·4390	·4411	·4431	2	4	6	8	10	12	15	17	19
24	0·4452	·4473	·4494	·4515	·4536	·4557	·4578	·4599	·4621	·4642	2	4	6	8	11	13	15	17	19
25	0·4663	·4684	·4706	·4727	·4748	·4770	·4791	·4813	·4834	·4856	2	4	6	9	11	13	15	17	19
26	0·4877	·4899	·4921	·4942	·4964	·4986	·5008	·5029	·5051	·5073	2	4	7	9	11	13	15	17	20
27	0·5095	·5117	·5139	·5161	·5184	·5206	·5228	·5250	·5272	·5295	2	4	7	9	11	13	16	18	20
28	0·5317	·5340	·5362	·5384	·5407	·5430	·5452	·5475	·5498	·5520	2	5	7	9	11	14	16	18	20
29	0·5543	·5566	·5589	·5612	·5635	·5658	·5681	·5704	·5727	·5750	2	5	7	9	12	14	16	18	21
30	0·5774	·5797	·5820	·5844	·5867	·5890	·5914	·5938	·5961	·5985	2	5	7	9	12	14	16	19	21
31	0·6009	·6032	·6056	·6080	·6104	·6128	·6152	·6176	·6200	·6224	2	5	7	10	12	14	17	19	22
32	0·6249	·6273	·6297	·6322	·6346	·6371	·6395	·6420	·6445	·6469	2	5	7	10	12	15	17	20	22
33	0·6494	·6519	·6544	·6569	·6594	·6619	·6644	·6669	·6694	·6720	3	5	8	10	13	15	18	20	23
34	0·6745	·6771	·6796	·6822	·6847	·6873	·6899	·6924	·6950	·6976	3	5	8	10	13	15	18	21	23
35	0·7002	·7028	·7054	·7080	·7107	·7133	·7159	·7186	·7212	·7239	3	5	8	11	13	16	18	21	24
36	0·7265	·7292	·7319	·7346	·7373	·7400	·7427	·7454	·7481	·7508	3	5	8	11	14	16	19	22	24
37	0·7536	·7563	·7590	·7618	·7646	·7673	·7701	·7729	·7757	·7785	3	6	8	11	14	17	19	22	25
38	0·7813	·7841	·7869	·7898	·7926	·7954	·7983	·8012	·8040	·8069	3	6	9	11	14	17	20	23	26
39	0·8098	·8127	·8156	·8185	·8214	·8243	·8273	·8302	·8332	·8361	3	6	9	12	15	18	21	23	26
40	0·8391	·8421	·8451	·8481	·8511	·8541	·8571	·8601	·8632	·8662	3	6	9	12	15	18	21	24	27
41	0·8693	·8724	·8754	·8785	·8816	·8847	·8878	·8910	·8941	·8972	3	6	9	12	16	19	22	25	28
42	0·9004	·9036	·9067	·9099	·9131	·9163	·9195	·9228	·9260	·9293	3	6	10	13	16	19	22	26	29
43	0·9325	·9358	·9391	·9424	·9457	·9490	·9523	·9556	·9590	·9623	3	7	10	13	17	20	23	27	30
44	0·9657	·9691	·9725	·9759	·9793	·9827	·9861	·9896	·9930	·9965	3	7	10	14	17	21	24	27	31
45	1·000	1·003	1·007	1·011	1·014	1·018	1·021	1·025	1·028	1·032	0	1	1	1	2	2	2	3	3
46	1·036	1·039	1·043	1·046	1·050	1·054	1·057	1·061	1·065	1·069	0	1	1	1	2	2	3	3	3
47	1·072	1·076	1·080	1·084	1·087	1·091	1·095	1·099	1·103	1·107	0	1	1	2	2	2	3	3	3
48	1·111	1·115	1·118	1·122	1·126	1·130	1·134	1·138	1·142	1·146	0	1	1	2	2	2	3	3	4
49	1·150	1·154	1·159	1·163	1·167	1·171	1·175	1·179	1·183	1·188	0	1	1	2	2	2	3	3	4
50	1·192	1·196	1·200	1·205	1·209	1·213	1·217	1·222	1·226	1·230	0	1	1	2	2	3	3	4	4
51	1·235	1·239	1·244	1·248	1·253	1·257	1·262	1·266	1·271	1·275	0	1	1	2	2	3	3	4	4
52	1·280	1·285	1·289	1·294	1·299	1·303	1·308	1·313	1·317	1·322	0	1	1	2	2	3	3	4	4
53	1·327	1·332	1·337	1·342	1·347	1·351	1·356	1·361	1·366	1·371	0	1	1	2	2	3	3	4	4
54	1·376	1·381	1·387	1·392	1·397	1·402	1·407	1·412	1·418	1·423	1	1	2	2	3	3	4	4	5
55	1·428	1·433	1·439	1·444	1·450	1·455	1·460	1·466	1·471	1·477	1	1	2	2	3	3	4	4	5
56	1·483	1·488	1·494	1·499	1·505	1·511	1·517	1·522	1·528	1·534	1	1	2	2	3	3	4	5	5
57	1·540	1·546	1·552	1·558	1·564	1·570	1·576	1·582	1·588	1·594	1	1	2	2	3	4	4	5	5
58	1·600	1·607	1·613	1·619	1·625	1·632	1·638	1·645	1·651	1·658	1	1	2	3	3	4	4	5	6
59	1·664	1·671	1·678	1·684	1·691	1·698	1·704	1·711	1·718	1·725	1	1	2	3	3	4	5	5	6
60	1·732	1·739	1·746	1·753	1·760	1·767	1·775	1·782	1·789	1·797	1	1	2	3	4	4	5	6	6
61	1·804	1·811	1·819	1·827	1·834	1·842	1·849	1·857	1·865	1·873	1	2	2	3	4	5	5	6	7
62	1·881	1·889	1·897	1·905	1·913	1·921	1·929	1·937	1·946	1·954	1	2	2	3	4	5	6	7	7
63	1·963	1·971	1·980	1·988	1·997	2·006	2·014	2·023	2·032	2·041	1	2	3	4	4	5	6	7	8
64	2·050	2·059	2·069	2·078	2·087	2·097	2·106	2·116	2·125	2·135	1	2	3	4	5	6	7	8	8
65	2·145	2·154	2·164	2·174	2·184	2·194	2·204	2·215	2·225	2·236	1	2	3	4	5	6	7	8	9
66	2·246	2·257	2·267	2·278	2·289	2·300	2·311	2·322	2·333	2·344	1	2	3	4	5	7	8	9	10
67	2·356	2·367	2·379	2·391	2·402	2·414	2·426	2·438	2·450	2·463	1	2	4	5	6	7	8	10	11
68	2·475	2·488	2·500	2·513	2·526	2·539	2·552	2·565	2·578	2·592	1	3	4	5	7	8	9	10	12
69	2·605	2·619	2·633	2·646	2·660	2·675	2·689	2·703	2·718	2·733	1	3	4	6	7	9	10	11	13
70	2·747	2·762	2·778	2·793	2·808	2·824	2·840	2·856	2·872	2·888	2	3	5	6	8	9	11	13	14
71	2·904	2·921	2·937	2·954	2·971	2·989	3·006	3·024	3·042	3·060	2	3	5	7	9	10	12	14	16
72	3·078	3·096	3·115	3·133	3·152	3·172	3·191	3·211	3·230	3·251	2	4	6	8	10	12	14	15	17
73	3·271	3·291	3·312	3·333	3·354	3·376	3·398	3·420	3·442	3·465	2	4	6	9	11	13	15	17	19
74	3·487	3·511	3·534	3·558	3·582	3·606	3·630	3·655	3·681	3·706	2	5	7	10	12	15	17	20	22
75	3·732	3·758	3·785	3·812	3·839	3·867	3·895	3·923	3·952	3·981	3	6	8	11	14	17	20	22	25
76	4·011	4·041	4·071	4·102	4·134	4·165	4·198	4·230	4·264	4·297	3	6	10	13	16	19	22	26	29
77	4·331	4·366	4·402	4·437	4·474	4·511	4·548	4·586	4·625	4·666									
78	4·705	4·745	4·787	4·829	4·872	4·915	4·959	5·005	5·050	5·097									
79	5·145	5·193	5·242	5·292	5·343	5·396	5·449	5·503	5·558	5·614									
80	5·671	5·730	5·789	5·850	5·912	5·976	6·041	6·107	6·174	6·243				differences unreliable:					
81	6·314	6·386	6·460	6·535	6·612	6·691	6·772	6·855	6·940	7·026									
82	7·115	7·207	7·300	7·396	7·495	7·596	7·700	7·806	7·916	8·028									
83	8·144	8·264	8·386	8·513	8·643	8·777	8·915	9·058	9·205	9·357									
84	9·514	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20									
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95									
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46									
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27									
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08									
89	57·29	63·66	71·62	81·85	95·46	114·6	143·2	191·0	286·5	573·0									
x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1	2	3	4	5	6	7	8	9

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