Free Quality School Education

Ministry of Basic and Senior Secondary Education

Pupils' Handbook for Senior Secondary *Mathematics*

SSS I

Term II

STRICTLY NOT FOR SALE

Foreword

These Lesson Plans and the accompanying Pupils' Handbooks are essential educational resources for the promotion of quality education in senior secondary schools in Sierra Leone. As Minister of Basic and Senior Secondary Education, I am pleased with the professional competencies demonstrated by the writers of these educational materials in English Language and Mathematics.

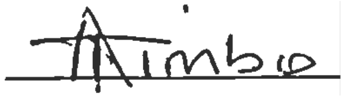
The Lesson Plans give teachers the support they need to cover each element of the national curriculum, as well as prepare pupils for the West African Examinations Council's (WAEC) examinations. The practice activities in the Pupils' Handbooks are designed to support self-study by pupils, and to give them additional opportunities to learn independently. In total, we have produced 516 lesson plans and 516 practice activities – one for each lesson, in each term, in each year, for each class. The production of these materials in a matter of months is a remarkable achievement.

These plans have been written by experienced Sierra Leoneans together with international educators. They have been reviewed by officials of my Ministry to ensure that they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognized techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make the best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new lesson plans. It is really important that the Lesson Plans and Pupils' Handbooks are used, together with any other materials they may have.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank our partners for their continued support. Finally, I also thank the teachers of our country for their hard work in securing our future



Mr. Alpha Osman Timbo

Minister of Basic and Senior Secondary Education

The policy of the Ministry of Basic and Senior Secondary Education, Sierra Leone, on textbooks stipulates that every printed book should have a lifespan of three years.

To achieve thus, DO NOT WRITE IN THE BOOKS.

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Introduction

to the Pupils' Handbook

These practice activities are aligned to the Lesson Plans, and are based on the National Curriculum and the West Africa Examination Council syllabus guidelines. They meet the requirements established by the Ministry of Basic and Senior Secondary Education.

1. The practice activities will not take the whole term, so use any extra time to revise material or re-do activities where you made mistakes.
2. Use other textbooks or resources to help you learn better and practise what you have learned in the lessons.
3. Read the questions carefully before answering them. After completing the practice activities, check your answers using the answer key at the end of the book.
4. Make sure you understand the learning outcomes for the practice activities and check to see that you have achieved them. Each lesson plan shows these using the symbol to the right. — Learning Outcomes
5. Organise yourself so that you have enough time to complete all of the practice activities. If there is time, quickly revise what you learned in the lesson before starting the practice activities. If it is taking you too long to complete the activities, you may need more practice on that particular topic.
6. Seek help from your teacher or your peers if you are having trouble completing the practice activities independently.
7. Make sure you write the answers in your exercise book in a clear and systematic way so that your teacher can check your work and you can refer back to it when you prepare for examinations.
8. Congratulate yourself when you get questions right! Do not worry if you do not get the right answer – ask for help and continue practising!

KEY TAKEAWAYS FROM SIERRA LEONE'S PERFORMANCE IN WEST AFRICAN SENIOR SCHOOL CERTIFICATE EXAMINATION – GENERAL MATHEMATICS[1]

This section, seeks to outline key takeaways from assessing Sierra Leonean pupils' responses on the West African Senior School Certificate Examination. The common errors pupils make are highlighted below with the intention of giving teachers an insight into areas to focus on, to improve pupil performance on the examination. Suggestions are provided for addressing these issues.

Common errors

1. Errors in applying principles of BODMAS
2. Mistakes in simplifying fractions
3. Errors in application of Maths learned in class to real-life situations, and vis-a-versa.
4. Errors in solving geometric constructions.
5. Mistakes in solving problems on circle theorems.
6. Proofs are often left out from solutions, derivations are often missing from quadratic equations.

Suggested solutions

1. Practice answering questions to the detail requested
2. Practice re-reading questions to make sure all the components are answered.
3. If possible, procure as many geometry sets to practice geometry construction.
4. Check that depth and level of the lesson taught is appropriate for the grade level.

[1] This information is derived from an evaluation of WAEC Examiner Reports, as well as input from WAEC examiners and Sierra Leonean teachers.

Lesson Title: Powers and roots of logarithms – Numbers greater than 1	**Theme:** Numbers and numeration
Practice Activity: PHM1-L049	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to calculate powers and roots of numbers greater than 1 using logarithms.

Overview

This lesson is on calculating powers and roots of numbers greater than 1 using logarithms. For example, consider the problems 138^2 and $\sqrt[3]{46.5}$. These seem difficult to calculate without using a calculator. However, we can calculate them using logarithms.

The **product rule** is applied to evaluate **powers** of numbers greater than 1:

Step 1. Find the logarithm of the number.
Step 2. Multiply the logarithm by the power.
Step 3. Find the antilog of the result in step 2.

Similarly, the **quotient rule** is applied to evaluate **roots** of numbers greater than 1:

Step 1. Find the logarithm of the number.
Step 2. Divide the logarithm by the root.
Step 3. Find the antilogarithm of the result in step 2.

You can apply logarithms to solve problems with more than one operation. For example, $245.5^2 \times 1.08^3$. This involves powers and multiplication. We apply the order of operations (BODMAS). Find the logarithms of the numbers first, then apply all of the operations necessary before applying the antilogarithm to the result.

Solved Examples

1. Evaluate 138^2

Solution:

Steps	Numbers	Logarithm
1. Find the logarithm	138	2.1399
2. Multiply by the power	138^2	$2.1399 \times 2 = 4.2798$
3. Find the antilog		$\text{antilog } 4.2798 = 19040$
Answer	$138^2 = 19040$	

2. Evaluate 3.12^5

Solution:

Steps	Numbers	Logarithm
1.	3.12	0.4942
2.	3.12^5	$0.4942 \times 5 = 2.471$
3.		antilog 2.471=295.8
Answer	$3.12^5 = 295.8$	

3. Evaluate $\sqrt{5}$

Solution:

This is a square root, so the root that we will divide by is 2.

Steps	Numbers	Logarithm
1. Find the logarithm	5	0.6990
2. Divide by the root	$\sqrt{5}$	$0.6990 \div 2 = 0.3495$
3. Find the antilog		antilog 0.3495=2.237
Answer	$\sqrt{5} = 2.237$	

4. Evaluate $\sqrt[3]{64.5}$

Solution:

This is a cubed root, so the root that we will divide by is 3.

Steps	Numbers	Logarithm
1.	64.5	1.8096
2.	$\sqrt[3]{64.5}$	$\frac{1.8096}{3} = 0.6032$
3.		antilog 0.6032= 4.011
Answer	$\sqrt[3]{64.5} = 4.011$	

5. Evaluate $\sqrt[4]{(25.1)^3}$

Solution:

This problem has both a square and a root on the number. Apply the square first using multiplication. Then, apply the root using division.

Steps	Number	Logarithm
Find the logarithm of $(25.1)^3$	25.1	1.3997
	$(25.1)^3$	$1.3997 \times 3 = 4.1991$
Find $\sqrt[4]{(25.1)^3}$	$\sqrt[4]{(25.1)^3}$	$4.1991 \div 4 = 1.0498$
	Answer	Antilog 1.0498 = 11.21

6. Evaluate $\sqrt{245.5} \times 8.01^2$

Solution:

In this case, roots and powers appear together. Find the logarithm of each separately. Then, add the logarithms to find the product of $\sqrt{245.5}$ and 8.01^2.

Steps	Number	Logarithm
Find the logarithm of $\sqrt{245.5}$	245.5	2.3901
	$\sqrt{245.5}$	$2.3901 \div 2 = 1.1951$
Find the logarithm of 8.01^2	8.01	0.9036
	8.01^2	$0.9036 \times 2 = 1.8072$
Multiply the 2 terms	$\sqrt{245.5} \times 8.01^2$	
Add their logarithms		1.1951 +1.8072
		3.0023
	Answer	Antilog 3.0023 = 1,006

Therefore, we have $\sqrt{245.5} \times 8.01^2 = 1{,}006$.

Practice

Use logarithm tables to:

1. Evaluate 31.1^2
2. Evaluate 3.39^4
3. Evaluate $\sqrt{225}$
4. Evaluate $\sqrt[4]{(62.5)^2}$
5. Evaluate $2.45^3 \times \sqrt[5]{611}$
6. Evaluate $\sqrt[3]{999} \div 2.33^2$

Lesson Title: Logarithms – Numbers less than 1	**Theme:** Numbers and numeration
Practice Activity: PHM1-L050	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to find the logarithms of numbers less than 1 using logarithm tables.

Overview

This lesson is on finding the logarithm of numbers less than 1 using logarithm tables. In other words, we are going to find the logarithm of decimal numbers which lie in the range 0 to 1.

An easy rule that works for decimals is that the characteristic is equal to the number of zeros trailing the decimal plus 1. Consider an example, 0.0314. The number has 1 zero after the decimal, so the characteristic is -2. The characteristic can also be found by writing the decimal number in standard form. Note that $0.0314 = 3.14 \times 10^{-2}$. The power of 10 gives the characteristic. When a decimal number should be expressed in its standard form $a \times 10^{-n}$, the characteristic of the logarithm is $\bar{n}$.

The characteristic is not written with a negative sign. We put the minus sign on top of the characteristic. Thus, we have $\bar{2},$ which is read as "**bar" 2**, but not "minus" 2.

The mantissa is found using a logarithm table, in the same way as with number greater than 1. Write the mantissa after the characteristic.

Solved Examples

1. Write the following numbers in standard form and derive the characteristics of their logarithms from the standard form.
   a. 0.121 b. 0.00212 c. 0.000561 d. 0.00410
   Solutions:

	Number	Standard form	Characteristics of logarithm
a.	0.121	1.21×10^{-1}	$\bar{1}$
b.	0.00212	2.12×10^{-3}	$\bar{3}$
c.	0.000561	5.61×10^{-4}	$\bar{4}$
d.	0.00410	4.10×10^{-3}	$\bar{3}$

2. Using the simple rule that the number of zeros trailing the decimal point plus 1 gives the characteristic, find the characteristic of the logarithm of each of the following numbers:
   a. 0.315 b. 0.00456 c. 0.000128 d. 0.0000100

Solutions:

	Number	Logarithm	Characteristics of logarithm
a.	0.315	$\log 0.315$	$-(0+1)=\bar{1}$
b.	0.00456	$\log 0.00456$	$-(2+1)=\bar{3}$
c.	0.000128	$\log 0.000128$	$-(3+1)=\bar{4}$
d.	0.0000100	$\log 0.0000100$	$-(4+1)=\bar{5}$

3. Find the logarithms of the following numbers using the logarithm table:

   a. 0.153 b. 0.00654 c. 0.000812 d. 0.0000210

   Solutions:

	Number	Logarithm
a.	0.153	$\log 0.153 = \bar{1}.1847$
b.	0.00654	$\log 0.00654 = \bar{3}.8156$
c.	0.000812	$\log 0.000812 = \bar{4}.9096$
d.	0.0000210	$\log 0.0000210 = \bar{5}.3222$

Practice

1. Write the following numbers in standard form. Then, write the characteristic of each.

   a. 0.4525 b. 0.0607 c. 0.00498 d. 0.000989

2. Using the simple rule that the number of zeros trailing the decimal point plus 1 gives the characteristic, find the characteristic of the logarithm of each of the following numbers:

   a. 0.135 b. 0.00546 c. 0.0786 d. 0.0000341

3. Find the logarithms of the following numbers:

   a. 0.123 b. 0.0987 c. 0.00654 d. 0.000899

Lesson Title: Antilogarithm – Numbers less than 0	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L051	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to find the antilogarithm of numbers less than 0 using antilogarithm tables.

Overview

Today's lesson is going to be on how to find the antilogarithms of the logarithms of numbers less than 0.

Antilogarithm is the exact opposite of the logarithm of a number. It brings the number back to its ordinary form from the logarithm form. So if $X = \log b$, then antilog $X = b$

To find the antilogarithms of numbers we use the antilogarithm tables, which give the inverse value of the mantissa to four figures. The characteristic is then used to position the decimal point in the value read from the mantissa. Write the number of zeros after the decimal based on the characteristic. Remember that there are 1 fewer zeros after the decimal than the value of the characteristic.

Solved Examples

1. Read the antilogarithms of the following logarithms of numbers.
   a. $\bar{1}.9178$ b. $\bar{2}.0451$ c. $\bar{6}.3251$ d. $\bar{3}.8547$
   e. $\bar{3}.2045$ f. $\bar{1}.3749$

   Solutions:

	Antilog	Value read from using mantissa antilog table	Number of zeros after the decimal	Answer
a.	$\bar{1}.9178$	8,275	0 zeros	0.8275
b.	$\bar{2}.0451$	1,109	1 zero	0.01109
c.	$\bar{6}.3251$	2,113	5 zeros	0.000002113
d.	$\bar{3}.8547$	7,157	2 zeros	0.007157
e.	$\bar{3}.2045$	1,602	2 zeros	0.001602
f.	$\bar{1}.3749$	2,371	0 zeros	0.2371

Practice

Find the antilogarithm of the following logarithms of numbers:

1. $\bar{2}.3067$
2. $\bar{4}.0101$
3. $\bar{1}.2011$

4. $\bar{4}.4145$
5. $\bar{6}.1101$
6. $\bar{5}.9999$
7. $\bar{2}.6303$
8. $\bar{3}.3344$

Lesson Title: Multiplication and division using logarithms – Numbers less than 1	**Theme:** Numbers and Numeration
Practice Activity: PHM1-052	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to multiply and divide numbers less than 1 using logarithms.

Overview

To find the **product** of numbers less than 1 using logarithm tables, follow these steps:

- Find the logarithms of the numbers
- Add the logarithms
- Find the antilog of the numbers resulting from the sum.

Similarly, to find the **quotient** of numbers less than 1 using logarithm tables, follow these steps:

- Find the logarithms of the numbers
- Subtract the logarithms
- Find the antilogarithm of the number resulting from the subtraction.

Solved Examples

1. Evaluate 0.056×0.95

   Solution:

Number	**Logarithm**
0.056 0.95	$\bar{2}.7482$ $+\bar{1}.9777$
Add logs for multiplication	$\bar{2}.7259$
$\text{Antilog } \bar{2}.7259 = 0.05320$	

2. Evaluate $\frac{0.0978}{0.06}$

   Solution:

Number	**Logarithm**
0.0978 0.06	$\bar{2}.9903$ $-\bar{2}.7782$
Subtract logs for division	0.2121
$\text{Antilog of } 0.2121 = 1.63$	

3. Evaluate $\frac{0.0067 \times 0.037}{0.57}$

Solution:

Apply the normal order of operations. Multiply the numbers in the numerator (using addition of logs) before dividing by the denominator (using subtraction of logs).

Number	**Logarithm**
0.0067 0.037	$\bar{3}.8261$ $+\bar{2}.5682$
Add logs for multiplication	$\bar{4}.3943$
0.57	$-\bar{1}.7559$
Subtract logs for division	$\bar{4}.6384$
Antilog $\bar{4}.6384 = 0.0004349$	

Practice

Evaluate the following using logarithmic tables:

1. $\frac{0.01875}{0.0015}$
2. 0.099×0.005
3. 0.491×0.23
4. $\frac{0.456 \times 0.56}{2.5}$
5. $\frac{2.5}{0.45 \times 0.613}$
6. $\frac{0.9981}{0.321}$

Lesson Title: Powers and roots of logarithms – Numbers less than 1	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L053	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to calculate powers and roots of numbers less than 1 using logarithms.

Overview

To evaluate the **powers** of numbers less than 1 using logarithm table, follow these steps:

- Find the logarithm of the number
- Multiply the logarithm found by the power
- Use the antilog table to find the antilog of the result to get your answer

To evaluate the **roots** of numbers less than 1 using logarithm tables, follow these steps:

- Find the logarithm of the number
- Divide the logarithm found by the root
- Use antilog table to find the antilog of the result to get your answer

Solved Examples

1. Evaluate: a. 0.0762^4; b. 0.063^2

   Solutions:

   a. 0.0762^4

Number	Logarithm
0.0762	$\bar{2}.8820$
Multiply log by power	$\times \quad 4$
	$\bar{5}.5280$
Antilog $\bar{5}.5280 = 0.00003373$	

Answer: $0.0762^4 = 0.00003373$

b. 0.063^2

Number	Logarithm
0.063	$\bar{2}.7993$
Multiply log by power	$\times \quad 2$
	$\bar{3}.5986$
Antilog $\bar{3}.5986 = 0.003968$	

Answer: $0.063^2 = 0.003968$

2. Evaluate: a. $\sqrt{0.0025}$; b. $\sqrt[3]{0.000216}$

Solutions:

a. $\sqrt{0.0025}$

Number	**Logarithm**
0.0025 For square root divide log by 2	$\bar{3}.979$ $\frac{\bar{3}.979}{2} = \frac{\bar{4}}{2} + \frac{1.3979}{2}$ $= \bar{2}.6999$
Antilog $\bar{2}.6999 = 0.0501$	

Answer: $\sqrt{0.0025} = 0.0501$

b. $\sqrt[3]{0.000216}$

Number	**Logarithm**
0.000216 For cube root divide log by 3	$\bar{4}.3341$ $\frac{\bar{4}.3341}{3} = \frac{\bar{6}}{3} + \frac{2.3341}{3}$ $= \bar{2}.7780$
Antilog $\bar{2}.7780 = 0.05998$	

Answer: $\sqrt[3]{0.000216} = 0.05998$

3. Evaluate: a. $0.00125^{\frac{1}{3}}$; b. $0.0081^{\frac{2}{3}}$

Solutions:

a. Note that $0.00125^{\frac{1}{3}} = \sqrt[3]{0.00125}$. Thus, divide the logarithm by 3:

Number	**Logarithm**
0.00125	$\bar{3}.0969$ $\div \quad 3$ $\bar{1}.0323$
Antilog $\bar{1}.0323 = 0.1077$	

Answer: $0.00125^{\frac{1}{3}} = 0.1077$

b. Note that $0.0081^{\frac{2}{3}} = \sqrt[3]{0.0081^2}$. Thus, multiply the logarithm by 2, then divide the result by 3.

Number	Logarithm
0.0081	$\bar{3}.9085$
Multiply	$\times \quad 2$ $\bar{5}.8170$
Divide	$\div \quad 3$ $\bar{2}.6057$
Antilog $\bar{2}.6057 = 0.0403$	

Answer: $0.0081^{\frac{2}{3}} = 0.0403$

Practice

1. Evaluate: a. 0.0459^2; b. 0.00125^3
2. Evaluate: a. $\sqrt[3]{0.625}$; b. $\sqrt{0.0109}$
3. Evaluate: a. $\frac{0.451^2}{\sqrt{0.516}}$; b. $\sqrt[4]{0.064} \times 0.27^2$
4. Evaluate: a. $0.0567^{\frac{1}{2}}$; b. $0.123^{\frac{4}{5}}$

Lesson Title: Laws of logarithms – Part 1	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L054	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to identify that $\log_{10}(pq) = \log_{10} p + \log_{10} q$.

Overview

This lesson is on the multiplication law of logarithms. Remember that with indices, when you multiply powers of the same base, you keep the base and add the exponent. For example: $y^2 \times y^3 = y^{2+3} = y^5$.

Note that logarithms are exponents, and when you multiply, you are going to add the logarithms. This means, the log of a product is the sum of the logs. For example, $\log_a xy = \log_a x + \log_a y$.

Note also that the log of a sum is NOT the sum of the logs. For example, $\log_a(x + y) \neq \log_a x + \log_a y$. The log of a sum cannot be simplified.

There are some other basic rules to keep in mind when simplifying logarithms. You may use these throughout the next few lessons:

- The logarithm of any number to the base of the same number is 1, that is $\log_a a = 1$.
- The logarithm of 1 to any base is 0, that is $\log_a 1 = 0$.

Solved Examples

1. Simplify the following:
   a. $\log 3 + \log 4$ b. $\log 9 + \log 2$ c. $\log 3 + \log 2 + \log 5$
   Solutions:
   a. $\log 3 + \log 4 = \log(3 \times 4)$
   $= \log 12$
   b. $\log 9 + \log 2 = \log(9 \times 2)$
   $= \log 18$
   c. $\log 3 + \log 2 + \log 5 = \log(3 \times 2 \times 5)$
   $= \log 30$
2. Expand the following:
   a. $\log_5 18$
   b. $\log_4 14$
   c. $\log_2 35$
   d. $\log_3(4m + 5)$

Solutions:

a. $\log_5 18 = \log_5(6 \times 3)$
$= \log_5 6 + \log_5 3$

or: $\log_5 18 = \log_5(9 \times 2)$
$= \log_5 9 + \log_5 2$

b. $\log_4 14 = \log_4(2 \times 7)$
$= \log_4 2 + \log_4 7$

c. $\log_2 35 = \log_2(5 \times 7)$
$= \log_2 5 + \log_2 7$

d. $\log_3(4m + 5)$: Cannot be expanded

3. Given that $\log_{10} 3 = 0.4771$, $\log_{10} 4 = 0.6021$, $\log_{10} 5 = 0.6990$ and $\log_{10} 6 = 0.7782$, find the values of the following:
   a. $\log_{10} 9$ b. $\log_{10} 24$ c. $\log_{10} 20$ d. $\log_{10} 30$ e. $\log_{10} 72$

   Solutions:

   a. $\log_{10} 9 = \log_{10}(3 \times 3)$
   $= \log_{10} 3 + \log_{10} 3$
   $= 0.4771 + 0.4771$
   $= 0.9542$

   b. $\log_{10} 24 = \log_{10}(6 \times 4)$
   $= \log_{10} 6 + \log_{10} 4$
   $= 0.7782 + 0.6021$
   $= 1.3803$

   c. $\log_{10} 20 = \log_{10}(4 \times 5)$
   $= \log_{10} 4 + \log_{10} 5$
   $= 0.6021 + 0.6990$
   $= 1.3011$

   d. $\log_{10} 30 = \log_{10}(5 \times 6)$
   $= \log_{10} 5 + \log_{10} 6$
   $= 0.6990 + 0.7782$
   $= 1.4772$

   e. $\log_{10} 72 = \log_{10}(3 \times 4 \times 6)$
   $= \log_{10} 3 + \log_{10} 4 + \log_{10} 6$
   $= 0.4771 + 0.6021 + 0.7782$
   $= 1.8574$

Practice

1. Expand the following:
   a. $\log_{10} 14$ b. $\log_{10} 15$ c. $\log 49$ d. $\log 25$
2. Simplify:
   a. $\log_2 6 + \log_2 8$
   b. $\log_5 4 + \log_5 2 + \log_5 3$
   c. $\log_4 8 + \log_4 8 + \log_4 2$
   d. $\log_3 9 + \log_3 2 + \log_3 2$

3. Given that $\log_{10} 2 = 0.3010$, $\log_{10} 5 = 0.6990$, and $\log_{10} 7 = 0.8451$, find the value of the following:
    a. $\log 28$
    b. $\log 56$
    c. $\log 70$

Lesson Title: Laws of logarithms – Part 2	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L055	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to identify that $\log_{10}\left(\frac{p}{q}\right) = \log_{10} p - \log_{10} q$.

Overview

This lesson is on the division law of logarithms. Remember that with indices, when you divide powers of the same base, you keep the base and subtract the exponent. For example: $y^7 \div y^3 = y^{7-3} = y^4$.

Logarithms are also a way of expressing exponents. So when you take the logarithm of the quotient of any two numbers, it is equal to simply subtracting their logarithms of the same base. That is, $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$.

Note that the log of a difference is NOT the difference of the logs. That is, $\log_a(x - y) \neq \log_a x - \log_a y$. The log of a difference cannot be simplified.

Solved Examples

1. Expand: a. $\log_{10} 3\frac{1}{3}$ b. $\log_{10} 9\frac{5}{4}$

   Solutions:

   First, write each mixed fraction as an improper fraction of the form $\frac{x}{y}$. Then expand using the division law of logarithms.

   a. $\log_{10} 3\frac{1}{3} = \log_{10}\left(\frac{10}{3}\right)$
   $= \log_{10} 10 - \log_{10} 3$

   b. $\log_{10} 9\frac{4}{5} = \log_{10}\left(\frac{49}{5}\right)$
   $= \log_{10} 49 - \log_{10} 5$

2. Simplify: a. $\log_3 21 - \log_3 7$ b. $\log_3 27 - \log_3 18$

   Solutions:

   a. $\log_3 21 - \log_3 7 = \log_3\left(\frac{21}{7}\right)$
   $= \log_3 3$

   b. $\log_3 27 - \log_3 18 = \log_3\left(\frac{27}{18}\right)$
   $= \log_3\left(\frac{3}{2}\right)$

3. Given that $\log_5 2 = 0.431$ and $\log_5 3 = 0.682$, find the value of

   a. $\log_5\left(1\frac{2}{3}\right)$ b. $\log_5\left(1\frac{1}{2}\right)$

Solutions:

First, rewrite each mixed fraction as an improper fraction. Then, expand and solve by substituting the given values.

a. $\log_5\left(1\frac{2}{3}\right) = \log_5\left(\frac{5}{3}\right)$
$= \log_5 5 - \log_5 3$
$= 1 - 0.682$
$= 0.318$

b. $\log_5\left(1\frac{1}{2}\right) = \log_5\left(\frac{3}{2}\right)$
$= \log_5 3 - \log_5 2$
$= 0.682 - 0.431$
$= 0.251$

4. Given that $\log_{10} 7 = 0.8451$, $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$, find the value of: a. $\log_{10}\left(\frac{9}{7}\right)$ b. $\log_{10} 0.21$

Solutions:

a. $\log_{10}\left(\frac{9}{7}\right)$
$= \log_{10} 9 - \log_{10} 7$
$= \log_{10} 3 \times 3 - \log_{10} 7$
$= \log_{10} 3 + \log_{10} 3 - \log_{10} 7$
$= 0.4771 + 0.4771 - 0.8451$
$=0.1091$

b. $\log_{10} 0.21 = \log\frac{21}{100}$
$= \log_{10} 21 - \log_{10} 100$
$= \log_{10}(7 \times 3) - \log_{10}(10 \times 10)$
$= \log_{10} 7 + \log_{10} 3 - (\log_{10} 10 + \log_{10} 10)$
$= 0.8451 + 0.4771 - (1 + 1)$
$= -0.6778$

Practice

1. Expand: a. $\log_7 5\frac{2}{3}$ b. $\log_9 7\frac{4}{5}$
2. Simplify: a. $\log_4 32 - \log_4 8$ b. $\log_3 48 - \log_3 27$
3. Given that $\log_5 2 = 0.431$ and $\log_5 3 = 0.682$ find:
   a. $\log_5 6\frac{2}{3}$ b. $\log_5 0.6$ c. $\log_5\left(\frac{9}{25}\right)$
4. Given that $\log_{10} 7 = 0.8451$, $\log_{10} 2 = 0.3010$, $\log_{10} 3 = 0.4771$ find:
   a. $\log_{10}\left(11\frac{4}{6}\right)$ b. $\log_{10} 6\frac{1}{8}$

Lesson Title: Laws of logarithms – Part 3	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L056	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to identify that $\log_{10}(P)^n = n\log_{10} P$.

Overview

When evaluating certain logarithmic expressions and equations, the logarithm power law can be used. The logarithm power/exponent law states that we can transfer the exponent of a number whose logarithm we are finding to the front of logarithm. The general rule is $\log_a m^r = r\log_a m$.

Along with the product and quotient rule, the logarithm power rule can be used for expanding and simplifying logarithmic expressions.

Note that if you have an entire logarithmic expression raised to an exponent, it cannot be simplified using this rule. In other words, $(\log_a m)^x \neq x\log_a m$.

Solved Examples

1. Simplify the following expressions:
   a. $\log_{10} 2^7$ b. $\log_5 a^3$ c. $\log_8 8^4$ d. $\log_2 4$

   Solutions:
   Rewrite each expression using the third law of logarithms:

   a. $\log_{10} 2^7 = 7\log_{10} 2$
   b. $\log_5 a^3 = 3\log_5 a$
   c. $\log_8 8^4 = 4\log_8 8 = 4 \times 1 = 4$
   d. For this example, first rewrite 4 as an index:
   $\log_2 4 = \log_2 2^2$
   Now apply the third law of logarithms:
   $\log_2 2^2 = 2\log_2 2 = 2 \times 1 = 2$

2. Given $\log_{10} 3 = 0.4771$ and $\log_{10} 2 = 0.3010$ evaluate:
   a. $\log_{10} 32$ b. $\log_{10} 27$ c. $\log_{10} 36$

   Solutions:
   a. $\log_{10} 32 = \log_{10} 2^5$
   $= 5\log_{10} 2$
   $= 5(0.3010)$
   $= 1.505$
   b. $\log_{10} 27 = \log_{10} 3^3$

$= 3\log_{10} 3$
$= 3(0.4771)$
$= 1.4313$

c. $\log_{10} 36 = \log_{10}(4 \times 9) = \log_{10} 4 + \log_{10} 9$
$= \log_{10} 2^2 + \log_{10} 3^2$
$= 2\log_{10} 2 + 2\log_{10} 3$
$= 2(0.3010) + 2(0.4771) = 1.5562$

3. Simplify:
   a. $\frac{\log_a 8+\log_a 16-\log_a 2}{\log_a 32}$
   b. $\frac{\log_2 8^3}{\log_2 8}$

   Solutions:

   a. $\frac{\log_a 8+\log_a 16-\log_a 2}{\log_a 32} = \frac{\log_a\left(\frac{8\times16}{2}\right)}{\log_a 32} = \frac{\log_a 64}{\log_a 32} = \frac{\log_a 2^6}{\log_a 2^5} = \frac{6(\log_a 2)}{5(\log_a 2)} = \frac{6}{5} = 1\frac{1}{5}$

   b. $\frac{\log_2 8^3}{\log_2 8} = \frac{3(\log_2 8)}{(\log_2 8)} = 3$

4. Simplify:
   a. $\log_{10} 25 + \log_{10} 4$
   b. $\log_3 15 + \log_3 9 - \log_3 5$

   Solutions:

   a.
   $$\begin{aligned} \log_{10} 25 + \log_{10} 4 &= \log_{10}(25 \times 4) \\ &= \log_{10} 100 \\ &= \log_{10} 10^2 \\ &= 2\log_{10} 10 \\ &= 2 \times 1 = 2 \end{aligned}$$

   b.
   $$\begin{aligned} \log_3 15 + \log_3 9 - \log_3 5 &= \log_3\left(\frac{15\times9}{5}\right) \\ &= \log_3 27 \\ &= \log_3 3^3 \\ &= 3(\log_3 3) \\ &= 3 \times 1 = 3 \end{aligned}$$

Practice

1. Simplify: a. $\log_3 x^8$ b. $\log_2 8$ c. $\log_3 3^{17}$
2. Given $\log_5 2 = 0.431$ and $\log_5 3 = 0.682$ find the values of:
   a. $\log_5 8$
   b. $\log_5\left(\frac{4}{5}\right)$
   c. $\log_5 18$

3. Simplify:
    a. $\frac{\log_{10} 625}{\log_{10} 25}$
    b. $\frac{\log_{10} a^4 - \log_{10} a^2}{\log_{10} a^3}$
    c. $\frac{\log_{10} 81}{\log_{10} 3}$
4. Evaluate: a. $3 + \log_3 81$ b. $(\log_5 125)^2 - \log_5 25$

Lesson Title: Define and describe sets and elements of a set	Theme: Numbers and Numeration
Practice Activity: PHM1-L057	Class: SSS 1

Learning Outcome
By the end of the lesson, you will be able to use various ways of writing and describing sets in terms of their numbers and elements.

Overview

A set is a collection of objects that have something in common or follow a common rule. For example, a set of fruits can be mangoes, oranges, guavas, pears, and so on.

The objects or things which make up a set are called **elements** of the set. In writing or defining sets, the symbol ∈ means "is a member of". For example, if we represent the set of even numbers between 0 and 10 by A, then we have A $= \{2, 4, 6, 8\}$. We can write 2∈A to show that 2 is a member of the set A.

The symbol ∉ means "is not a member of" so 3∉A means 3 is not a member of set A. We usually use a capital letter for the name of the set (A, B, C, …) and sometimes use lowercase letters (a, b, c, …) for the elements of the set.

The "cardinality of a set" or the "cardinal number of a set" is the number of elements in the set. It is denoted with the letter n and brackets. For example, consider $n(A) = 4$. This statement says that the set A has 4 elements.

Sometimes the members of a set may be too many for us to list them all. For example, the set B of integers from 1 to 1000. It is tedious to list all its members so we usually list few of the first elements and then continue with three dots $(\dots)$ and write the last number. The set B can be written $B = \{1, 2, 3, 4, \dots, 1000\}$

Solved Examples

1. Define each of the following sets of things by their common property
   a. Pots, spoons, frying pan, cups, plates.
   b. Paper clip, paper, stapler, pen, pencil, envelope.
   c. 1, 4, 9, 16, 25, 36.
   Solutions:
   a. Pots, spoons, frying pan, cups, plates are a set of eating utensils.
   b. Paper clip, paper, stapler, pen, pencil, envelope are a set of stationary or office materials.
   c. 1, 4, 9, 16, 25, 36 are a set of integers of perfect squares between 0 and 40.

2. List the elements of the following sets:
   a. {The months of the years that have 30 days}
   b. {All the even numbers between 0 and 100}
   c. {The days of the week}

   Solutions:

   a. {April, June, September, November}
   b. {2, 4, 6…..98}
   c. {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

3. Describe each set in words:
   a. A = {a, b, c, d, e, f, g, h, i, j}
   b. B = {2, 3, 5, 7, 11, 13, 17, ….., 29}
   c. C = {Divider, protractor, compass, ruler}

   Solutions:

   a. A is the set of the first 10 letters of the alphabet.
   b. B is the set of prime numbers between 1 and 30.
   c. C is a set of mathematical instruments.

4. Let C be the set of countries in Africa. Insert either $\in$ or $\notin$ in the blank spaces:
   a. Sierra Leone ____ C
   b. Spain _____ C
   c. United Kingdom _____ C

   Solutions:

   a. Sierra Leone $\in$ C
   b. Spain $\notin$ C
   c. United Kingdom $\notin$ C

5. Complete the following:
   a. If D = {the days of the week} find the cardinality of D.
   b. If Y = {The months of the year that have 31 days} find the cardinality of Y.
   c. If W = {whole numbers greater than 1 and less than 10} find $n(W)$.

   Solutions:

   a. D = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}
      There are 7 elements in the set, so $n(D) = 7$
   b. Y = { January, March, May, July, August, October, December}
      There are 7 elements in the set D, so $n(Y) = 7$
   c. W = {2, 3, 4, 5, 6, 7, 8, 9}
      There are 8 elements in the set W, so $n(W) = 8$

Practice

1. Define each of the following sets of things by their common property:

a. {London, Madrid, Moscow, Washington DC, Freetown}
b. {Bee, Frog, Monkey, Wasp}
c. {2, 4, 6, 8, 10, 12, 14, 16, 18}

2. List the elements of the following sets:
    a. The set of months of the years with the first letter J
    b. The set of odd numbers between 0 and 20
    c. The set of even numbers less than 10
3. Describe the following sets in words:
    a. {Freetown, Conakry, Monrovia, Abuja}
    b. {Cow, chicken, pig, goat, sheep}
    c. {Red, orange, yellow, green, blue, indigo, violet}
4. Make each of the statements true or false by replacing the symbol $*$ by either $\in$ or $\notin$.
    a. $6 * \{4, 9, 16, 25 \ldots ..81\}$
    b. $\frac{4}{5} *$ {Whole numbers}
    c. $\sqrt{2} *$ {Irrational numbers}
5. Complete the following:
    a. If D = { 4, 5, 6, 7, 8, 9} then n (D) =
    b. If X = {1, 2, 3, 4, 5, 6} then n(X) =
    c. If Y = {0} then n(Y) =

Lesson Title: Set-builder notation	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L058	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to write and interpret sets of values using set-builder notation.

Overview

Set-builder notation involves describing a set in terms of its characteristic or property, where only those with that specific property are considered elements of the set.

For example, consider $Q = \{x \mid x$ is a district in Sierra Leone$\}$. This statement says that "Q is the set of x such that x is a district in Sierra Leone". Here, the set Q is given by a property, instead of listing the members. The members of set Q are all of the districts in Sierra Leone. In this example, x is used as a symbol to represent a member of the set under consideration.

In set-builder notation, the vertical line | is read as "such that". You will sometimes see a colon (:) used instead of a vertical line.

Solved Examples

1. List the elements of the set: A = $\{x: x$ is multiples of 3 less than 20}
   Solution:
   A = {3, 6, 9, 12, 15, 18}

2. List the elements of the set B = $\{x| \ 1 \leq x \leq 5\}$
   Solution:
   B = {1, 2, 3, 4, 5}

3. Write in set-builder notation: D = $\{-4, -3, -2, -1 \ldots .4\}$
   Solution:
   D = $\{x: x - 5 < x < 5\}$

4. Write in set-builder notation: E = {4, 8, 12, 16, 20, 24, 28}
   Solution:
   E = $\{x|x$ is multiples of 4 between 1 and 30}

Practice

1. List the elements of the set: A = $\{x: x$ is factor of 24}
2. List the elements of the set: B = $\{x: 10 \leq x \leq 15\}$

3. List the element of the set: D = $\{x: x$ is a prime number between 2 and 20$\}$.
4. List the elements of the set $B = \{x: x$ is a month of the year that has 31 days$\}$
5. Write in set-builder notation: F = {10, 100, 1,000, 10,000, 100,000, …}
6. Write in set-builder notation: G = {1, 3, 5, 7, 9}

Lesson Title: Finite and infinite sets	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L059	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to define and identify finite and infinite set.

Overview

A finite set is a set that comes to an end. For example, consider the set of even numbers less than 10. The elements of this set are {2, 4, 6, 8}. The members of a finite set are countable and can be listed.

An infinite set is uncountable, and the last element cannot be found. In an infinite set we add ellipses (…) at the end to show that the elements go on forever. For example, consider the set of all multiples of 3. There are an infinite number of multiples of 3, and they are {3, 6, 9, 12, 15, 18, 21…}.

Solved Examples

1. List the elements of each set, and state whether it is finite or infinite. Give the cardinality of each finite set.
   a. $A = \{x|x$ is a factor of 12}
   b. $B = \{x: x$ is a multiple of 5}
   c. $C = \{x|x$ is a multiple of 6 less than 50}
   d. $D = \{x \mid x$ is a prime number}
   e. $E = \{x: x$ is an even number less than 30}
   Solutions:
   a. A = {1, 2, 3, 4, 6, 12}; finite; $n(A) = 6$
   b. B = {5, 10, 15, 20, 25, …}; infinite
   c. C = {6, 12, 18, …, 48}; finite; $n(C) = 8$
   d. D = { 2, 3, 5, 7, 11, 13, …}; infinite
   e. E = {2, 4, 6, 8, 10, …, 28}; finite; $n(E) = 14$

2. State which of the sets is finite or infinite. List the elements of each.
   a. Days of the week
   b. The alphabet
   c. Odd numbers
   d. Whole numbers greater than 100
   e. Prime numbers less than 35
   Solutions:
   a. Finite; {Sunday, Monday, …, Saturday}.
   b. Finite; {a, b, c, …, z}
   c. Infinite; {1, 3, 5, 7, 9, …}

d. Infinite; {101, 102, 103, 104, …}
e. Finite; {2, 3, 5, 7, …, 31}

Practice

State whether each set is finite or infinite. List its members, using ellipses where necessary.

1. Factors of 18
2. Natural numbers
3. Months of the year
4. Multiples of 8 less than 70
5. Multiples of 7
6. Even numbers between 10 and 40
7. Prime numbers greater than 50

Lesson Title: Null/empty, unit and universal sets	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L060	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to define and identify null/empty sets, unit sets, and universal sets.

Overview

This lesson is on how to define and identify null/empty sets, unit sets and universal sets.

A set that does not contain any element is called a **null** or **empty set**. Such a set is represented by a curly bracket with nothing inside { } or the null symbol $\{\emptyset\}$. For example, the set A of cats with horns is a null set. We can write $A = \{\ \}$ or $A = \{\emptyset\}$.

A **unit set** is a set that has only one element, no more or less. Its cardinality is always one. For example, consider the set F = $\{x: x$ is a month with less than 30 days$\}$. We know that F = {February}, which is a unit set. The cardinality is $n(F) = 1$.

A **universal set** is the set of all elements or objects under consideration. For example, consider a school. We can use sets to discuss different groups of pupils in the school. For example, we have SSS1, SSS2, SSS3, and SSS4 pupils in the school. We have female and male pupils in the school. The universal set is the set of all pupils under consideration; in other words, all pupils in the school. A universal set is denoted by $\mathcal{E}\ or\ U$.

Solved Examples

1. Identify the sets below as empty/null sets or not:
   a. A = {Goats with five legs}
   b. B = {Months of the year with 45 days}
   c. C = {Families with 2 children}
   Solutions:
   a. A is an empty set.
   b. B is an empty set.
   c. C is not an empty set.

2. Decide whether the following sets are unit sets or not:
   a. X = {Countries with capital city London}
   b. Y = $\{x: x$ is the solution of $x + 1 = 2\}$
   c. Z = $\{x: x$ is the value of $\sqrt{1}\}$

Solutions:

a. X = {United Kingdom}.

$n(X) = 1$, so X is a unit set.

b. Y = {1};

$n(Y) = 1$, so X is a unit set.

c. Z = $\{+1, -1\}$

$n(Z) = 2$, so Z is not a unit set.

3. State the universal set of the following sets:
   a. C = {Consonants} and V = {Vowels}
   b. F = {Female Sierra Leoneans} and M= {Male Sierra Leoneans}

   Solutions:

   a. U = {x: x is a letter of the alphabet}

   = {a, b, c, d, e, f, g, h, i, j, k, l, m , n, o, p, q, r, s, t, u, v, w, x, y, z}

   b. U = { x: x is a Sierra Leonean}

Practice

1. Decide whether each of the following sets are empty sets or not.
   a. G = {Goats that can swim}
   b. R = {Reptiles that can breast feed}
   c. M = {Countries that have landed men on the moon}
   d. P = {Female Presidents of Sierra Leone}
2. State the elements of each of the following unit sets:
   a. D = {Month of the years with three letters}
   b. E = {x: x is the solution of $x - 2 = 5$}
   c. W = {The first day of the week}
3. State the universal set of the sets E = {x: x is an even number} and O = {x: is an odd number}
4. Given that U = {All quadrilaterals}, list the elements of the following:
   a. C = {Cyclic quadrilaterals}
   b. E = {Quadrilateral with two equal opposite sides}

Lesson Title: Equivalent and equal sets	Theme: Numbers and Numeration
Practice Activity: PHM1-L061	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to define and identify equivalent and equal sets.

Overview

To be equivalent any pair of sets should have the same cardinality. In other words, sets are equivalent if they have the same number of elements. This implies that there is one- to-one correspondence between the elements of any two sets for them to be equivalent. For example, consider 2 sets, $A = \{1,2,3,4,5\}$ and $B = \{a, e, i, o, u\}$; $n(A) = n(B) = 5$. Thus, A and B are equivalent.

Note that all null/empty sets are equivalent. Similarly, all unit sets are equivalent.

Two sets are equal if and only if they have the same elements. This implies that for any pair of sets to be equal, they must first be equivalent and then have the same elements regardless of the order of occurrence of the elements. For example, if P = {1, 2, 3} and Q = {2, 3, 1} then P = Q. P and Q are equal sets.

Note that the order in which the members of a set are written does not matter in determining whether they are equal.

Solved Examples

1. Consider the following sets. Establish whether they are equivalent or not.
   a. P = { 5, 4, 3, 2, 1} and Q = {J, U, L, Y, S}
   b. A = {a, b, c, d, e} and R = {m, n, o, p}
   c. F = {Hen, goat, cow, pig, cock} and G = $\{x : x > 5\}$
   Solutions:
   a. Since n(P) = n(Q) = 5, P is equivalent to Q.
   b. n(A) = 5 and n(R) = 4. Since n(A) ≠ n(R) then set A is not equivalent to R.
   c. n(F) = 5 and n(G) is infinity so set F is not equivalent to G.

2. Consider the following sets and establish whether they are equal or not.
   a. W = $\{x : 0 < x < 4\}$ and V = $\{x : 0 \leq x \leq 4\}$
   b. C = {spoon, saucer, plate, pot}, and S = {pot, spoon, plate, saucer}
   c. D = $\{x : x = \sqrt{4}\}$ and E =$\{ x : x^2 - 4 = 0\}$
   d. X = $\{ x : 2x = -16\}$ and Y = {8}
   Solutions:
   a. W = {1, 2, 3} and V ={0, 1, 2, 3, 4}, since the elements in W and V are not all identical, then W ≠ V.

b. The elements in C and S are identical, so C = S.
c. Note that D = $\{+2, -2\}$ and E = $\{+2, -2\}$. D = E since both sets have identical elements
d. Note that X = $\{-8\}$ and Y = $\{8\}$. Since $-8 \neq 8$, the elements in X and Y are not identical, so X ≠ Y.

Practice

1. Consider the following pairs of sets. Establish whether they are equivalent or not.
    a. K = {0, -1, 1, 5, 6} and L = $\{\pi, \Phi, \rho, \gamma, \theta\}$
    b. A = {u, v, w, x, y} and B = {9, 10, 11}
    c. C = {Toyota, Benz, Lexus, Nissan} and T = {Chrysler, Dodge, Pontiac}
    d. N = {Lucy, Brima, Joe, Mary} and M = $\{x: 4 < x < 9\}$

2. Consider the following pairs of sets and establish whether they are equal or not.
    a. V = $\{x: x > 0\}$ and W = {x: $x < 0$}
    b. C = {January, February, March, April} and D = {December, November, October, September}
    c. A = { $x: x$ is an odd number between 0 and 10} and $B = \{1, 3, 5, 7, 9\}$
    d. X = $\{x: x = \sqrt{25}\}$ and Y = $\{x: 5x - 25 = 0\}$
    e. R = $\{x: (x+1)(x-1) = 0\}$ and S = $\{x: -1 \leq x \leq 1\}$
    f. I = {1} and J = $\{x: x - 1 = 0\}$

Lesson Title: Subsets	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L062	**Class:** SSS 1

Learning Outcomes
By the end of the lesson, you will be able to:
1. Describe and identify the subsets of a given set.
2. Represent subsets with Venn diagrams.
3. Use correct symbols to demonstrate subsets.

Overview

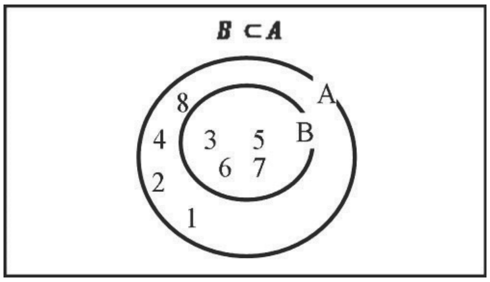
For any two sets A and B, if every element in set B is present in set A, then B is a subset of A.

A special notation is used to represent the relationships between subsets. In the case of A and B we say $B \subset A$. This is read as "set B is a subset of set A" or "B contained A". The symbol ⊂ means "is a subset of" or "is found in". The notation ⊄ means "not a subset of".

If two sets are equal then, then they are also subsets of each other. For example, consider sets A = {1, 2, 3, 4} and B = {4, 3, 2, 1}. They have the same elements, so $B \subset A$ and $A \subset B$.

A Venn diagram is a diagram that shows the relationship between sets. It is most commonly drawn with circles and squares. A circle inside of another circle commonly shows subsets.

For example, the Venn diagram below shows $B \subset A$, where $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $B = \{3, 5, 6, 7\}$.



Note that the empty set is a subset of every set, and every set is a subset of itself.

Solved Examples

1. Given the set X = {m, n. o, p, q}, list any ten subsets of X.
   Solution:
   Given X = {m, n, o, p, q}, ten subsets are:
   X_1 = {m, n, o, p}, X_2 = {m, n, o, q}, X_3 = {m, n, p, q}, X_4 = {m, o, p, q}

X_5 = {n, o, p, q}, X_6 = {m, n}, X_7 = {m, o}, X_8 = {m, p}, X_9 = {m, q}, X_{10} = {m, n, o}

2. If A = {a, b, c, d, e, f, g, h} identify whether the following sets are subsets of A:
   a. B = {b, d, f, h}
   b. C = {a, c, e, g}
   c. D = {a, b, c, d, e}
   d. E = {a, c, l, g}

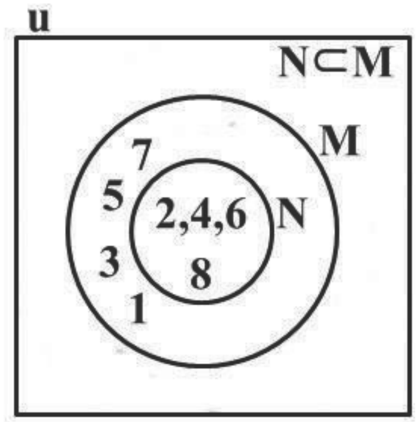
   Solutions:
   a. B is a subset of A; all elements of B are found in the set A.
   b. C is a subset of A; all the elements in set C are found in set A.
   c. D is a subset of A; all the elements in the set D are found in set A.
   d. E is not a subset of A; only the elements in E are found in A, l is not in A.

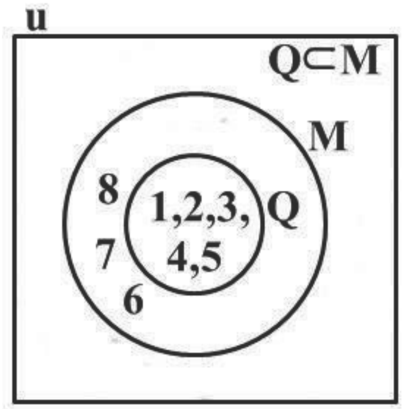
3. Given the set M = {1, 2, 3, 4, 5, 6, 7, 8} which of the following sets are subsets of M?
   a. i. N = {2, 4, 6, 8}
      ii. P = {1, 2, 5, 7, 9}
      iii. Q = {1, 2, 3, 4 5}

   b. Draw Venn diagrams to illustrate the relationships between set M and each of its subsets in part a.

   Solutions:
   a. i. N is a subset of M; all the elements in N are in M.
      ii. P is not a subset of M; 9 is an element of p but not in M.
      iii. Q is a subset of M; all the elements in Q are in M.

   b. Diagrams showing that N and Q are subsets of M:





4. List all the subsets of the set Z = {U, V, W}.

   Solution:

   The subsets of Z = {U, V, W} are:

   Z_1 = {U, V, W}, Z_2 = {U, V}, Z_3 = {U, W}, Z_4 = {V, W}
   Z_5 = {U}, Z_6 = {V}, Z_7 = {W} and Z_8= { }.

Practice

1. Given the set S = {a, b, c, d} list all its subsets.
2. Given the set X = {x: $0 \leq x \leq 20$, where x is an integer}, identify which of the following sets are subsets of X:
    a. W = {x: $3 < x < 9$ where x is an integer}
    b. V = {x: $0 \leq x \leq 5$, where x is an integer}
    c. T = {x: $x > 0$}
3. Represent the relationship between the following set B and its subsets V and C on a Venn diagram.

    B = {x: x is an alphabet between a and f}

    V = {x: x is a vowel}

    C = {x: x is a consonant}
4. List all the subsets of the set M = {x: x is a multiple of 2 up to 6}

Lesson Title: Intersection of 2 sets	Theme: Numbers and numeration
Practice Activity: PHM1-T2-W16-L063	**Class:** SSS 1

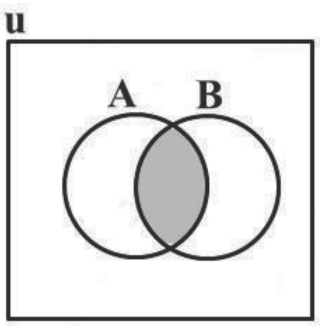
Learning Outcomes
By the end of the lesson, you will be able to:
1. Describe and identify the intersection of 2 sets.
2. Represent the intersection of two sets with Venn diagrams.
3. Use the correct symbols for intersection.

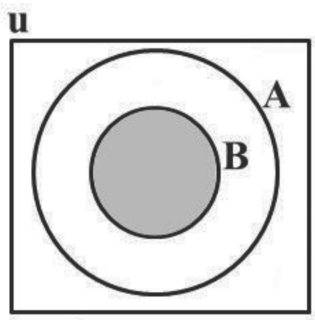
Overview

The intersection of two sets A and B is the set that contains all the elements of A that also belong to B (or all the elements of B that also belong to A) but no other elements. For example, take set A to be the colours of the flag of Sierra Leone: A = {green, white, blue} and set B to be the colours of the flag of Ghana: B = {red, yellow, green}.

The intersection of set A and B is the element(s) that is/are common to both sets A and B. It is denoted by A ∩ B and is read as "A intersection B".

The intersection of two sets can be drawn using a Venn diagram. A Venn diagram is an important tool allowing relationships between sets to be visualised graphically. In the Venn diagrams below, the shaded portion shows the intersection of the two sets A and B.

Ⅰ


Ⅱ


In Ⅰ we have A ∩ B.

In Ⅱ note that if B ⊂ A, then B ∩ A = B.

Solved Examples

1. Consider the two sets A and B:

   A = {2, 4, 6, a, b}

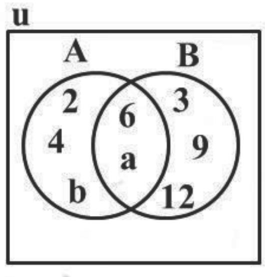
   B = {3, 6, 9, a, 12}

   a. Find A ∩ B
   b. Draw a Venn diagram to illustrate both sets

Solutions:

a. A ∩ B = {6, a}, because 6 and a are common to both sets A and B.

b. Venn diagram:



2. Consider two sets P and Q:

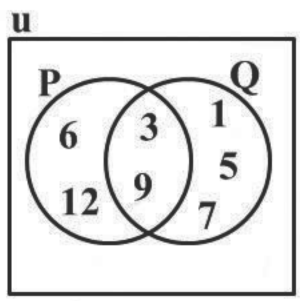
   P = {multiples of 3 less than 13]

   Q = {Odd numbers less than 10}

   a. List the members of both sets P and Q.
   b. Find P ∩ Q.
   c. Draw a Venn diagram to illustrate P ∩ Q.

   Solutions:

   a. P = {3, 6, 9, 12} and Q = {1, 3, 5, 7, 9}
   b. P ∩ Q = {3, 9}, because 3 and 9 are common to both sets.
   c. Venn diagram:



3. Set A is prime numbers less than 15, and set B is even numbers less than 15.

   a. Find A ∩ B.
   b. Draw a Venn diagram to illustrate set A and set B.

   Solutions:

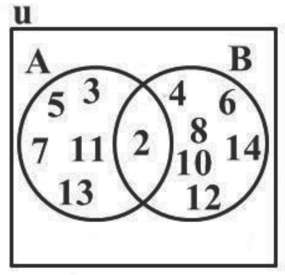
   a. List the elements of set A and set B:

   A = {prime numbers less than 15}
   = {2, 3, 5, 7, 11, 13}

   B = {even numbers less than 15}
   = {2, 4, 6, 8, 10, 12, 14}

   Therefore, we have A ∩ B = {2}, because 2 is the only common element.

b. Venn diagram:



Practice

1. Consider two sets P and Q:

   P = {6, 12, 18, 24}

   Q = {12, 24, 36, 48}

   a. Find P ∩ Q.
   b. Draw a Venn diagram to illustrate set P and set Q.

2. Consider two sets R and S:

   R = {factors of 12}

   S = {prime numbers less than 4}

   a. List the elements of sets P and set S.
   b. Find R ∩ S.
   c. Illustrate set R and set S in Venn diagram.

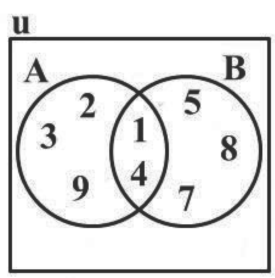
3. Consider two sets A and B:

   A = {multiples of 4 less than 19}

   B = {even numbers greater than 7 but less than 19}

   a. List the elements of set A and set B.
   b. Find A ∩ B.
   c. Illustrate set A and set B on a Venn diagram.

4. From the Venn diagram below:
   a. List the elements of set A and set B.
   b. Identify A ∩ B.



Lesson Title: Intersection of 3 sets	Theme: Numbers and Numeration
Practice Activity: PHM1-L064	**Class:** SSS 1

Learning Outcomes

By the end of the lesson, you will be able to:

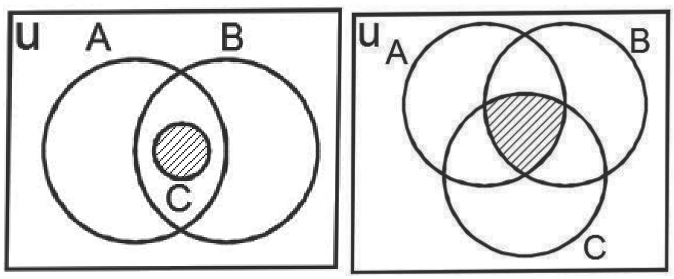
1. Describe and identify the intersection of 3 sets.
2. Represent the intersection of 3 sets with a Venn diagram.

Overview

This lesson is on identifying the intersection of 3 sets and representing them by a Venn diagram.

The intersection of three sets X, Y and Z is the set of elements that are common to sets X, Y and Z. It is denoted by $X \cap Y \cap Z$ which is read as "X intersection Y intersection Z".

The Venn diagram drawn below shows the intersection of 3 sets. The intersection $A \cap B \cap C$ is the shaded area in each diagram.



The elements of the sets can be shown in a Venn diagram. To draw a Venn diagram, start by filling in the elements in the intersection first. Work outwards, filling in the elements that are in only one set last.

Solved Examples

1. For the sets:

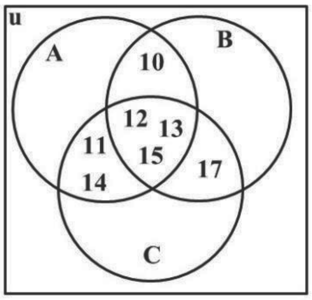
   $A = \{10, 11, 12, 13, 14, 15\}$
   $B = \{10, 12, 13, 15, 16, 17\}$
   $C = \{11, 12, 13, 14, 15, 17\}$

   a. Find $A \cap B$, $B \cap C$, $A \cap C$, and $A \cap B \cap C$
   b. Illustrate the sets on a Venn diagram

   Solutions:

   a. $A \cap B = \{10, 12, 13, 15\}$
   $B \cap C = \{12, 13, 15, 17\}$
   $A \cap C = \{11, 12, 13, 14, 15\}$
   $A \cap B \cap C = \{12, 13, 15\}$

b.



2. For the sets:

   X = {4, 6, a, b, 8}

   Y = {4, a, b, c, 5}

   Z = {5, a, b, d, 8, 9}

   a. Find X∩Y, Y∩Z, X∩Z, and X∩Y∩Z.
   b. Illustrate your answer with a Venn diagram.

   Solutions:

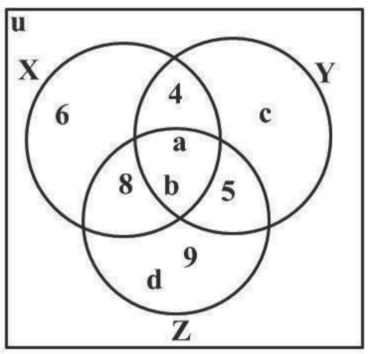
   a. $X \cap$ Y = {4, a, b}

      Y∩ Z = {5, a, b}

      X∩Z = {8, a, b}

      X∩Y∩Z = {a, b}

   b.



3. U is the universal set consisting of all positive integers x such that $\{22 < x < 42\}$. P, Q and R are subsets of U such that P = $\{x: x$ prime numbers}, Q = $\{x: x$ is an odd numbers above 30} and R = {23, 31, 32, 37, 41}

   a. Find P∩Q, Q∩R, P∩R, P∩Q∩R.
   b. Illustrate your answer in a Venn diagram.

   Solutions:

   First, list the elements of each set:

   U = {23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41}

   P = {23, 29, 31, 37, 41}

   Q={31,33,35,37,39,41}.

   R = {23, 31, 32, 37, 39}

a. Find the intersections, P∩Q, Q∩R, P∩R and P∩Q∩R:

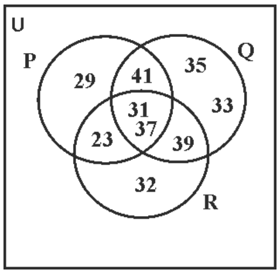
P∩Q = {31, 37, 41}

Q∩R = {31, 37, 39}

P∩R = {23, 31, 37}

P∩Q∩R = {31, 37}

b. Draw the Venn diagram:



Practice

1. Consider the following sets, where U is the universal set:

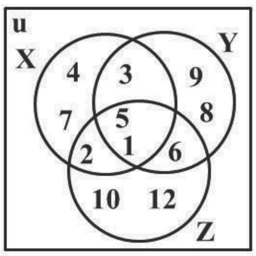
   U = {2, 4, 6, 8...20}

   A = {multiples of 4 less than 20}

   B = {even numbers less than 14}

   C = {8, 12, 16, 18, 20}

   a. Find: i. A∩B; ii. B∩C; iii. A∩C, iv. A∩B∩C
   b. Draw a Venn diagram to illustrate A∩ B∩ C
2. If U = {1, 2, 3, 4, 5, 6, 7} and P,Q, and R are subsets, such that P = {1, 3, 6}, Q = {3, 4, 6},and R = {2, 3, 7},
   a. Find, P∩Q, P∩R, Q∩R, P∩Q∩R.
   b. Illustrate the intersections with a Venn diagram.
3. M, L, and T are subsets of the universal set U, where U = $\{x: 1 \leq x \leq 15\}$ and x is an integer. M consists of odd numbers, L consists of even numbers and T consists of multiples of 3.
   a. Find M∩L, M∩T, L∩T, and M∩L∩T.
   b. Draw a Venn diagram to illustrate U, M, L, and T.
4. The Venn diagram shown is a universal set U with subsets X,Y, and Z. Use it to find: X∩Y, X∩Z, Y∩Z, X∩Y∩Y.



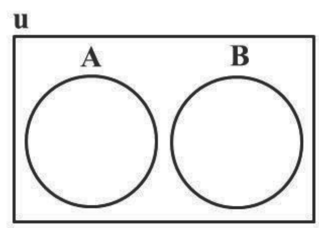
Lesson Title: Disjoint set	Theme: Numbers and Numeration
Practice Activity: PHM1-L065	**Class:** SSS 1

Learning Outcomes
By the end of the lesson, you will be able to:
1. Describe and identify disjoint sets.
2. Represent disjoint sets with Venn diagram.

Overview

This lesson is on describing and identifying disjoint sets. Any two sets are said to be disjoint if they have no elements in common. That is, their intersection is the empty/null set. In symbols, $A \cap B = \emptyset$ means that A and B are disjoint sets.

Disjoint sets A and B can be illustrated using a Venn diagram as shown below:



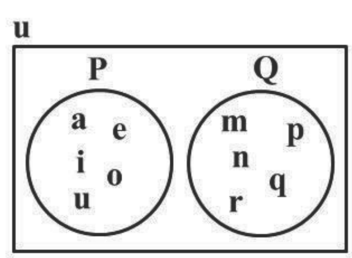
Events are often used in probability. In terms of events, disjoint events cannot happen at the same time (they are mutually exclusive).

The following are considered as disjoint events:

(i) Laughing and whistling.
(ii) A football game and a lawn tennis match on the same pitch.
(iii) Going North and South at the same time.
(iv) Day and night occurring at the same time.
(v) Getting a head and a tail in a single toss of a fair coin.

Solved Examples

1. Given the sets, P = {a, e, i, o, u} and Q = {m, n, p, q, r} show that P and Q are disjoint sets and illustrate the two sets on a Venn diagram.
   Solution:
   P = {a, e, i, o, u},
   Q = {m, n, p, q, r}
   P∩Q = Φ



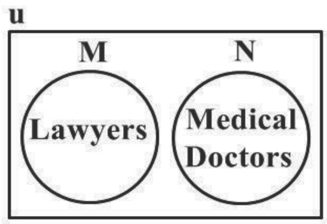
2. Given the sets: M = {set of lawyers in Sierra Leone}, N = {Set of medical doctors in Sierra Leone}, establish that the sets M and N are disjoint sets and illustrate on a Venn diagram.

   Solution:

   M = {set of lawyers in Sierra Leone}

   N = {Set of medical doctors in Sierra Leone}

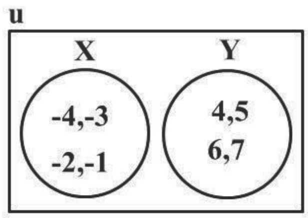
   M∩N = Φ



3. Given the set X = $\{x: -4 \leq x < 0\}$, Y = $\{x: 4 \leq x < 8\}$
   a. List the elements of sets X and Y
   b. Show that sets X and Y are disjoint sets
   c. Show X and Y on a Venn diagram

   Solutions:

   a. X = $\{x: -4 \leq x < 0\}$ = $\{-4, -3, -2, -1\}$
      Y = $\{x: 4 \leq x < 8\}$ = {4, 5, 6, 7}
   b. Sets X and Y do not have any element in common
      X∩Y = Φ
   c. Venn diagram:



Practice

1. Given the sets M = $\{-1, -2, -3, -4\}$ and N = $\{\sqrt{2}, \pi, \sqrt{3}, \sqrt{5}\}$ show that M and N are disjoint sets. Show them on a Venn diagram.
2. Given W = {Breed of goat} and N = {Breed of chicken}, show that sets M and N are disjoint sets and show them on a Venn diagram.
3. Given sets X = $\{x: 3 \leq x \leq 6\}$ and Y = $\{x: -3 \leq x \leq 0\}$
   a. List the elements of sets X and Y.
   b. Establish whether sets X and Y are disjoint sets.
   c. Show X and Y on a Venn diagram.
4. If set P = {i, j, k, l} and R = {4, 6, 8, 10} show that sets P and Q are disjoint sets and illustrate them on a Venn diagram.

Lesson Title: Union of two sets	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L066	**Class:** SSS 1

Learning Outcomes

By the end of the lesson, you will be able to:

1. Describe and identify the union of two sets.
2. Represent the union of the two sets with a Venn diagram.
3. Use the correct symbol for union.

Overview

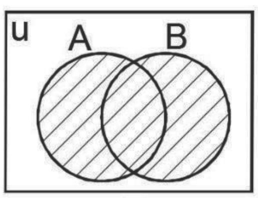
The union of any two sets A and B is the set that contains elements or objects that belong to either sets A or B or both.

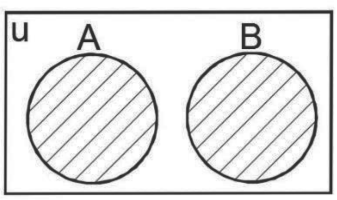
The union of two sets A and B is the set formed by putting the two sets together.

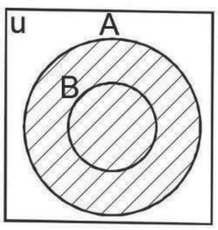
The union of 2 sets A and B is written in symbols as $A \cup B$. It is read $'A$ union B'.

If a member appears in both sets, it is listed only once in the union. Also note that when the members of sets are numbers, they are listed in ascending order (smallest member first).

These 3 diagrams show the union of sets. The shaded portion shows the union of the two sets A and B.





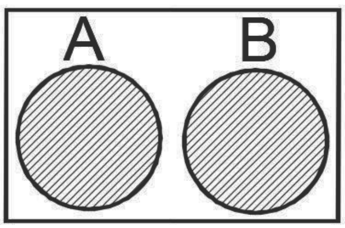


Solved Examples

1. Given the sets A = {orange, pineapple, apple, mango, banana} and B = {plate, spoon, knife, cup}; find the union of the two sets and show the union on a Venn diagram.

   Solution:

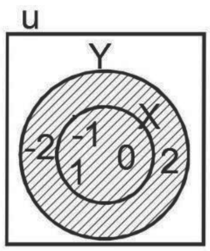
   A∪B = {plate, spoon, knife, cup, orange, pineapple, apple, mango, banana}



2. Given the set X = $\{x: -2 < x < 2, x$ is an integer$\}$ and Y = $\{x: -2 \leq x \leq 2, x$ is an integer$\}$
    a. List the elements of X and Y
    b. Find the union of X and Y
    c. Show the union of X and Y in a Venn diagram

    Solutions:

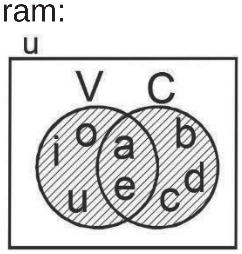
    a. X = $\{x: -2 < x < 2; x$ is an integer$\}$ = $\{-1, 0, 1\}$
       Y = $\{x: -2 \leq x \leq 2, x$ is an integer$\}$ = $\{-2, -1, 0, 1, 2\}$
    b. X∪Y = $\{-2, -1, 0, 1, 2,\}$ = Y, note that X⊂Y.
    c. Venn diagram:



3. Given the sets V = {a, e, i, o, u} and C = {a, b, c, d, e}:
    a. Find the union of V and C
    b. Show the union of V and C in a Venn diagram

    Solutions:

    a. V∪C = {a, b, c, d, e, i, o, u}
    b. Venn diagram:



4. Given the sets:

    M = $\{x: x$ is a prime number up to 10$\}$
    N = $\{x: x$ is an odd positive integer up to 10$\}$
    P = $\{x: x$ is an even positive integer up to 10$\}$

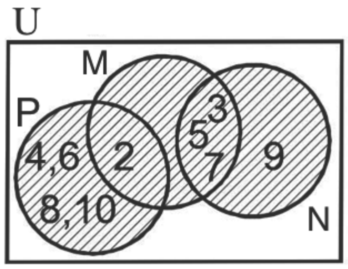
    a. List the elements of sets M, N and P
    b. Find the union of sets M, N and P.
    c. Show the union of the sets in a Venn diagram

    Solutions:

    a. List the elements of the sets:

       M = $\{x: x$ is a prime number up to 10$\}$ = {2, 3, 5, 7}
       N = $\{x: x$ is an odd positive integer up to 10$\}$ = {3, 5, 7, 9]
       P = $\{x: x$ is an even positive integer up to 10$\}$ = {2, 4, 6, 8, 10}

    b. M∪N∪P = {2, 3, 4, 5, 6, 7, 8, 9, 10}
    c. Venn diagram:



Practice

1. If M = {5, 7, 8, 10, 11} and N = {4, 5, 6, 7} write the set M∪N. Show M and N on a Venn diagram.
2. Given U = {a, b, c, d, e, f} and:

   X = {a, b, c, d}

   Y = {c, d, e}

   Z = {b, d, f},

   a. List the members of the following: i. X∪Y   ii. Y∪Z   iii. X∪Z
   b. Show X∪Y and X∪Z in Venn diagrams.
3. The members of a quiz team at a school for a competition form a universal set U where U = {Alice, Brima, David, Grace, Moses, Sama Yomba, Sia, Lamin, Kandeh}. If:

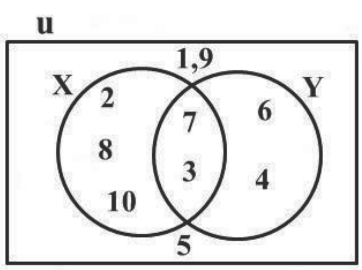
   A = {members whose name end in **a**}

   B = {Members with three letter names}

   C = {members whose names begin with letter **S**}.

   List the following sets:

   a. A∪B   b. B∪C   c. A∪C
4. The figure below is a Venn diagram representing a Universal set U and the subsets X and Y.



List the elements of the following sets:

a. X∪Y
b. X∪U
c. Y∪U

Lesson Title: Complement of a set	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L067	**Class:** SSS 1

Learning Outcomes

By the end of the lesson, you will be able to:

1. Describe and identify the complement of a set.
2. Represent the compliment of a set with a Venn diagram.

Overview

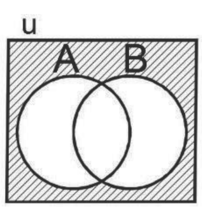
If U is the universal and A is a subset of U, then the complement of A is the set of members which belong to the universal set U but do not belong to A. In other words, the complement of a set is equal to elements in the universal set minus elements in the set.

The complement of a set A is written as A’ and is read as “A prime”. For example, if U = {3, 7, 9, 11, 13}, and A = {3, 9, 11}. Then A’ = {7, 13}

The following are the properties of the complement of a set:

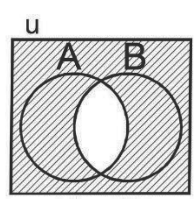
- The complement of the universal set is the empty set. Complement of U is the set which does not contain any element ($\mathrm{U}' = \{\ \}$).
- The complement of an empty set is the universal set under consideration. ($\emptyset' = \mathrm{U}$).
- The union of a set and its complement give the universal set ($\mathrm{A} \cup \mathrm{A}' = \mathrm{U}$).
- The intersection of a set and its complement is the empty set; there are no elements that exist in both sets ($\mathrm{A} \cap \mathrm{A}' = \emptyset$).

The diagram below illustrate complements of some sets. They are shown in symbols, and how to read each set of symbols is given.



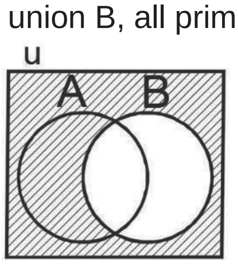
$(\mathrm{A} \cup \mathrm{B})'$

“A union B, all prime”



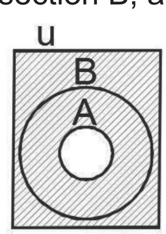
$(\mathrm{A} \cap \mathrm{B})'$

“A intersection B, all prime”



B'

“B prime”



(A⊂B)’

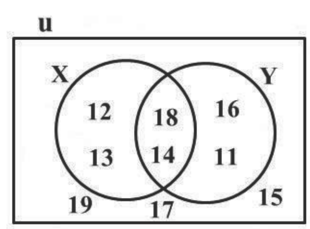
“prime of A subset B”

Solved Examples

1. If U = $\{-3 \leq x < 3\}$ and A = $\{-1 \leq x \leq 2\}$,
   a. List the elements of U and A
   b. Find A'
   Solutions:
   a. U = $\{-3, -2, -1, 0, 1, 2\}$
      A = $\{-1, 0, 1, 2\}$
   b. A'= $\{-3, -2\}$

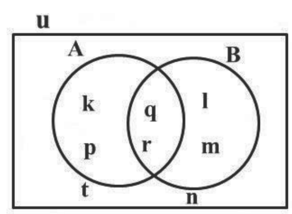
2. If A and B are subsets of the universal set U such that A = {1, 3, 4, 6}, B = {2, 4, 7, 8}, and U = {1,2, 3, 4, 5, 6, 7, 8, 9, 10}, list the elements of:
   a. A'
   b. B'
   c. A'∩B'
   Solutions:
   a. A' = {2, 5, 7, 8, 9, 10}
   b. B' = {1, 3, 5, 6, 9, 10}
   c. A'∩B' = {5, 9, 10}

3. Use the Venn diagram below to identify and list the following sets.
   a. U, X, Y
   b. X'
   c. Y'
   d. X'∩Y'



   Solutions:
   a. U = {11, 12, 13, 14, 15, 16, 17, 18, 19}
      X = {12, 13, 14, 18}
      Y = {11, 14, 16, 18}
   b. X' = {11, 15, 16, 17, 19}
   c. Y' = {12, 13, 15, 17, 19}
   d. X'∩Y' = {15, 17, 19}

4. Use the Venn diagram below to find the following sets.
   a. A'
   b. B'
   c. A'∩B'



   Solutions:
   First, identify the sets:

U = {k, l, m, n, p, q, r, s, t}
A = {k, p, q, r}
B = {l, m, q, r}

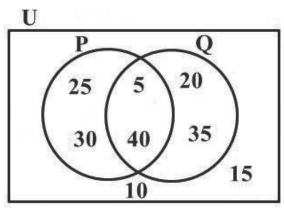
a. A' = {l, m, n, t}
b. B' = {k, p, n, t}
c. A'∩B' ={n, t}

Practice

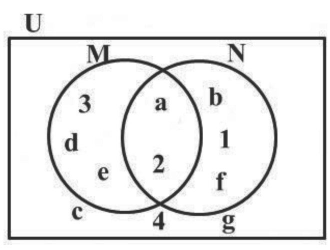
1. U = $\{4 \leq x \leq 12\}$ and x is an integer. P is a subset of U, such that P = $\{6 \leq x < 10\}$
    a. List the element of sets U and P
    b. Find P'

2. U = {10, 20, 30, 40, 50, 60}. A and B are subsets of U such that A = {20, 30, 60}, B = {20, 30, 50}, find:
    a. A'
    b. B'
    c. A'∩B'

3. Using the Venn diagram below, identify and list the following sets:
    a. U, P and Q
    b. P'
    c. Q'
    d. P'∩Q'



4. Use the Venn diagram below to find the following sets:
    a. U, M, and N
    b. M'
    c. N'
    d. M'∩N'



Lesson Title: Problem solving with 2 sets	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L068	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to diagram and solve problems involving 2 sets, including real-life problems.

Overview

When we have two sets A and B then:

- $n(\mathrm{A} \cup \mathrm{B})$ is the number of elements present in either of the sets A or B.
- $n(\mathrm{A} \cap \mathrm{B})$ is the number of elements present in both the sets A and B.
- For two sets A and B: $n(\mathrm{A} \cup \mathrm{B}) = n(\mathrm{A}) + n(\mathrm{B}) - n(\mathrm{A} \cap \mathrm{B})$

This is one formula that is often used in problem solving with sets. The formula states that to find the total number of elements in A and B, then we add the number of elements in each set, and subtract the number of elements in their intersection. This is because we do not want to count the elements in their intersection twice.

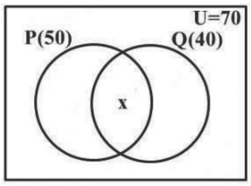
Solved Examples

1. If A and B are two finite sets such that n(A) = 32, n(B) = 18 and n(A∪B) = 42, find n(A∩B).
   Solution:
   $n(A\cup B) = n(A) + n(B) - n(A\cap B)$
   $n(A\cap B) = n(A) + n(B) - n(A\cup B)$
   $= 32+18 - 42$
   $= 50 - 42$
   $= 8$

2. A test was conducted for a class of 70 pupils. Fifty passed Mathematics and 40 passed English. Each pupil passed at least one subject.
   a. Illustrate this information on a Venn diagram.
   b. How many pupils passed both Mathematics and English?
   Solutions:
   Let U = {number of pupils that took the test}. Then, n{U} = 70
   P = {number of pupils who passed Mathematics}, and n{P} = 50
   Q = {number of pupils who passed English}, and n{Q} = 40
   x = number of pupils who passed both Maths and English = P∩Q

n(x) = n(P∩Q)

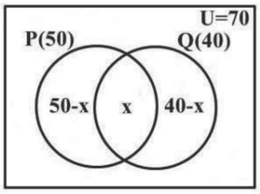
a. The Venn diagram shows the above information. →



Number of pupils who passed Mathematics only= $50 - x$

Number of pupils who passed English only= $40 - x$

This is shown in the Venn diagram. →



b. To find x =n(P∩Q), add the number of elements in the three regions and equate it to n{U}. Solve the equation for x.

$$
\begin{aligned}
(50 - x) + x + (40 - x) &= 70 \\
50 - x + x + 40 - x &= 70 \\
50 + 40 - x &= 70 \\
90 - x &= 70 \\
x &= 90 - 70 \\
&= 20
\end{aligned}
$$

3. In a school with a total population of 320 pupils, 118 of them play football only and 102 play volleyball only. Each pupil plays at least one of the two games.
   a. Draw a Venn diagram to illustrate this information.
   b. How many pupils play:
      i. Both games
      ii. Football

Solutions:

Let U = {pupil population in the school}. Then n{U} = 320.

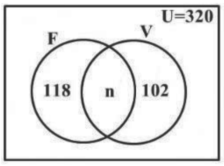
F = Set of pupils who play Football

V =Set of pupils who play Volleyball

n = number of pupils who play both games

Note: Here, do not subtract n from the other regions because of the word **only.**

a. Venn diagram →



b. i. Those who play both games = n. Add the 3 regions and equate to U, then solve for n.

$$
\begin{aligned}
118 + n + 102 &= 320 \\
220 + n &= 320 \\
n &= 320 - 220 = 100
\end{aligned}
$$

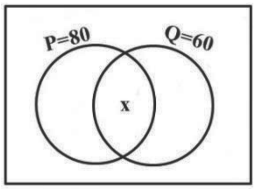
ii. Number of pupils who play football

$= n + 118$

$= 100 + 118$

$= 218$

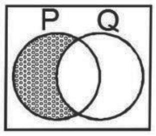
4. In the Venn diagram, set P has 80 members, set Q has 60 members while there are members in P∩Q and 120 members in P∪Q. use the Venn diagram to answer the following questions



    a. Shade the region which shows P only
    b. Find the number of members in P∩Q
    c. Find the number of members in Q only

    Solutions:

    a.



    b. P∩Q $= x$

    Region for P only $= 80 - x$

    Region for Q only $= 60 - x$

    Add the three regions and equate to n(P∪Q)$= 120$

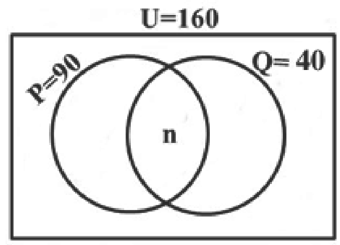
$$80 - x + 60 - x + x = 120$$
$$80 + 60 - x = 120$$
$$140 - x = 120$$
$$x = 140 - 120$$
$$= 20$$

    c. Members in Q only $= 60 - x = 60 - 20 = 40$

Practice

1. In a school of 120 pupils, 70 are in the Lawn tennis team and 80 are in the Volleyball team. Each pupil is in at least one team.
    a. Illustrate this information on a Venn diagram.
    b. Find how many pupils are in both teams.
    c. Find how many pupils are in Lawn tennis team only.
2. In a village of 180 adult inhabitants, 60 speaks only Mende fluently and 70 speaks only Kono fluently. Each adult speaks at least one of the two languages
    a. Draw a Venn diagram to illustrate this information
    b. How many adults speak: i. Both languages fluently; ii. Kono; iii. Mende
3. In a class of 49 pupils, 27 offer French and 32 offer Krio. Nine pupils do not offer any of the languages.
    a. Draw a Venn diagram to illustrate this information.
    b. How many stunts offer: i. Both languages; ii. Only French; iii. Only Krio

4. Use the Venn diagram below to answer the following questions:
    a. Find the number of members in P∩Q.
    b. Find the number of members in:
        i. P only
        ii. Q only



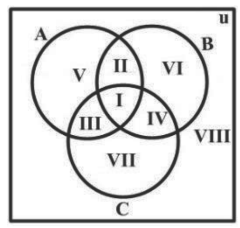
Lesson Title: Problem solving with 3 sets – Part 1	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L069	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to diagram and solve problems involving 3 sets, including real-life problems.

Overview

If A, B and C are subsets of U the universal set then the Venn diagram below shows the various regions of a problem with 3 sets.

The following are elements in each section of the diagram.



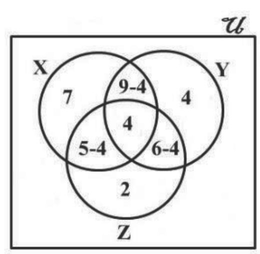
- I is the intersection of all the sets. $(A \cap B \cap C)$
- II is the elements of A and B only. $(C' \cap (B \cap A))$
- III is the elements of A and C only. $(B' \cap (A \cap C))$
- IV is the elements of B and C only. $(A' \cap (B \cap C))$
- V is the elements in A only. $(A \cap (B \cup C)'$ or $A \cap B' \cap C')$
- VI is the elements in B only. $(B \cap (C \cup A)'$ or $B \cap C' \cap A')$
- VII is the elements in C only. $(C \cap (B \cup A)'$ or $C \cap B' \cap A')$
- VIII is the elements that are not in any of the 3 sets.

This is the first of 2 lessons on solving problems involving 3 sets.

Solved Examples

1. Given three sets X, Y and Z such that $n(X \cap Y \cap Z) = 4$, $n(X \cap Y) = 9$, $n(X \cap Z) = 5$, $n(Y \cap Z) = 6$, $n(X \cap Y' \cap Z') = 7$, $n(X' \cap Y' \cap Z) = 2$ and $n(X' \cap Y \cap Z') = 4$,
   a. Draw the Venn diagram
   b. Find: i. n(X)   ii. n(Y)   iii. n(Z)

   Solutions:

   a. See Venn diagram on the right.
   b. i. Find n(X) by adding all elements in set X:
   $$n(X) = 7 + (9 - 4) + 4 + (5 - 4)$$
   $$= 7 + 5 + 4 + 1$$
   $$= 17$$
   ii. Find n(Y) by adding all elements in set Y:
   $$n(Y) = 4 + (9 - 4) + (6 - 4) + 4$$
   $$= 4 + 5 + 2 + 4$$
   $$= 15$$
   iii. Find n(Z) by adding all elements in set Z:
   $$n(Z) = 4 + (5 - 4) + 2 + (6 - 4)$$
   $$= 4 + 1 + 2 + 2$$
   $$= 9$$

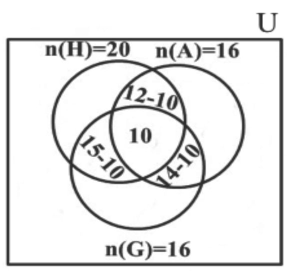
2. There are 73 pupils in a class. Twenty of them study history, 18 study art and 16 study government. Ten study all three subjects, 15 study both history and government, 12 study both history and art, while 14 study both art and government.
   a. Illustrate the information on a Venn diagram.
   b. Using the Venn diagram determine the number of pupils who study at least two subjects.

   Solutions:

   Let the set of pupils in the class be U; so total number of pupils will be n(U). Let the set of pupils who study history be H this implies that n(H) = number of pupils who study history. Let the set of pupils who study art be A; so n(A) = number of pupils who study art. Let the set of pupils who study government be G; so n(G) = number of pupils who study government.

   From the problem, we have n(U) = 73, n(H) = 20, n(A) = 16, n(G) = 16, n(H∩A∩G) = 10, n(H∩G) = n(H∩A) = 12 and n(A∩G) = 14.

   a. See Venn diagram at right.



   b. The number of pupils that study at least two subjects will be represented by adding each segment in the intersections of the sets, as follows:

n(at least two subjects) = n(H∩A∩G') + n(H∩G∩A') + n(A∩G∩H') + n(H∩A∩G)

$= (15 - 10) + (14 - 10) + (12 - 10) + 10$

= 5 + 4 + 2 + 10

= 21

3. A group of prospective voters were asked to choose at least one candidate of three, represented by the colours red, green and white. Twenty of them preferred green, 17 preferred red and 16 preferred white. Eight preferred a coalition of white and red, 9 preferred green and red 12 preferred green and white and 5 preferred all three colours.
   a. Draw a Venn diagram to illustrate the data.
   b. From your diagram find:
      i. The total number of prospective voters surveyed.
      ii. The total number of voters that preferred only one candidate (colour).

   Solutions:

   All prospective voters prefer at least one candidate, therefore there is no complement. So:

   Let U = {set of prospective voters}

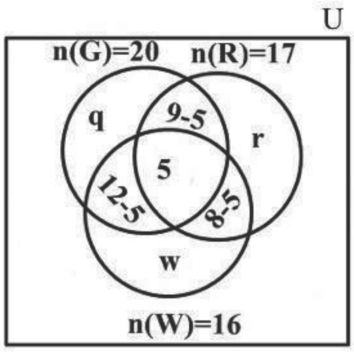
   G = {those who prefer green}, implying n(G) = 20

   R = {those who prefer red}, implying n(R) = 17

W = {those who prefer white}, implying n(W) = 16

n(W∩R) = 8, n(G∩R) = 9, n(W∩G) = 12, n(G∩R∩W) = 5

a. See Venn diagram at right.



b. Let the number of those voters that prefer green only be g, those who prefer red only be r and those who prefer white only be w.

n(G) $= g + (9-5) + 5 + (12-5)$

$20 = g + 4 + 5 + 7$

$20 = g + 16$

$g = 20 - 16 = 4$

n(W) $= w + (8-5) + 5 + (12-5)$

$16 = w + 3 + 5 + 7$

$16 = w + 15$

$w = 16 - 15 = 1$

n(R) $= r + (9-5) + 5 + (8-5)$

$17 = r + 4 + 5 + 3$

$17 = r + 12$

$r = 17 - 12 = 5$

i. Total number of prospective voters:

n(G∪R∪W) $= g + (9-5) + 5 + (12-5) + (8-5) + r + w$

$= 4 + 4 + 5 + 7 + 3 + 5 + 1 = 19$

ii. Number of voters that preferred only one candidate:

$= g + r + w = 4 + 5 + 1 = 10$

Practice

1. In a class of 45 pupils, it is known that 24 study Music, 20 study Chemistry and 22 study Biology. All the pupils study at least one subject of the three. Three do all three subjects while 7 do Music and Biology. Six do Music and Chemistry but not Biology and 8 do Chemistry and Biology. How many pupils study: a. Only one subject; b. Two subjects only.
2. At a Junior Secondary School there are 12 teachers. Of these, 6 teach Mathematics, 5 teach Integrated Science, and 4 teach Social Studies, 2 teach Mathematics and Integrated Science but no one teaches both Mathematics and Social Studies.
   a. Draw a Venn diagram to illustrate the data.
   b. Find: i. The number of teachers who teach Integrated Science and Social Studies; ii. The number of teachers who teach Social Studies only.

Lesson Title: Problem solving with 3 sets – Part 2	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L070	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to diagram and solve problems involving 3 sets, including real-life problems.

Overview

This is the second of 2 lessons on problem solving with 3 sets.

If 3 sets A, B and C are subsets of a universal set U, then the following rule holds:

$n(U) = n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

This rule is used throughout this lesson.

Solved Examples

1. A group of people were asked whether they liked soccer, basketball or rugby. Their responses are as shown in the table below;

Game liked	No. of people
All three games	7
Soccer and rugby	11
Soccer and basketball	10
Basketball and rugby	9
Soccer	34
Rugby only	18
Basketball only	16
None of the three games	3

   a. Represent the data in a Venn diagram
   b. How many people liked soccer only?
   c. How many people were surveyed?
   d. How many people liked exactly two games?

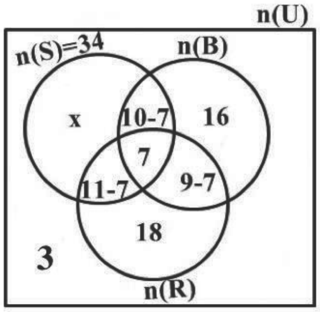
   Solutions:

   Let U = set of people surveyed
   S = set of soccer lovers
   B = set of basketball lovers
   R = set of Rugby lovers

   Then we have the following:

   $n(S \cap B \cap R) = 7$ $\quad$ $n(S) = 34$
   $n(S \cap R) = 11$ $\quad$ $n[R \cap (B \cup S)'] = 18$
   $n(S \cap B) = 10$ $\quad$ $n[B \cap (S \cup R)'] = 16$
   $n(B \cap R) = 9$ $\quad$ $n(B \cap R \cap S)' = 3$

a. See Venn diagram on the right.



b. Let the number of soccer lovers only be x, so that n[S∩(B∪R)'] $= x$.
Create an equation using the total number of people who like soccer, and solve for x:

n(S) = $x + (10 - 7) + 7 + (11 - 7)$

$34 = x + 3 + 7 + 4$

$34 = x + 14$

$x = 34 - 14 = 20$

c. Find the sum of all of the segments:

n(U) $= x + (10 - 7) + 16 + (11 - 7) + 7 + (9 - 7) + 18 + 3$

n(U) $= 20 + 3 + 16 + 4 + 7 + 2 + 18 + 3 = 73$

d. To find the number of people who liked exactly two games, add segments that represent exactly 2 games:

$= (10 - 7) + (11 - 7) + (9 - 7)$

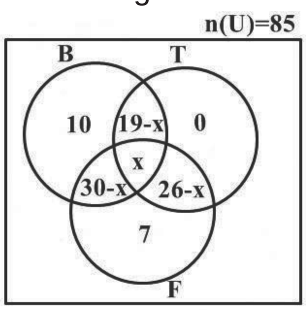
$= 3 + 4 + 2 = 9$

2. In a survey, 74 out of 88 tourists interviewed said they had visited at least the Freetown cotton tree (T), Lumley Beach (B) or old Fourah Bay College (F). Of these, 19 had visited only Lumley Beach and Cotton Tree, 30 visited Lumley Beach and old Fourah Bay College, 26 visited old Fourah Bay College and the cotton tree. No one had visited only the Cotton tree, 10 visited only Lumley Beach, 7 visited only Old Fourah Bay College and x had visited all three sites.
   a. Draw a Venn diagram to illustrate the information.
   b. Write a suitable equation in x and solve it.
   c. Find i. n[(B∩T) ∪F] ii. n[(B∩F)'∪T] iii. n(T)

Solutions:

Note that: n(U) = 88, n(T∪B∪F) = 74, n(B∩T) = 19, n(B∩F) = 30, n(F∩T) = 26, n[T∩(B∪F)'] = 0, n[B∩(T∪F)']=10, n[F∩(B∪T)'] = 7, and n(B∩T∩F) $= x$

a. Venn diagram:



b. n(B∪T∪F) $= 10 + (19 - x) + x + (30 - x) + 0 + (26 - x) + 7$

$74 = 10 + 19 - x + x + 30 - x + 26 - x + 7$ (equation in x)

$74 = 92 - 2x$

$2x = 92 - 74$

$x = \frac{18}{2} = 9$

c. (i) n[(B∩T)∪F] $= [(19 - x) + x] + (30 - x + 26 - x + 7)$

$= [(19 - 9) + 9] + (30 - 9 + 26 - 9 + 7)$

$= (10 + 9 + 21 + 17 + 7)$

$= 64$

(ii) n[(BUF)'∩T] = 0

(iii) n(T) $= 19 - x + 0 + x + 26 - x$

$= 19 + 26 - x$

$= 19 + 26 - 9$

$= 36$

Practice

1. In a sample survey of 47 people about the local languages they can speak in Sierra Leone, 31 were found to speak Krio, 26 Temne and 16 Mende. Three were found to speak all three languages, 2 spoke Temne and Mende only, and 6 spoke Mende only.
    a. Illustrate the information on a Venn diagram.
    b. Find the number of people who speak:
        i. Temne only; ii. Only one language; iii. Only two languages
2. There are 70 women in a club. Each plays at least one of the following games: Ludo (L), cards (C) and dice (D). If 20 play Ludo and an equal number play two games only:
    a. Illustrate the information on a Venn diagram.
    b. Find the number of women who play Ludo.
3. The Leone star coach invited 20 players for a soccer match. Eight of them were attackers, 12 of them were defenders and 12 were midfielders. The coach realised that 5 could play in the attack and in midfield, 6 defence and midfield, 2 only in attack and 3 could play all three roles.
    a. Illustrate the information on a Venn diagram.
    b. Find the number of players that can play:
        i. Only midfield; ii. Exactly two roles

Lesson Title: Use of variables	Theme: Algebraic Processes
Practice Activity: PHM1-L071	**Class:** SSS 1

Learning Outcomes
By the end of the lesson, you will be able to:
1. Identify that variables represent unknown numbers.
2. Identify the values of variables in simple algebraic equations (e.g. $2 + x = 5$).

Overview

This lesson is on how to identify the values of variables in simple algebraic expressions.

A variable is a letter that is used to represent a number. For example, consider the expression $2 + x = 5$. The variable x represents a specific number that will sum to 5 when 2 is added to it.

While x is the most common variable used in algebra, any letter can be used.

An algebraic expression is a collection of letters and symbols combined with at least one or more of the mathematical operations, $+$, $-$, $\times$ or $\div$. For example, these are some algebraic expressions: $3m + n - 5$, $5pq^3$, and $3pst \div 9p^2t^2s$

The **coefficient** of an algebraic term is the number in front of a variable. For example, in $8m, 8$ is the coefficient of m. The coefficient is multiplied by the variable, so that $8m = 8 \times m$.

A number on its own (without any variable) is called a constant term in an algebraic expression. For example, in $3x + 5$, 5 is a constant term.

Solved Examples

1. Identify the variables in the algebraic expressions:
   a. $5\pi r^2 h$ b. $x + y - 5$ c. $v = u + 9.8t$

   Solutions:
   a. r and h are variables. Recall that π has a known value, which is 3.14….
   b. x and y are variables
   c. u, v and t are variables

2. Identify the coefficients of x and y and the constant term in the following algebraic expressions:
   a. $-6x + 5y + 7$ b. $\frac{6}{7}y - x - 1$ c. $x + y + 10$

Solutions:

a. $-6x + 5y + 7$:
The coefficient of x is -6
The coefficient of y is 5
The constant term is 7

b. $\frac{6y}{7} - x - 1$:
The coefficient of x is -1
The coefficient of y is $\frac{6}{7}$
The constant term is -1

c. $x + y + 10$:
The coefficient of x is 1
The coefficient of y is 1
The constant term is 10

3. Write the following statements as algebraic expressions:
    a. 8 less than a certain number
    b. 7 more than a certain number
    c. Half a certain number
    d. y times a certain number
    e. 3 less than 2 times a certain number

Solutions:

Let x be the "certain number" in each example. Then we have:

a. $x - 8$
b. $x + 7$
c. $\frac{x}{2}$ or $\frac{1}{2}x$ or $0.5x$
d. $y \times x = xy$
e. $2x - 3$

4. Find the value of the variable in each of the following expressions:
    a. $x - 2 = 3$
    b. $2y = 10$
    c. $3a = 6$
    d. $3 + b = 7$

Solutions:

a. $x = 5$, because $5 - 2 = 3$
b. $y = 5$, because $2 \times 5 = 10$
c. $a = 2$, because $3 \times 2 = 6$
d. $b = 4$, because $3 + 4 = 7$

Practice

1. Identify the variables in the algebraic expressions:
    a. $\frac{4}{3}\pi r^3$
    b. $m + n - 4$
    c. $x = ut + \frac{1}{2}at^2$
2. Identify the coefficients and constants in each expression:
    a. $-5a + 6y - 10$
    b. $\frac{4}{5}y + 7 - x$
    c. $x - y + 10$
3. Write the following statements as algebraic expressions:
    a. A certain number times 60.
    b. 100 times as big as y
    c. y more than two times x
4. Find the value of the variable in each of the following expressions:
    a. $x + 1 = 14$
    b. $4a = 16$
    c. $2b = 7 - 1$
    d. $z - 4 = 2$

Lesson Title: Simplification – grouping terms	**Theme:** Algebraic Processes
Practice Activity: PHM1-L072	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to simplify algebraic expressions by grouping like terms.

Overview

In algebra, like terms are terms that have the same variable, and the variables have the same power. For example, $2a, 3a, 5a$ are all like terms, with the variable a to the power 1. As another example, $5p^2$ and $8p^2$ are like terms.

Like terms can be combined by adding or subtracting to give a single term. The result will have the same variable with the same power. The coefficients of the terms are added or subtracted. For example, consider these combinations:

$$a + a = (1 + 1)a = 2a$$

$$a + a + a = (1 + 1 + 1)a = 3a$$

$$4a - a = (4 - 1)a = 3a$$

Unlike terms are two or more terms that are not like terms. That is, they do not have the same variables or powers. Examples of unlike terms are:

$$2a + 3b + 4c$$
$$3m^2 + 2m^3$$

Unlike terms cannot be combined.

Solved Examples

1. Simplify: $8x - 2y - 4x + 5y$
   Solution:

$8x - 2y - 4x + 5y$	$=$	$8x - 4x + 5y - 2y$	Collect like terms
	$=$	$(8 - 4)x + (5 - 2)y$	Combine like terms
	$=$	$4x + 3y$	

2. Simplify: $7p - 3q + 3p + 2q$
   Solution:

$7p - 3q + 3p + 2q$	$=$	$7p + 3p + 2q - 3q$	Collect like terms
	$=$	$(7 + 3)p + (2 - 3)q$	Combine like terms
	$=$	$10p - q$	

3. Simplify the following algebraic expressions:
   a. $12e + 5f - 4e - 2f$
   b. $2m + 5n - 3m - 4n$
   c. $11x - 10y - 10x + 12y$
   d. $3u - 3 + 4v - 2u + 7 - 2v$

   Solutions:

   a.

$12e + 5f - 4e - 2f$	$=$	$12e - 4e + 5f - 2f$	Collect like terms
	$=$	$(12 - 4)e + (5 - 2)f$	Combine like terms
	$=$	$8e + 3f$	

   b.

$2m + 5n - 3m - 4n$	$=$	$2m - 3m + 5n - 4n$	Collect like terms
	$=$	$(2 - 3)m + (5 - 4)n$	Combine like terms
	$=$	$-m + n$	

   c.

$$
\begin{aligned}
11x - 10y - 10x + 12y &= 11x - 10x + 12y - 10y \\
&= (11 - 10)x + (12 - 10)y \\
&= x + 2y
\end{aligned}
$$

   d.

$$
\begin{aligned}
3u - 3 + 4v - 2u + 7 - 2v &= 3u - 2u + 4v - 2v + 7 - 3 \\
&= (3 - 2)u + (4 - 2)v + (7 - 3) \\
&= u + 2v + 4
\end{aligned}
$$

4. Simplify the following algebraic expressions:
   a. $pq + 12xp + 5pq - 3xp$
   b. $a + 3ab + 6 + 7a - 2ab - 1$

   Solutions:

   a.

$$
\begin{aligned}
pq + 12xp + 5pq - 3xp &= pq + 5pq + 12xp - 3xp \\
&= (1 + 5)pq + (12 - 3)xp \\
&= 6pq + 9xp
\end{aligned}
$$

   b.

$$
\begin{aligned}
a + 3ab + 6 + 7a - 2ab - 1 &= a + 7a + 3ab - 2ab + 6 - 1 \\
&= (1 + 7)a + (3 - 2)ab + (6 - 1) \\
&= 8a + ab + 5
\end{aligned}
$$

Practice

Simplify the following algebraic expressions:

1. $4y - 3x + 5x - 3y$
2. $9a + 4b - 11a + 3b$
3. $6m + 11n - 4m + 2n - m + n$
4. $2xy - 4xp + 3xy + 3xp$
5. $8mn + 9m + 4mn - 10m$
6. $4pq - 5pq + 8qr + 4qr + 3pq - 4qr$
7. $12p^2q - 4pq^2 + pq^2 - 4p^2q$
8. $3xyz - 4xz + 6xyz + 3xz$
9. $4ab + 7 - 3a - 3 - 2ab + 7a$

Lesson Title: Simplification – removing brackets	**Theme:** Algebraic processes
Practice Activity: PHM1-L073	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to simplify algebraic expressions by removing brackets.

Overview

In today's lesson, we are looking at algebraic expressions that contain brackets. For example, $2(5x-4)$. In order to simplify such expressions, we must first remove the brackets. In removing brackets, multiply the term outside the bracket by each of the terms inside the bracket.

We must be very careful with signs when removing brackets. When there is a (+) sign before the bracket, the sign inside the brackets does not change when the brackets are removed. When there is a negative sign (–) in front of the brackets, the signs inside the bracket change when the brackets are removed. This is because of the rules of multiplication. Remember that multiplying a negative by a positive gives a negative. Multiplying a negative and a negative gives a positive.

After removing brackets, collect any like terms together and combine them.

Solved Examples

1. Simplify: $2(5x-4)$
   Solution:
   $2(5x-4) = (2)(5x)+(2)(-4)$ Multiply each term by 2
   $= 10x-8$

2. Simplify: $-4(2y-3)$
   Solution:
   $-4(2y-3) = (-4)(2y)+(-4)(-3)$ Multiply each term by –4
   $= -8y+12$

3. Simplify: $8p-3(2q+2p)$
   Solution:
   $8p-3(2q+2p) = 8p+(-3)(2q)+(-3)(2p)$ Remove the brackets
   $= 8p-6q-6p$
   $= 8p-6p-6q$ Collect like terms
   $= 2p-6q$

4. Simplify: $5n - 3(2m + 3n)$

Solution:

$$\begin{aligned} 5n - 3(2m + 3n) &= 5n - 6m - 9n && \text{Remove the brackets} \\ &= 5n - 9n - 6m && \text{Collect like terms} \\ &= -4n - 6m \end{aligned}$$

5. Remove the brackets and simplify $3(y + 5) + 6(y + 2)$.

Solution:

$$\begin{aligned} 3(y + 5) + 6(y + 2) &= 3y + 15 + 6y + 12 && \text{Remove the brackets} \\ &= 3y + 6y + 15 + 12 && \text{Collect like terms} \\ &= 9y + 27 \end{aligned}$$

6. Remove the brackets and simplify $7(2p + 3) - 4(p - 2)$.

Solution:

$$\begin{aligned} 7(2p + 3) - 4(p - 2) &= 14p + 21 - 4p + 8 && \text{Remove the brackets} \\ &= 14p - 4p + 21 + 8 && \text{Collect like terms} \\ &= 10p + 29 \end{aligned}$$

7. Remove the brackets and simplify $4(b - 2a) + 3(a - 3b) - 4(b - a)$.

Solution:

$$\begin{aligned} 4(b - 2a) + 3(a - 3b) - 4(b - a) &= 4b - 8a + 3a - 9b - 4b + 4a \\ &= -8a + 3a + 4a + 4b - 9b - 4b \\ &= -a - 9b \end{aligned}$$

Practice

Remove brackets and simplify the following algebraic expressions:

1. $5(x - 4)$
2. $-7(3y - 4)$
3. $9m - 2(m + n)$
4. $11u - 3u(2v + 3)$
5. $-(2x + 4y) - (x - y)$
6. $8(-3m + 2n) - 2(m + n)$

Lesson Title: Simplification – expanding brackets	**Theme:** Algebraic Processes
Practice Activity: PHM1-L074	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to simplify algebraic expressions by expanding brackets.

Overview

In expanding an expression containing brackets, we rewrite the expression in an equivalent form without any brackets. Expanding brackets is the same as removing brackets. For example, to expand $3(x+2)$ means we multiply each of the terms inside the bracket by 3. Thus, $3(x+2) = 3x + 6$.

This lesson is on expanding brackets when there are 2 sets of brackets, for example, $(x+5)(x+10)$. In this expression, 2 binomials are multiplied. Binomials are algebraic expressions with 2 terms. Multiply each term in one bracket by each term in the other bracket. After removing the brackets, always collect like terms together and simplify.

Solved Examples

1. Expand $(x+5)(x+10)$
   Solution:

$(x+5)(x+10)$	$= x(x+10) + 5(x+10)$	Multiply each term of the first binomial by the second binomial
	$= x^2 + 10x + 5x + 50$	Remove the brackets
	$= x^2 + 15x + 50$	Combine like terms

2. Expand $(y-4)(y-3)$
   Solution:

$(y-4)(y-3)$	$= y(y-3) - 4(y-3)$	Multiply
	$= y^2 - 3y - 4y + 12$	Remove the brackets
	$= y^2 - 7y + 12$	Combine like terms

3. Expand $(3a+b)(2a-b)$
   Solution:

$(3a+b)(2a-b)$	$= 3a(2a-b) + b(2a-b)$	Multiply
	$= 6a^2 - 3ab + 2ab - b^2$	Remove the brackets
	$= 6a^2 - ab - b^2$	Combine like terms

4. Expand $(x-5)^2$

Solution:

$$\begin{aligned}(x-5)^2 &= (x-5)(x-5) && \text{Rewrite}\\ &= x(x-5)-5(x-5) && \text{Multiply}\\ &= x^2-5x-5x+25 && \text{Remove the brackets}\\ &= x^2-10x+25 && \text{Combine like terms}\end{aligned}$$

5. Expand $(3mn-4)(2mn+3)$

Solution:

$$\begin{aligned}(3mn-4)(2mn+3) &= 3mn(2mn+3)-4(2mn+3) && \text{Multiply}\\ &= 6m^2n^2+9mn-8mn-12 && \text{Remove the brackets}\\ &= 6m^2n^2+mn-12 && \text{Combine like terms}\end{aligned}$$

6. Expand and simplify $(5m+6n)(u-v)+(3m+2n)(2u+v)$

Solution:

Step 1. Expand the first two brackets:

$$\begin{aligned}(5m+6n)(u-v) &= u(5m+6n)-v(5m+6n) && \text{Multiply}\\ &= 5mu+6nu-5mv-6nv && \text{Remove the brackets}\end{aligned}$$

Step 2. Expand the last two brackets:

$$\begin{aligned}(3m+2n)(2u+v) &= 2u(3m+2n)+v(3m+2n) && \text{Multiply}\\ &= 6mu+4nu+3mv+2nv && \text{Remove the brackets}\end{aligned}$$

Step 3. Collect like terms after the two expansions:

$$5mu+6mu+6nu+4nu-5mv+3mv-6nv+2nv$$
$$= 11mu+10nu-2mv-4nv$$

7. Expand and simplify $2p(2p-4)^2$

Solution:

Step 1. Expand the brackets:

$$\begin{aligned}(2p-4)^2 &= (2p-4)(2p-4)\\ &= 2p(2p-4)-4(2p-4)\\ &= 4p^2-8p-8p+16\\ &= 4p^2-16p+16\end{aligned}$$

Step 2. Multiply by $2p$:

$$\begin{aligned}2p(2p-4)^2 &= 2p(4p^2-16p+16)\\ &= 8p^3-32p^2+32p\end{aligned}$$

Practice

Expand and simplify the following:

1. $(7x + 2)(x + 4)$
2. $(p - 5)(p - 6)$
3. $(m + 3)(m - 2)$
4. $(x - 4)^2$
5. $(n + 3)^2$
6. $(3m + 2)(n + 3) + (5m - 4)(n + 2)$
7. $3x(x - 2)^2$
8. $4(x + 6)^2$

Lesson Title: Factoring - common factors	**Theme:** Algebraic Processes
Practice Activity: PHM1-L075	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to factorise algebraic expressions by determining common factors.

Overview

The focus of this lesson is how to factorise algebraic expressions by determining common factors. Factorising in algebra is the reverse of expanding.

Consider $2(a+b)=2a+2b$. You have expanded $2(a+b)$ by removing brackets.

If you switch sides in the above equation you will get $2a+2b=2(a+b)$. We can factorise $2a+2b$ by taking the common factor 2 out of each term in $2a+2b$ to get 2 multiplied by $(a+b)$.

The general method used to factorise terms with common factors in algebraic expressions is to find the greatest common factor (GCF) to all terms and bring it outside the bracket.

Variables can also be common factors. If every term of an expression has the same variable, it can be factorised. Remember that factoring is the same as dividing. If you factor a variable from an expression, subtract the exponents.

If you choose a factor that is not the GCF, you might factor a number or variable, and then realise there is more that you can factor. It is fine to factorise an expression more than once, just continue with the next factor.

To check for the correctness of your factorisation, you can always expand the factorised expression. If you get the original expression, then you are right. Otherwise, check your factorisation again.

Solved Examples

1. Factorise the expression $10a-15b+5$
   Solution:
   Notice that 5 is a common factor in each term of the expression. That is, each can be divided by 5: $\frac{10a}{5}=2a$, $\frac{15b}{5}=3b$, $\frac{5}{5}=1$
   Factor 5 from each term:
   $$\begin{aligned} 10a-15b+5 &= 5(2a)-5(3b)+5(1) \\ &= 5(2a-3b+1) \end{aligned}$$

2. Factorise: $4a-ab$

Solution:

a is a common factor of each term. Take a out of the bracket:

$4a - ab = a(4 - b)$

3. Factorise :$8x^3 + 4x^2 + 2x$

   Solution:

   $2x$ is a common factor of each term:

   $8x^3 + 4x^2 + 2x = 2x(4x^2 + 2x + 1)$

4. Factorise $2a^7 + 3a^6 + a^5 + 4a^3$

   Solution:

   a^3 is a common factor.

   $2a^7 + 3a^6 + a^5 + 4a^3 = a^3(2a^4 + 3a^3 + a^2 + 4)$

5. Factorise:
   a. $64x^3 + 16x^2$
   b. $5a^4 - 10a^3 + 15ab$

   Solutions:
   a. $16x^2$ is a common factor: $64x^3 + 16x^2 = 16x^2(4x + 1)$
   b. $5a$ is a common factor: $5a^4 - 10a^3 + 15ab = 5a(a^3 - 2a^2 + 3b)$

6. Factorise:
   a. $15p^6q^4 - 6p^4q^3$
   b. $3xy^3 - 3xy^2 + 6x^2y^2$

   Solutions:
   a. $3p^4q^3$ is a common factor:

      $15p^6q^4 - 6p^4q^3 = 3p^4q^3(5p^2q - 2)$
   b. $3xy^2$ is a common factor:

      $3xy^3 - 3xy^2 + 6x^2y^2 = 3xy^2(y + 2x - 1)$

Practice

Factorise the following algebraic expressions:

1. $9ab - 27$
2. $x^2 - 4x$
3. $a^3 + ab$
4. $16x^4 - 8x^3 + 4x^2$
5. $81a^3 + 9a^2$
6. $12x^4 - 36x^3 + 48x^2$
7. $18s^5t^6 - 6s^4t^3 + 3s^2t^2$
8. $5uv^3 - 25uv^2 + 15u^2v^2$

Lesson Title: Factoring – grouping	**Theme:** Algebraic processes
Practice Activity: PHM1-L076	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to factorise algebraic expressions by grouping common terms.

Overview

This lesson focuses on factorising algebraic expression by grouping terms with common factors. We often have expressions with four terms which can be factored by grouping the terms that have common factors.

For example, consider the algebraic expression: $x^2 - ax + bx - ab$

Notice that the 4 terms do not have 1 common factor. The absence of a common factor to all four terms does not mean the expression cannot be factorised. There are common factors to some pairs of terms which can be grouped for factorisation.

Note that you can change the order of the terms based on which terms have common factors. For the example expression, we can form a group with a common factor of x, and another group with a common factor of b. We then factor x and b.

Form the two groups: $x^2 - ax + bx - ab = (x^2 - ax) + (bx - ab)$

Factor x and b: $x(x - a) + b(x - a)$

We now have 2 terms in the expression, $x(x - a)$ and $b(x - a)$. Notice that $(x - a)$ is a common factor in these terms. It can be factored out of the expression, leaving $(x + b)$:

$$x^2 - ax + bx - ab = x(x - a) + b(x - a) = (x - a)(x + b)$$

Solved Examples

1. Factorise completely: $2a^3 + 4a^2 - 3a - 6$
   Solution:

$2a^3 + 4a^2 - 3a - 6$	$= (2a^3 + 4a^2) + (-3a - 6)$	Create smaller groups
	$= 2a^2(a + 2) - 3(a + 2)$	Factor out each GCF
	$= (a + 2)(2a^2 - 3)$	Factor out $(a + 2)$

2. Factorise completely: $2x^3 - 6x^2 + 10x - 30$
   Solution:
   In this case, all 4 terms have a common factor, 2. When there are common factors, we should first factor them out. We can then proceed with the steps described above on the 4 terms inside brackets.

$2x^3 - 6x^2 + 10x - 30$	$= 2(x^3 - 3x^2 + 5x - 15)$	Factor out 2
	$= 2[(x^3 - 3x^2) + (5x - 15)]$	Create smaller groups
	$= 2[x^2(x - 3) + 5(x - 3)]$	Factor out each GCF
	$= 2(x - 3)(x^2 + 5)$	Factor out $(x - 3)$

3. Factorise completely: $18mn + 4m^2 - 27pn - 6pm$
   Solution:

$18mn + 4m^2 - 27pn - 6pm$	$= 18mn - 27pn + 4m^2 - 6pm$	Reposition terms
	$= (18mn - 27pn) + (4m^2 - 6pm)$	Create groups
	$= 9n(2m - 3p) + 2m(2m - 3p)$	Factor GCFs
	$= (2m - 3p)(9n + 2m)$	Factor $(2m - 3p)$

4. Factorise completely: $xz + wy + xy + wz$
   Solution:

$xz + wy + xy + wz$	$= xz + wz + wy + xy$	Reposition terms
	$= (xz + wz) + (wy + xy)$	Create groups
	$= z(x + w) + y(x + w)$	Factor out GCFs
	$= (x + w)(z + y)$	Factor out $(x + w)$

5. Factorise the expression $4uz - 6uv - 12vz + 2u^2$
   Solution:

$4uz - 6uv - 12vz + 2u^2$	$= 4uq + 2u^2 - 6uv - 12vz$	Reposition terms
	$= (4uq + 2u^2) + (-6uv - 12vz)$	Create groups
	$= 2u(2z + u) - 6v(2z + u)$	Factor out GCFs
	$= (2z + u)(2u - 6v)$	Factor out $(2z + u)$

Practice

Factorise the following expressions completely:

1. $ax - a + x - 1$
2. $12eg - 4eh - 6fg + 2fh$
3. $cd - ce - d^2 + de$
4. $p + q + 5ap + 5aq$
5. $x^2 + 2x - 5x - 10$
6. $6a^2 - 3a + 4a - 2$
7. $x^2 + 2x + 3x + 6$
8. $3x - 2dy + 3y - 2dx$

Lesson Title: Substitution of values	**Theme:** Algebraic Processes
Practice Activity: PHM1-L077	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to substitute values into given algebraic expressions.

Overview

This lesson is on substituting values into given algebraic expressions.

In substitution, variables are replaced by numbers, and we evaluate the expression to find a final result. Substitution is a useful tool in algebra to find a value or to rewrite equations in terms of a single variable.

In substitution, you will need to observe the following rules:

- When substituting negative numbers put them in brackets () so that you can get the calculation right.
- Two like signs (–)(–) or (+)(+) become a positive sign and two unlike signs (+)(–) or (–)(+) become a negative sign. This follows the rule for multiplying negative numbers.
- Remember to use the correct order of operations (BODMAS).

Solved Examples

1. Find the value of $x - 2$ if $x = 6$.
   Solution:
   We can substitute 6 for x. Instead of $x - 2$, we will have $6 - 2$.
   Answer: $6 - 2 = 4$.

2. If $x = 3$ and $y = 4$, what is $x + xy$?
   Solution:
   Remember that two variables written together in a term (as in xy) means they are multiplied together.

   $x + xy = 3 + (3)(4)$ Substitute 3 for x and 4 for y
   $= 3 + 12$ Simplify
   $= 15$

3. Evaluate the expression $x^2 + xy$ given $x = 3$ and $y = 4$
   Solution:

   $x^2 + xy = 3^2 + (3)(4)$ Substitute 3 for x and 4 for y
   $= 9 + 12$ Simplify
   $= 21$

4. Evaluate $x^2 + 2x + 1$ given $x = 5$.
   Solution:

$$x^2 + 2x + 1 = 5^2 + 2(5) + 1 \quad \text{Substitute 5 for } x$$
$$= 25 + 10 + 1 \quad \text{Simplify}$$
$$= 36$$

5. Evaluate $2 - a + a^2$ given $a = -2$
   Solution:

$$2 - a + a^2 = 2 - (-2) + (-2)^2 \quad \text{Substitute } -2 \text{ for } a$$
$$= 2 + 2 + 4 \quad \text{Simplify}$$
$$= 8$$

6. Evaluate $\frac{2y-z}{2x^2yz}$ when $x = 1, y = -4$ and $z = -10$
   Solution:

$$\frac{2y-z}{2x^2yz} = \frac{2(-4)-(-10)}{2(1)^2(-4)(-10)} \quad \text{Substitute } x = 1, y = -4, z = -10$$
$$= \frac{-8+10}{2(1)(40)} \quad \text{Simplify}$$
$$= \frac{2}{2\times 40}$$
$$= \frac{2}{80} = \frac{1}{40}$$

7. Find the value of x in the equation $x = ut + \frac{1}{2}gt^2$, given $u = 2,\ t = 5$ and $g = 10$.
   Solution:

$$x = ut + \frac{1}{2}gt^2 \quad \text{Substitute } u = 2,\ t = 5 \text{ and } g = 10$$
$$= 2(5) + \frac{1}{2}(10)(5)^2 \quad \text{Simplify}$$
$$= 10 + 5(25)$$
$$= 10 + 125$$
$$= 135$$

Practice

1. What is the value of $x^2y - y^2x$ if $x = 3$ and $y = -1$?
2. If $a = -7$ and $b = 3$, calculate the value of:
   a. $\left(\frac{a+b}{a-b}\right)^2$
   b. $2a^2b + b^2a$
   c. $27a^2 \div 12b^2$
3. What is the value of $\frac{str}{st-tr}$ if $s = -3,\ t = 2$ and $r = -22$?
4. Evaluate the value of C from the equation $C = \frac{5}{9}(F - 32)$ given:
   a. $F = 59$
   b. $F = 95$

Lesson Title: Addition of algebraic fractions	**Theme:** Algebraic processes
Practice Activity: PHM1-L078	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to add algebraic expressions.

Overview

Recall the rules for adding fractions. Fractions with the same denominator can be added by simply adding the numerators. When fractions have different denominators, we determine the LCM of the denominators and change the denominators to their LCM before adding.

This lesson is on the addition of algebraic fractions. Algebraic fractions are fractions with a variable term in the numerator, in the denominator, or in both. Examples of algebraic fractions are: $\frac{x}{4}$, $\frac{2}{x+1}$, $\frac{x}{2+x}$. This lesson is on the addition of simple algebraic fractions, with variables in the denominators.

To add algebraic fractions, you will follow similar rules to adding numerical fractions. To add algebraic fractions, follow these steps:

- Find the lowest common multiple (LCM) of the denominators.
- Express all fractions with the LCM in the denominator.
- Once the fractions have the same denominator, they are **like fractions** and can be added.
- Add the fractions, and combine like terms if possible.
- If the answer can be simplified, simplify it.

Solved Examples

1. Simplify: $\frac{x}{3}+\frac{x}{2}$

   Solution:

   Find the LCM of the denominators (3 and 2). Their LCM is 6, so we will change the denominator of each fraction to 6 before adding.

   $\frac{x}{3}+\frac{x}{2} = \frac{2\times x}{6}+\frac{3\times x}{6}$ Express denominators as the LCM

   $= \frac{2x}{6}+\frac{3x}{6}$ Simplify the numerators

   $= \frac{2x+3x}{6}$ Add the numerators

   $= \frac{5x}{6}$

2. Simplify: $\frac{2x}{9}+\frac{5x}{6}$

Solution:

Find the LCM of the denominators, which is 18.

$$\frac{2x}{9}+\frac{5x}{6} = \frac{2(2x)}{18}+\frac{3(5x)}{18}$$ Express denominators as the LCM

$$= \frac{4x}{18}+\frac{15x}{18}$$ Simplify the numerators

$$= \frac{4x+15x}{18}$$ Add

$$= \frac{19x}{18}$$

3. Simplify: $\frac{4x-1}{5}+\frac{2x+2}{5}$

Solution:

The denominators are the same, so proceed with addition:

$$\frac{4x-1}{5}+\frac{2x+2}{5} = \frac{4x-1+2x+2}{5}$$ Add the numerators

$$= \frac{4x+2x+2-1}{5}$$ Collect like terms

$$= \frac{6x+1}{5}$$ Combine like terms

4. Simplify: $\frac{4x+3}{12}+\frac{2x+5}{8}$

Solution:

Find the LCM of their denominators, which is 24.

$$\frac{4x+3}{12}+\frac{2x+5}{8} = \frac{2(4x+3)}{24}+\frac{3(2x+5)}{24}$$ Express the denominators as the LCM

$$= \frac{8x+6+6x+15}{24}$$ Simplify and add the numerators

$$= \frac{8x+6x+6+15}{24}$$ Collect like terms

$$= \frac{14x+21}{24}$$ Combine like terms

5. Simplify: $\frac{3x+6}{24}+\frac{2x-1}{8}$

Solution:

Find the LCM of their denominators, which is 24.

$$\frac{3x+6}{24}+\frac{2x-1}{8} = \frac{3x+6}{24}+\frac{3(2x-1)}{24}$$ Express the denominators as the LCM

$$= \frac{3x+6+3(2x-1)}{24}$$ Add

$$= \frac{3x+6+6x-3}{24}$$ Simplify

$$= \frac{3x+6x+6-3}{24}$$ Collect like terms

$$= \frac{9x+3}{24}$$ Combine like terms

$$= \frac{3(3x+1)}{24}$$ Factor the numerator

$$= \frac{3x+1}{8}$$ Simplify

Practice

1. Express as a single fraction:
   a. $\frac{a}{5}+\frac{4a}{5}$
   b. $\frac{2m}{9}+\frac{5m}{9}$
   c. $\frac{5x}{7}+\frac{3x}{7}$
   d. $\frac{x}{2}+1$
2. Add:
   a. $\frac{a}{5}+\frac{a}{3}$
   b. $\frac{x}{4}+x$
   c. $\frac{n}{10}+\frac{n}{1}$
   d. $\frac{x}{4}+\frac{x}{8}$
3. Simplify:
   a. $\frac{3a+1}{4}+\frac{a-1}{3}$
   b. $\frac{2n+2}{7}+\frac{3n}{4}$
   c. $\frac{3x}{9}+\frac{x-1}{6}$
   d. $\frac{x+1}{5}+\frac{x}{10}$

Lesson Title: Subtraction of algebraic fractions	**Theme:** Algebraic fractions
Practice Activity: PHM1-L079	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to subtract algebraic fractions.

Overview

This lesson is on subtraction of algebraic fractions. Recall that in the addition of algebraic fractions we find the LCM of the denominator, express the fraction in terms of the LCM and simplify to get a single fraction as the answer. We will follow similar steps to subtract algebraic fractions.

To subtract algebraic fractions, follow the steps:

- Find the lowest common multiple (LCM) of the denominators.
- Express all fractions with the LCM in the denominator.
- Once the fractions have the same denominator, they are **like fractions** and can be added or subtracted.
- Subtract the fractions. Subtract each of the like terms in the numerator.
- If the answer can be simplified, simplify it.

Be careful of a minus sign before a bracket. Where a minus sign occurs before a bracket, the signs of the terms inside the bracket change. For example, $-(a-b)$ becomes $-a+b$. The negative sign is distributed to each term inside the brackets.

Solved Examples

1. Simplify: $\frac{3x}{4}-\frac{x}{3}$
   Solution:
   Find the LCM of the denominators, which is 12.

$\frac{3x}{4}-\frac{x}{3}$	$= \frac{3(3x)}{12}-\frac{4x}{12}$	Express denominators as the LCM
	$= \frac{9x}{12}-\frac{4x}{12}$	Simplify numerator
	$= \frac{9x-4x}{12}$	Subtract the numerators
	$= \frac{5x}{12}$	Combine like terms

2. Simplify: $\frac{4a+1}{5}-\frac{4}{5}$
   Solution:
   The denominators are the same, so proceed with subtraction:

$$\frac{4a+1}{5}-\frac{4}{5} = \frac{4a+1-4}{5}$$
$$= \frac{4a-3}{5}$$

3. Simplify: $\frac{a-4}{2} - \frac{a-2}{6}$

Solution:

Find the LCM of the denominators, which is 6.

$\frac{a-4}{2} - \frac{a-2}{6}$	$= \frac{3(a-4)}{6} - \frac{a-2}{6}$	Express denominators as the LCM
	$= \frac{3a-12}{6} - \frac{a-2}{6}$	Simplify the numerator
	$= \frac{3a-12-(a-2)}{6}$	Subtract the numerators
	$= \frac{3a-12-a+2}{6}$	Distribute the negative sign
	$= \frac{3a-a-12+2}{6}$	Collect like terms
	$= \frac{2a-10}{6}$	Combine like terms
	$= \frac{2(a-5)}{2(3)}$	Factor out 2
	$= \frac{a-5}{3}$	Simplify

4. Simplify: $\frac{m+4}{3} - \frac{2-m}{5}$

Solution:

Find the LCM of the denominators, which is 15.

$\frac{m+4}{3} - \frac{2-m}{5}$	$= \frac{5(m+4)}{15} - \frac{3(2-m)}{15}$	Express denominators as the LCM
	$= \frac{5(m+4)-3(2-m)}{15}$	
	$= \frac{5m+20-6+3m}{15}$	
	$= \frac{5m+3m+20-6}{15}$	Collect like terms in the numerator and simplify
	$= \frac{8m+14}{15}$	
	$= \frac{2(4m+7)}{15}$	

5. Simplify: $\frac{8x-8}{6} - \frac{2x-5}{7}$

Solution:

First, notice that we can simplify the fraction on the left. We can factor 2 from the numerator and denominator.

$\frac{8x-8}{6} - \frac{2x-5}{7}$	$= \frac{2(4x-4)}{2(3)} - \frac{2x-5}{7}$	Factor the left fraction
	$= \frac{4x-4}{3} - \frac{2x-5}{7}$	Simplify
	$= \frac{7(4x-4)}{21} - \frac{3(2x-5)}{21}$	Express denominators as the LCM
	$= \frac{7(4x-4)-3(2x-5)}{21}$	Subtract the numerators
	$= \frac{28x-28-6x+15}{21}$	Simplify
	$= \frac{28x-6x-28+15}{21}$	Collect like terms

$= \frac{22x-13}{21}$ Combine like terms

Practice

1. Simplify:
    a. $\frac{3x}{4} - \frac{2x}{4}$
    b. $\frac{2y}{3} - 1$
    c. $\frac{2x+8}{5} - \frac{7}{5}$
    d. $\frac{a}{3} - \frac{4a}{3}$
2. Subtract:
    a. $\frac{a}{3} - \frac{a}{4}$
    b. $\frac{4x}{5} - x$
    c. $\frac{5x}{2} - \frac{2x}{3}$
    d. $\frac{x}{5} - \frac{x}{4}$
3. Express as a single fraction in its lowest terms:
    a. $\frac{5y}{2} - \frac{2y+6}{6}$
    b. $\frac{a-4}{2} - \frac{a-3}{3}$
    c. $\frac{2y+2}{3} - \frac{y-1}{4}$
    d. $\frac{2y+3}{3} - \frac{7}{12}$

Lesson Title: Linear equations	Theme: Algebraic processes
Practice Activity: PHM1-L080	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to solve linear equations using the balance method.

Overview

A linear equation in one variable is an equation that can be written in the form $ax + b = c$ where a, b and c are real numbers, x is the variable and $a \neq 0$. The highest power of the variable in a linear equation is 1, so linear equations are also called first-degree equations.

To use the balancing method, we solve by applying the same operation to both sides of an equal to sign. Our goal is to get the variable by itself.

These are possible steps in using the balancing method:

- Add the same quantity to each side.
- Subtract the same quantity from each side.
- Multiply each side by the same quantity.
- Divide each side by the same quantity.

You often need to perform multiple operations to get the answer. When balancing, perform addition or subtraction before multiplication and division.

Solved Examples

1. Solve for x in the equation $x + 5 = 7$.
   Solution:

$$\begin{aligned} x + 5 &= 7 \\ x + 5 - 5 &= 7 - 5 && \text{Subtract 5 from both sides} \\ x + 0 &= 2 \\ x &= 2 \end{aligned}$$

2. Solve $8 = 3x - 7$.
   Solution:

$$\begin{aligned} 8 &= 3x - 7 \\ 8 + 7 &= 3x - 7 + 7 && \text{Add 7 to both sides} \\ 15 &= 3x \\ \tfrac{15}{3} &= \tfrac{3x}{3} && \text{Divide both sides by } 3 \\ 5 &= x \end{aligned}$$

3. Solve $5y = 40$.

Solution:

$$\begin{aligned} 5y &= 40 \\ \frac{5y}{5} &= \frac{40}{5} && \text{Divide both sides by } 5 \\ y &= 8 \end{aligned}$$

4. Solve for x if $5x - 3 = 3x + 7$.
   Solution:

$$\begin{aligned} 5x - 3 &= 3x + 7 \\ 5x - 3 - 3x &= 3x + 7 - 3x && \text{Subtract } 3x \text{ from both sides} \\ 2x - 3 &= 7 \\ 2x - 3 + 3 &= 7 + 3 && \text{Add 3 to both sides} \\ 2x &= 10 \\ \frac{2x}{2} &= \frac{10}{2} && \text{Divide both sides by 2} \\ x &= 5 \end{aligned}$$

5. Solve $7y = 2y + 20$.
   Solution:

$$\begin{aligned} 7y &= 2y + 20 \\ 7y - 2y &= 2y - 2y + 20 && \text{Subtract } 2y \text{ from both sides} \\ 5y &= 20 \\ \frac{5y}{5} &= \frac{20}{5} && \text{Divide both sides by 5} \\ y &= 4 \end{aligned}$$

6. Solve $4 - 7y = 3y + 4$.
   Solution:

$$\begin{aligned} 4 - 7y &= 3y + 4 \\ 4 - 7y - 3y &= 3y - 3y + 4 && \text{Subtract } 3y \text{ from both sides} \\ 4 - 10y &= 4 \\ 4 - 4 - 10y &= 4 - 4 && \text{Subtract 4 from both sides} \\ -10y &= 0 \\ \frac{-10y}{-10} &= \frac{0}{-10} && \text{Divide both sides by} -10 \\ y &= 0 \end{aligned}$$

Practice

Solve the following equations:

1. $11 + 5m = -4$
2. $9y + 1 = 7y$
3. $3n - 15 = 45$

4. $5y - 2 = 8y - 7$
5. $x + 4 = 7 - 2x$
6. $0 = 10 - 8y$
7. $3 - 4x = 5x + 12$
8. $1 + 7m = 5m + 1$
9. $5y + 6 = 3y - 5$

Lesson Title: Linear equations with brackets	**Theme:** Algebraic Processes
Practice Activity: PHM1-L081	**Class:** SSS 1

Learning Outcome By the end of the lesson, you will be able to solve linear equations that contain brackets.

Overview

This lesson is on solving linear equations that contain brackets.

To solve the equation containing brackets, first remove the brackets. Then balance the equation, as in the previous lesson.

Remember that when there is a negative sign outside the bracket, the signs inside the brackets change.

Solved Examples

1. Solve $2(p+1) = 18$.

   Solution:

$$\begin{aligned} 2(p+1) &= 18 && \\ 2p+2 &= 18 && \text{Removing the brackets} \\ 2p+2-2 &= 18-2 && \text{Subtract 2 from both sides} \\ 2p &= 16 && \\ \frac{2p}{2} &= \frac{16}{2} && \text{Divide both sides by 2} \\ p &= 8 && \end{aligned}$$

2. Solve $3(m+6) = 2(m-3)$.

   Solution:

$$\begin{aligned} 3(m+6) &= 2(m-3) && \\ 3m+18 &= 2m-6 && \text{Remove brackets} \\ 3m-2m+18 &= 2m-2m-6 && \text{Subtract } 2m \text{ from both sides} \\ m+18 &= -6 && \\ m+18-18 &= -6-18 && \text{Subtract 18 from both sides} \\ m &= -24 && \end{aligned}$$

3. Solve $5y-3 = 3(y+4)$.

   Solution:

$$\begin{aligned} 5y-3 &= 3(y+4) && \\ 5y-3 &= 3y+12 && \text{Remove the brackets} \\ 5y-3y-3 &= 3y-3y+12 && \text{Subtract } 3y \text{ from both sides} \end{aligned}$$

$$\begin{aligned} 2y &= 12 \\ \frac{2y}{2} &= \frac{12}{2} && \text{Divide both sides by 2} \\ y &= 6 \end{aligned}$$

4. Solve $8m - 4(m + 3) = 2m + 6$.

Solution:

$$\begin{aligned} 8m - 4(m+3) &= 2m + 6 \\ 8m - 4m - 12 &= 2m + 6 && \text{Remove the brackets} \\ 4m - 12 &= 2m + 6 && \text{Simplify} \\ 4m - 2m - 12 &= 2m - 2m + 6 && \text{Subtract } 2m \text{ from both sides} \\ 2m - 12 &= 6 \\ 2m - 12 + 12 &= 6 + 12 && \text{Add 12 to both sides} \\ 2m &= 18 \\ \frac{2m}{2} &= \frac{18}{2} && \text{Divide both sides by 2} \\ m &= 9 \end{aligned}$$

5. Solve $5(y - 4) - 4(2y + 1) = 0$

Solution:

$$\begin{aligned} 5(y-4) - 4(2y+1) &= 0 \\ 5y - 20 - 8y - 4 &= 0 && \text{Remove brackets} \\ 5y - 8y - 20 - 4 &= 0 && \text{Collect like terms} \\ -3y - 24 &= 0 \\ -3y - 24 + 24 &= 0 + 24 && \text{Add 24 to both sides} \\ -3y &= 24 \\ \frac{-3y}{-3} &= \frac{24}{-3} && \text{Divide both sides by } -3 \\ y &= -8 \end{aligned}$$

6. Solve $2(3m + 5) - 3(m - 4) = 1$

Solution:

$$\begin{aligned} 2(3m+5) - 3(m-4) &= 1 \\ 6m + 10 - 3m + 12 &= 1 && \text{Remove bracket} \\ 6m - 3m + 10 + 12 &= 1 && \text{Collect like terms} \\ 3m + 12 &= 1 \\ 3m + 12 - 12 &= 1 - 12 && \text{Subtract 12 from both side} \\ 3m &= -11 \\ \frac{3m}{3} &= \frac{-11}{3} && \text{Divide both sides by 3} \\ m &= \frac{-11}{3} \\ m &= -3\tfrac{2}{3} \end{aligned}$$

Practice

Solve the following equations:

1. $2(a + 1) = 12$
2. $2(m + 4) = 9$
3. $4(x + 7) + 12 = 0$
4. $3(y + 2) = 2 - y$
5. $4(2a - 7) = 3(4a - 9)$
6. $3(6 + 7x) + 2(1 - 5x) = 42$
7. $4(3 - 5p) - 7(5 - 4p) + 3 = 0$

Lesson Title: Linear equations with fractions	**Theme:** Algebraic Processes
Practice Activity: PHM1-L082	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to solve linear equations that contain fractions.

Overview

This lesson is on solving linear equations that contain fractions, for example: $\frac{x}{6} = 8$.

When an equation contains a fraction, we can multiply to cancel the denominator. Remember to multiply both sides of the equation by the same number.

When there is more than 1 fraction in an equation, they can be cleared by multiplying throughout by the LCM of the denominators. After multiplying by the LCM, follow the steps from the previous 2 lessons. Remove any brackets, and balance the equation to solve for the variable.

Remember that you can check your solution by substituting the value of the variable into the equation. If both sides of the equation are equal, then the solution is correct.

Solved Examples

1. Solve $\frac{x}{6} = 8$

   Solution:

$$\begin{aligned} \frac{x}{6} &= 8 \\ 6\left(\frac{x}{6}\right) &= 6(8) && \text{Multiply by 6} \\ x &= 48 \end{aligned}$$

2. Solve: $\frac{x}{2} + 3 = 17$

   Solution:

$$\begin{aligned} \frac{x}{2} + 3 &= 17 \\ 2\left(\frac{x}{2}\right) + 2(3) &= 2(17) && \text{Multiply by 2} \\ x + 6 &= 34 \\ x + 6 - 6 &= 34 - 6 && \text{Subtract 6} \\ x &= 28 \end{aligned}$$

3. Solve for x: $\frac{x}{6} + \frac{x}{4} = 10$

Solution:

$$\begin{aligned}
\frac{x}{6}+\frac{x}{4} &= 10 \\
12\left(\frac{x}{6}\right)+12\left(\frac{x}{4}\right) &= 12(10) && \text{Multiply each term by the LCM, 12} \\
2x+3x &= 120 && \text{Simplify} \\
5x &= 120 && \text{Combine like terms} \\
\frac{5x}{5} &= \frac{120}{5} && \text{Divide both sides by 5} \\
x &= 24
\end{aligned}$$

4. Solve the equation for x: $\frac{7}{24}=\frac{x}{8}+\frac{1}{6}$

Solution:

$$\begin{aligned}
\frac{7}{24} &= \frac{x}{8}+\frac{1}{6} \\
24\left(\frac{7}{24}\right) &= 24\left(\frac{x}{8}\right)+24\left(\frac{1}{6}\right) && \text{Multiply by 24, the LCM} \\
7 &= 3x+4 \\
7-4 &= 3x+4-4 && \text{Subtract 4} \\
3 &= 3x \\
\frac{3}{3} &= \frac{3x}{3} && \text{Divide by 3} \\
1 &= x
\end{aligned}$$

5. Solve for y in the equation: $\frac{3}{4}y-2\frac{1}{3}=\frac{2}{3}y$

Solution:

$$\begin{aligned}
\frac{3}{4}y-2\frac{1}{3} &= \frac{2}{3}y \\
\frac{3y}{4}-\frac{7}{3} &= \frac{2y}{3} && \text{Convert mixed to improper fraction} \\
12\left(\frac{3y}{4}\right)-12\left(\frac{7}{3}\right) &= 12\left(\frac{2y}{3}\right) && \text{Multiply each term by the LCM, 12} \\
9y-28+28 &= 8y+28 && \text{Add 28} \\
9y &= 8y+28 \\
9y-8y &= 8y-8y+28 && \text{Subtract } 8y \\
y &= 28
\end{aligned}$$

6. Solve for x: $\frac{x-1}{3}-\frac{x+1}{10}=5$

Solution:

$$\begin{aligned}
\frac{x-1}{3}-\frac{x+1}{10} &= 5 \\
30\left(\frac{x-1}{3}\right)-30\left(\frac{x+1}{10}\right) &= 30(5) && \text{Multiply each term by the LCM, 30} \\
10(x-1)-3(x+1) &= 150 \\
10x-10-3x-3 &= 150 && \text{Remove the brackets (expand)} \\
7x-13 &= 150 && \text{Combine like terms} \\
7x-13+13 &= 150+13 && \text{Add 13}
\end{aligned}$$

$$
\begin{aligned}
7x &= 163 \\
\frac{7x}{7} &= \frac{163}{7} \qquad \text{Divide by 7} \\
x &= 23\frac{2}{7}
\end{aligned}
$$

Practice

Solve the following equations:

1. $\frac{1}{5}x = 3$
2. $\frac{2a}{5} + 1 = 9$
3. $\frac{y}{3} + \frac{y}{2} = 4$
4. $\frac{1}{5}x + \frac{1}{4}x = \frac{1}{2}$
5. $\frac{2}{3}(x - 6) = 8$
6. $\frac{k-3}{3} - \frac{k+2}{5} = 3$
7. $\frac{2p+7}{6} = \frac{2p-1}{3}$

Lesson Title: Word problems	Theme: Algebraic Processes
Practice Activity: PHM1-L083	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to create and solve equations from word problems.

Overview

This lesson is on solving equations derived from word (story) problems. There are many types of word problems which involve relations among known and unknown numbers. These can be written in the form of equations.

To solve word problems you will need the following steps:

- Read the problem carefully and note what is given and what is required.
- Assign a variable to represent each unknown.
- Identify any value that is multiplied by the variable (the coefficient).
- Identify any value that is constant.
- Write the algebraic expression representing the situation.
- Solve the equation for the unknown variable.
- Check whether the answer satisfies the conditions of the problem.

Solved Examples

1. The sum of two numbers is 25. One of the numbers exceeds the other by 9. Find the numbers.
   Solution:
   Let one of the numbers be y. Then the other number is $y+9$.
   The sum of the two numbers is 25. Therefore, in this situation, $y+y+9=25$.
   Solve the equation for y:
   $$\begin{aligned} y+y+9 &= 25 \\ 2y+9 &= 25 \\ 2y+9-9 &= 25-9 && \text{Subtract 9 from both sides} \\ 2y &= 16 \\ \frac{2y}{2} &= \frac{16}{2} && \text{Divide both sides by 2} \\ y &= 8 \end{aligned}$$
   We can find the other number using the formula: $y+9=8+9=17$

   Therefore, the two numbers are 8 and 17.

2. The difference between two numbers is 10. If one of the numbers is 4, what is the other number?

Solution:

Let the other number be y. The difference between 4 and y can either be $4 - y$ or $y - 4$.

If the difference is 10, then either:

$$\begin{aligned} 4 - y &= 10 \\ 4 - 10 &= y \\ -6 &= y \\ y &= -6 \end{aligned} \quad \text{OR} \quad \begin{aligned} y - 4 &= 10 \\ y &= 10 + 4 \\ y &= 14 \end{aligned}$$

3. A girl is 5 years old and her mother is 27 years old. In how many years' time will the daughter be half her mother's age?

Solution:

Age of girl now $= 5$

Age of mother now $= 27$

Let x years be the number of years in which daughter will be $\frac{1}{2}$ her mother's age.

Age of daughter in x years $= (5 + x)$

Age of mother in x years $= (27 + x)$

For the daughter's age to be $\frac{1}{2}$ her mother's age, we will have $(5 + x) = \frac{1}{2}(27 + x)$. Solve the equation for x:

$$\begin{aligned} (5 + x) &= \tfrac{1}{2}(27 + x) \\ 2(5 + x) &= 27 + x && \text{Multiply throughout by 2} \\ 10 + 2x &= 27 + x \\ 2x - x &= 27 - 10 && \text{Collect like terms} \\ x &= 17 \text{ years} \end{aligned}$$

In 17 years' time, the girl will be half her mother's age.

4. One farmer has 119 chickens and another has 73. After they each sell the same number of chickens, the first farmer is left with three times as many chickens as the other. How many chickens did each sell?

Solution:

If each farmer sells y chickens, the first farmer will be left write $(119 - y)$ chickens and the second farmer will be left with $(73 - y)$ chickens.

If the first farmer is left with 3 times that of the second after selling y chickens, then we have the equation $(119 - y) = 3(73 - y)$. Solve for y to find how many chickens they each sold:

$$
\begin{aligned}
(119 - y) &= 3(73 - y) \\
119 - y &= 219 - 3y && \text{Distribute 3 on the right-hand side} \\
3y - y &= 219 - 119 && \text{Collect like terms} \\
2y &= 100 \\
y &= \frac{100}{2} \\
y &= 50 \text{ chickens}
\end{aligned}
$$

5. The sum of 6 and one-third of a certain number is one less than twice the number.
    a. Express this statement in algebraic terms.
    b. What is the value of the number?

    Solution:

    a. Let the number be x. The sum of 6 and one-third of x is $(6 + \frac{x}{3})$.
    Twice the number x is $2x$. One less twice x is $(2x - 1)$.
    Therefore, we have the equation $6 + \frac{x}{3} = 2x - 1$.

    b. Solve for x:

$$
\begin{aligned}
6 + \frac{x}{3} &= 2x - 1 \\
3\left(6 + \frac{x}{3}\right) &= 3(2x - 1) && \text{Multiply throughout by 3 to eliminate the fraction} \\
18 + x &= 6x - 3 \\
18 + 3 &= 6x - x && \text{Collect like terms} \\
21 &= 5x \\
x &= \frac{21}{5} = 4\frac{1}{5}
\end{aligned}
$$

Practice

1. The result of adding 15 to a certain number and dividing the answer by 4 is the same as taking the number from 80.
    a. Express this statement as an algebraic equation.
    b. Find the value of the number.
2. A given number of matches are needed to fill 20 match boxes with the same number of matches in each box.
    a. How many matches are there in each box?
    b. If each box has three less match sticks, there will be enough sticks for 32 boxes. What is the total number of match sticks?
3. Bockarie is 11 years older than Kallon. In 5 years, Bockarie will be twice as old as Kallon. Find their present ages.
4. Sia and Kumba share Le7350.00 between them such that Sia gets Le950.00 more than Kumba. Find out how much money each gets.
5. Divide 59 mangoes into two parts so that one part is 7 mangoes less than five times the other part. How many mangoes are in the two parts?

Lesson Title: Substitution in formulae	**Theme:** Algebraic Processes
Practice Activity: PHM1-L084	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to substitute given values into a formula.

Overview

This lesson is on substituting given values into a formula. A formula is an equation in which variables represent quantities. For example, we have a formula for finding the perimeter P, of a rectangle. For a rectangle with length l and width w, perimeter is given by the formula $P = 2(l + w)$, where l and w are in the same units. In this example, P is the subject of the formula. By substituting values of l and w into the formula, the corresponding value of P can be found.

Solved Examples

1. Calculate the perimeter of a rectangle with a length of 8 cm and width of 6 cm.
Solution:

P	$= 2(l + w)$	Formula
	$= 2(8 \text{ cm} + 6 \text{ cm})$	Substitute $l = 8$ *and* $w = 6$
	$= 2(14 \text{ cm})$	Simplify
	$= 28 \text{ cm}$	

1. Given that $a = 3, b = -2$, find the value of the algebraic expression $2a + b$.
Solution:
Substitute the given values into the formula and simplify:

$$2a + b = 2(3) + (-2) = 6 - 2 = 4$$

2. Given that $m = -5, n = 4$, find the value of the algebraic expression $\frac{2m-n}{m^2}$.
Solution:
Substitute the given values into the formula and simplify:

$$\frac{2m-n}{m^2} = \frac{2(-5)-(4)}{(-5)^2} = \frac{-10-4}{25} = \frac{-14}{25}$$

3. The circumference C of a circle is given by the formula $C = 2\pi r$, where r is the radius and π (pi) is a constant. Calculate the circumference of a circle with a radius of 28 cm. Use $\frac{22}{7}$ for π.
Solution:

C	$= 2\pi r$	Formula
	$= 2 \times \frac{22}{7} \times 28 \text{ cm}$	Substitute $\pi = \frac{22}{7}$ and $r = 28$

$= 2 \times 22 \times 4$ cm Simplify
$= 176$ cm

4. The total surface area of a closed cylinder is given by $A = 2\pi r^2 + 2\pi rh$, where r is the radius of the circular faces, h is the height. Use the formula to calculate the area of a cylinder with a radius of 8 cm and height of 20 cm. Use 3.14 for π.
   Solution:
   $A = 2\pi r^2 + 2\pi rh$ Formula
   $= 2 \times 3.14 \times 8^2 + 2 \times 3.14 \times 8 \times 20$ Substitute $\pi = 3.14, r = 8, h = 20$
   $= 401.92 + 1{,}004.8$ Simplify
   $= 1{,}406.72$ cm²

5. The volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius of the circular base and h is the height. Use $\pi = \frac{22}{7}$ to find the volume of a cylinder with the base radius of 7 cm and height of 40 cm.
   Solution:
   $V = \pi r^2 h$ Formula
   $= \frac{22}{7} \times 7 \text{ cm} \times 7 \text{ cm} \times 40 \text{ cm}$ Substitute $\pi = \frac{22}{7}, r = 7, h = 40$
   $= 22 \times 7 \text{ cm} \times 40 \text{ cm}$ Simplify
   $= 6{,}160$ cm³

6. Given that $E = v + \frac{1}{2}mv^2$, find the value of E if $v = 10$ and $m = 4$.
   Solution:
   $E = v + \frac{1}{2}mv^2$ Formula
   $= 10 + \frac{1}{2} \times 4 \times 10^2$ Substitute $v = 10, m = 4$
   $= 10 + 2 \times 100$ Simplify
   $= 10 + 200$
   $= 210$

Practice

1. Given that $x = -1, y = 3$, and $z = 5$ find the value of the algebraic expressions:
   a. $y - 3x$
   b. $xy + z$
   c. $2y + z^2$
   d. $x^2 - 5x + 2$
2. The relationship between the Celsius (C) and Fahrenheit (F) scales of temperature is given by $C = \frac{5}{9}(F - 32)$. Find the temperature in Celsius if $F = 42°\text{F}$.

3. Given that $v = u + at$, find the value of v if $u = 10$, $a = 30$, $t = \frac{1}{6}$
4. The length of a rectangle is given by the formula $l = \frac{P}{2} - w$, where P is the perimeter and w is the width. Find the value of l when $P = 74$ cm and $w = 15$ cm.
5. The formula $I = \frac{P \times R \times T}{100}$, find I if $P = 450{,}000$; $R = 4\%$ and $T = 2$.
6. The curved surface area of a cylinder is given by $A = 2\pi rh$, where r is a radius of circular base and h is the height. Find the area if the radius is 15 cm and height is 25 cm. Use 3.14 for π.

Lesson Title: Change of subject – Part 1	**Theme:** Algebraic Processes
Practice Activity: PHM1-L085	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to change the subject of a formula.

Overview

A formula is a type of equation which shows the relationship between variables such as x and y. For an equation to be a formula, it must have more than one variable. For example, $m+n=9$ is a formula, because it shows the relationship between m and n. $3n-4=0$ is not a formula because it only has one variable, n.

The subject of a formula is the single variable to which everything else in the formula is equal. The subject of a formula will usually be positioned to the left of the equals sign. For example, x is the subject of the formula $x=3+y$.

We can change the subject of a formula. For example, consider $x=3+y$. We could make y the subject. Simply subtract 3 from both sides, and we have $x-3=y$. This can also be written as $y=x-3$. This is the same formula as $x=3+y$, but we have changed the subject from x to y.

We change the subject by getting a variable by itself on one side of the equals sign. We may use addition, subtraction, multiplication, or division to do this. Remember the following rules when changing the subject of an equation:

- Adding and subtracting are the opposite of one another
- Multiplying and dividing are the opposite of one another
- You must perform the same method on both sides of the equation.

If you are trying to change the subject to a variable that appears more than once in a formula, first collect all terms that contain that variable on one side of the equation. Combine like terms or factor if possible, and solve.

Solved Examples

1. Make x the subject of each formula:
   a. $y=4x$; b. $y=x+2$ c. $y=\frac{1}{2}x$ d. $y=\frac{x-1}{2}$

   Solutions:

   a. To get x by itself, divide throughout by 4:

$$\begin{aligned} y &= 4x \\ \frac{y}{4} &= \frac{4x}{4} \\ \frac{y}{4} &= x \end{aligned}$$

b. To solve for x, subtract 2 from both sides:

$$\begin{aligned} y &= x+2 \\ y-2 &= x+2-2 \\ y-2 &= x \end{aligned}$$

c. To solve for x, multiply both sides by 2:

$$\begin{aligned} y &= \tfrac{1}{2}x \\ 2y &= 2\left(\tfrac{1}{2}x\right) \\ 2y &= x \end{aligned}$$

d. To write the formula with x as the subject, we must take 2 steps:

$$\begin{aligned} y &= \tfrac{x-1}{2} && \\ 2y &= 2\left(\tfrac{x-1}{2}\right) && \text{Multiply both sides by 2} \\ 2y &= x-1 && \\ 2y+1 &= x-1+1 && \text{Add 1 to both sides} \\ 2y+1 &= x && \end{aligned}$$

2. Make x the subject of the formula $z = 4x + 4y$.
Solution:
We must take multiple steps to get x by itself:

$$\begin{aligned} z &= 4x+4y && \\ z-4y &= 4x+4y-4y && \text{Subtract } 4y \text{ from both sides} \\ z-4y &= 4x && \\ \tfrac{z-4y}{4} &= \tfrac{4x}{4} && \text{Divide both sides by 4} \\ \tfrac{z-4y}{4} &= x && \end{aligned}$$

3. Make y the subject of $3y + 3x = 2y + 1$.
Solution:
The variable y appears twice. We must collect all terms containing y on one side of the equation before solving.

$$\begin{aligned} 3y+3x &= 2y+1 && \\ 3y-2y &= -3x+1 && \text{Group terms containing } y \\ y &= -3x+1 && \text{Combine like terms} \end{aligned}$$

4. Make x the subject of $ny + 3x = tx + 2$.
Solution:
The variable x appears twice. We must collect all terms containing x on one side of the equation before solving.

$$\begin{aligned} ny+3x &= tx+2 && \\ 3x-tx &= 2-ny && \text{Group terms containing } x \\ x(3-t) &= 2-ny && \text{Factorise } x \end{aligned}$$

$$\frac{x(3-t)}{3-t} = \frac{2-ny}{3-t} \quad \text{Divide both sides by } 3-t$$

$$x = \frac{2-ny}{3-t}$$

5. Make c the subject of the formula $A = \frac{1}{2}(b+c)h$

   Solution:

$$A = \frac{1}{2}(b+c)h$$

$$2A = 2 \times \frac{1}{2}(b+c)h \quad \text{Multiply both sides by 2}$$

$$2A = (b+c)h$$

$$\frac{2A}{h} = \frac{(b+c)h}{h} \quad \text{Divide both sides by } h$$

$$\frac{2A}{h} = b+c$$

$$\frac{2A}{h} - b = b - b + c \quad \text{Subtract } b \text{ from both sides}$$

$$\frac{2A}{h} - b = c$$

6. Making y the subject of $m + n + ey = 2m - y$

   Solution:

$$m + n + ey = 2m - y$$

$$ey + y = 2m - m - n \quad \text{Group terms containing } y$$

$$y(e+1) = 2m - m - n \quad \text{Factorise } y$$

$$y(e+1) = m - n \quad \text{Combine like terms with } m$$

$$\frac{y(e+1)}{e+1} = \frac{m-n}{e+1} \quad \text{Divide both sides by } e+1$$

$$y = \frac{m-n}{e+1}$$

Practice

1. Make b the subject of each formula:
   a. $a = 13b$; b. $a = b - 3$ c. $a = \frac{1}{4}b$ d. $a = 2b - 8$
2. Make a the subject in the formula $p = 2(a+b)$.
3. Make y the subject in the following relations:
   a. $4d + 5cy = qy + rt$
   b. $3py - ay = r + s$
   c. $ky + 6x = 4y - 3x$
4. Make u the subject of the following:
   a. $v = \frac{u+1}{u+2}$
   b. $hu = \frac{w+3u}{m}$
   c. $n = \frac{2uk}{v} + 2$
5. Make F the subject of the formula $C = \frac{5}{9}(F - 32)$.

Lesson Title: Change of subject – Part 2	**Theme:** Algebraic Processes
Practice Activity: PHM1-L086	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to change the subject of a formula.

Overview

This lesson is on changing the subject of more complicated relations, involving powers and roots.

You must always ensure that whatever you do in changing the subject of a formula, you maintain balance in your equation by carrying out the same operations on both sides of the equal sign. For example:

- Adding or subtracting the same quantity on both sides
- Multiplying or dividing both sides by the same quantity
- Applying the same power or root to both sides.

Note that squaring "undoes" a square root, and taking the square root "undoes" a square.

Solved Examples

1. Make q the subject of the relation $p = \sqrt{mq}$
   Solution:
   $p = \sqrt{mq}$
   $p^2 = \left(\sqrt{mq}\right)^2$ Square both sides to remove the square root
   $p^2 = mq$
   $\frac{p^2}{m} = \frac{mq}{m}$ Divide both sides by m to isolate q as subject
   $q = \frac{p^2}{m}$

2. Make b the subject of the formula $p = \sqrt{a+b}$
   Solution:
   $p = \sqrt{a+b}$
   $p^2 = \left(\sqrt{a+b}\right)^2$ Square both sides
   $p^2 = a+b$
   $p^2 - a = b$

3. Make i the subject of the formula $R = (1+i)^2$.
   $R = (1+i)^2$

$$\sqrt{R} = \sqrt{(1+\mathrm{i})^2} \quad \text{Take the square root of both sides}$$
$$\sqrt{R} = 1+i$$
$$i = \sqrt{R}-1 \quad \text{Subtract 1 from each side}$$

4. Make R the subject of the relation $q = \frac{m}{R^2} + k$

Solution:

$$q = \frac{m}{R^2} + k$$
$$q - k = \frac{m}{R^2} \quad \text{Subtract } k \text{ from both sides}$$
$$R^2 q - kR^2 = R^2\left(\frac{m}{R^2}\right) \quad \text{Multiply both sides by } R^2 \text{ to remove fraction}$$
$$R^2(q-k) = m \quad \text{Factorise } R^2$$
$$\frac{R^2(q-k)}{q-k} = \frac{m}{q-k} \quad \text{Divide both sides by } q-k$$
$$R^2 = \frac{m}{q-k}$$
$$R = \; = \pm\sqrt{\frac{m}{q-k}} \quad \text{Take the square root of both sides}$$

Remember that when you find the square root of a number, the result can be either positive or negative.

5. Make y the subject of the relation, $N + \sqrt{y} = q$. Then, find the value of y when $q = 5$, $N = 3$.

Solution:

Step 1. Change the subject:

$$N + \sqrt{y} = q$$
$$\sqrt{y} = q - N \quad \text{Subtract } N \text{ from both sides}$$
$$\left(\sqrt{y}\right)^2 = (q-N)^2 \quad \text{Square both sides}$$
$$y = (q-N)^2$$

Step 2. Substitute the given values:

$$y = (5-3)^2 \quad \text{Substitute } q = 5 \text{ and } N = 3$$
$$y = 2^2 = 4$$

6. Make x the subject of the relation $y = ax^2 + bd + c$

Solution:

$$y = ax^2 + bd + c$$
$$y - bd - c = ax^2 \quad \text{Subtract } bd \text{ and } c \text{ from both sides}$$
$$\frac{y-bd-c}{a} = \frac{ax^2}{a} \quad \text{Divide both sides by } a$$
$$\frac{y-bd-c}{a} = x^2$$
$$x = \pm\sqrt{\frac{y-bd-c}{a}} \quad \text{Take the square root of both sides}$$

Practice

1. Solve for r: $p = r^2 + 1$
2. Make t the subject of the formula $x = \sqrt{t + u}$.
3. Make x the subject of the relation $y = \frac{1}{3}\sqrt{x + 1}$. Then, find the value of x when $y = -2$.
4. Solve for u: $F = \frac{mv^2}{u}$
5. Make t the subject of the formula $x = uf + \frac{1}{2}gt^2$.
6. Make r the subject of the relation $\frac{1}{3}\pi r^2 = v$.
7. Solve for u: $c = \sqrt{\frac{u-a}{u-b}}$

Lesson Title: Reduction to basic form of surds	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L087	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to reduce surds to basic forms.

Overview

Surds are numbers that we cannot find a whole number square root of. They can be left in square root form to express their exact values. For example, consider these square roots: $\sqrt{2}, \sqrt{3}, \sqrt{5}$. These are examples of surds, because we cannot simplify them to whole numbers.
Surds are left in root form (with √) to express their exact value. When calculated, surds have an infinite number of non-recurring decimals. Therefore, surds are irrational numbers. The square roots of all prime numbers are surds. They give decimals that never repeat and never end.

In simplifying surds to basic forms, it is often necessary to find the largest perfect square factor. You will use the fact that $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$. For example, consider $\sqrt{8}$. This is a surd that can be simplified. Note that 4 divides 8, and 4 is a perfect square.

Simplify $\sqrt{8}$ as follows: $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$.

Always remember some common perfect squares. This will help you solve problems with surds. Some common perfect squares are listed here:

Number	Square
2	4
3	9
4	16
5	25
6	36

Number	Square
7	49
8	64
9	81
10	100
11	121

Number	Square
12	144
13	169
14	196
15	225

Solved Examples

1. Reduce each of the following surds to its basic form:
   a. $\sqrt{8}$ b. $\sqrt{12}$ c. $\sqrt{18}$ d. $\sqrt{24}$

   Solutions:

   a. $\sqrt{8} = \sqrt{4 \times 2}$
   $= \sqrt{4} \times \sqrt{2}$
   $= 2 \times \sqrt{2}$

   b. $\sqrt{12} = \sqrt{4 \times 3}$
   $= \sqrt{4} \times \sqrt{3}$
   $= 2 \times \sqrt{3}$

$= 2\sqrt{2}$ $= 2\sqrt{3}$

c. $\sqrt{18} = \sqrt{9 \times 2}$
$= \sqrt{9} \times \sqrt{2}$
$= 3 \times \sqrt{2}$
$= 3\sqrt{2}$

d. $\sqrt{24} = \sqrt{4 \times 6}$
$= \sqrt{4} \times \sqrt{6}$
$= 2 \times \sqrt{6}$
$= 2\sqrt{6}$

2. Reduce each surd to its basic form:
   a. $\sqrt{288}$ b. $\sqrt{192}$ c. $\sqrt{48}$ d. $\sqrt{125}$

   Solutions:

   a. $\sqrt{288} = \sqrt{144 \times 2}$
   $= \sqrt{144} \times \sqrt{2}$
   $= 12 \times \sqrt{2}$
   $= 12\sqrt{2}$

   b. $\sqrt{192} = \sqrt{64 \times 3}$
   $= \sqrt{64} \times \sqrt{3}$
   $= 8 \times \sqrt{3}$
   $= 8\sqrt{3}$

   c. $\sqrt{48} = \sqrt{16 \times 3}$
   $= \sqrt{16 \times 3}$
   $= \sqrt{16} \times \sqrt{3}$
   $= 4 \times \sqrt{3}$
   $= 4\sqrt{3}$

   d. $\sqrt{125} = \sqrt{25 \times 5}$
   $= \sqrt{25} \times \sqrt{5}$
   $= 5 \times \sqrt{5}$
   $= 5\sqrt{5}$

3. Simplify: $\sqrt{16{,}000}$

   Solution:

   When you encounter a large number in a surd, try to break it down into factors as usual. You may have to try factoring it a few different ways before you find the way that gives perfect squares and allows you to simplify.

   $\sqrt{16{,}000} = \sqrt{1{,}600 \times 10}$
   $= \sqrt{16 \times 100 \times 10}$
   $= \sqrt{16} \times \sqrt{100} \times \sqrt{10}$
   $= 4 \times 10 \times \sqrt{10}$
   $= 40\sqrt{10}$

Practice

1. Reduce the following surds to their basic forms:
   a. $\sqrt{32}$ b. $\sqrt{40}$ c. $\sqrt{50}$ d. $\sqrt{27}$
2. Reduce the following surds to their basic forms:
   a. $\sqrt{432}$ b. $\sqrt{180}$ c. $\sqrt{1,250}$ d. $\sqrt{240}$
3. Simplify the following:
   a. $\sqrt{6,250}$ b. $\sqrt{243}$ c. $\sqrt{9,000}$ d. $\sqrt{800}$

Lesson Title: Addition and subtraction of surds – Part 1	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L088	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to solve simple problems involving addition and subtraction of surds.

Overview

Surds that are in basic form can be added or subtracted. We can only add or subtract like surds. Like surds have the same surd part. To add them, keep the same surd part, and add the coefficients. For example, $m\sqrt{k} + n\sqrt{k} = (m+n)\sqrt{k}$.

Mixed surds cannot be added or subtracted. That is, they cannot be simplified further. For example, $m\sqrt{k} + n\sqrt{p}$ or $m\sqrt{k} - n\sqrt{p}$ cannot be simplified further.

To add or subtract two or more surds, follow the steps below:

- **Step 1.** Convert each surd to its simplest form.
- **Step 2.** Add or subtract any like surds.

Solved Examples

1. Simplify: $3\sqrt{2} + 7\sqrt{2}$
   Solution:
   These are like surds, so the coefficients can be added:
   $$\begin{aligned} 3\sqrt{2} + 7\sqrt{2} &= (3+7)\sqrt{2} \\ &= 10\sqrt{2} \end{aligned}$$

2. Simplify: $\sqrt{8} + \sqrt{32}$
   Solution:
   $$\begin{aligned} \sqrt{8} + \sqrt{32} &= \sqrt{4 \times 2} + \sqrt{16 \times 2} && \text{Simplify each surd} \\ &= \sqrt{4} \times \sqrt{2} + \sqrt{16} \times \sqrt{2} \\ &= 2\sqrt{2} + 4\sqrt{2} \\ &= (2+4)\sqrt{2} && \text{Add like surds} \\ &= 6\sqrt{2} \end{aligned}$$

3. Simplify: $2\sqrt{3} - \sqrt{3}$
   Solution:
   These are like surds, so the coefficients can be subtracted:
   $$\begin{aligned} 2\sqrt{3} - \sqrt{3} &= (2-1)\sqrt{3} \\ &= 1\sqrt{3} \end{aligned}$$

$= \sqrt{3}$

4. Simplify: $\sqrt{27} - \sqrt{12}$

   Solution:

   $\sqrt{27} - \sqrt{12} = \sqrt{9 \times 3} - \sqrt{4 \times 3}$ Simplify each surd
   $= \sqrt{9} \times \sqrt{3} - \sqrt{4} \times \sqrt{3}$
   $= 3\sqrt{3} - 2\sqrt{3}$
   $= (3 - 2)\sqrt{3}$ Subtract like surds
   $= \sqrt{3}$

5. Simplify: $\sqrt{24} - 3\sqrt{6}$

   Solution:

   $\sqrt{24} - 3\sqrt{6} = \sqrt{4 \times 6} - 3\sqrt{6}$ Simplify $\sqrt{24}$
   $= \sqrt{4} \times \sqrt{6} - 3\sqrt{6}$
   $= 2\sqrt{6} - 3\sqrt{6}$
   $= (2 - 3)\sqrt{6}$ Subtract like surds
   $= -1\sqrt{6}$
   $= -\sqrt{6}$

6. Simplify $\sqrt{98} - 2\sqrt{8}$

   Solution:

   $\sqrt{98} - 2\sqrt{8} = \sqrt{49 \times 2} - 2\sqrt{4 \times 2}$ Simplify $\sqrt{24}$
   $= \sqrt{49} \times \sqrt{2} - 2\sqrt{4} \times \sqrt{2}$
   $= 7\sqrt{2} - 2(2)\sqrt{2}$
   $= 7\sqrt{2} - 4\sqrt{2}$
   $= (7 - 4)\sqrt{2}$ Subtract like surds
   $= 3\sqrt{2}$

Practice

Simplify the following:

1. $5\sqrt{2} - \sqrt{2}$
2. $4\sqrt{5} + 3\sqrt{5}$
3. $\sqrt{3} + \sqrt{3}$
4. $\sqrt{24} - 2\sqrt{6}$
5. $\sqrt{50} - \sqrt{8}$
6. $\sqrt{75} - 2\sqrt{12}$
7. $3\sqrt{8} + \sqrt{50}$

Lesson Title: Addition and subtraction of surds – Part 2	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L089	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to solve more complicated problems involving addition and subtraction of surds.

Overview

Remember that we can only add or subtract surds which are like, or have the same surd part. First simplify surds to their basic form, then add or subtract if possible.

Solved Examples

1. Simplify $\sqrt{147}-\sqrt{75}+\sqrt{27}$
   Solution:
   $\sqrt{147}-\sqrt{75}+\sqrt{27} = \sqrt{49\times 3}-\sqrt{25\times 3}+\sqrt{9\times 3}$ Simplify surds
   $= \sqrt{49}\times\sqrt{3}-\sqrt{25}\times\sqrt{3}+\sqrt{9}\times\sqrt{3}$
   $= 7\sqrt{3}-5\sqrt{3}+3\sqrt{3}$
   $= (7-5+3)\sqrt{3}$ Add/subtract coefficients
   $= 5\sqrt{3}$

2. Simplify $\sqrt{180}-\sqrt{45}+\sqrt{20}$
   Solution:
   $\sqrt{180}-\sqrt{45}+\sqrt{20} = \sqrt{36\times 5}-\sqrt{9\times 5}+\sqrt{4\times 5}$ Simplify surds
   $= \sqrt{36}\times\sqrt{5}-\sqrt{9}\times\sqrt{5}+\sqrt{4}\times\sqrt{5}$
   $= 6\sqrt{5}-3\sqrt{5}+2\sqrt{5}$
   $= (6-3+2)\sqrt{3}$ Add/subtract
   $= 5\sqrt{5}$

3. Simplify $\sqrt{50}-3\sqrt{2}+\frac{1}{2}\sqrt{8}$
   Solution:
   $\sqrt{50}-3\sqrt{2}+\frac{1}{2}\sqrt{8} = \sqrt{25\times 2}-3\sqrt{2}+\frac{1}{2}\sqrt{4\times 2}$ Simplify surds
   $= 5\sqrt{2}-3\sqrt{2}+\frac{2}{2}\sqrt{2}$
   $= 5\sqrt{2}-3\sqrt{2}+\sqrt{2}$
   $= (5-3+1)\sqrt{2}$ Add/subtract
   $= 3\sqrt{2}$

4. Simplify $\frac{5}{2}\sqrt{32}-\frac{1}{5}\sqrt{125}-\frac{1}{3}\sqrt{72}+\frac{3}{2}\sqrt{180}$
   Solution:

$$\frac{5}{2}\sqrt{32} - \frac{1}{5}\sqrt{125} - \frac{1}{3}\sqrt{72} + \frac{3}{2}\sqrt{180} \quad = \frac{5}{2}\sqrt{16\times 2} - \frac{1}{5}\sqrt{25\times 5} - \frac{1}{3}\sqrt{36\times 2} + \frac{3}{2}\sqrt{36\times 5}$$
$$= \frac{5}{2}\times 4\sqrt{2} - \frac{1}{5}\times 5\sqrt{5} - \frac{1}{3}\times 6\sqrt{2} + \frac{3}{2}\times 6\sqrt{5}$$
$$= 10\sqrt{2} - \sqrt{5} - 2\sqrt{2} + 9\sqrt{5}$$
$$= 10\sqrt{2} - 2\sqrt{2} + 9\sqrt{5} - \sqrt{5}$$
$$= (10-2)\sqrt{2} + (9-1)\sqrt{5}$$
$$= 8\sqrt{2} + 8\sqrt{5}$$
$$= 8\left(\sqrt{2} + \sqrt{5}\right)$$

5. If $\sqrt{294} + \sqrt{216} - \sqrt{486} = m\sqrt{6}$. Find the value of m.
   Solution:
   Simplify the left-hand side, then add and subtract the surds. Based on the right-hand side, it looks like we will have $\sqrt{6}$ in the surds on the left-hand side.

$$\sqrt{294} + \sqrt{216} - \sqrt{486} \quad = \sqrt{49\times 6} + \sqrt{36\times 6} - \sqrt{81\times 6}$$
$$= \sqrt{49}\times\sqrt{6} + \sqrt{36}\times\sqrt{6} - \sqrt{81}\times\sqrt{6}$$
$$= 7\sqrt{6} + 6\sqrt{6} - 9\sqrt{6}$$
$$= (7+6-9)\sqrt{6} = 4\sqrt{6}$$

Hence $m = 4$

Practice

Simplify the following surds as far as possible.

1. $\frac{2}{3}\sqrt{63} + \frac{1}{2}\sqrt{28} - \frac{1}{10}\sqrt{175}$
2. $\frac{5}{4}\sqrt{32} - \frac{1}{3}\sqrt{72}$
3. $2\sqrt{18} - \frac{1}{2}\sqrt{128} + \frac{3}{2}\sqrt{72}$
4. $\frac{5}{2}\sqrt{48} + \frac{1}{3}\sqrt{216} - \frac{2}{5}\sqrt{75} - \frac{1}{2}\sqrt{24}$
5. $5\sqrt{125} - 2\sqrt{500} + 2\sqrt{20}$
6. $3\sqrt{7} - \sqrt{343} + 4\sqrt{28}$
7. If $\sqrt{175} - \sqrt{448} + \sqrt{112} = n\sqrt{7}$, find the value of n.

Lesson Title: Properties of surds	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L090	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to identify properties of surds.

Overview

Recall that surds are square roots of numbers that are imperfect squares. They are irrational numbers. The following are examples of surds: $\sqrt{2},\ \sqrt{3},\ \sqrt{5},\ \sqrt{6},\ \sqrt{7}$

In operating with surds, please note carefully the following properties of surds:

1. $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$
2. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
3. $\frac{b}{\sqrt{a}} = \frac{b}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}} = \frac{b\sqrt{a}}{a}$ (This is known as rationalising the denominator, and it will be covered in a later lesson.)
4. $a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}$
5. $\sqrt{a^2} = \sqrt{a} \times \sqrt{a} = a$
6. $a\sqrt{b} \times \sqrt{c} = a\sqrt{bc}$
7. $a \times \sqrt{b} = a\sqrt{b}$
8. $a \times b\sqrt{c} = ab\sqrt{c}$

Solved Examples

1. Simplify $\sqrt{27}$
   Solution:

$$\sqrt{27} = \sqrt{9 \times 3}$$
$$= \sqrt{9} \times \sqrt{3} \qquad \text{Using 1) } \sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$
$$= 3 \times \sqrt{3}$$
$$= 3\sqrt{3} \qquad \text{Using 7) } a \times \sqrt{b} = a\sqrt{b}$$

2. Simplify $\sqrt{3\frac{3}{8}}$
   Solution:
   First, change the mixed fraction to an improper fraction so that property 2. can be applied.

$$\sqrt{3\frac{3}{4}} = \sqrt{\frac{15}{4}}$$
$$= \frac{\sqrt{15}}{\sqrt{4}} \qquad \text{Using 2) } \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$= \frac{\sqrt{15}}{2}$

3. Simplify $5\sqrt{5} + 4\sqrt{5}$

   Solution:

   $5\sqrt{5} + 4\sqrt{5} = (5+4)\sqrt{5}$ Using 4) $a\sqrt{c} + b\sqrt{c} = (a+b)\sqrt{c}$

   $= 9\sqrt{5}$

4. Simplify:

   a. $3 \times 2\sqrt{5}$ c. $\sqrt{7^2}$

   b. $2\sqrt{5} \times \sqrt{2}$ d. $\sqrt{x^3}$

   Solutions:

   a. $3 \times 2\sqrt{5} = (3 \times 2)\sqrt{2}$ Using 8) $a \times b\sqrt{c} = ab\sqrt{c}$

   $= 6\sqrt{2}$

   b. $2\sqrt{5} \times \sqrt{2} = 2\sqrt{5 \times 2}$ Using 6) $a\sqrt{b} \times \sqrt{c} = a\sqrt{bc}$

   $= 2\sqrt{10}$

   c. $\sqrt{7^2} = \sqrt{7} \times \sqrt{7}$ Using 5) $\sqrt{a^2} = \sqrt{a} \times \sqrt{a} = a$

   $= 7$

   d. $\sqrt{x^3} = \sqrt{x} \times \sqrt{x} \times \sqrt{x}$ Using 5) $\sqrt{a^2} = \sqrt{a} \times \sqrt{a} = a$

   $= x \times \sqrt{x}$

   $= x\sqrt{x}$

Practice

1. Simplify: a. $\sqrt{32}$ b. $\sqrt{48}$ c. $\sqrt{24}$
2. Simplify: a. $\sqrt{2\frac{1}{4}}$ b. $\sqrt{5\frac{5}{9}}$ c. $\sqrt{1\frac{9}{16}}$
3. Simplify: a. $a\sqrt{6} + 2a\sqrt{6} - 4a\sqrt{6}$

   b. $5\sqrt{2} + 6\sqrt{2} - 10\sqrt{2}$

   c. $\sqrt{12} + \sqrt{27} - \sqrt{48}$
4. Simplify: a. $4 \times 2\sqrt{2}$ b. $4\sqrt{3} \times 2\sqrt{2}$ c. $\sqrt{a^5}$ d. $\sqrt{x^4}$

Lesson Title: Multiplication of surds – Part 1	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L091	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to multiply surds.

Overview

In the multiplication of surds you need to carefully follow the following steps:

1. Express each surd in its basic or simplest form.
2. Multiply the coefficients, and multiply the surd parts.
3. Make sure the result is in its simplest form, and simplify it if possible.

The general rules for multiplying surds are:

$$\sqrt{a} \times \sqrt{\mathrm{b}} = \sqrt{ab}$$
$$a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$$

Solved Examples

1. Find the product of $7\sqrt{2}$ and $5\sqrt{3}$.
   Solution:
   $7\sqrt{2} \times 5\sqrt{3} = (7 \times 5)\sqrt{2 \times 3}$ Multiply coefficients and numbers in surds
   $= 35\sqrt{6}$
   The result is already in its simplest form, so the answer is $35\sqrt{6}$.

2. Simplify: $\sqrt{5} \times \sqrt{15}$
   Solution:
   $\sqrt{5} \times \sqrt{15} = \sqrt{5 \times 15}$ Multiply numbers in surds
   $= \sqrt{75}$
   $= \sqrt{25 \times 3}$ Simplify $\sqrt{75}$
   $= \sqrt{25} \times \sqrt{3}$
   $= 5\sqrt{3}$

3. Find the product of $4\sqrt{7}$ and $3\sqrt{5}$.
   Solution:
   $4\sqrt{7} \times 3\sqrt{5} = (4 \times 3)\sqrt{7 \times 5}$ Multiply coefficients and numbers in surds
   $= 12\sqrt{35}$

4. Simplify: $\sqrt{3} \times \sqrt{18}$
   Solution:
   $\sqrt{3} \times \sqrt{18} = \sqrt{3} \times \sqrt{9 \times 2}$ Simplify $\sqrt{18}$

$= \sqrt{3} \times (\sqrt{9} \times \sqrt{2})$
$= \sqrt{3} \times 3\sqrt{2}$
$= 3\sqrt{3 \times 2}$ Multiply numbers in surds
$= 3\sqrt{6}$

5. Multiply: $\sqrt{6} \times \sqrt{18}$
Solution:
$\sqrt{6} \times \sqrt{18} = \sqrt{6} \times \sqrt{9 \times 2}$ Simplify $\sqrt{18}$
$= \sqrt{6} \times (\sqrt{9} \times \sqrt{2})$
$= \sqrt{6} \times 3\sqrt{2}$
$= 3\sqrt{6 \times 2}$ Multiply numbers in surds
$= 3\sqrt{12}$
$= 3\sqrt{4 \times 3}$ Simplify $3\sqrt{12}$
$= 3\sqrt{4}\sqrt{3}$
$= 3 \times 2\sqrt{3}$
$= 6\sqrt{3}$

6. Multiply: $3\sqrt{24} \times 2\sqrt{12}$
Solution:
$3\sqrt{24} \times 2\sqrt{12} = 3\sqrt{4 \times 6} \times 2\sqrt{4 \times 3}$ Simplify both surds
$= 3\sqrt{4} \times \sqrt{6} \times 2 \times \sqrt{4} \times \sqrt{3}$
$= 3 \times 2 \times \sqrt{6} \times 2 \times 2 \times \sqrt{3}$
$= 6\sqrt{6} \times 4\sqrt{3}$
$= 6 \times 4\sqrt{6 \times 3}$ Multiply
$= 24\sqrt{18}$
$= 24 \times \sqrt{9 \times 2}$ Simplify
$= 24 \times 3\sqrt{2}$
$= 72\sqrt{2}$

7. Simplify: $\left(2\sqrt{3}\right)^2$
Solution:
$\left(2\sqrt{3}\right)^2 = 2\sqrt{3} \times 2\sqrt{3}$ Multiply
$= (2 \times 2)\sqrt{3 \times 3}$ Multiply coefficients and numbers in surds
$= 4\sqrt{9}$
$= 4 \times 3$
$= 12$

Practice

1. Find the product of:
   a. $7\sqrt{3}$ and $5\sqrt{3}$
   b. $\sqrt{32}$ and $\sqrt{80}$
2. Simplify:
   a. $4\sqrt{2} \times 2\sqrt{8}$
   b. $4\sqrt{12} \times 6\sqrt{18}$
3. Multiply:
   a. $\sqrt{64} \times \sqrt{48}$
   b. $2\sqrt{75} \times \sqrt{125}$
4. Simplify:
   a. $\left(3\sqrt{2}\right)^2$
   b. $\sqrt{54} \times \sqrt{128}$

Lesson Title: Multiplication of surds – Part 2	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L092	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to multiply surds.

Overview

The lesson today is on the simplification of more complicated problems involving the multiplication of surds.

Remember the basic laws involving multiplication of surds. The coefficient parts are multiplied, and the surd parts are multiplied. For example:

a. $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b} = \sqrt{ab}$
b. $a \times b\sqrt{c} = ab\sqrt{c}$
c. $a\sqrt{b} \times c\sqrt{d} = ab\sqrt{bd}$

Remember the order of operations (BODMAS) and apply it when there are multiple operations in one problem.

Solved Examples

1. Simplify:
   a. $\sqrt{24} \times \left(\sqrt{3}\right)^2$
   b. $10(\sqrt{3} + 2)$

   Solution:

a. $\sqrt{24} \times \left(\sqrt{3}\right)^2$
$= \sqrt{4 \times 6} \times (\sqrt{3} \times \sqrt{3})$
$= \sqrt{4} \times \sqrt{6} \times 3$
$= 2 \times 3 \times \sqrt{6}$
$= 6\sqrt{6}$

b. $10(\sqrt{3} + 2)$
$= 10\sqrt{3} + 10(2)$
$= 10\sqrt{3} + 20$
$= 20 + 10\sqrt{3}$

2. Simplify: $\sqrt{10}(\sqrt{8} + 2)$

   Solution:

$$\begin{aligned}
\sqrt{10}(\sqrt{8} + 2) &= \sqrt{10} \times \sqrt{8} + 2 \times \sqrt{10} \\
&= \sqrt{10 \times 8} + 2\sqrt{10} \\
&= \sqrt{80} + 2\sqrt{10} \\
&= \sqrt{16 \times 5} + 2\sqrt{10} \\
&= \sqrt{16} \times \sqrt{5} + 2\sqrt{10} \\
&= 4\sqrt{5} + 2\sqrt{10}
\end{aligned}$$

3. Simplify: $\sqrt{48}-3\sqrt{2}\left(2\sqrt{3}-4\right)-4\sqrt{48}$

Solution:

$$\begin{aligned}\sqrt{48}-3\sqrt{2}\left(2\sqrt{3}-4\right)-4\sqrt{48} &= \sqrt{16\times 3}-3\times 2\sqrt{2\times 3}+3\times 4\sqrt{2}-4\sqrt{16\times 3}\\ &= \sqrt{16}\times\sqrt{3}-6\sqrt{6}+12\sqrt{2}-4\times\sqrt{16}\times\sqrt{3}\\ &= 4\sqrt{3}-6\sqrt{6}+12\sqrt{2}-4\times 4\sqrt{3}\\ &= 4\sqrt{3}-6\sqrt{6}-12\sqrt{2}-16\sqrt{3}\\ &= -12\sqrt{2}-12\sqrt{3}-6\sqrt{6}\end{aligned}$$

4. Simplify $(\sqrt{2}-1)(3-\sqrt{2})$ and write the result in the form $a+b\sqrt{c}$, stating the value of a, b and c.

Solution:

$$\begin{aligned}\left(\sqrt{2}-1\right)\left(3-\sqrt{2}\right) &= \sqrt{2}\left(3-\sqrt{2}\right)-1(3-\sqrt{2})\\ &= 3\sqrt{2}-\sqrt{2}\times\sqrt{2}-3+\sqrt{2}\\ &= 3\sqrt{2}-2-3+\sqrt{2}\\ &= -5+4\sqrt{2}\end{aligned}$$

Hence $a=-5, b=4$ and $c=2$

5. Simplify: $\frac{\sqrt{125}\times\sqrt{63}}{\sqrt{28}\times\sqrt{80}}$.

Solution:

$$\begin{aligned}\frac{\sqrt{125}\times\sqrt{63}}{\sqrt{28}\times\sqrt{80}} &= \frac{\sqrt{25\times 5}\times\sqrt{9\times 7}}{\sqrt{4\times 7}\times\sqrt{16\times 5}}\\ &= \frac{\sqrt{25}\times\sqrt{5}\times\sqrt{9}\times\sqrt{7}}{\sqrt{4}\times\sqrt{7}\times\sqrt{16}\times\sqrt{5}}\\ &= \frac{5\sqrt{5}\times 3\sqrt{7}}{2\sqrt{7}\times 4\sqrt{5}}\\ &= \frac{5\times 3\sqrt{7}\sqrt{5}}{2\times 4\sqrt{5}\sqrt{7}}\\ &= \frac{15\sqrt{35}}{8\sqrt{35}}\\ &= \frac{15}{8} = 1\frac{7}{8}\end{aligned}$$

6. Simplify $(2-3\sqrt{3})\,(3+2\sqrt{3})$ and write the result in the form $a+b\sqrt{c}$.

Solution:

Expand the brackets, then simplify:

$$\begin{aligned}(2-3\sqrt{3})\,(3+2\sqrt{3}) &= 2\left(3+2\sqrt{3}\right)-3\sqrt{3}\,(3+2\sqrt{3})\\ &= 6+4\sqrt{3}-9\sqrt{3}-6(3)\\ &= (6-18)+(4-9)\sqrt{3}\\ &= -12-5\sqrt{3}\end{aligned}$$

Practice

1. Simplify: $\sqrt{20} \times \left(\sqrt{5}\right)^2$
2. Simplify: $\sqrt{2}(3 - 2\sqrt{2})$
3. Simplify: $\sqrt{18}\left(\sqrt{27} - \frac{2}{\sqrt{2}}\right)$
4. Simplify: $2\sqrt{5}(6 - 3\sqrt{5})$
5. Simplify $\sqrt{18}\left(3 + 2\sqrt{8}\right) + 3\sqrt{2}$ and write your result in the form $a + b\sqrt{c}$.
6. Simplify: $\frac{\sqrt{80}\times\sqrt{63}}{\sqrt{125}\times\sqrt{28}}$
7. Simplify $\left(2 + 3\sqrt{3}\right)^2$and write the result in the form $a + b\sqrt{c}$

Lesson Title: Rationalisation of denominators of surds – Part 1	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L093	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to rationalise the denominators of surds.

Overview

Rationalising the denominator of a surd is the process of changing the denominator from a surd to a rational number.

The denominator of a fraction should not be an irrational number. To make the irrational number in the denominator a rational number, we multiply both the numerator and denominator by the denominator.

The general rule is $\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$.

Remember that when you multiply a fraction that has the same numerator and denominator, you are actually multiplying by 1. We can have irrational numbers in the numerator of a fraction.

Solved Examples

1. Simplify: $\frac{1}{\sqrt{2}}$

   Solution:

   $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ Multiply the top and bottom by $\sqrt{2}$

   $= \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$

   $= \frac{\sqrt{2}}{2}$ Note: $\sqrt{2} \times \sqrt{2} = 2$

2. Simplify the following:

   a. $\frac{\sqrt{3}}{\sqrt{7}}$ b. $\frac{10}{\sqrt{5}}$

   Solutions:

   a. $\frac{\sqrt{3}}{\sqrt{7}} = \frac{\sqrt{3}}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

   $= \frac{\sqrt{21}}{7}$

   b. $\frac{10}{\sqrt{5}} = \frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

   $= \frac{10\sqrt{5}}{5}$

   $= 2\sqrt{5}$

3. Simplify the following:

   a. $\frac{4}{\sqrt{24}}$ b. $\frac{2\sqrt{2}}{\sqrt{12}}$ c. $\frac{3}{\sqrt{27}}$ d. $\frac{9}{\sqrt{27}}$

Solutions:

a. $\frac{4}{\sqrt{24}} = \frac{4}{2\sqrt{6}}$
$= \frac{2}{\sqrt{6}}$
$= \frac{2}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$
$= \frac{2\sqrt{6}}{6}$
$= \frac{\sqrt{6}}{3}$

b. $\frac{2\sqrt{2}}{\sqrt{12}} = \frac{2\sqrt{2}}{2\sqrt{3}}$
$= \frac{\sqrt{2}}{\sqrt{3}}$
$= \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$
$= \frac{\sqrt{6}}{3}$

c. $\frac{3}{\sqrt{27}} = \frac{3}{3\sqrt{3}}$
$= \frac{1}{\sqrt{3}}$
$= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$
$= \frac{\sqrt{3}}{3}$

d. $\frac{9}{\sqrt{27}} = \frac{9}{3\sqrt{3}}$
$= \frac{3}{\sqrt{3}}$
$= \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$
$= \frac{3\sqrt{3}}{3}$
$= \sqrt{3}$

4. Simplify the following:

a. $\frac{3\sqrt{3}}{2\sqrt{5}}$ b. $\frac{5}{3\sqrt{5}}$ c. $\frac{3\sqrt{3}}{2\sqrt{5}}$

Solutions:

a. $\frac{3\sqrt{3}}{2\sqrt{5}} = \frac{3\sqrt{3}}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$
$= \frac{3\sqrt{15}}{2\times5}$
$= \frac{3\sqrt{15}}{10}$

b. $\frac{5}{3\sqrt{5}} = \frac{5}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$
$= \frac{5\sqrt{5}}{3\times5}$
$= \frac{5\sqrt{5}}{15}$
$= \frac{\sqrt{5}}{3}$

c. $\frac{3\sqrt{3}}{2\sqrt{5}} = \frac{3\sqrt{3}}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$
$= \frac{3\sqrt{15}}{2\times5}$
$= \frac{3\sqrt{15}}{10}$

Practice

Simplify the following surds:

1. $\frac{3}{\sqrt{8}}$ 2. $\frac{\sqrt{5}}{\sqrt{7}}$ 3. $\frac{16}{\sqrt{128}}$ 4. $\frac{\sqrt{27}}{\sqrt{72}}$ 5. $\frac{3\sqrt{2}}{\sqrt{27}}$ 6. $\frac{2\sqrt{5}}{3\sqrt{2}}$
7. $\frac{24}{\sqrt{24}}$ 8. $\frac{7\sqrt{2}}{\sqrt{14}}$ 9. $\frac{18}{\sqrt{162}}$ 10. $\frac{4\sqrt{3}}{3\sqrt{2}}$

Lesson Title: Rationalisation of the denominator of surds – Part 2	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L094	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to rationalise the denominator of surds.

Overview

This is the second lesson on rationalising the denominator of surds. It covers problems where there are 2 terms in the numerator or denominator, and at least one of them is a surd.

We use a special process when there are 2 terms in the denominator and one of them is a surd. For example, consider $\frac{1}{3-\sqrt{2}}$. Such problems require you to multiply the numerator and denominator by the **conjugate** of the binomial. To find the conjugate, change the sign in the middle. The conjugate of $3-\sqrt{2}$ is $3+\sqrt{2}$.

When a binomial is multiplied by its conjugate, simply square each term to save time. For example, $(3-\sqrt{2})(3+\sqrt{2}) = 3^2 - \sqrt{2}^2$. Observe why this is true. The middle terms will always cancel:

$(3-\sqrt{2})(3+\sqrt{2})$	$=$	$3(3+\sqrt{2}) - \sqrt{2}(3+\sqrt{2})$	Multiply
	$=$	$3^2 + 3\sqrt{2} - 3\sqrt{2} - \sqrt{2}^2$	Remove brackets
	$=$	$3^2 - \sqrt{2}^2$	Middle terms cancel

You will also see problems with 2 terms in the numerator, such as $\frac{\sqrt{3}+\sqrt{7}}{\sqrt{21}}$. These problems are solved using the process from the previous class. Multiply the numerator and denominator by $\sqrt{21}$ to rationalise the denominator, then simplify.

Solved Examples

1. Simplify: $\frac{1}{3-\sqrt{2}}$
   Solution:
   Note that you will need to multiply the numerator and denominator by $3+\sqrt{2}$, the conjugate of $3-\sqrt{2}$.

$\frac{1}{3-\sqrt{2}}$	$=$	$\frac{1}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$	Multiply top and bottom by $3+\sqrt{2}$
	$=$	$\frac{3+\sqrt{2}}{3^2-\sqrt{2}^2}$	
	$=$	$\frac{3+\sqrt{2}}{9-2}$	Simplify
	$=$	$\frac{3+\sqrt{2}}{7}$	

2. Simplify: $\frac{3}{2+\sqrt{5}}$

Solution:

$$\begin{aligned}\frac{3}{2+\sqrt{5}} &= \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} \\ &= \frac{3(2-\sqrt{5})}{(2)^2-(\sqrt{5})^2} \\ &= \frac{6-3\sqrt{5}}{4-5} \\ &= \frac{6-3\sqrt{5}}{-1} \\ &= -(6-3\sqrt{5}) \\ &= -6+3\sqrt{5}\end{aligned}$$

3. Simplify the following:

a. $\frac{2}{3+2\sqrt{2}}$ b. $\frac{5}{9-3\sqrt{5}}$

Solutions:

a. $$\begin{aligned}\frac{2}{3+2\sqrt{2}} &= \frac{2}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} \\ &= \frac{2(3-2\sqrt{2})}{(3)^2-(2\sqrt{2})^2} \\ &= \frac{6-4\sqrt{2}}{9-8} \\ &= \frac{6-4\sqrt{2}}{1} \\ &= 6-4\sqrt{2}\end{aligned}$$

b. $$\begin{aligned}\frac{5}{9-3\sqrt{5}} &= \frac{5}{9-3\sqrt{5}} \times \frac{9+3\sqrt{5}}{9+3\sqrt{5}} \\ &= \frac{5(9+3\sqrt{5})}{9^2-(3\sqrt{5})^2} \\ &= \frac{45+15\sqrt{5}}{81-45} \\ &= \frac{45+15\sqrt{5}}{36} \\ &= \frac{45}{36}+\frac{15}{36}\sqrt{5} \\ &= \frac{5}{4}+\frac{5}{12}\sqrt{5}\end{aligned}$$

4. Express $\frac{\sqrt{3}+\sqrt{7}}{\sqrt{21}}$ in the form $a\sqrt{3}+b\sqrt{7}$, where a and b are rational numbers.

Solution:

$\frac{\sqrt{3}+\sqrt{7}}{\sqrt{21}} = \frac{\sqrt{3}+\sqrt{7}}{\sqrt{21}} \times \frac{\sqrt{21}}{\sqrt{21}}$ Rationalise the denominator

$= \frac{\sqrt{21}(\sqrt{3}+\sqrt{7})}{\sqrt{21}\times\sqrt{21}}$

$= \frac{\sqrt{63}+\sqrt{147}}{21}$ Multiply the numerator

$= \frac{\sqrt{9\times7}+\sqrt{49\times3}}{21}$ Simplify surds

$= \frac{3\sqrt{7}+7\sqrt{3}}{21}$

$= \frac{7\sqrt{3}+3\sqrt{7}}{21}$ Change the order of the numerator

$= \frac{7\sqrt{3}}{21} + \frac{3\sqrt{7}}{21}$ Simplify

$= \frac{1}{3}\sqrt{3} + \frac{1}{7}\sqrt{7}$ where $a = \frac{1}{3}$ and $b = \frac{1}{7}$

5. Simplify: $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{15}}$

   Solution:

$$\frac{\sqrt{3}+\sqrt{5}}{\sqrt{15}} = \frac{\sqrt{3}+\sqrt{5}}{\sqrt{15}} \times \frac{\sqrt{15}}{\sqrt{15}}$$
$$= \frac{\sqrt{15}(\sqrt{3}+\sqrt{5})}{\sqrt{15}\times\sqrt{15}}$$
$$= \frac{\sqrt{45}+\sqrt{75}}{15}$$
$$= \frac{\sqrt{9\times5}+\sqrt{25\times3}}{15}$$
$$= \frac{3\sqrt{5}+5\sqrt{3}}{15}$$
$$= \frac{3\sqrt{5}}{15} + \frac{5\sqrt{3}}{15}$$
$$= \frac{\sqrt{5}}{5} + \frac{\sqrt{3}}{3}$$
$$= \frac{1}{5}\sqrt{5} + \frac{1}{3}\sqrt{3}$$

6. Simplify $\frac{1+\sqrt{2}}{\sqrt{2}}$ to the form $m + n\sqrt{2}$.

   Solution:

$$\frac{1+\sqrt{2}}{\sqrt{2}} = \frac{1+\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$= \frac{\sqrt{2}(1+\sqrt{2})}{\sqrt{2}\times\sqrt{2}}$$
$$= \frac{\sqrt{2}+2}{2}$$
$$= \frac{\sqrt{2}}{2} + \frac{2}{2}$$

$= 1 + \frac{1}{2}\sqrt{2}$ where $m = 1$ and $n = \frac{1}{2}$

Practice

1. Simplify:

   a. $\frac{3}{4-\sqrt{3}}$ b. $\frac{1}{3+\sqrt{2}}$ c. $\frac{5}{5+\sqrt{2}}$ d. $\frac{6}{7-2\sqrt{2}}$ e. $\frac{7}{8-2\sqrt{3}}$

2. Simplify the following:

   a. $\frac{9-\sqrt{108}}{\sqrt{72}}$ b. $\frac{\sqrt{5}+\sqrt{75}}{\sqrt{27}}$ c. $\frac{8+\sqrt{12}}{2\sqrt{5}}$

3. Simplify: $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{24}}$

Lesson Title: Expansion and simplification of surds	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L095	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to expand and simplify expressions involving surds.

Overview

In expanding a bracket that involve surds you use the distributive law. For example, consider $\sqrt{5}\left(\sqrt{45}-\frac{8}{\sqrt{80}}\right)$. The term in front of the bracket ($\sqrt{5}$) will be multiplied by each term inside the brackets.

You will also see problems that involve the multiplication of 2 binomials, such as $(1+\sqrt{5})(1-\sqrt{5})$. Distribute as usual. Multiply each term in the first bracket by the expression in the second bracket.

Remember to follow the rules for multiplying surds. Rational numbers can be multiplied by each other, and surds can be multiplied by each other. However, numbers and surds behave like numbers and variables in algebra and cannot be combined (For example, $2\times\sqrt{5}=2\sqrt{5}$, and this cannot be simplified further).

Solved Examples

1. Expand and simplify $(1+\sqrt{5})(1-\sqrt{5})$
   Solution:

$$\begin{aligned}
(1+\sqrt{5})(1-\sqrt{5}) &= 1(1-\sqrt{5})+\sqrt{5}(1-\sqrt{5}) && \text{Expand (distribute)}\\
&= 1-\sqrt{5}+\sqrt{5}-\left(\sqrt{5}\right)^2 && \text{Simplify}\\
&= 1-\sqrt{5}+\sqrt{5}-5 && \text{Note that } \left(\sqrt{5}\right)^2=\sqrt{5}\times\sqrt{5}=5\\
&= 1-5-\sqrt{5}+\sqrt{5} && \text{Collect like terms}\\
&= -4
\end{aligned}$$

2. Expand and simplify $\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$
   Solution:

$$\begin{aligned}
\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right) &= \sqrt{5}\left(\sqrt{5}+\sqrt{2}\right)+\sqrt{2}\left(\sqrt{5}+\sqrt{2}\right)\\
&= \left(\sqrt{5}\right)^2+\sqrt{5\times 2}+\sqrt{2\times 5}+\left(\sqrt{2}\right)^2\\
&= 5+\sqrt{10}+\sqrt{10}+2\\
&= 5+2+\sqrt{10}+\sqrt{10}\\
&= 7+2\sqrt{10}
\end{aligned}$$

3. Expand and simplify $\sqrt{5}\left(\sqrt{45}-\frac{8}{\sqrt{80}}\right)$

Solution:

$\sqrt{5}\left(\sqrt{45}-\frac{8}{\sqrt{80}}\right)$	$=$	$\sqrt{5\times45}-\sqrt{5}\times\frac{8}{\sqrt{80}}$	Multiply $\sqrt{5}$ by each term in the bracket
	$=$	$\sqrt{225}-\frac{8\sqrt{5}}{\sqrt{80}}$	
	$=$	$15-\frac{8\sqrt{5}}{\sqrt{16\times5}}$	
	$=$	$15-\frac{8\sqrt{5}}{4\sqrt{5}}$	
	$=$	$15-\frac{8}{4}$	Cancel $\sqrt{5}$
	$=$	$15-2$	
	$=$	13	

4. Simplify: $\sqrt{2}\left(\sqrt{20}+\frac{4}{\sqrt{40}}\right)$

   Solution:

   Note that it is easier to simplify $\sqrt{20}$ and $\sqrt{40}$ first. It is possible to solve the problem by multiplying $\sqrt{2}$ first, but this will result in larger numbers within surds.

$\sqrt{2}\left(\sqrt{20}+\frac{4}{\sqrt{40}}\right)$	$=\sqrt{2}\left(\sqrt{4\times5}+\frac{4}{\sqrt{4\times10}}\right)$	Simplify $\sqrt{20}$ and $\sqrt{40}$
	$=\sqrt{2}\left(2\sqrt{5}+\frac{4}{2\sqrt{10}}\right)$	
	$=2\sqrt{2\times5}+\frac{4\sqrt{2}}{2\sqrt{10}}$	Multiply each term by $\sqrt{2}$
	$=2\sqrt{10}+\frac{2\sqrt{2}}{\sqrt{10}}$	Simplify
	$=2\sqrt{10}+\frac{2\sqrt{2}}{\sqrt{10}}\times\frac{\sqrt{10}}{\sqrt{10}}$	Rationalise the denominator
	$=2\sqrt{10}+\frac{2\sqrt{20}}{10}$	
	$=2\sqrt{10}+\frac{\sqrt{4\times5}}{5}$	Simplify
	$=2\sqrt{10}+\frac{2\sqrt{5}}{5}$	
	$=2\sqrt{10}+\frac{2}{5}\sqrt{5}$	

Practice

1. Simplify: $\sqrt{2}(\sqrt{10}+\sqrt{2})$
2. Expand and simplify: $\sqrt{18}(2\sqrt{5}-\sqrt{3})$
3. Expand and simplify:
   a. $(1+\sqrt{2})(1-\sqrt{2})$
   b. $(2-3\sqrt{2})(3\sqrt{2}+1)$
4. Expand and simplify: $\left(1+\sqrt{2}\right)^2$
5. Simplify: $\sqrt{5}\left(\sqrt{50}+\frac{5}{\sqrt{50}}\right)$

Lesson Title: Practice of surds	**Theme:** Numbers and Numeration
Practice Activity: PHM1-L096	**Class:** SSS 1

Learning Outcome
By the end of the lesson, you will be able to apply various operations to simplify expressions involving surds.

Overview

This lesson is on evaluating various expressions involving surds. The rules from previous lessons will be used to solve problems, including those for operating on and rationalising surds.

Solved Examples

1. Evaluate $\frac{\sqrt{75}\times\sqrt{50}}{\sqrt{80}\times\sqrt{180}}$

   Solution:

   $\frac{\sqrt{75}\times\sqrt{50}}{\sqrt{80}\times\sqrt{180}} = \frac{\sqrt{25\times3}\times\sqrt{25\times2}}{\sqrt{16\times5}\times\sqrt{36\times5}}$ Reduce each surd to basic form

   $= \frac{5\sqrt{3}\times5\sqrt{2}}{4\sqrt{5}\times6\sqrt{5}}$

   $= \frac{(5\times5)\sqrt{3\times2}}{(4\times6)\sqrt{5}\times\sqrt{5}}$ Multiply

   $= \frac{25\sqrt{6}}{24\times5} = \frac{5}{24}\sqrt{6}$ Simplify

2. If $x\sqrt{45} - \sqrt{125} + 2\sqrt{5} = 0$, find x

   Solution:

   $x\sqrt{45} - \sqrt{125} + 2\sqrt{5} = 0$

   $x\sqrt{9\times5} - \sqrt{25\times5} + 2\sqrt{5} = 0$ Reduce all surds to basic form

   $3x\sqrt{5} - 5\sqrt{5} + 2\sqrt{5} = 0$

   $3x\sqrt{5} - 3\sqrt{5} = 0$ Combine like terms

   $3x\sqrt{5} = 3\sqrt{5}$ Transpose $-3\sqrt{5}$

   $x = \frac{3\sqrt{5}}{3\sqrt{5}}$ Divide throughout by $3\sqrt{5}$

   $x = 1$

3. Given $\sqrt{2} = 1.414$, evaluate $\sqrt{72} + \frac{2}{\sqrt{8}}$.

   Solution:

   $\sqrt{72} + \frac{2}{\sqrt{8}} = \sqrt{36\times2} + \frac{2}{\sqrt{4\times2}}$ Reduce the surds to basic form

   $= 6\sqrt{2} + \frac{2}{2\sqrt{2}}$

   $= \frac{6\sqrt{2}}{1} + \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{2}(6\sqrt{2})+1}{\sqrt{2}}$ Add

$= \frac{6\times2+1}{\sqrt{2}}$

$= \frac{12+1}{\sqrt{2}}$

$= \frac{13}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ Rationalise the denominator

$= \frac{13\sqrt{2}}{2}$ Substitute $\sqrt{2} = 1.414$

$= \frac{13\times1.414}{2}$

$= 9.191$

4. Simplify $\frac{1+\sqrt{2}}{1-\sqrt{2}}$

   Solution:

   $\frac{1+\sqrt{2}}{1-\sqrt{2}} = \frac{1+\sqrt{2}}{(1-\sqrt{2})} \times \frac{1+\sqrt{2}}{(1+\sqrt{2})}$ Rationalise the denominator

   $= \frac{(1+\sqrt{2})(1+\sqrt{2})}{(1-\sqrt{2})(1-\sqrt{2})}$

   $= \frac{1(1+\sqrt{2})+\sqrt{2}(1+\sqrt{2})}{1^2-(\sqrt{2})^2}$ Expand and simplify the numerator

   $= \frac{1+\sqrt{2}+\sqrt{2}+(\sqrt{2})^2}{1-2}$ Simplify

   $= \frac{1+2\sqrt{2}+2}{-1}$

   $= \frac{3+2\sqrt{2}}{-1}$

   $= -3 - 2\sqrt{2}$

5. Simplify $\frac{2+\sqrt{3}}{2-\sqrt{3}} - \frac{2-\sqrt{3}}{2+\sqrt{3}}$ and write your answer in the form $a + b\sqrt{c}$. Identify a, b and c in your result.

   Solution:

   $\frac{2+\sqrt{3}}{2-\sqrt{3}} - \frac{2-\sqrt{3}}{2+\sqrt{3}} = \frac{(2+\sqrt{3})(2+\sqrt{3})-(2-\sqrt{3})(2-\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$ Subtract the fractions

   $= \frac{[4+4\sqrt{3}+(\sqrt{3})^2]-[4-4\sqrt{3}+(\sqrt{3})^2]}{(2)^2-(\sqrt{3})^2}$ Expand each part of the numerator

   $= \frac{(4+4\sqrt{3}+3)-(4-4\sqrt{3}+3)}{4-3}$ Simplify

   $= \frac{7+4\sqrt{3}-7+4\sqrt{3}}{1}$

   $= 8\sqrt{3}$

   Writing result in the form $a + b\sqrt{c}$ we have result as $0 + 8\sqrt{3}$, $a = 0, b = 8$ and $c = 3$

6. Given $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$, evaluate:
   a. $\sqrt{125}(6 - \sqrt{5})$

b. $(\sqrt{8}+\sqrt{12})-(\sqrt{18}-\sqrt{12})$

Solution:

a. $\sqrt{125}(6-\sqrt{5})$
$= \sqrt{25\times 5}(6-\sqrt{5})$
$= \sqrt{25}\times\sqrt{5}(6-\sqrt{5})$
$= 5\sqrt{5}(6-\sqrt{5})$
$= 5\times 6\sqrt{5} - 5\times\sqrt{5}\times\sqrt{5}$
$= 30\sqrt{5} - 5\times 5$
$= 3\sqrt{5} - 25$
$= 30(2.236) - 25 = 67.08 - 25$
$= 42.08$

b. $(\sqrt{8}+\sqrt{12})-(\sqrt{18}-\sqrt{12})$
$= (\sqrt{4\times 2}+\sqrt{4\times 3})-(\sqrt{9\times 2}-\sqrt{4\times 3})$
$= (2\sqrt{2}+2\sqrt{3})-(3\sqrt{2}-2\sqrt{3})$
$= 2\sqrt{2}+2\sqrt{3}-3\sqrt{2}+2\sqrt{3}$
$= 2\sqrt{2}-3\sqrt{2}+2\sqrt{3}+2\sqrt{3}$
$= -\sqrt{2}+4\sqrt{3}$
$= -1.414+4(1.732)$
$= 5.514$

7. Evaluate: $\sqrt{0.48}$

Solution:

Note that when a decimal is in a surd, it can be evaluated by first converting it to a fraction.

$\sqrt{0.48}$	$=$	$\sqrt{\frac{48}{100}}$	Convert to a fraction
	$=$	$\frac{\sqrt{48}}{\sqrt{100}}$	Simplify
	$=$	$\frac{\sqrt{16\times 3}}{10}$	
	$=$	$\frac{\sqrt{16}\times\sqrt{3}}{10}$	
	$=$	$\frac{4\times\sqrt{3}}{10}$	
	$=$	$\frac{2}{3}\sqrt{3}$	

Practice

1. Evaluate $\frac{\sqrt{72}\times\sqrt{48}}{\sqrt{27}\times\sqrt{128}}$
2. If $a\sqrt{40}+\sqrt{90}-\sqrt{10}=\sqrt{1000}$, find the value of a
3. Given $\sqrt{5}=2.236$, Evalute $\sqrt{125}-\frac{10}{\sqrt{500}}$
4. Simplify $\frac{3+\sqrt{5}}{3-\sqrt{5}}$
5. Simplify $\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}$
6. Evaluate $\sqrt{0.08}$

Answer Key – Term 2

Lesson Title:	Powers and roots of logarithms – Numbers greater than 1
Practice Activity:	PHM1-L049

1. 967.4
2. 132.0
3. 15
4. 7.907
5. 53.07
6. 1.841

Lesson Title:	Logarithm of Numbers less than 1
Practice Activity:	PHM1-L050

1. a. 4.525×10^{-1}, $\bar{1}$; b. 6.07×10^{-2}, $\bar{2}$; c. 4.98×10^{-3}, $\bar{3}$; d. 9.89×10^{-4}, $\bar{4}$
2. a. $\bar{1}$ b. $\bar{3}$ c. $\bar{2}$ d. $\bar{5}$
3. a. $\bar{1}.0899$ b. $\bar{2}.9943$ c. $\bar{3}.8156$ d. $\bar{4}.9538$

Lesson Title:	Antilogarithm-Numbers less than 1
Practice Activity:	PHM1-L051

1. 0.02026
2. 0.0001023
3. 0.1589
4. 0.0002597
5. 0.000001288
6. 0.00009997
7. 0.04269
8. 0.002160

Lesson Title:	Multiplication and division of numbers less than 1-using logarithmic tables
Practice Activity:	PHM1-L052

1. 12.5
2. 0.0004950
3. 0.1129
4. 0.1021
5. 9.063
6. 3.109

Lesson Title: Powers and roots of logarithms – numbers less than 1	
Practice Activity: PHM1-L053	

1. a. 0.002107 b. 1.953×10^{-9}
2. a. 0.8548 b. 0.1044
3. a. 0.2832 b. 0.0366
4. a. 0.2381 b. 0.1870

Lesson Title: Law of logarithms – Part 1	
Practice Activity: PHM1-L054	

1. a. $\log_{10} 2 + \log_{10} 7$, b. $\log_{10} 3 + \log_{10} 5$, c. $\log 7 + \log 7$, d. $\log 5 + \log 5$
2. a. $\log_2 48$, b. $\log_5 24$, c. $\log_4 128$, d. $\log_3 36$
3. a. 1.4771, b. 1.7481, c. 1.8451

Lesson Title: Laws of logarithms – Part 2	
Practice Activity: PHM1-L055	

1. a. $\log_7 17 - \log_7 3$; b. $\log_7 39 - \log_7 5$
2. a. 1; b. $\log_3 \frac{16}{9}$
3. a. 1.18; b. -0.318; c. -0.636
4. a. 1.067; b. 0.7872

Lesson Title: Law of logarithms – Part 3	
Practice Activity: PHM1-L056	

1. a. $8 \log_3 x$; b. 3; c. 17
2. a. 1.293; b. -0.138; c. 1.795
3. a. 2; b. $\frac{2}{3}$; c. 4
4. a. 7; b. 7

Lesson Title: Define and describe sets and elements of set	
Practice Activity: PHM1-L057	

1. a. Set of capital cities b. Set of living things c. Set of even numbers between 1 and 20,
2. a. {January, June, July} b. {1, 3, 5, 7, 9, 11, 13, 15, 17, 19} c. {2, 4, 6, 8},
3. a. Set of capital cities in West Africa b. Set of Farm animals c. Set of colours in the rainbow
4. a. $6 \notin \{4, 9, 16, 25.....81\}$ b. $\frac{4}{5} \notin$ {Whole numbers} c. $\sqrt{2} \in$ {irrational numbers}
5. a. $n(D) = 6$ b. $n(X) = 6$ c. $n(Y) = 1$

Lesson Title: Set Notation	
Practice Activity: PHM1-L058	

1. A={1, 2, 3, 4, 6, 8, 12, 24}
2. B={10, 11, 12, 13, 14, 15}
3. D={ 3, 5, 7, 11, 13, 17, 19}
4. B={January, March, May, July, August, October, December}
5. F={x: x is a multiple of 10}
6. G={ x: x is an odd number between 0 and 10}

Lesson Title: Finite and infinite set	
Practice Activity: PHM1-L059	

1. finite; {1, 2, 3, 6, 9, 18}
2. infinite; {0, 1, 2, 3, 4, …}
3. finite; {January, February, …, December}
4. finite; {8, 16, 24, …, 64}
5. infinite; {7, 14, 21, 28, …}
6. finite; {12, 14, 16, …, 38}
7. infinite; {51, 53, 59, …}

Lesson Title: Null/empty, unit and universal set	
Practice Activity: PHM1-L060	

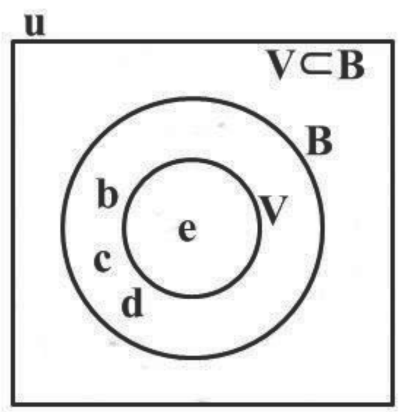
1. (a) Empty set (b) Empty set (c) not an empty set (d) Empty set
2. (a) D = {May} (b) E = {7} (c) W = {Sunday}
3. U = {x: x is an integer}
4. a. C = { Square, Rectangle, trapezium, Rhombus, kite, trapezoid}
   b. E = {Square, Rectangle, Rhombus, Parallelogram}

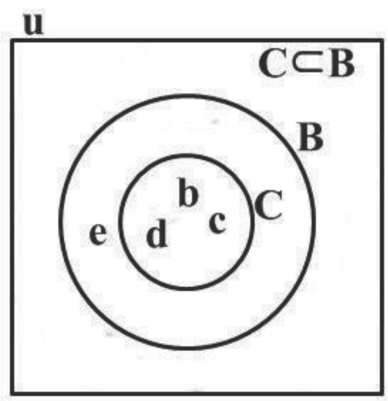
Lesson Title: Equivalent and equal sets
Practice Activity: PHM1-L061

1. a. K and L are equivalent b. A and B are not equivalent c. C and T are not equivalent d. M and N are equivalent
2. a. V and W are not equal b. C and D are not equal c. A and B are equal d. X and Y are not equal e. R and S are not equal f. I and J are equal

Lesson Title: Subsets
Practice Activity: PHM1-L062

1. $S_0 = \{a, b, c, d\}$, $S_1 = \{a, b, c\}$, $S_2 = \{b, c, d\}$, $S_3 = \{c, d, a\}$, $S_4 = \{d, a, b\}$, $S_5 = \{a, b\}$, $S_6 = \{a, c\}$ $S_7 = \{a, d\}$ $S_8 = \{b, c\}$ $S_9 = \{b, d\}$ $S_{10} = \{c, d\}$ $S_{11} = \{a\}$ $S_{12} = \{b\}$ $S_{13} = \{c\}$ $S_{14} = \{d\}$, $S_{15} = \Phi = \{\}$
2. a. W is a subset of X; b. V is a subset of X; c. T is a subset of X.
3. Venn diagrams:

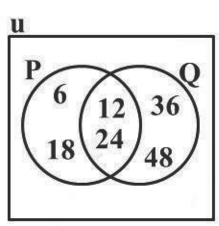




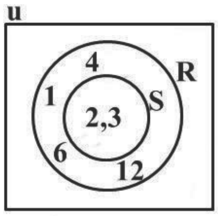
4. $M_1 = \{2, 4, 6\}$, $M_2 = \{2, 4\}$, $M_3 = \{2, 6\}$, $M_4 = \{4, 6\}$, $M_5 = \{2\}$, $M_6 = \{4\}$, $M_7 = \{6\}$, $M_8 = \{\ \}$

Lesson Title: Intersection of 2 sets
Practice Activity: PHM1-L063

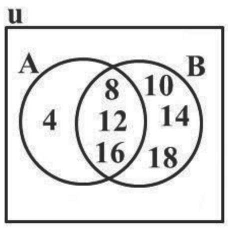
1. a. $P \cap Q$ {12, 24}; b. Venn diagram:



2. a. R = {1, 2, 3, 4, 6, 12}, S = {2, 3}; b. R∩S = {2, 3}; c. Venn diagram:



3. a. A = {4, 8, 12, 16} B = {8, 10, 12, 14, 16, 18}; b. A ∩ B = {8, 12, 16}; c. Venn diagram:

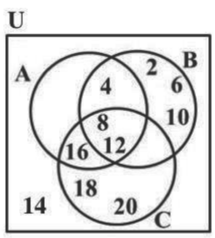


4. a. A = {1, 2, 3, 4, 9}, B = {1, 4, 5, 7, 8}; b. A ∩ B = {1, 4}

Lesson Title:	Intersection of three sets
Practice Activity:	PHM1-L064

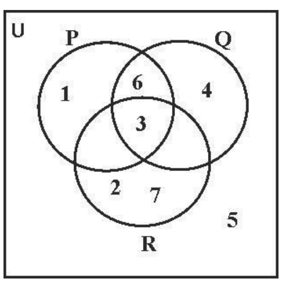
1. a. i{4, 8, 12}; ii. {8, 12}; iii. {8, 12, 16}; iv. {8, 12}

   b. Venn diagram:



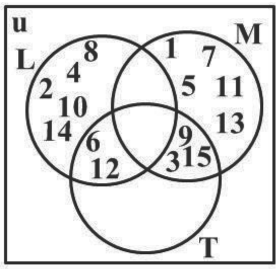
2. a. P∩Q = {3, 6}, P∩R = {3}, Q∩R = {3}, P∩Q∩R = {3},

b. Venn diagram:



3. a. M∩L = { }, M∩T = {3, 9, 15}, L∩T = {6, 12}, M∩L∩T = { },

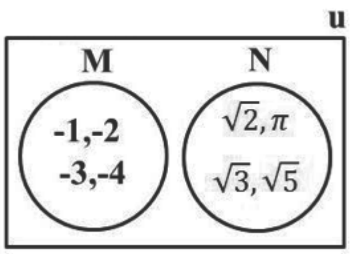
b. Venn diagram:



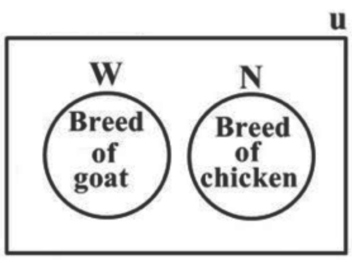
4. X∩Y = {1, 3, 5}, X∩Z = {1, 2, 5}, Y∩Z = {1, 5, 6}, X∩Y∩Z = {1, 5}

Lesson Title:	Disjoint sets
Practice Activity:	PHM1-L065

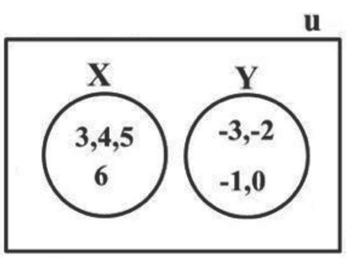
1. $M \cap N = \emptyset$



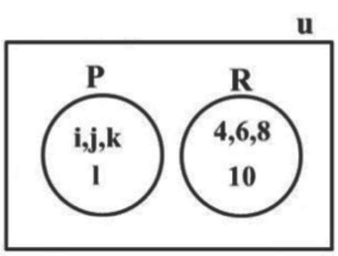
2. $W \cap N = \emptyset$



3. a. X={3,4,5,6}, Y={-3,-2,-1,0}; b. $X \cap Y = \emptyset$; c. Diagram:

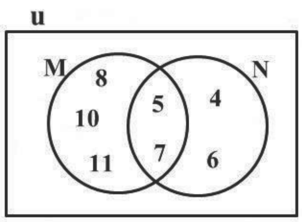


4. $P \cap R = \emptyset$

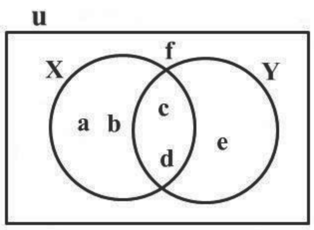


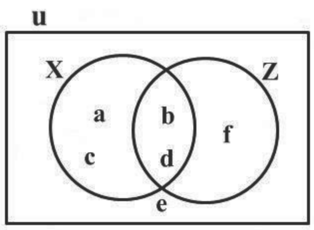
Lesson Title: Union of two sets	
Practice Activity: PHM1-L066	

1. M ∪ N = {4, 5, 6, 7, 8, 10, 11}



2. a. i. {a, b, c, d, e,}; ii. {b, c, d, e, f}; iii. {a, b, c, d, f}; b. Venn diagrams:





3. a. A∪B {Brima, Sama, Yomba, sia}; b. B∪C = {Sia, Sama}; c. A∪C = {Brima, Sama, Sia, Yomba}
4. a. {2, 3, 4, 6, 7, 8, 10}; b. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} = U; c. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} = U

Lesson Title: Complement of a set	
Practice Activity: PHM1-L067	

1. a. U = {4, 5, 6, 7, 8, 9, 10, 11, 12} P = {6, 7, 8, 9}

b. P' = {4, 5, 10, 11, 12}

2. a. A' = {10, 40, 50}; b. B' = {10, 40, 60}; c. A'∩B' = {10, 40}

3. a. U= {5, 10, 15, 20, 25, 30, 35, 40} P = {5, 25, 30, 40} Q = {5, 20, 35, 40}

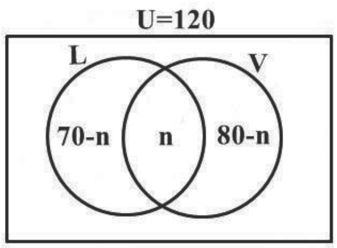
b. P' = {10, 15, 20, 35}; c. Q' = {10, 15, 25, 30}; d. P'∩Q' = {10, 15}

4.a. U = {1, 2, 3, 4, a, b, c, d, e, f, g} M = {a, 2, 3, d, e} N ={a, 1, 2, 3, b, f}

b. M' = {1, b, c, f, 4, g}; c. N' = {3, 4, c, d, e, g}; d. M'∩N' = {c, g, 4}

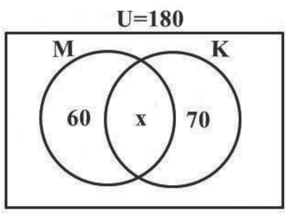
Lesson Title: Real life problems involving 2 sets	
Practice Activity: PHM1-L068	

1. a. Venn diagram:



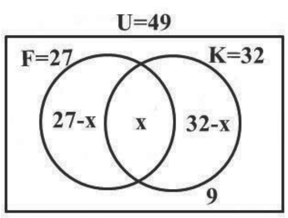
b. 30 pupils c. 40 played Lawn tennis only

2. a. Venn diagram:



b. i. 50 ii. 120 iii. 110

3. a. Venn diagram:



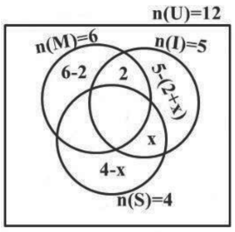
b. i. 19 ii. 8 iii. 13

4. a. 30 b. i. 60 ii. 10

Lesson Title: Real life problems involving 3 sets – Part 1	
Practice Activity: PHM1-L069	

1. a. 33; b. 12

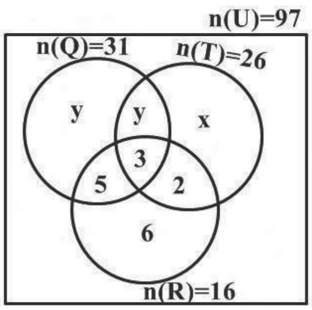
2. a. Venn diagram:



b. i. 1 ii. 3

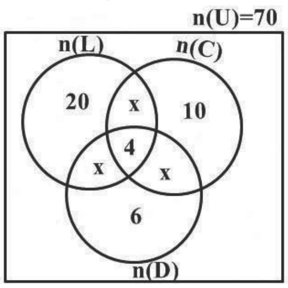
Lesson Title:	Real life problems involving 3 sets – Part 2
Practice Activity:	PHM1-L070

1. a. Venn diagram:



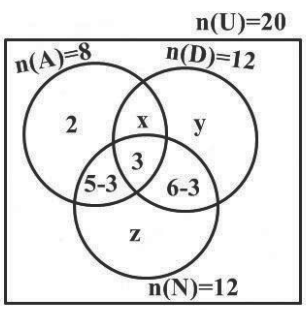
b. i. 8; ii. 24; iii. 20

2. a. Venn diagram:



b. 44

3. a. Venn diagram:



b. i. 4; ii. 6

Lesson Title: Use of variables	
Practice Activity: PHM1-L071	

1. a. r b. m, n c. x u, t and a
2. a. coefficients are -5 and 6; constant is -10

   b. coefficients are $\frac{4}{5}$ and -1; constant is 7

   c. coefficients are 1 and -1; constant is 10

3. a. $60x$; b. 100y, c. $y + 2x$ or $2x + y$,

4. a. $x = 13$; b. $a = 4$; c. $b = 3$, d. $z = 6$

Lesson Title: Simplification – grouping terms	
Practice Activity: PHM1-L072	

1. $2x + y$
2. $-2a + 7b$
3. $m + 14n$
4. $5xy - xp$
5. $12mn - m$
6. $2pq + 8qr$
7. $8p^2q - 3pq^2$
8. $9xyz - xz$
9. $2ab + 4a + 4$

Lesson Title: Simplification – removing brackets	
Practice Activity: PHM1-L073	

1. $5x - 20$
2. $-21y + 28$
3. $7m - 2n$
4. $2u - 6uv$
5. $-3x - 3y$
6. $-26m + 14n$

Lesson Title: Simplification – expanding brackets	
Practice Activity: PHM1-L074	

1. $7x^2 + 30x + 8$
2. $p^2 - 11p + 30$

3. $m^2 + m - 6$
4. $x^2 - 8x + 16$
5. $n^2 + 6n + 9$
6. $8mn - 2n + 19m - 2$
7. $3x^3 - 12x^2 + 12x$
8. $4x^2 + 48x + 144$

Lesson Title:	Factoring – common factors
Practice Activity:	PHM1-L075

1. $9(ab - 3)$
2. $x(x - 4)$
3. $a(a^2 + b)$
4. $4x^2(4x^2 - 2x + 1)$
5. $9a^2(9a + 1)$
6. $12x^2(x^2 - 3x + 4)$
7. $3s^2t^2(6s^3t^4 - 2s^2t + 1)$
8. $5uv^2(v - 5 + 3u)$

Lesson Title:	Factoring – grouping
Practice Activity:	PHM1-L076

1. $(x - 1)(a + 1)$,
2. $2(3g - h)(2e - f)$,
3. $(d - e)(c - d)$,
4. $(1 + 5a)(p + q)$,
5. $(x + 2)(x - 5)$,
6. $(2a - 1)(3a + 2)$,
7. $(x + 2)(x + 3)$,
8. $(3 - 2d)(x + y)$

Lesson Title:	Substitution of values
Practice Activity:	PHM1-L077

1. -12,
2. a. $\frac{4}{25}$, b. 231, c. $12\frac{1}{4}$,
3. $3\frac{9}{19}$,
4. a. 15, b. 35

Lesson Title: Addition of algebraic fractions
Practice Activity: PHM1-L078

1. a. a b. $\frac{7m}{9}$, c. $\frac{8x}{7}$, d. $\frac{x+2}{2}$
2. a. $\frac{8a}{15}$, b. $\frac{5x}{4}$, c. $\frac{11n}{10}$, d. $\frac{3x}{8}$
3. a. $\frac{13a-1}{12}$, b. $\frac{29n+8}{28}$, c. $\frac{3x-1}{6}$, d. $\frac{3x+2}{10}$

Lesson Title: Subtractions of algebraic fractions
Practice Activity: PHM1-L079

1. a. $\frac{x}{4}$, b. $\frac{2y-3}{3}$, c. $\frac{2x+1}{5}$, d. $-a$
2. a. $\frac{a}{12}$, b. $-\frac{x}{5}$, c. $\frac{11x}{6}$, d. $-\frac{x}{20}$
3. a. $\frac{13y-6}{6}$, b. $\frac{a-6}{6}$, c. $\frac{5y+11}{12}$, d. $\frac{8y+5}{12}$

Lesson Title: Linear equations
Practice Activity: PHM1-L080

1. $m=-3$
2. $y=-\frac{1}{2}$
3. $n=20$
4. $y=1\frac{2}{3}$
5. $x=1$
6. $y=1\frac{1}{4}$
7. $x=-1$
8. $m=0$
9. $y=-5\frac{1}{2}$

Lesson Title: Linear equations with brackets
Practice Activity: PHM1-L081

1. $a=5$
2. $m=\frac{1}{2}$
3. $x=-10$
4. $y=-1$

5. $a = -\frac{1}{4}$
6. $x = 2$
7. $p = 2\frac{1}{2}$

Lesson Title:	Linear equations with fractions
Practice Activity:	PHM1-L082

1. $x = 15$
2. $a = 20$
3. $y = 4\frac{4}{5}$
4. $x = 1\frac{1}{9}$
5. $x = 18$
6. $k = 33$
7. $p = 4\frac{1}{2}$

Lesson Title:	Word problems
Practice Activity:	PHM1-L083

1. a. $\frac{15+x}{4} = 80 - x$; b. 61

2. a. $\frac{x}{20}$; b. 160

3. 6 years, 17 years

4. Le 3,200.00, Le 4,150.00

5. 11 mangoes, 48 mangoes

Lesson Title:	Substitution in formulae
Practice Activity:	PHM1-L084

1. a. 6; b. 2; c. 31; d. 8
2. 5.56 or $5\frac{5}{9}$
3. 15
4. 22cm
5. 36,000
6. 2,355 cm^2

Lesson Title:	Change of subject- Part 1
Practice Activity:	PHM1-L085

1. a. $b = \frac{a}{13}$; b. $b = a + 3$; c. $b = 4a$; d. $b = \frac{a}{2} + 4$
2. $a = \frac{p}{2} - b$,
3. a. $y = \frac{rt-4d}{5c-q}$, b. $y = \frac{r+s}{3p-a}$, c. $y = \frac{-9x}{k-4}$
4. a. $u = \frac{1+2v}{v-1}$, b. $u = \frac{w}{mh-3}$, c. $u = \frac{nv-2v}{2k}$
5. $F = \frac{9}{5}c + 32$

Lesson Title:	Change of Subject – Part 2
Practice Activity:	PHM1-L086

1. $x = 144$
2. $r = \sqrt{p-1}$
3. $x = 9y^2 - 1$; $x = 35$ when $y = -2$.
4. $u = \frac{mv^2}{F}$
5. $t = \pm\sqrt{\frac{2x-2uf}{g}}$
6. $r = \pm\sqrt{\frac{3v}{\pi}}$
7. $u = \frac{c^2b-a}{c^2-1}$

Lesson Title:	Reduction to Base Forms of Surds
Practice Activity:	PHM1-L087

1. a. $4\sqrt{2}$; b. $2\sqrt{10}$; c. $5\sqrt{2}$; d. $3\sqrt{3}$

2. a. $12\sqrt{3}$; b. $6\sqrt{5}$; c. $25\sqrt{2}$; d. $4\sqrt{15}$

3. a. $25\sqrt{10}$; b. $9\sqrt{3}$; c. $30\sqrt{10}$; d. $20\sqrt{2}$

Lesson Title:	Addition and Subtraction of Surds - Part 1
Practice Activity:	PHM1-L088

1. $4\sqrt{2}$
2. $7\sqrt{5}$
3. $2\sqrt{3}$

4. 0
5. $3\sqrt{2}$
6. $\sqrt{3}$
7. $11\sqrt{2}$

Lesson Title:	Addition and Subtraction of Surds – Part 2
Practice Activity:	PHM1-L089

1. $\frac{5}{2}\sqrt{7}$
2. $3\sqrt{2}$
3. $11\sqrt{2}$
4. $8\sqrt{3} + \sqrt{6}$
5. $9\sqrt{5}$
6. $4\sqrt{7}$
7. $n = 1$

Lesson Title:	Properties of Surds
Practice Activity:	PHM1-L090

1. a. $4\sqrt{2}$; b. $4\sqrt{3}$; c. $2\sqrt{6}$

2. a. $1\frac{1}{2}$; b. $\frac{5\sqrt{2}}{3}$; c. $1\frac{1}{4}$

3. a. $-a\sqrt{6}$; b. $\sqrt{2}$; c. $\sqrt{3}$

4. a. $8\sqrt{2}$; b. $8\sqrt{6}$; c. $a^2\sqrt{a}$ (d) x^2

Lesson Title:	Multiplication of Surds – Part 1
Practice Activity:	PHM1-L091

1. a. 105; b. $16\sqrt{10}$
2. a. 32; b. $144\sqrt{6}$
3. a. $32\sqrt{3}$; b. $50\sqrt{15}$
4. a. 18; b. $48\sqrt{3}$

Lesson Title: Multiplication of Surds – Part 2	
Practice Activity: PHM1-L092	

1. $10\sqrt{5}$
2. $3\sqrt{2}-4$
3. $9\sqrt{6}-6$
4. $12\sqrt{5}-30$
5. $24+12\sqrt{2}$
6. $1\frac{1}{5}$
7. $31+12\sqrt{3}$

Lesson Title: Rationalisation of the Denominator of Surds – Part 1	
Practice Activity: PHM1-L093	

1. $\frac{3\sqrt{2}}{4}$
2. $\frac{\sqrt{35}}{7}$
3. $\sqrt{2}$
4. $\frac{\sqrt{6}}{4}$
5. $\frac{\sqrt{6}}{3}$
6. $\frac{\sqrt{10}}{3}$
7. $2\sqrt{6}$
8. $\sqrt{7}$
9. $\sqrt{2}$
10. $\frac{2\sqrt{6}}{3}$

Lesson Title: Rationalisation of the Denominator of Surds – Part 2	
Practice Activity: PHM1-L094	

1. a. $\frac{12+3\sqrt{3}}{13}$; b. $\frac{3-\sqrt{2}}{7}$; c. $\frac{25-5\sqrt{2}}{23}$; d. $\frac{42+12\sqrt{2}}{41}$; e. $\frac{14}{13}-\frac{7}{26}\sqrt{3}$,
2. a. $\frac{3}{4}\sqrt{2}-\frac{1}{2}\sqrt{6}$; b. $\frac{5}{3}+\frac{\sqrt{15}}{9}$; c. $\frac{4}{5}\sqrt{5}+\frac{\sqrt{15}}{5}$,
3. $\frac{1}{4}\sqrt{2}+\frac{1}{6}\sqrt{3}$

Lesson Title: Expansion and Simplification of Surds	
Practice Activity: PHM1-L095	

1. $2 + 2\sqrt{5}$
2. $6\sqrt{10} - 3\sqrt{6}$
3. a. -1; b. $-16 + 3\sqrt{2}$
4. $3 + 2\sqrt{2}$
5. $\frac{11}{2}\sqrt{10}$

Lesson Title: Practice of Surds	
Practice Activity: PHM1-L096	

1. 1
2. 4
3. 10.7328
4. $\frac{7}{2} + \frac{3}{2}\sqrt{5}$
5. 14
6. $\frac{1}{5}\sqrt{2}$ *or* $\frac{\sqrt{2}}{5}$

Appendix I: Logarithm Table

COMMON LOGARITHM TABLE

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Appendix II: Anti-Logarithm Table

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0 0 1	1 1 1	2 2 2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 1	1 1 1	2 2 2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1	1 1 1	2 2 2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0 1 1	1 1 2	2 2 3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1	1 2 2	2 2 3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0 1 1	1 2 2	2 3 3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0 1 1	1 2 2	2 3 3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0 1 1	1 2 2	2 3 3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0 1 1	1 2 2	2 3 3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0 1 1	1 2 2	2 3 3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0 1 1	1 2 2	2 3 3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0 1 1	1 2 2	3 3 3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0 1 1	1 2 2	3 3 3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1696	0 1 1	2 2 2	3 3 3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0 1 1	2 2 2	3 3 3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0 1 1	2 2 2	3 3 4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1	2 2 2	3 3 4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1	2 2 3	3 3 4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1	2 2 3	3 3 4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0 1 1	2 2 3	3 4 4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1	2 2 3	3 4 4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0 1 1	2 2 3	3 4 4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1	2 2 3	3 4 4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1 1 2	2 3 3	4 4 5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1 1 2	2 3 3	4 4 5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1 1 2	2 3 3	4 4 5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2	2 3 3	4 4 5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1 1 2	2 3 3	4 4 5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2	2 3 3	4 5 5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2	2 3 4	4 5 5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
·47	2951	2958	2965	2972	2970	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 4 4	5 6 6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 4 4	5 6 6

FRITHLOGARTAIM ANTI-LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1 1 2	3 4 4	5 6 7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1 2 2	3 4 5	5 6 7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1 2 2	3 4 5	5 6 7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1 2 2	3 4 5	6 6 7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1 2 2	3 4 5	6 6 7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1 2 2	3 4 5	6 7 7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1 2 3	3 4 5	6 7 8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1 2 3	3 4 5	6 7 8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1 2 3	4 4 5	6 7 8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1 2 3	4 5 5	6 7 8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1 2 3	4 5 6	6 7 8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1 2 3	4 5 6	7 8 9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1 2 3	4 5 6	7 8 9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1 2 3	4 5 6	7 8 9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1 2 3	4 5 6	7 8 9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1 2 3	4 5 6	7 8 9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1 2 3	4 5 6	7 9 10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1 2 3	4 5 7	8 9 10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1 2 3	4 6 7	8 9 10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1 2 3	5 6 7	8 9 10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1 2 4	5 6 7	8 9 11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1 2 4	5 6 7	8 10 11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1 2 4	5 6 7	9 10 11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1 3 4	5 6 8	9 10 11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1 3 4	5 6 8	9 10 12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1 3 4	5 7 8	9 10 12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1 3 4	5 7 8	9 11 12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1 3 4	5 7 8	10 11 12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1 3 4	6 7 8	10 11 13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1 3 4	6 7 9	10 11 13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1 3 4	6 7 9	10 12 13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2 3 5	6 8 9	11 12 14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2 3 5	6 8 9	11 12 14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2 3 5	6 8 9	11 13 14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2 3 5	6 8 10	11 13 15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2 3 5	7 8 10	12 13 15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2 3 5	7 8 10	12 13 15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2 3 5	7 9 10	12 14 16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2 4 5	7 9 11	12 14 16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2 4 5	7 9 11	13 14 16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2 4 6	7 9 11	13 15 17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2 4 6	8 9 11	13 15 17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2 4 6	8 10 12	14 15 17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2 4 6	8 10 12	14 16 18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2 4 6	8 10 12	14 16 18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2 4 6	8 10 12	15 17 19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2 4 6	8 11 13	15 17 19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2 4 7	9 11 13	15 17 20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2 4 7	9 11 13	16 18 20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2 5 7	9 11 14	16 18 20

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