

**Free Quality
School
Education**

Ministry of
Basic and Senior
Secondary
Education

Pupils' handbook for

JSS Mathematics

**JSS
3**

**Term
3**

STRICTLY NOT FOR SALE

FOREWORD

The production of Teachers' Guides and Pupils' handbooks in respect of English and Mathematics for Junior Secondary Schools (JSSs) in Sierra Leone is an innovation. This would undoubtedly lead to improvement in the performance of pupils in the Basic Education Certificate Examination in these subjects. As Minister of Basic and Senior Secondary Education, I am pleased with this development in the educational sector.

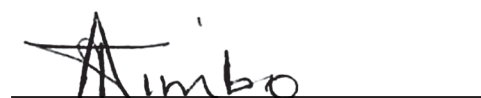
The Teachers' Guides give teachers the support they need to utilize appropriate pedagogical skills to teach; and the Pupils' Handbooks are designed to support self-study by the pupils, and to give them additional opportunities to learn independently.

These Teachers' Guides and Pupils' Handbooks had been written by experienced Sierra Leonean and international educators. They have been reviewed by officials of my Ministry to ensure that they meet specific needs of the Sierra Leonean population.

I call on the teachers and pupils across the country to make the best use of these educational resources.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd. Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank the Department for International Development (DFID) for their continued support. Finally, I also thank the teachers of our country - for their hard work in securing our future.

A handwritten signature in black ink, appearing to read 'Timbo', is written over a horizontal line. The signature is stylized and includes a star-like symbol above the first letter.

Mr. Alpha Osman Timbo

Minister of Basic and Senior Secondary Education

The Ministry of Basic and Senior Secondary Education,
Sierra Leone, policy stipulates that every printed book
should have a lifespan of 3 years.

To achieve this DO NOT WRITE IN THE BOOKS.

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Introduction

to the Pupils' Handbook

These practice activities are aligned to the lesson plans in the Teachers' Guide, and are based on the National Curriculum and the West Africa Examination Council syllabus guidelines. They meet the requirements established by the Ministry of Education, Science and Technology.

-  The practice activities will not take the whole term, so use any extra time to revise material or re-do activities where you made mistakes.
-  Use other textbooks or resources to help you learn better and practise what you have learned in the lessons.
-  Read the questions carefully before answering them. After completing the practice activities, check your answers using the answer key at the end of the book.
-  Make sure you understand the learning outcomes for the practice activities and check to see that you have achieved them. Each lesson plan shows these using the symbol to the right.
-  Organise yourself so that you have enough time to complete all of the practice activities. If there is time, quickly revise what you learned in the lesson before starting the practice activities. If it is taking you too long to complete the activities, you may need more practice on that particular topic.
-  Seek help from your teacher or your peers if you are having trouble completing the practice activities independently.
-  Make sure you write the answers in your exercise book in a clear and systematic way so that your teacher can check your work and you can refer back to it when you prepare for examinations.
-  Congratulate yourself when you get questions right! Do not worry if you do not get the right answer – ask for help and continue practising!



Learning Outcomes

Lesson Title: Review of the Number Line	Theme: Algebra
Practice Activity: PHM-09-106	Class: JSS 3



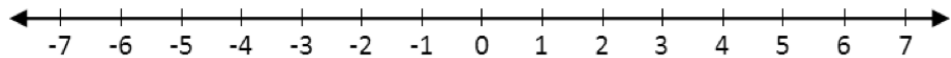
Learning Outcomes

By the end of the lesson, you will be able to:

1. Illustrate positive and negative numbers on the number line.
2. Compare and order numbers on the number line.

Overview

A number line is a way to see the relationship between numbers. It looks like this:

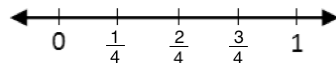


This number line shows negative and positive numbers between -7 and 7. The arrows on the ends of the number line show that it goes forever in both directions. Number lines do not end. Number lines always increase, or go up, from left to right. This means that a number to the right is greater than any number to its left.

Number lines include marks to show where the numbers are on the line. These marks are all the same distance apart.

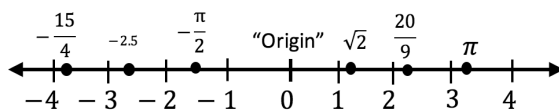
Number lines can show different ranges of numbers by using a different **scale**. For example, we could create a number line that shows certain fractions from 0 to 1 like the one below.

This number line has a scale of $\frac{1}{4}$.



Number lines can show any real number, including integers, fractions or decimals.

An **x** or a dot can be used to show numbers on a number line. For example, the numbers $-\frac{15}{4}$, -2.5 , $-\frac{\pi}{2}$, $\sqrt{2}$, $\frac{20}{9}$, and π can be shown on the number line as follows:



We can estimate the locations of fractions and decimal numbers. For example, π is approximately 3.14, which is a little more than 3. Therefore, we graph it slightly to the right of 3 on the number line.

The number line can be used to compare and order numbers. If a number is farther to the right on the number line, it is a larger number. When ordering numbers, 'ascending' means from smallest to largest and 'descending' means from largest to smallest.

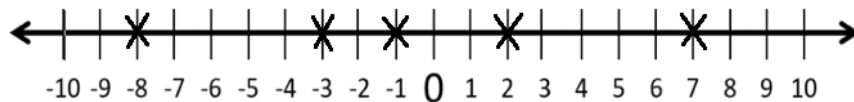
Solved Examples

1. Represent on a number line: $-3, -8, 2, -1, 7$

Solution

First, choose an appropriate size and scale for the number line. The numbers given are positive and negative whole numbers between -10 and 10 . Therefore, a number line showing whole numbers between -10 and 10 is best.

Draw the number line, and draw an x at each number listed:

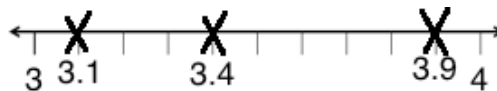


2. Represent the following on a number line: $3.1, 3.4, 3.9$.

Solution

First, choose an appropriate size and scale for the number line. These numbers are between 3 and 4 , and have decimal places with tenths. Therefore, a number line showing tenths from 3 to 4 is best.

Draw the number line, and draw an x at each number listed:



3. Represent the following on the number line:

a. $-\frac{2}{5}$ b. $5\frac{1}{10}$ c. 0.6 d. -8 e. -1.7

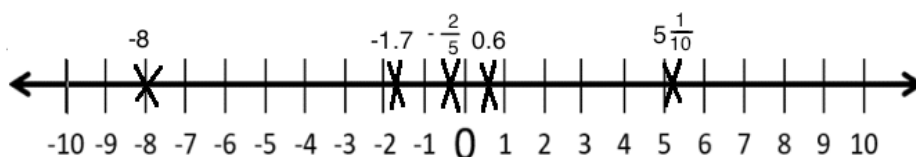
Solutions

We can show all of these on a number line from -10 to 10 . For the decimals and fractions, we will show approximately where they are.

First, convert fractions to decimals so they are easy to plot:

a. $-\frac{2}{5} = -0.4$ b. $5\frac{1}{10} = 5.1$

Draw the number line, and draw an x at each number listed:



4. Circle the smaller number in each set:

- a. $-6.8, -8.6$ b. $3, -8$ c. $-1.5, 1.5$

Solutions

The smaller number is the one to the left on the number line. Look at a number line to find the answers. Remember that negative numbers are always less than positive numbers. The following answers should be circled:

- a. -8.6 b. -8 c. -1.5

5. Write the numbers in descending order:

- a. $-4, 3, -6, 1, -2$
b. $-10, -2, -8, -4$

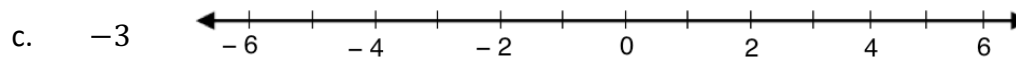
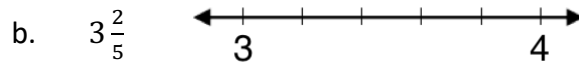
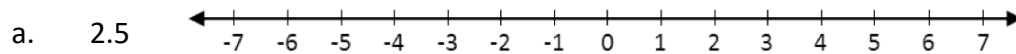
Solutions

Descending order means largest to smallest. The numbers farthest to the right on the number line come first. The answers are:

- a. $3, 1, -2, -4, -6$
b. $-2, -4, -8, -10$

Practice

1. Draw an **x** to show each number on the given number line:



2. Represent the following number on a number line:

- a. -4 , b. 4 , c. 0 , d. 6 , e. -8 , f. -1

3. Represent the following numbers on a number line:

- a. $-\frac{3}{2}$, b. $\frac{5}{2}$, c. $-\frac{1}{2}$, d. $-2\frac{1}{2}$


4. Circle the larger number in each set:

- a. $-4, 4$ b. $-3, -1$ c. $4, -5$

5. Write the numbers in ascending order:

- a. $-1, 1, 2, -2, -3, 3$
b. $\frac{1}{2}, -\frac{1}{2}, 0, 1\frac{1}{2}, -1\frac{1}{2}$

Lesson Title: Introduction to Inequality	Theme: Algebra
Practice Activity: PHM-09-107	Class: JSS 3

	<p>Learning Outcomes</p> <p>By the end of the lesson, you will be able to:</p> <ol style="list-style-type: none"> 1. Identify the 'greater than' and 'less than' symbols and use them to compare positive and negative numbers. 2. Show 'greater than' and 'less than' on the number line.
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Overview

In this lesson, you will use the 'greater than' and 'less than' symbols to compare positive and negative numbers. We will also show 'greater than' and 'less than' on the number line.

These are maths symbols that are used to compare 2 numbers:

- = is equal to
- \neq is not equal to
- < is less than
- > is greater than

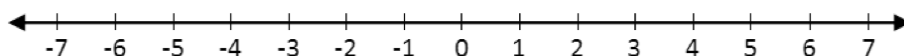
The symbol = shows that 2 expressions or numbers are equal. The other 3 symbols are called inequality symbols.

The symbols < and > are used to compare 2 numbers. Both symbols show which number is larger and which is smaller. The point on the symbol always points to the smaller number. The symbol always opens to the larger number.

When comparing numbers, you may look at a number line to determine which number is greater.

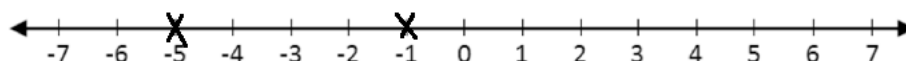
Solved Examples

1. Use one of the symbols <, > or = to make the statement true. Show your answer on the number line below: $-1 \square -5$



Solution

-1 is greater than -5 because it is to the right of -5 on the number line. The answer is: $-1 > -5$. On the number line, this is:



2. Use one of the symbols $<$, $>$ or $=$ to make the statement true:

a. $3 \square 5$

b. $0.7 \square \frac{7}{10}$

c. $-2 \square -6$

Solutions

For each problem, your $>$ or $<$ symbol should open to the larger number. If the numbers are equal, use $=$. The answers are:

a. $3 < 5$

b. $0.7 = \frac{7}{10}$

c. $-2 > -6$

Note that when 2 numbers are negative, the number that looks smaller is actually larger. Answer c. is an example of this: $-2 > -6$.

3. Use one of the symbols $<$, $>$ or $=$ to make the statement true:

a. $-2 \square 4$

b. $1 \square -1$

c. $-16 \square 10$

Solutions

Note that positive numbers are always larger than negative numbers. The answers are:

a. $-2 < 4$

b. $1 > -1$

c. $-16 < 10$

4. Use one of the symbols $<$, $>$ or $=$ to make the statement true:

a. $1 - 3 \square -4 + 2$

b. $2 - 6 \square 1 + 1$

c. $-3 - 3 \square 3 - 3$

d. $2 + 8 \square -3 + 10$

Solutions

Evaluate both sides of each problem using addition and subtraction. Then compare the 2 sides with one of the symbols.

a. $1 - 3 \square -4 + 2$

$-2 \square -2$

$-2 = -2$

b. $2 - 6 \square 1 + 1$

$-4 \square 2$

$-4 < 2$

c. $-3 - 3 \square 3 - 3$

$-6 \square 0$

$-6 < 0$

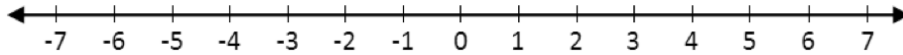
d. $2 + 8 \square -3 + 10$

$10 \square 7$

$10 > 7$

Practice

1. Use one of the symbols $<$, $>$ or $=$ to make the statement true. Show your answer on the number line below: $-6 \square 3$



2. Use one of the symbols $<$, $>$ or $=$ to make the statement true:

a. $9 \square -2$

b. $0.8 \square -\frac{1}{5}$

c. $-1 \square -12$

3. Use one of the symbols $<$, $>$ or $=$ to make the statement true:

a. $-13 \square 2$

b. $8 \square -3$

c. $-4 \square -7$

4. Use one of the symbols $<$, $>$ or $=$ to make the statement true:


a. $2 + 9 \square -10 + 2$

b. $1 - 9 \square 3 - 2$

c. $-3 + 8 \square 10 - 5$

d. $6 + 3 \square -1 + 6$

Lesson Title: Linear Inequality	Theme: Algebra
Practice Activity: PHM-09-108	Class: JSS 3

	<p>Learning Outcomes By the end of the lesson, you will be able to:</p> <ol style="list-style-type: none"> 1. Interpret simple linear inequalities. 2. Represent simple linear inequalities on the number line.
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Overview

Inequalities in one variable are any inequalities that include a variable, inequality symbol (< or >) and one or more numbers.

For example, these are inequalities in one variable:

$$x > 2 \qquad y < -6 \qquad x + 3 > 4 \qquad 4 < b - 9$$

Inequalities in one variable can be shown on a number line using circles and arrows. These are the arrows that show each inequality symbol:



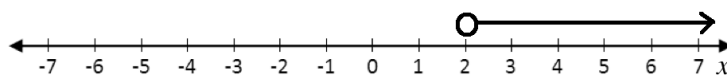
The circle should be on or above the number part of the inequality. The arrow points in the direction of the numbers that make the inequality true.

Solved Examples

1. Show $x > 2$ on a number line.

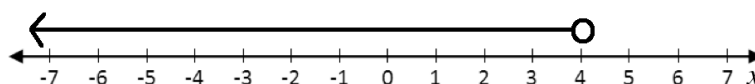
Solution

The circle is drawn above positive 2. The arrow is used to show values of x , and it should point in the direction that makes the statement $x > 2$ true. The statement is true for values of x greater than 2, so it points to the right.

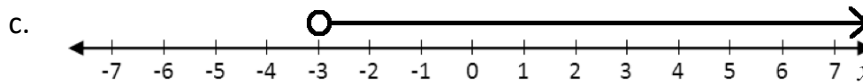
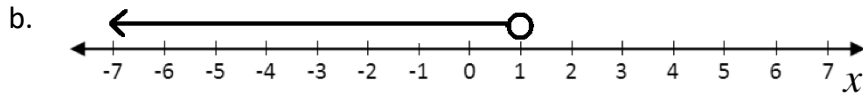
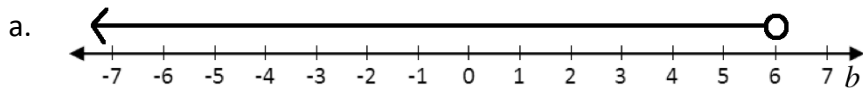


2. Illustrate on a number line: $x < 4$

Solution



3. Write the inequality for each number line:



Solutions

a. The circle is above 6, and the variable next to the number line is b . The arrow points to the left, which tells us to use the $<$ symbol.

Answer: $b < 6$

b. The circle is above 1, and the variable next to the number line is x . The arrow points to the left, which tells us to use the $<$ symbol.

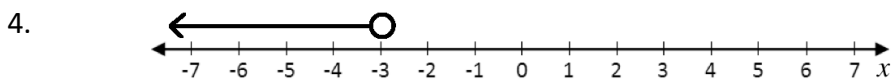
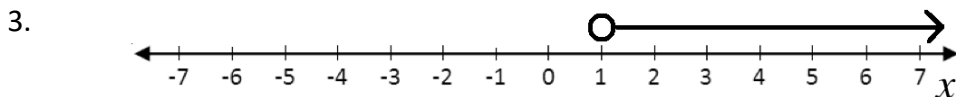
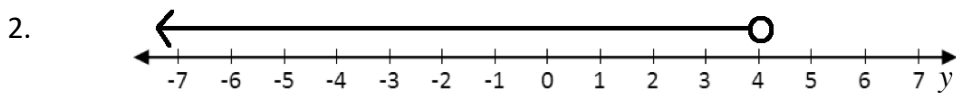
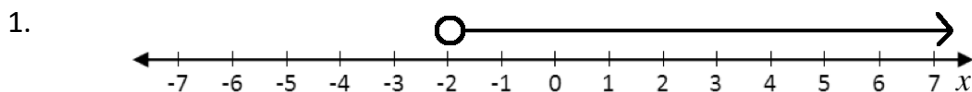
Answer: $x < 1$

c. The circle is above -3 , and the variable next to the number line is x . The arrow points to the right, which tells us to use the $>$ symbol.

Answer: $x > -3$

Practice


Write the inequality for each number line:



Draw each inequality on a number line:

5. $x > 4$
6. $x < -2$
7. $y > 3$
8. $x > -3$

Lesson Title: Solving Linear Inequalities in One Variable I	Theme: Algebra
Practice Activity: PHM-09-109	Class: JSS 3

	<p>Learning Outcome By the end of the lesson, you will be able to solve linear inequalities in one variable using addition and subtraction.</p>
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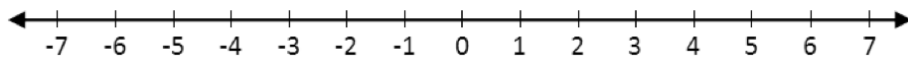
Overview

Inequalities can be solved using nearly the same process that is used for equations. You want to get the variable alone on one side of the inequality symbol.

To solve an inequality, you may add or subtract a number from both sides of the inequality symbol. The inequality symbol will not change.

Solved Examples

1. Solve $x + 3 > 5$ and show your answer on the number line below:

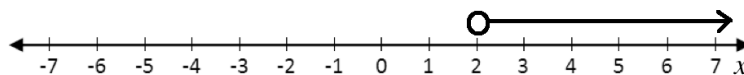


Solution

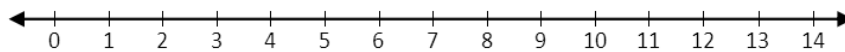
To get x alone on one side of the $>$ symbol, subtract 3 from both sides.

$$\begin{array}{rcl}
 x + 3 & > & 5 \\
 x + 3 - 3 & > & 5 - 3 & \text{Subtract 3 from both sides} \\
 x & > & 2
 \end{array}$$

The answer is $x > 2$. This can be drawn on the number line:



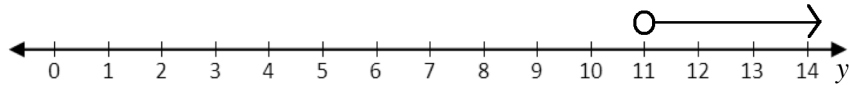
2. Solve $y - 4 > 7$ and show your answer on the number line:



Solution

$$\begin{array}{rcl}
 y - 4 & > & 7 \\
 y - 4 + 4 & > & 7 + 4 & \text{Add 4 to both sides} \\
 y & > & 11
 \end{array}$$

The answer is $y > 11$. This can be drawn on the number line:

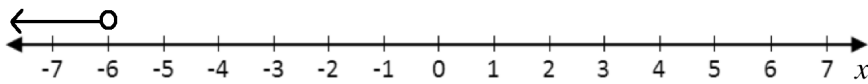


3. Solve $x + 2 < -4$ and show your answer on the number line.

Solution

$$\begin{aligned}x + 2 &< -4 \\x + 2 - 2 &< -4 - 2 && \text{Subtract 2 from both sides} \\x &< -6\end{aligned}$$

The answer is $x < -6$. Draw a number line that extends past -6 and use it to show the inequality:




Practice

Solve each inequality below and show your answers on the number line.

1. $y + 7 < 12$
2. $x - 1 > -2$
3. $y + 2 > -3$
4. $x + 3 < -1$
5. $x - 4 > 0$
6. $y + 8 < 3$

Lesson Title: Solving Linear Inequalities in One Variable II	Theme: Algebra
Practice Activity: PHM-09-110	Class: JSS 3

	<p>Learning Outcome By the end of the lesson, you will be able to solve linear inequalities in one variable using multiplication and division.</p>
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Overview

In this lesson, you will solve linear inequalities in one variable using multiplication and division. You will use the exact same method to solve for x in the linear inequality as if it was a linear equation.

Remember that multiplication and division are the opposite of each other, and they can be used to cancel. Remember to take the same action on both sides of the inequality sign.

If there is a coefficient on a variable, **divide** throughout by that number to cancel it. For example, these inequalities require **division**:

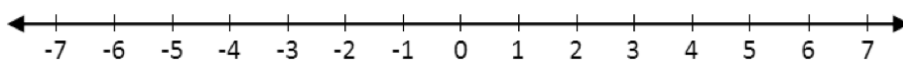
$$4x > 8 \qquad 12 > 2x \qquad 3x < 30$$

If the variable is in the numerator of a fraction, **multiply** throughout by the denominator of the fraction to cancel it. For example, these inequalities require **multiplication**:

$$\frac{x}{2} > 8 \qquad 12 > \frac{x}{4} \qquad \frac{1}{2}x < 3$$

Solved Examples

1. Solve $4x > 8$ and show the answer on the number line below:

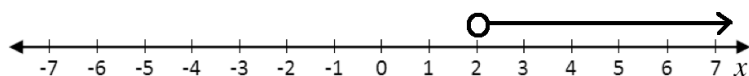


Solution

Divide throughout by the coefficient on x , which is 4.

$$\begin{array}{rcl} 4x & > & 8 \\ \frac{4x}{4} & > & \frac{8}{4} & \text{Divide both sides by 4} \\ x & > & 2 \end{array}$$

On the number line, this is:

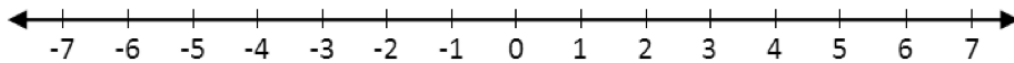


2. Solve: $12y > 24$

Solution

$$\begin{array}{rcl} 12y & > & 24 \\ \frac{12y}{12} & > & \frac{24}{12} \\ y & > & 2 \end{array} \quad \text{Divide both sides by 12}$$

3. Solve $\frac{x}{2} < -2$ and show the answer on the number line below:

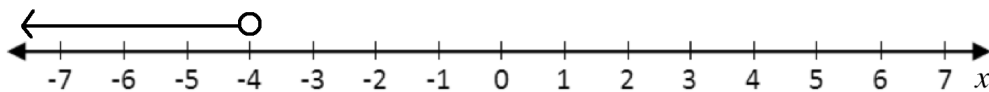


Solution

Multiply throughout by the numerator of the fraction on x , which is 2.

$$\begin{array}{rcl} \frac{x}{2} & < & -2 \\ \frac{2x}{2} & < & -2 \times 2 \\ x & < & -4 \end{array} \quad \text{Multiply both sides by 2}$$

On the number line, this is:



4. Solve: $\frac{1}{4}y > 3$

Solution

Note that $\frac{1}{4}y = \frac{y}{4}$. We can cancel the 4 in either expression by multiplying by 4. Multiply the inequality by 4:

$$\begin{array}{rcl} \frac{1}{4}y & > & 3 \\ \frac{4}{4}y & > & 3 \times 4 \\ y & > & 12 \end{array} \quad \text{Multiply both sides by 4}$$

Practice

1. Solve each inequality for y :

- $18 < 6y$
- $2y > 20$
- $3y < 4 + 5$

2. Solve each inequality for x :

a. $\frac{1}{2}x > 10$


b. $\frac{x}{10} < -1$

c. $15 > \frac{x}{3}$

3. Solve $4y > 16$ and show the solution on a number line.

4. Solve $2 < \frac{y}{2}$ and show the solution on a number line.

Lesson Title: Solving Linear Inequalities in One Variable III	Theme: Algebra
Practice Activity: PHM-09-111	Class: JSS 3

	<p>Learning Outcome By the end of the lesson, you will be able to apply algebraic principles to solve a linear inequality and illustrate the answer on the number line.</p>
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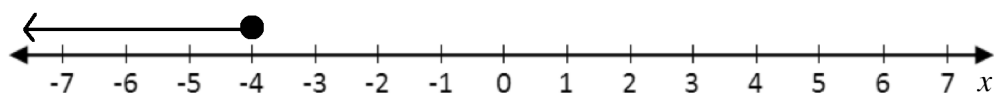
Overview

In this lesson, you will continue to solve linear inequalities. You will solve inequalities that require multiple steps, such as $5x + 3 \leq 18$.

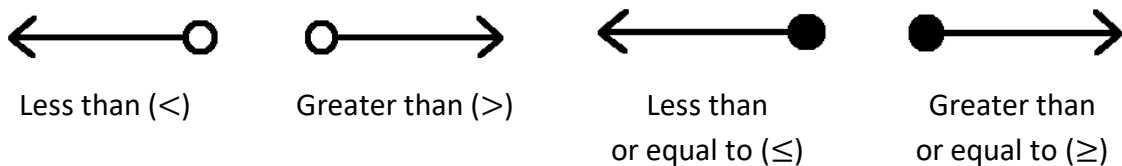
Remember that to solve an inequality, you may add, subtract, multiply or divide a number from both sides. Follow the same process that you have used to solve equations.

Notice that the inequality symbol used above looks different. The symbol \leq means **less than or equal to**, and \geq means **greater than or equal to**.

For example, consider $x \leq 4$. It means that $x = 4$ is a possible solution to the linear inequality $x \leq 4$. This is displayed on the number line below. The circle is shaded to show that $x = 4$ is a possible solution.



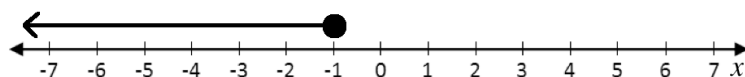
These are the arrows that show each inequality symbol:



Solved Examples

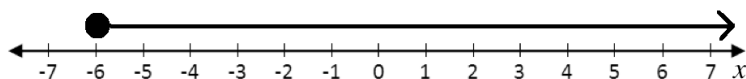
1. Show the truth set of $x \leq -1$ on a number line.

Solution

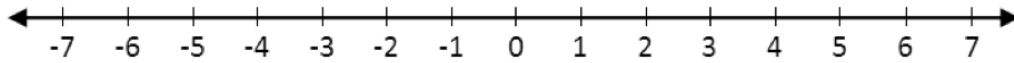


2. Illustrate $x \geq -6$ on a number line.

Solution



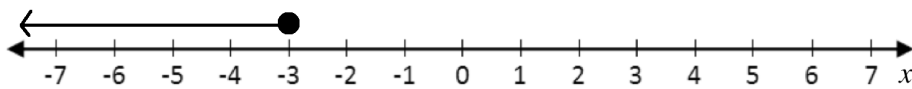
3. Solve: $5x + 3 \leq 18$ and show the solution on the number line:



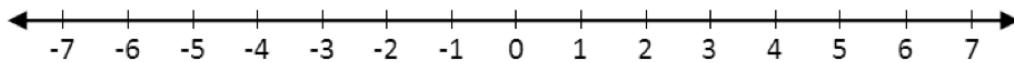
Solution

$$\begin{aligned}
 5x + 3 &\leq 18 \\
 5x + 3 - 3 &\leq 18 - 3 && \text{Subtract 3} \\
 5x &\leq 15 \\
 \frac{5x}{5} &\leq \frac{15}{5} && \text{Divide by 5} \\
 x &\leq 3
 \end{aligned}$$

On the number line, this is:



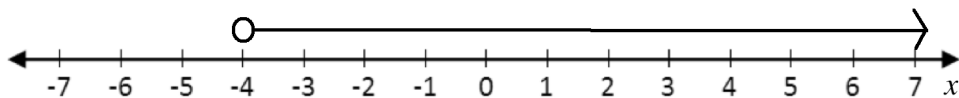
4. Solve $2(x + 1) > -6$ and display your answer on the number line:



Solution

$$\begin{aligned}
 2(x + 1) &> -6 \\
 \frac{2(x+1)}{2} &> \frac{-6}{2} && \text{Divide by 2} \\
 x + 1 &> -3 \\
 x + 1 - 1 &> -3 - 1 && \text{Subtract 1} \\
 x &> -4
 \end{aligned}$$

On the number line, this is:



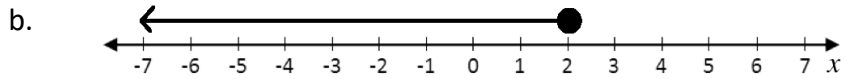
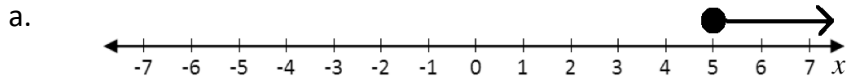
5. Solve: $\frac{y}{2} + 1 \geq -5$

Solution

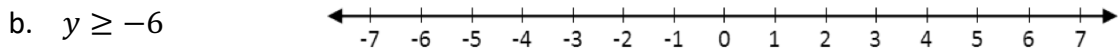
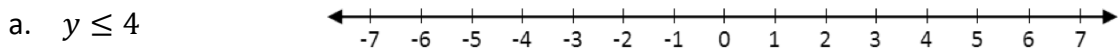
$$\begin{aligned}
 \frac{y}{2} + 1 &\geq -5 \\
 \frac{y}{2} + 1 - 1 &\geq -5 - 1 && \text{Subtract 1} \\
 \frac{y}{2} &\geq -6 \\
 \frac{2y}{2} &\geq 2(-6) && \text{Multiply by 2} \\
 y &\geq -12
 \end{aligned}$$

Practice

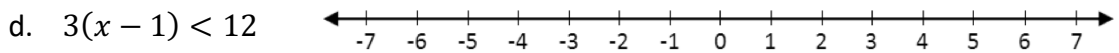
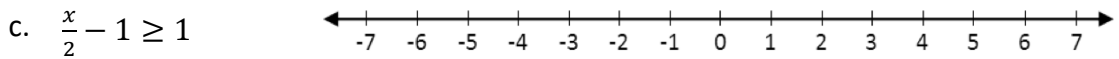
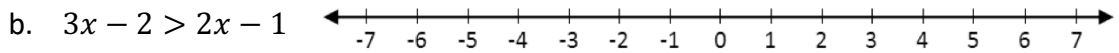
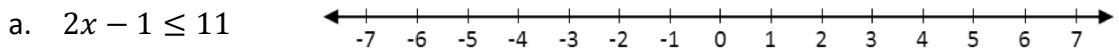
1. Write the inequality for each number line:




2. Illustrate each inequality on the number line:



3. Solve each inequality for x and show the solution on the number line:



Lesson Title: Creating Inequalities from Story Problems	Theme: Algebra
Practice Activity: PHM-09-112	Class: JSS 3

 <p>Learning Outcomes By the end of the lesson, you will be able to:</p> <ol style="list-style-type: none"> 1. Identify the unknown variable in a story problem. 2. Identify an inequality problem and apply the appropriate inequality symbol.

Overview

In this lesson, you will write inequalities for story problems. You will identify the unknown variable in a story problem, and represent it with a letter. You will also apply the appropriate inequality symbol.

The following are some common terms that tell you which inequality symbol to use:

- **Greater than ($>$):** more than, over
- **Less than ($<$):** fewer than, under
- **Greater than or equal to (\geq):** at least
- **Less than or equal to (\leq):** at most

There are other words that compare amounts and can be used to describe an inequality. Use your problem-solving skills to think about each statement and decide which symbol to use.

Solved Examples

1. Write an inequality for each statement:
 - a. The classroom can accommodate at most 45 pupils.
 - b. Fatu will buy a chicken only if the weight is at least 2 kg.
 - c. To attend a certain school, pupils must be 12 years or older.
 - d. She earns Le 50,000.00 or less per day.

Solutions

When you use terms “at most” or “at least” you are including the given number. The word “or” with a comparison word (older, less, and so on) can also tell you to include the given number.

You may use any variable in your inequalities.

- a. “At most” 45 pupils means the classroom can accommodate any number less than or equal to 45. The inequality is $x \leq 45$.
- b. “At least” 2 kg means Fatu will buy a chicken if it weighs greater than or equal to 2 kg. The inequality is $c \geq 2$.
- c. Twelve years “or older” means that pupils can be greater than or equal to 12 years old. The inequality is $p \geq 12$.

- d. Le 50,000.00 “or less” means that she earns less than or equal to Le 50,000.00 per day. The inequality is $e \leq 50,000$.

2. Write an inequality for each statement:

- Children must be over 3 years old to participate.
- She spends less than Le 5,000.00 on transport each day.
- He must sell more than 50 kg of rice to make a profit.
- Only children under 18 are eligible for the programme.

Solutions

Words like “over” and “under” tell us that the given number is not included.


- “Over” 3 years old tells us that 3-year-old children are not eligible, but any child older is eligible. The inequality is $c > 3$.
- “Less than” Le 5,000.00 tells us that her transportation costs never reach Le 5,000.00. The inequality is $t < 5,000$.
- “More than” 50 kg tells us that he must sell an amount greater than 50. The inequality is $r > 50$.
- “Under” 18 tells us that 18-year-old people are not eligible, but anyone younger than 18 is eligible. The inequality is $c < 18$.

Practice

Write an inequality for each statement:

- Pupils must score more than 70 marks to pass.
- He is younger than 20 years old.
- She buys at least 5 kg of rice each week.
- This car can accommodate at most 8 people.
- The pupils in the drama club are all older than 10 years old.
- Children must be taller than 120 cm to join this football team.
- You must be at least 40 years old to be elected President of Sierra Leone.
- She sells more than 20 kg of sugar each day.
- He sells 40 mangos per day at most.
- You must be at least 18 years old to vote.
- All of the chickens on his farm weigh at least 1 kg.
- Frozen chicken is stored at 0°C or colder.

Lesson Title: Solving Inequality Story Problems I	Theme: Algebra
Practice Activity: PHM-09-113	Class: JSS 3

	<p>Learning Outcome By the end of the lesson, you will be able to create and solve an inequality from a story problem.</p>
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Overview

In this lesson, you will create and solve linear inequalities from story problems. You will assign a variable to each unknown value. You may review the terms from the previous lesson to decide which inequality symbol to use.

The stories in this lesson will give more inequalities that involve operations. For example, $x + 2 < 50$ or $3x > 18$. You will use the skills you learned in previous lessons to solve for the variable in such expressions.

Solved Examples

- Hawa and Alice play football. In the last game, Hawa scored 2 more goals than Alice. Together, they scored less than 8 goals. Find an expression for how many goals Alice scored.

Solution

Let x be the number of goals Alice scored. Then Hawa scored $x + 2$ goals. Together, the goals they scored are less than 8. This is $x + x + 2 < 8$. Simplify this expression and solve for x :

$x + x + 2 < 8$	Expression from the problem
$2x + 2 < 8$	Simplify
$2x + 2 - 2 < 8 - 2$	Subtract 2
$2x < 6$	
$\frac{2x}{2} < \frac{6}{2}$	Divide by 2
$x < 3$	

The expression for the goals Alice scored is $x < 3$. Alice scored fewer than 3 goals.

- Fatu plans to spend at most Le 15,000.00 on a new maths book. She saves Le 3,000.00 each week toward the cost of the book. Write a linear inequality for the number of weeks that Fatu needs to save.

Solution

Let w be the number of weeks that Fatu needs to save. Multiply by the amount she saves each week to give the amount she has after w weeks: $3,000w$. If she spends at most Le 15,000.00, this means that she can spend less than or equal to Le 15,000.00. This is $3,000w \leq 15,000$. Solve for w :

$$\begin{array}{rcl} 3,000w & \leq & 15,000 & \text{Expression from the problem} \\ \frac{3,000w}{3,000} & \leq & \frac{15,000}{3,000} & \text{Divide by 3,000} \\ w & \leq & 5 & \end{array}$$

Fatu will save for 5 weeks or fewer.

- Mustapha buys rice to sell in his shop. The rice costs Le 2,000.00 per kilogramme, and he wants to spend at least Le 60,000.00. Write this information as a linear inequality. How much rice can he buy?

Solution

Let r be the rice that he can buy in kilogrammes. Multiply by the cost per kilogramme to find how much he spends on r kg: $2,000r$. If he spends at least Le 60,000.00, this means that he spends greater than or equal to Le 60,000.00. This is $2,000r \geq 60,000$. Solve for r :


$$\begin{array}{rcl} 2,000r & \geq & 60,000 & \text{Expression from the problem} \\ \frac{2,000r}{2,000} & \geq & \frac{60,000}{2,000} & \text{Divide by 2,000} \\ r & \geq & 30 & \end{array}$$

Mustapha will buy 30 kg of rice or more.

Practice

- Sia and Michael solved math problems. Together, they solved at least 20 problems. Sia solved 4 more problems than Michael did. Write an expression for how many problems Michael solved.
- Foday and Mustapha shared a basket of mangos. Foday ate twice as much as Mustapha. Together, they ate more than 15 mangos! Write an expression for how many mangos Mustapha ate.
- Aminata sells pumpkins in the market. The price of each pumpkin is Le 3,000.00. Aminata needs to earn at least Le 36,000.00 to buy a new school uniform for her daughter. Write this information as a linear inequality. How many pumpkins does she need to sell?
- David plans to spend at most Le 200,000.00 on his birthday party. He saves Le 10,000.00 each week. Write a linear inequality for the number of weeks that David saves for his party.

Lesson Title: Solving Inequality Story Problems II	Theme: Algebra
Practice Activity: PHM-09-114	Class: JSS 3

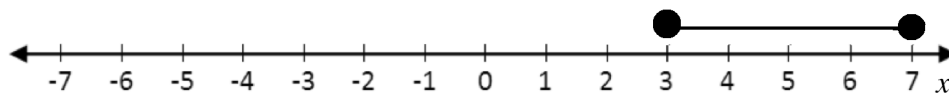
	<p>Learning Outcome By the end of the lesson, you will be able to create and solve an inequality from more complicated story problems.</p>
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Overview

In this lesson, you will create and solve an inequality from a different type of story problems.

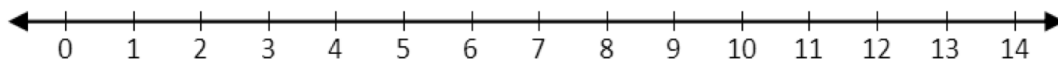
Some of the story problems will involve **compound inequalities**. These are inequalities with two inequality symbols. The variable is in the middle. For example: $3 \leq x \leq 7$. This inequality has two parts, $x \geq 3$ and $x \leq 7$. This means that x could be equal to 3, 4, 5, 6 or 7.

This is shown on the number line with a line segment connecting 3 and 7:



Solved Examples

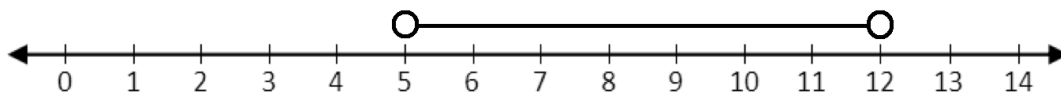
- To be eligible for an educational activity, children must be older than 5 but younger than 12. Write an inequality to show the age of eligible children. Illustrate the inequality on the number line below.



Solution

Let x be the age of eligible children. From the problem, we have $x > 5$ and $x < 12$. Note that children who are 5 are not eligible, because they must be older than 5. Therefore, we use the symbol $>$. Similarly, children who are 12 are not eligible, so we use the symbol $<$.

Combining the two inequalities, we have $5 < x < 12$. This is shown on the number line with open circles at 5 and 12:



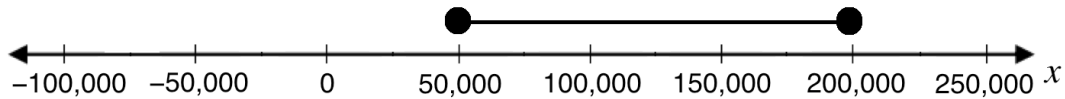
- Hawa is shopping for new shoes. She is planning to spend a minimum of Le 50,000.00 and a maximum of Le 200,000.00 on the new shoes. Write an inequality for the amount she is willing to spend on shoes. Display the inequality on a number line.

Solution

Let x be the amount that Hawa is willing to spend. From the problem, we have $x \geq 50,000$ and $x \leq 200,000$.

Combining the two inequalities, we have $50,000 \leq x \leq 200,000$.

We can show this on a number line that counts by 50,000:



3. Hawa will spend between Le 30,000.00 and Le 50,000.00 on new clothes for her daughter. She has already saved Le 5,000.00. If x is the amount she still has to save, write this information as a linear inequality. Simplify your answer.

Solution

Hawa has already saved Le 5,000.00, and she needs to save x more. This gives $5,000 + x$ as the amount that Hawa will have to spend on clothes.

She will spend between Le 30,000.00 and Le 50,000.00, which gives the inequality $30,000 \leq 5,000 + x \leq 50,000$.

To simplify this inequality, subtract 5,000 from each part of the inequality. This will get x alone in the middle.

$$\begin{array}{rcl} 30,000 & \leq & 5,000 + x & \leq & 50,000 \\ 30,000 - 5,000 & \leq & 5,000 - 5,000 + x & \leq & 50,000 - 5,000 \\ 25,000 & \leq & x & \leq & 45,000 \end{array}$$

The simplified inequality is $25,000 \leq x \leq 45,000$.

4. Fatu harvests watermelon from her garden each day. She will go to sell them in the market when she has between 20 and 25 watermelons. She has already harvested 4 watermelons. If x is the amount she needs to harvest, write this information as a linear inequality. Simplify your answer, then display the simplified inequality on a number line.

Solution

Fatu has already harvested 4 watermelons, and she needs to harvest x more. This gives $4 + x$ as the amount that Fatu will harvest.

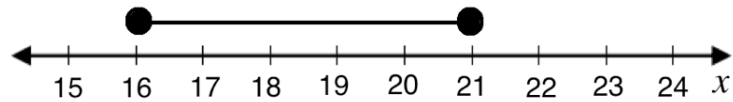
She will go to the market when she has between 20 and 25, which gives the inequality $20 \leq 4 + x \leq 25$.

To simplify this inequality, subtract 4 from each part. This will get x alone in the middle.

$$\begin{array}{rccccccc} 20 & \leq & 4 + x & \leq & 25 & & \\ 20 - 4 & \leq & 4 - 4 + x & \leq & 25 - 4 & & \\ 16 & \leq & x & \leq & 21 & & \end{array}$$

The simplified inequality is $16 \leq x \leq 21$.

Draw a number line that includes 16 and 21, and represent the inequality as shown:



Practice

1. The temperature in Freetown ranged from 28°C to 35°C on a certain day. Write this information as a linear inequality, and display the inequality on a number line.
2. To be eligible for a certain programme, children should be taller than 120 cm and shorter than 150 cm. Write this information as a linear inequality. Illustrate the inequality on a number line.
3. Mustapha weaves baskets and sells them. The minimum number of baskets he weaves in a day is 3, and the maximum number of baskets he weaves in a day is 9. Write this information as a linear inequality, and display the inequality on a number line.
4. John will spend between 20,000.00 and 30,000.00 on a gift for his mother. He has already saved Le 10,000.00. If x is the amount he still has to save, write this information as a linear inequality. Simplify your answer, then display the simplified inequality on a number line.
5. Mustapha sells the baskets that he weaves in the market. He will go to sell them in the market when he has between 30 and 40 baskets. He has already made 5 baskets. If x is the number he still needs to weave, write this information as a linear inequality. Simplify your answer, then display the simplified inequality on a number line.

Lesson Title: Practice with Inequalities	Theme: Algebra
Practice Activity: PHM-09-115	Class: JSS 3



Learning Outcome

By the end of the lesson, you will be able to solve various linear inequality problems and represent answers on the number line.

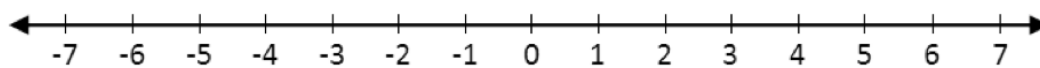
Overview

In this lesson, you will solve various linear inequality problems and represent answers on the number line. You will use the skills that you learned in the previous lessons.

You will also use one new piece of information. If you **divide** or **multiply** an inequality by a **negative** number, **change** the direction of the inequality symbol. For example, consider $-4x > 8$. You need to divide both sides by -4 to get x alone. You will change the $>$ symbol to $<$. See Solved Example 1 for the solution to this inequality.

Solved Examples

1. Solve $-4x > 8$ and display your answer on the number line:

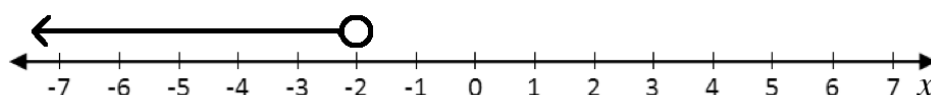


Solution

To solve for x , divide throughout by -4 and reverse the direction of the inequality)

$$\begin{array}{rcl}
 -4x & > & 8 \\
 \frac{-4x}{-4} & < & \frac{8}{-4} & \text{Divide throughout by } -4 \\
 x & < & -2
 \end{array}$$

Display the inequality on the number line:



2. Solve: $-5x + 3 \leq 18$

Solution

Follow these steps to solve for x :

$$\begin{array}{rcl}
 -5x + 3 & \leq & 18 \\
 -5x + 3 - 3 & \leq & 18 - 3 & \text{Subtract 3} \\
 -5x & \leq & 15 \\
 \frac{-5x}{-5} & \geq & \frac{15}{-5} & \text{Divide throughout by } -5 \text{ (reverse the} \\
 x & \geq & -3 & \text{direction of the inequality)}
 \end{array}$$

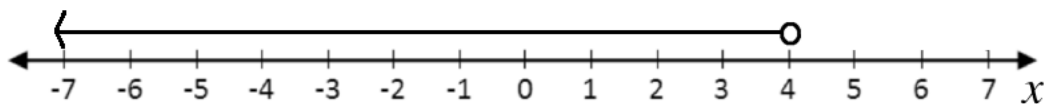
3. Solve $3(x + 2) < 2(13 - x)$ and display your answer on a number line.

Solution

Follow these steps to solve for x :

$$\begin{array}{rcl} 3(x + 2) < 2(13 - x) & & \\ 3x + 6 < 26 - 2x & & \text{Multiply through each side} \\ 3x + 2x + 6 < 26 - 2x + 2x & & \text{Add } 2x \\ 5x + 6 < 26 & & \\ 5x + 6 - 6 < 26 - 6 & & \text{Subtract 6} \\ 5x < 20 & & \\ \frac{5x}{5} < \frac{20}{5} & & \text{Divide throughout by 5} \\ x < 4 & & \end{array}$$

Display your answer on a number line:




Practice

Solve each inequality and display your answer on a number line:

1. $-2x > 12$
2. $-3x + 2 \geq 11$
3. $2 - 5x \leq x - 4$
4. $18 < -6y$
5. $2(y + 1) \leq 4y + 1$
6. $3x - 2 > 2x - 1$

Lesson Title: Data Collection	Theme: Statistics and Probability
Practice Activity: PHM-09-116	Class: JSS 3

	<p>Learning Outcome By the end of the lesson, you will be able to collect data from class members and display it in lists and pictograms.</p>
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Overview

In this lesson, you will learn how to collect and display data. Data are any numerical facts, information, or measurements of something.

Writing a list is one form of collecting data. For example, this is a list of the ages of Hawa’s family members: 4, 28, 2, 32, 8.

A list is very basic and does not give us much information. We can organise our list by writing the numbers in rank order. This means we will write the list in ascending or descending order. In this case, we have:

- Ascending order: 2, 4, 8, 28, 32
- Descending order: 32, 28, 8, 4, 2

Symbols or pictures can also be used to represent a certain number of items. For example, if you are counting people, you may draw pictures of the people to keep count of them. This is called a **pictogram**. A pictogram is a diagram that uses pictures or drawings to display the data collected. See Solved Examples 2 and 3 for examples of a pictograms.

Solved Examples

1. Foday collected the ages of his classmates in a survey. They are written on his paper below. Use Foday’s results to complete the following questions.

12	11	12	12	10	13	11	10
10	10	12	12	11	9	13	12
9	13	11	12	11	9	12	13

- a. Write the ages of his classmates in ascending order.
- b. What is the age of Foday’s oldest classmate?
- c. What is the age of Foday’s youngest classmate?
- d. How many of his classmates are 12?


Solutions


- a. The age of each classmate should appear in your list. Numbers may appear more than once. Write down the numbers exactly as many times as they appear. The list is:
9, 9, 9, 10, 10, 10, 10, 11, 11, 11, 11, 11, 12, 12, 12, 12, 12, 12, 12, 12, 13, 13, 13, 13
 - b. The age of Foday’s oldest classmate is the last number in the list, 13 years old.
 - c. The age of Foday’s youngest classmate is the first number in the list, 9 years old.
 - d. Count the number of times 12 appears in the list. It appears 8 times, so 8 of Foday’s classmates are 12 years old.
2. Sia counted the members of her family. She counted 7 females and 6 males. Help her draw a pictogram to show the number of family members with symbols. Use the following symbols for male and female:

Male:  Female: 

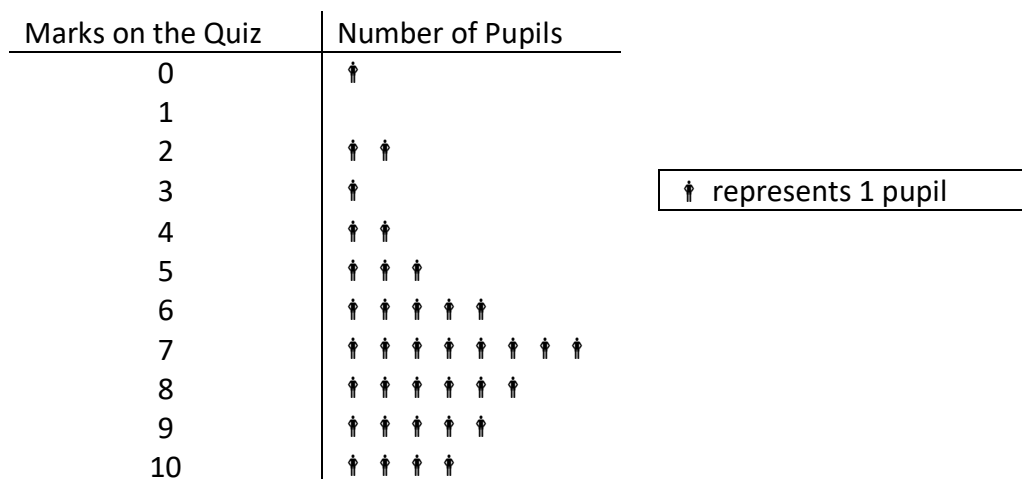
Solution

There are 7 females, so the female symbol should be drawn 7 times. There are 6 males, so the male symbol should be drawn 6 times.

Females: 

Males: 

3. The pupils in one class took a quiz worth 10 marks in total. The pictogram below shows the marks they obtained. Use the pictogram to answer the questions below.



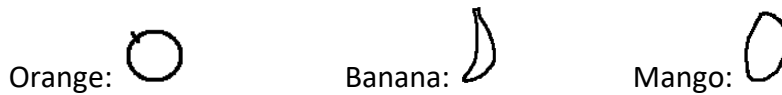
- a. How many pupils sat for the quiz?
- b. What mark did the greatest number of pupils obtain?
- c. What mark did the lowest number of pupils obtain?
- d. How many pupils obtained 4 marks?
- e. If 7 marks is passing, how many pupils passed?

Solutions

- To find out how many pupils sat for the quiz, count the number of † symbols in the pictogram. There are 37 symbols, so 37 pupils sat the quiz.
- The greatest number of pupils obtained 7 marks. This has the most † symbols: 8.
- The lowest number of pupils obtained 1 mark. No pupils obtained 1 mark.
- There are 2 † symbols next to 4, so 2 pupils obtained 4 marks.
- Count the number of † that obtained 7 or more marks. That is, the number of symbols after 7, 8, 9 and 10. There are 23 symbols, so 23 pupils passed.

Practice

- Martin is carrying a basket of fruit home for his family. He has 5 oranges, 8 bananas and 3 mangos. Record the number of each fruit that he has using a pictogram. Use the following symbols for the fruit:




- The school will buy new shoes for all of the football players on the team. The shoe sizes of the football players are given in the pictogram. Use the pictogram to answer the questions below.

Shoe Size	Number of Players
38	†
39	† †
40	† † † †
41	† † † † † †
42	† †
43	†

† represents 1 player

- How many football players are there?
 - How many of the players wear size 39?
 - How many players wear size 40 or smaller?
 - The shop has 3 pairs of shoes in size 40. Is it enough?
- The coach took the height measurements of the football players to buy them new uniforms. Their heights (in cm) are: 178, 170, 167, 172, 173, 177, 172, 170, 172, 173, 177, 169, 170, 168, 172, 173.
 - Write the heights of the players in ascending order.
 - What is the height of the tallest player?
 - What is the height of the shortest player?
 - If players taller than 175 cm wear a large uniform, how many large uniforms are needed?

Lesson Title: Frequency Tables	Theme: Statistics and Probability
Practice Activity: PHM-09-117	Class: JSS 3

	<p>Learning Outcome By the end of the lesson, you will be able to organise and display collected data in a frequency table.</p>
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Overview

In this lesson, you will learn how to organise and display collected data in a **frequency table**.

To get more information out of data, we need to collect it in a more organised way. Once we have collected it we need to display it in a way that is easy for people to understand. A frequency table is one way to organize data.

In a frequency table, you will have 1 column for tally marks. This is to keep count while you are collecting data. The next column is for the number that you counted in with tally marks.

For example, say you want to take a survey of your classmates to understand what subjects are their favourite. You will draw a table with a row for each subject:

SUBJECT	TALLY MARKS	NUMBER OF PUPILS
Mathematics		
English Language		
Social Studies		
Integrated Studies		
Total		

As you survey your classmates, you will keep count of their answers in the “tally marks” column. Remember that tally marks are grouped in fives. After you complete the survey, you will write the number of pupils who prefer each subject in the “number of pupils” column. You can find the total number of pupils surveyed, and write the numbers in the last column, for “Total”.

Here is an example of a completed table, for a classroom with 40 pupils:

SUBJECT	TALLY MARKS	NUMBER OF PUPILS
Mathematics		6
English Language		10
Social Studies		15
Integrated Studies		9
Total	40	40

Solved Examples

1. The ages of 15 pupils in a class were recorded as follows: 12, 14, 15, 13, 13, 12, 14, 14, 13, 15, 13, 13, 14, 12, 15. Use this data to complete the table below:

AGES	TALLY MARKS	NUMBER OF PUPILS
12		
13		
14		
15		
Total		

Solution

For each number in the list, make 1 tally mark in the table. It can help to cross off the ages in the list as you record them. After you record all of the pupils with tally marks, write the total number of pupils of each age in the “number of pupils” column.

12, 14, 15, 13, 13, 12, 14, 14, 13, 15, 13, 13, 14, 12, 15

AGES	TALLY MARKS	NUMBER OF PUPILS
12		3
13	 	5
14		4
15		3
Total	15	15

2. Below is a list of the marks obtained by pupils in a mathematics quiz worth 10 possible points. Organise the data in a table, then answer the questions.

Marks: 9, 10, 10, 7, 8, 6, 6, 7, 5, 9, 10, 9, 9, 7, 7, 7, 6, 8, 10, 5, 6, 4, 7, 8, 9, 6, 8, 8.

- How many pupils took the maths quiz?
- How many pupils obtained 10 marks?
- What is the lowest mark obtained by a pupil?
- What mark did the greatest number of pupils achieve?

Solutions

First, draw the table. You will have columns for “quiz marks”, “tally marks”, and “number of pupils”. Record the marks in the list in your table:

QUIZ MARKS	TALLY MARKS	NUMBER OF PUPILS
4		1
5		2
6		5
7		6
8		5
9		5
10		4
Total	28	28

Use the information in the table to answer the questions:

- Twenty-eight pupils took the maths quiz.
- Four pupils obtained 10 marks.
- The lowest mark obtained is 4.
- The mark with the greatest number of pupils is 7 marks.

Practice

- Fatu is a nurse in a hospital. This morning, she recorded the weights of the babies in the hospital. Help her organise this data in a table, then answer the questions below.


Weights of babies (kg): 3, 4, 4, 5, 3, 4, 6, 5, 7, 4, 3, 5, 5, 6, 6, 7, 5, 5, 3

- How many babies did Fatu weigh?
 - What is the weight of the heaviest baby?
 - How many babies weigh 4 kg?
 - What is the most common weight of the babies?
- Some pupils in a JSS1 class measured their height to the nearest 5 cm. Their heights are recorded below. Organise this data in a table, then answer the questions below.

Height of pupils (cm): 160, 155, 155, 170, 165, 165, 155, 150, 160, 155, 165, 170.

- How many pupils measured their heights?
- What is the height of the tallest pupil?
- How many pupils are 160 cm tall?

Lesson Title: Bar Charts	Theme: Statistics and Probability
Practice Activity: PHM-09-118	Class: JSS 3

	<p>Learning Outcome By the end of the lesson, you will be able to display collected data in a bar chart.</p>
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Overview

This lesson is on drawing bar charts based on data. The data is given in a table similar to the frequency tables you created in the previous lesson. The numbers in the table give the frequency, or quantity, of each item.

In a bar chart, the height of each bar tells the frequency. Bar charts are used to compare different quantities.

When drawing a bar chart, draw and label the axes first, and give your chart a title. Then you will draw the bars. It is important to use an appropriate scale on your y-axis. It should extend to at least the greatest frequency in the table.

Solved Examples

1. A teacher surveyed a class to learn the favourite fruits of the class members. The result of the survey is show in the frequency table below. Draw a bar chart for the data.

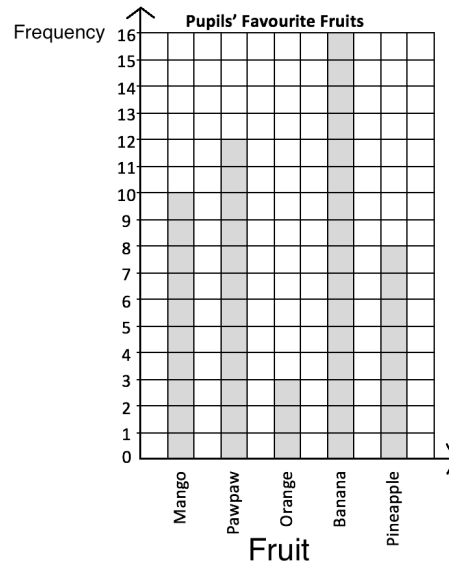
Favourite Fruits	
Fruit	Frequency
Mango	10
Pawpaw	12
Orange	3
Banana	16
Pineapple	8

Solution

Each row on the table is 1 bar on the chart.

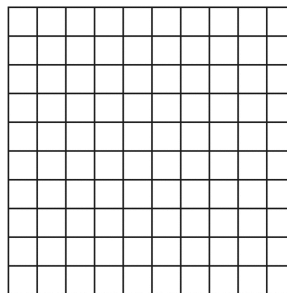
Draw the axes. The 'Frequency' axis should extend to at least 16. The 'Fruit' axis should have the 5 fruits.

Draw the bars. The height of each bar is given in the frequency table.



2. The table below shows the number of JSS pupils that play football on the school team. Using the grid below, draw a bar chart to present the data.

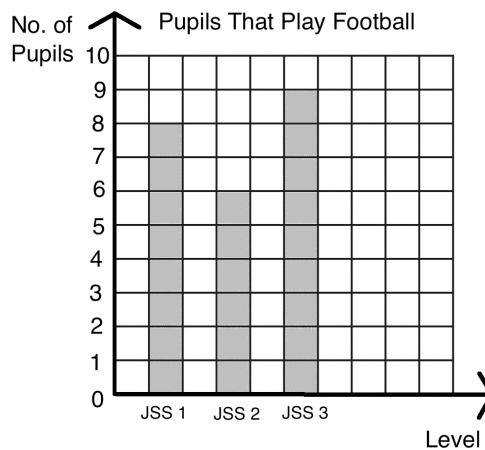
Level	JSS 1	JSS 2	JSS 3
No. of Pupils	8	6	9



Solution

This frequency table is drawn horizontally. It is the same as a frequency table drawn vertically. In this case, each column gives information for a bar on the chart.

In this case, you are given a grid for the bar chart. Draw the axes on it, and label each bar with the correct JSS level.



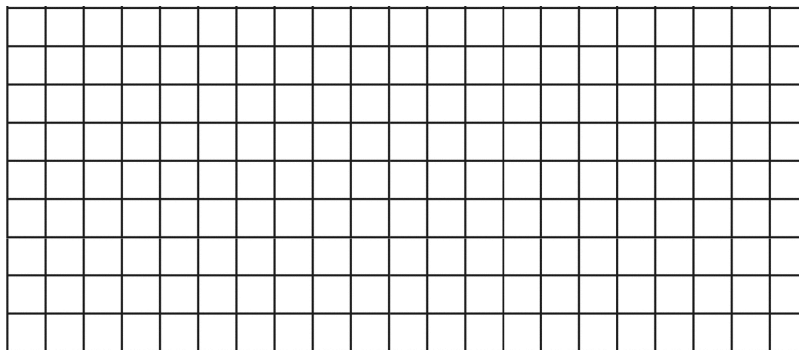
Practice

1. The following is the result of a survey conducted by an NGO on the jobs worked by adults in a community in Kailahun district. Draw a bar chart to show the information.


Job	No.
Farmers	8
Traders	2
Health workers	3
Teachers	5
Other	4

2. The table below shows the marks of pupils on a test. No pupil scored lower than 40% or higher than 85%. Draw a bar chart for the information using the grid below.

Marks	40%	45%	50%	55%	60%	65%	70%	75%	80%	85%
No. of Pupils	1	2	4	3	0	2	5	8	1	2



Lesson Title: Line Graphs	Theme: Statistics and Probability
Practice Activity: PHM-09-119	Class: JSS 3

	Learning Outcome By the end of the lesson, you will be able to display collected data in a line graph.
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Overview

In this lesson, you will learn to display collected data in another type of chart, called a **line graph**. To draw a line graph, you need a collection of data that has changed over time. A line graph is a graph that uses points connected by a line to show how something changes in value as time goes by.

In a line graph, the x -axis is time. It can be measured in any unit of time, including minutes, hours, days, weeks, months, or years. The y -axis shows the quantity of something that changes.

Solved Examples

- Mariama wants to show how much weight her dog Billy has gained from when he was a puppy to when he was fully grown. Here is the data she collected on her notepad:

Months	1	2	3	4	5	6	7	8	9
Weight in lbs	10	15	20	25	30	35	40	45	50

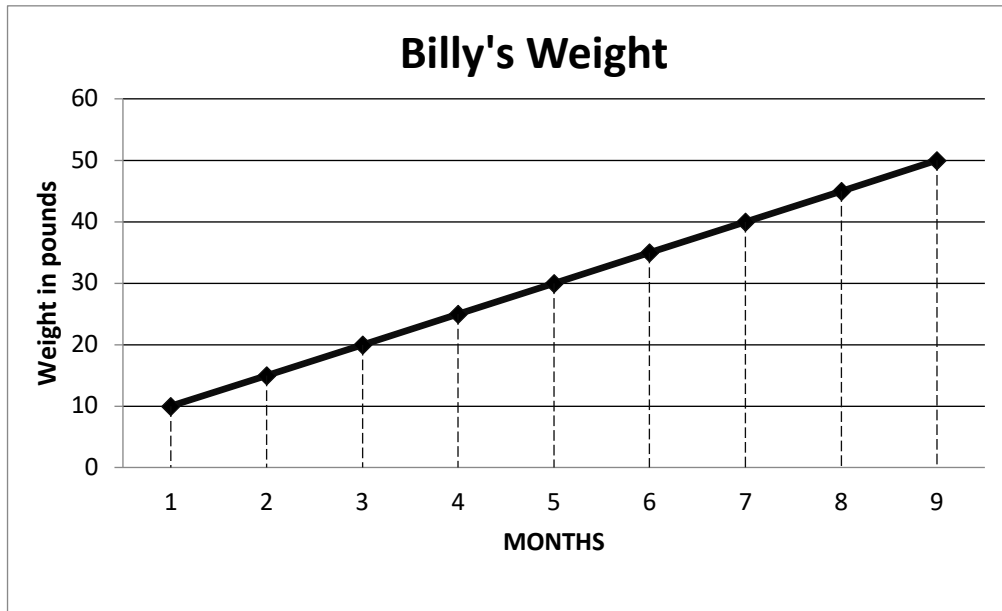
Display the data in a line graph.

Solution

Like other graphs, line graphs need two axes, one vertical (y -axis), and one horizontal (x -axis). In this case, the x -axis is the time in months. Each number in the “Months” row of the table should be on the x -axis.

The y -axis represents the range of Billy’s weight in pounds. The dog’s lowest weight was 10 lbs and his highest weight was 50 lbs. We will make our y -axis range from 0 to 60 lbs to cover all of his weight. You may count by 5s or 10s on the y -axis.

Plot each point from the table. Connect each point to the next one with a straight line. You may use a ruler or straight edge.



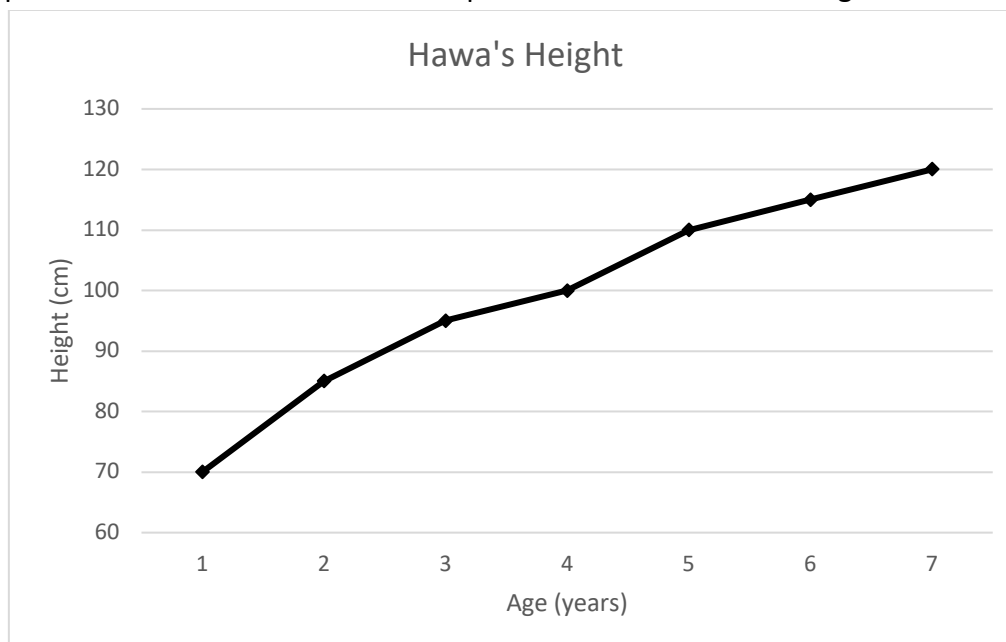
Make sure your line graph has a title, and that each axis is labeled.

2. Hawa's parents measured her height each year on her birthday. Her height for 7 years is given in the table below, in cm. Draw a line graph for the data.

Age (years)	1	2	3	4	5	6	7
Height (cm)	70	85	95	100	110	115	120

Solution

Draw the two axes. The x -axis is age in years, and the y -axis is height in centimetres. Plot each point from the table. Connect each point to the next with a straight line.



Note that it is not necessary to draw lines from the axes to each point. However, you may draw the lines (as in Solved Example 1) with a straight edge. This can help you plot the points accurately.

Practice

1. The table below shows daily temperatures for London, recorded for 6 days in degrees Celsius. Display the data in a line graph with a y -axis ranging from 15 to 25 degrees.

Day	1	2	3	4	5	6
Temperature ($^{\circ}\text{C}$)	17	19	18	16	21	23

2. Mustapha sells rice in his shop. He keeps track of the amount of rice he sells each day, in kilogrammes. This helps him to know when to buy more rice for his shop. The amount of rice he sold each day for one week is in the table below. Display the data in a line graph.

Day	1	2	3	4	5	6	7
Rice sold (kg)	3	4	2.5	3.5	5	5.5	3

3. Fatu travels each day for her job. She kept track of the money she spent on transportation this week. Display the data below in a line graph.

Day	1	2	3	4	5
Leones (Le)	4,000.00	2,000.00	1,000.00	6,000.00	5,000.00

Lesson Title: Interpreting Charts and Graphs	Theme: Statistics and Probability
Practice Activity: PHM-09-120	Class: JSS 3



Learning Outcomes

By the end of the lesson, you will be able to:

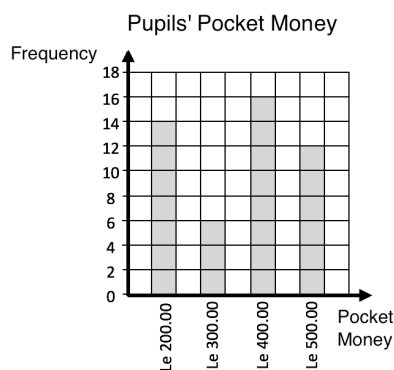
1. Make comparisons using pictograms, bar charts, and line graphs.
2. Draw conclusions from charts and graphs.

Overview

In this lesson, you will make comparisons and draw conclusions using pictograms, bar charts, and line graphs.

Solved Examples

1. The bar chart below shows the pocket money received by pupils in a class in a week. Use the bar chart to answer the questions below.

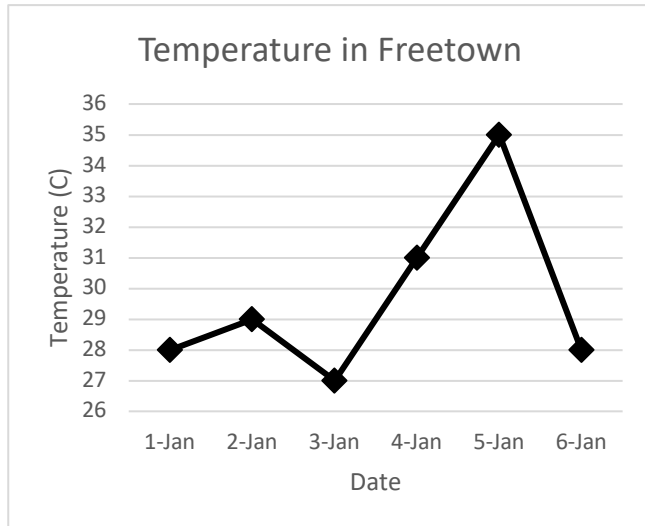


- a. Find the total number of pupils in the class.
- b. What is the maximum amount of pocket money that any pupil received?
- c. Which amount of money did the highest number of pupils receive?
- d. How many pupils received Le 200.00 for a week?
- e. How many pupils received either Le 300.00 or Le 500.00 for a week?

Solutions

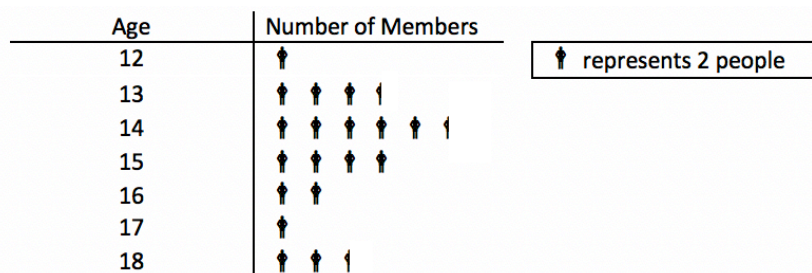
- a. Add the heights of the bars: $14 + 6 + 16 + 12 = 48$ pupils
 - b. The maximum amount of money that any pupil received is Le 500.00. It is the greatest number on the "Pocket Money" axis.
 - c. Sixteen pupils received Le 400.00 for a week. This highest number of pupils received Le 400.00 because it has the tallest bar.
 - d. Fourteen pupils.
 - e. We want to include all of the pupils who received Le 300.00 and all of the pupils who received Le 500.00. Add the heights of the bars for Le 300.00 and Le 500.00: $6 + 12 = 18$ pupils
2. The line graph below gives the temperature in Freetown in degrees Celsius during the first 6 days of the year. Use the graph to answer the questions below.

- Which day was the hottest?
- Which day was the coldest?
- What was the temperature difference between the hottest day and coldest day?
- Were any 2 dates the same temperature? If so, which dates?
- What was the greatest temperature change from one day to the next?



Solutions

- 5th January was the hottest day, because it is the highest on the graph at 35 degrees.
 - 3rd January was the coldest day, because it is the lowest on the graph at 27 degrees.
 - Subtract to find the temperature difference: $35 - 27 = 8$ degrees.
 - Yes, 1st and 6th January were the same temperature, 28 degrees.
 - The greatest temperature change was from 5th January to 6th January. The temperature dropped from 35 to 28 degrees, a change of 7 degrees.
3. The pictogram below gives the ages of members of the school drama club. Use the pictogram to answer the questions.



- How many people are in the drama club?
- How many drama club members are 14 years old?
- How many drama club members are either 12 or 13?
- The drama club will go on a trip to Freetown. Only members aged 16 or older are eligible to go. How many members can go to Freetown?

Solutions

In this case, note that ↑ represents 2 people. This means that half of the ↑ symbol represents 1 person. You can count by 2's or multiply by 2 to find how many people are each age. There are two 12-year-old members, seven 13-year-old member, and so on.

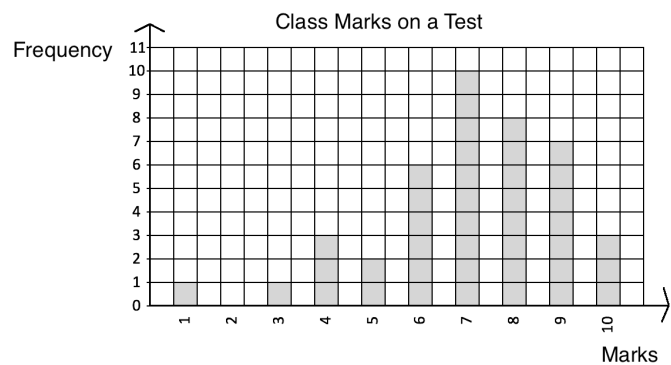
- Count the number of symbols and multiply by 2. There are 19.5 symbols, so there are $19.5 \times 2 = 39$ members of the club.

- b. Count the number of symbols for 14 years old, and multiply by 2. There are 5.5 symbols, so there are $5.5 \times 2 = 11$ members.
- c. To find the number of members that are 12 or 13, add the number of members in the 2 categories. There are 4.5 symbols, which makes $4.5 \times 2 = 9$ members.
- d. Add the number of members who are 16, 17 or 18. There are 5.5 symbols, which makes $5.5 \times 2 = 11$ members.

Practice

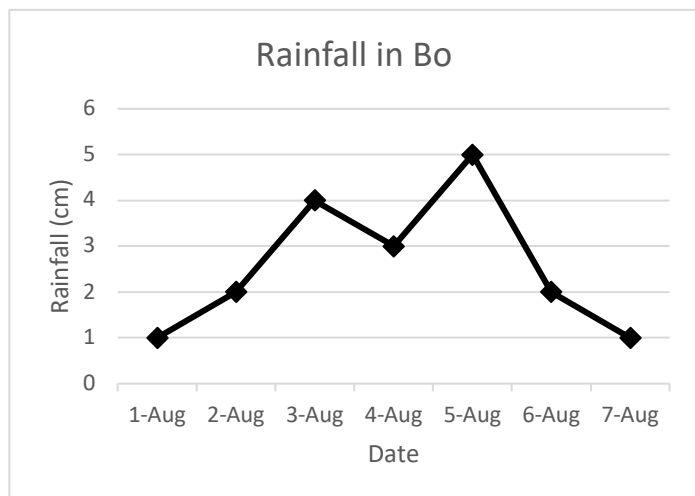
1. The bar chart below shows the marks that a class of pupils received on a test. Use the bar chart to answer the questions.

- a. How many pupils sat the test?
- b. What was the lowest score on the test?
- c. How many pupils scored 8 marks?
- d. If pupils need 7 marks or higher to pass, how many pupils passed?




2. Emmanuel likes to study the weather. He measured the rainfall in Bo during 1 week in August and recorded it in the graph below. Use the graph to answer the questions.

- a. How much did it rain on 6 August?
- b. On which date did it rain the most?
- c. On which date did it rain the least?
- d. How much more did it rain on 5 August than on 4 August?
- e. How much less did it rain on 7 August than on 5 August?



Lesson Title: Interpreting Pie Charts	Theme: Statistics and Probability
Practice Activity: PHM-09-121	Class: JSS 3

	<p>Learning Outcomes</p> <p>By the end of the lesson, you will be able to:</p> <ol style="list-style-type: none"> 1. Interpret information from a pie chart. 2. Find the sectoral angles of a pie chart and relate them to the whole (360°).
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Overview

This lesson is on interpreting pie charts and solving problems related to pie charts. A pie chart is a type of graph in which a circle is divided into sectors that each represents a portion of the whole.

In this lesson, parts of the whole will be given as percentages. The percentages in a pie chart always add up to 100%. Remember that 100% is the same as one whole. A bigger percentage takes up more space in the pie chart.

You can find the number of items represented by each sector of a pie chart using the percentage of the whole. You also need to know the total number represented in the chart. See parts c. and d. of Solved Example 1.

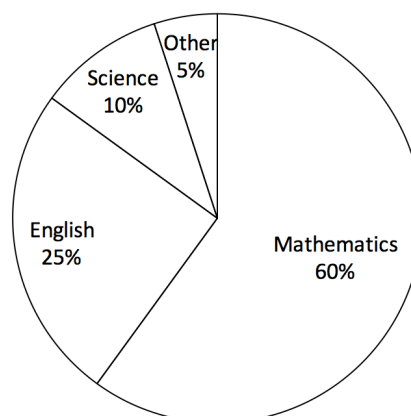
When you know the percentages for each sector, you can also figure out how many degrees each angle at the centre of each sector is. Being able to calculate the angles allows us to construct our own pie charts.

To find the degrees for the angle at the centre of each sector, we can multiply the percentage for each subject by 360° . Remember that a full rotation (or a full circle) is 360° .

Solved Examples

1. The principal of a school conducted a survey of 200 pupils to learn their favourite subjects. The result of the survey is in the pie chart at right. Use the pie chart to answer the questions:
 - a. Which subject was the most popular?
 - b. What percentage of pupils prefer English?
 - c. How many pupils prefer English?
 - d. How many pupils prefer mathematics?

Pupils' Favourite Subjects



Solutions

- a. Mathematics was the most popular subject. It has the largest sector in the pie chart.
- b. 25% of pupils prefer English.

- c. To find the number of pupils who prefer English, find 25% of 200, the number of pupils surveyed. The number who prefer English = $\frac{25}{100} \times 200 = 25 \times 2 = 50$ pupils.
- d. To find the number who prefer mathematics, find 60% of 200. The number who prefer maths = $\frac{60}{100} \times 200 = 60 \times 2 = 120$ pupils.

2. Find the sectoral angle of each sector of the pie chart in Solved Example 1.

Solution

There are 4 sectors in the pie chart in Solved Example 1, and we want to find the angle of each of them. Multiply each percentage by 360° .

Mathematics: $\frac{60}{100} \times 360^\circ = 216^\circ$

English: $\frac{25}{100} \times 360^\circ = 90^\circ$

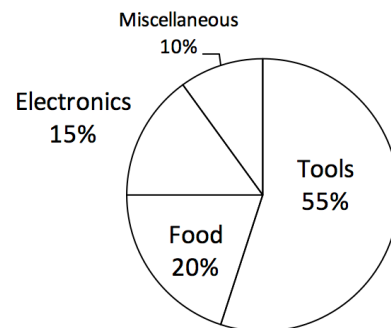
Science: $\frac{10}{100} \times 360^\circ = 36^\circ$

Other: $\frac{5}{100} \times 360^\circ = 18^\circ$

The 4 sectors together should make 360° , a full rotation. Check your answer by adding them together: $216^\circ + 90^\circ + 36^\circ + 18^\circ = 360^\circ$

3. This week, Aminata earned Le 2,000,000.00 by selling goods in her shop. The pie chart at right shows the percentage that Aminata has earned this week in each category of goods that she sells. Use the pie chart to answer the questions.

Goods Aminata Sells



- From which category of goods did Aminata earn the most money?
- From which category of goods did Aminata earn the least amount of money?
- How much did Aminata earn from electronics?
- How much did Aminata earn from food?
- How much more did Aminata earn from tools than from electronics?
- Do you have any business ideas for Aminata?

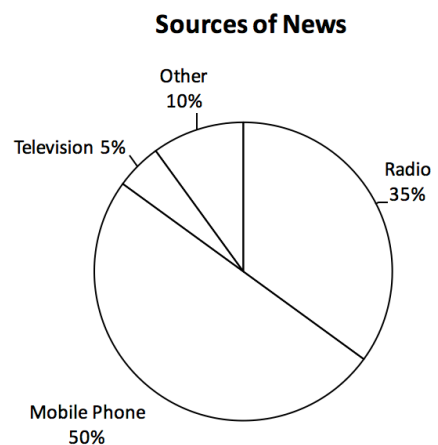
Solutions

- Aminata earned the most money from tools, which has the largest sector.
- Aminata earned the least amount of money from “miscellaneous”, which has the smallest sector. This is a category that includes everything she sold that is not tools, food, or electronics.

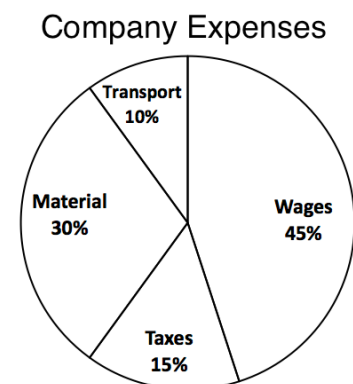
- c. To find how much she earned from electronics, find 15% of Le 2,000,000.00. Earned from electronics = $\frac{15}{100} \times \text{Le } 2,000,000 = 15 \times 20,000 = \text{Le } 300,000.00$.
- d. Earned from food = $\frac{20}{100} \times \text{Le } 2,000,000 = 20 \times 20,000 = \text{Le } 400,000.00$.
- e. To find how much more she earned from tools than electronics, we need to first know how much she earned from tools.
 Earned from tools = $\frac{55}{100} \times \text{Le } 2,000,000 = 55 \times 20,000 = \text{Le } 1,100,000.00$.
 Remember that she earned Le 300,000.00 from electronics (part c.).
 Subtract to find the difference: $1,100,000 - 300,000 = \text{Le } 800,000$.
 Aminata earned Le 800,000.00 more from tools than from electronics.
- f. Example advice: Aminata should buy more tools to sell and reduce the sales of miscellaneous.

Practice


- Fatu conducted a survey of 300 members of her community to learn how people get news. The results of her survey are in the pie chart. Use the chart to answer the questions:
 - How do most people get news?
 - What is the least common news source?
 - How many people get news by radio?
 - How many people get news by mobile phone?



- Find the sectoral angle of each sector of the pie chart in problem 1.
- Abu started a company. He created a pie chart to show his company expenses this week. His company spent a total of Le 4,000,000.00 this week. Use this information to answer the following questions:
 - What was the company's greatest expense this week?
 - How much did the company spend on transport?
 - How much did the company spend on taxes?
 - How much more money did the company spend on wages than materials?



Lesson Title: Creating Pie Charts	Theme: Statistics and Probability
Practice Activity: PHM-09-122	Class: JSS 3

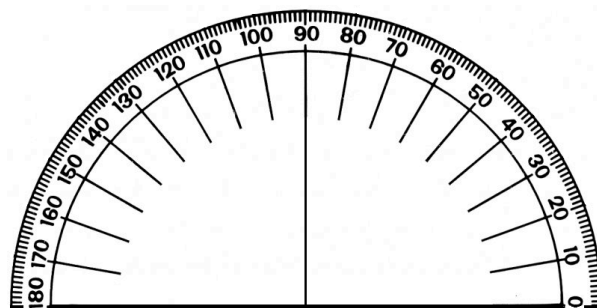
	<p>Learning Outcome By the end of the lesson, you will be able to display data collected from the class in a pie chart.</p>
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Overview

In this lesson, you will display data in a pie chart.

To draw a pie chart accurately, we must use a protractor. The entire circle is 360°, and each sector is one part of the whole. We must find the size of each sector in degrees. This is given by the angle it forms at the centre. We use a protractor to draw angles at the centre of the pie chart with the measurements we find.

If you do not have a protractor, you can make one with paper. Trace this protractor with a pen onto another piece of paper.



Solved Examples

1. A group of pupils was surveyed to find their favourite fruits. The data is in the table below. Create a pie chart for this data.

Favourite Fruit	Frequency	Percentage
Banana	16	40%
Mango	10	25%
Orange	6	15%
Pineapple	8	20%
TOTAL	40	100%

Solution

First, calculate the degree measure for each fruit. Give each frequency as a fraction of the whole (40), and multiply that fraction by 360°.

$$\text{Banana} = \frac{16}{40} \times 360^\circ = 144^\circ$$

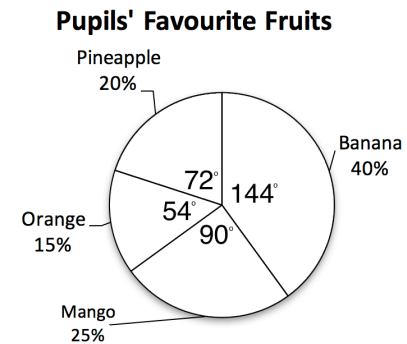
$$\text{Mango} = \frac{10}{40} \times 360^\circ = 90^\circ$$

$$\text{Orange} = \frac{6}{40} \times 360^\circ = 54^\circ$$

$$\text{Pineapple} = \frac{8}{40} \times 360^\circ = 72^\circ$$

Now, draw the pie chart using the degrees you found:

- Draw the empty circle and heading, 'Pupils' Favourite Fruits.'
- Draw each sector using a protractor. For example, here are the steps for drawing the 'Banana' sector:
 - Place the centre of the protractor on the centre of the pie chart and place the bottom of the protractor exactly along one radius of the circle.
 - Find the angle measurement for banana, 144° .
 - Use a straightedge to draw another radius from the centre at 144° .
- Label each sector as shown.



2. The table below shows how a family spends their money in one day. Represent the information in a pie chart.

Items	Amount Spent
Food	Le 15,000
House rent	Le 9,000
Electricity	Le 10,000
Transportation	Le 24,000
Other	Le 2,000

Solution

- a. **Step 1.** Find each quantity as a fraction of the whole, and multiply it by 360 to find the degree of its sector.

Calculate the total: $15,000 + 9,000 + 10,000 + 24,000 + 2,000 = 60,000$

Calculate the measures of the sectors:

$$\text{Food} = \frac{15,000}{60,000} \times 360^\circ = 90^\circ$$

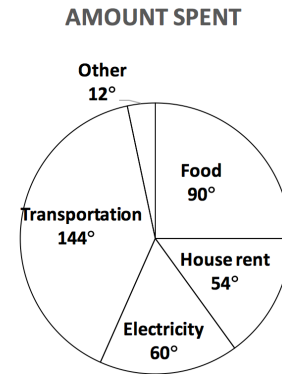
$$\text{House rent} = \frac{9,000}{60,000} \times 360^\circ = 54^\circ$$

$$\text{Electricity} = \frac{10,000}{60,000} \times 360^\circ = 60^\circ$$

$$\text{Transportation} = \frac{24,000}{60,000} \times 360^\circ = 144^\circ$$

$$\text{Other} = \frac{2,000}{60,000} \times 360^\circ = 12^\circ$$

Step 2. Use a protractor to draw the pie chart:



Practice


- In a secondary school, a census was taken regarding the favourite subjects of pupils. The results are in the table below. Represent the information on a pie chart.

Subjects	Number of pupils
Mathematics	200
English Language	50
Literature	90
Biology	100
Government	60
History	80
Economics	140

- The cost incurred on infrastructural development by a mining company is as follows: Wages: 45%; Taxes: 15%; Material: 30%; Transport: 10%. Show this information in a pie chart.
- In a secondary school in Sierra Leone, a census taken regarding the various ethnic groups gives the following results. Represent the data on a pie chart.

Ethnic group	Number of pupils
Temne	80
Mende	100
Susu	60
Krio	200
Fullah	50
Limba	90
Others	140

Lesson Title: Choosing a Graph or Chart	Theme: Statistics and Probability
Practice Activity: PHM-09-123	Class: JSS 3

	<p>Learning Outcome By the end of the lesson, you will be able to collect data and decide on the best type of graph or chart to represent it.</p>
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Overview

In this lesson, you will practise the previous lessons on bar charts, line graphs, and pie charts. Remember that these are used to show different types of data. Review their definitions:

- A **bar chart** is used to compare different amounts.
- A **line graph** is used to display and compare information that changes over time.
- A **pie chart** is a type of graph in which a circle is divided into sectors that each represents a proportion of the whole.

In this lesson, you will be given data in a table. You will decide which type of graph or chart is the best for representing the data. Use the definition of each type of chart above to help you decide.

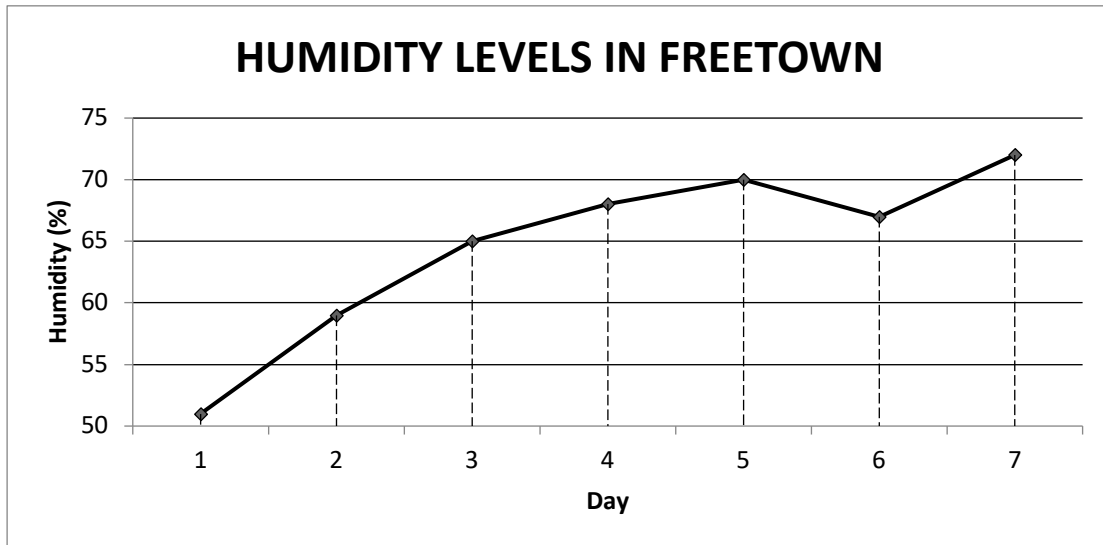
Solved Examples

1. The table below shows the humidity level recorded in Freetown for 7 days. Construct a graph which best demonstrates the humidity level for each day.

HUMIDITY LEVELS IN FREETOWN	
Day	Humidity Level (%)
1	51
2	59
3	65
4	68
5	70
6	67
7	72

Solution

Humidity tells us the amount of water vapour, or moisture, in the air. We want to compare different values, and we can see that the data is changing over time. A line graph would be the best choice for displaying this data.



2. Martin took a survey of his classmates to understand what field they want to work in when they finish school. He found that 25% want to work in education, 20% chose health, 30% chose agriculture, 15% chose law, and the rest chose other fields. Display this information in a chart or graph.

Solution

We are dealing with percentages as parts of a whole. We will use a pie chart to represent all of Martin's survey participants. There will be 5 sectors: education, health, agriculture, law and other.

Find the size of 'other' by subtracting the known percentages from 100%: $100 - 25 - 20 - 30 - 15 = 10\%$

Find the measure of the angle in each sector, and use a protractor to draw them accurately.

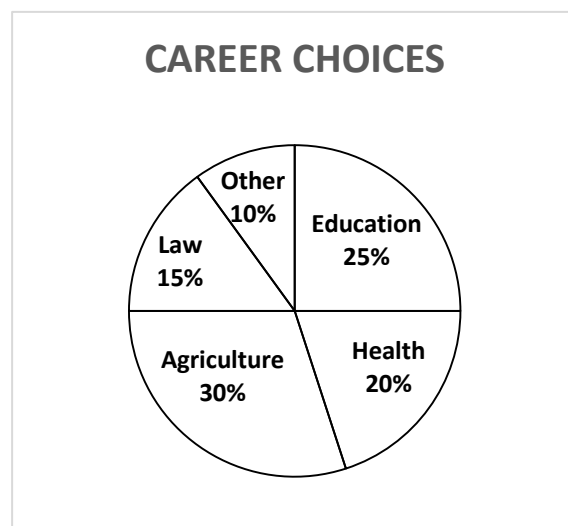
$$\text{Education: } \frac{25}{100} \times 360^\circ = 90^\circ$$

$$\text{Health: } \frac{20}{100} \times 360^\circ = 72^\circ$$

$$\text{Agriculture: } \frac{30}{100} \times 360^\circ = 108^\circ$$

$$\text{Law: } \frac{15}{100} \times 360^\circ = 54^\circ$$

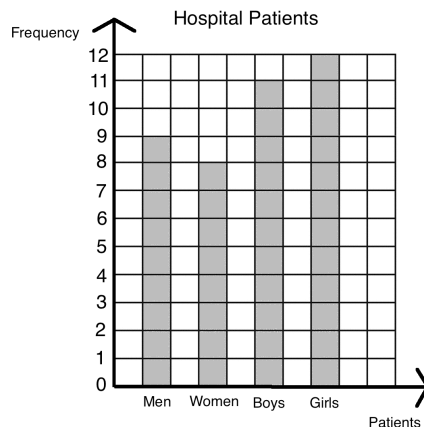
$$\text{Other: } \frac{10}{100} \times 360^\circ = 36^\circ$$



3. Dr. Bangura wants to create a chart or graph to show the patients admitted to the hospital this week. This week there were 9 men, 8 women, 11 boys and 12 girls admitted to the hospital. Display this information for her.

Solution

Dr. Bangura wants to compare different amounts. A bar chart is the best option. A pie chart could also be used to show parts of the whole. We would need to calculate percentages for each type of patient. The bar chart is shown below:



Practice


- David kept track of his spending this month. He found that he spent 20% on transportation, 30% on food, 35% on rent, and the rest on entertainment. Draw a graph or chart to show David’s spending.
- The principal of a secondary school collected the data in the table below. Help her display her data in a graph or chart.

Pupil Population	
Class	Number of Pupils
JSS 1	35
JSS 2	40
JSS 3	38
SSS 1	34
SSS 2	30
SSS 3	28
SSS 4	25

- Hawa is interested in studying the weather. She measured the temperature in her village for 6 days and recorded it in the table below. Help her display her data in a graph or chart.

Date	8 May	9 May	10 May	11 May	12 May	13 May
Temperature (°C)	30	31	33	29	30	35

Lesson Title: Mean	Theme: Statistics and Probability
Practice Activity: PHM-09-124	Class: JSS 3

	<p>Learning Outcomes</p> <p>By the end of the lesson, you will be able to:</p> <ol style="list-style-type: none"> 1. Calculate the mean of a set of data from a list, chart, or graph. 2. Interpret mean.
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Overview

The **mean** is a number that can tell us where the middle of the data is. It is also commonly known as the ‘average’. To find the mean of a **list** of data, add the numbers together and divide the total by the number of items.

To find the mean from a frequency table, add the frequencies given and divide by the total number of rows. You will follow a similar process to find the mean from a bar chart. The heights of the bars give the frequencies, so you will add the bar heights and divide by the number of bars.

You will also follow a similar process to find the mean from a line graph. Add the value of each point on the *y*-axis, and divide by the number of points.

Solved Examples

1. Mustapha has 5 children aged 10, 6, 7, 4, and 13. Find the mean of their ages.

Solution

Add the numbers: $10 + 6 + 7 + 4 + 13 = 40$

Divide by the number of children: $40 \div 5 = 8$

The mean age of Mustapha’s children is 8 years old.

2. Ten pupils received the following scores on their maths exam: 87, 100, 76, 92, 90, 95, 85, 67, 99 and 95. Find the mean of the scores.

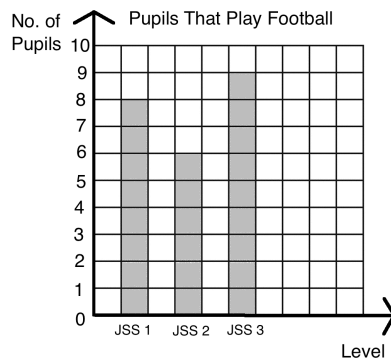
Solution

Add the scores and divide by the number of pupils, 10:

$$\frac{87+100+76+92+90+95+85+67+99+95}{10} = \frac{886}{10} = 88.6$$

The mean score is 88.6.

3. The bar chart below shows the number of JSS pupils that play football on the school team from each class. Find the mean number of players in each class.



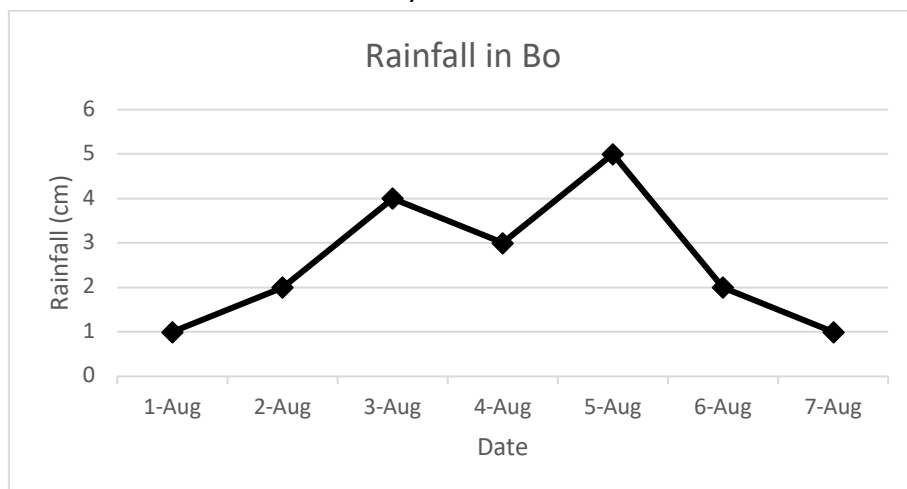
Solution

The heights of the bars give the number of pupils on the football team in each class. You may record them in a list: 8, 6, 9.

Find the mean by adding the number of pupils per class, and dividing by the number of classes: $\frac{8+6+9}{3} = \frac{23}{3} = 7\frac{2}{3}$

The mean number of pupils on the team from each class is 8 pupils. We can round it to a whole number because we are dealing with human beings.

4. The line graph below shows the rainfall in Bo over a 1-week period. Find the mean amount of rain that Bo received each day.



Solution

You may record the rainfall for each day in a list: 1, 2, 4, 3, 5, 2, 1

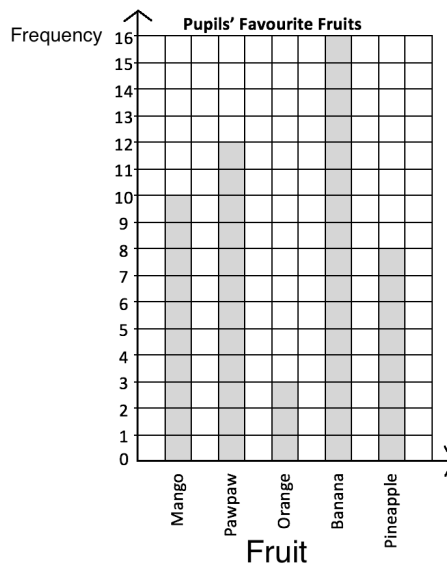
Find the mean by adding the rainfall for each day, and dividing by the number of days:

$$\frac{1+2+4+3+5+2+1}{7} = \frac{18}{7} = 2\frac{4}{7}$$

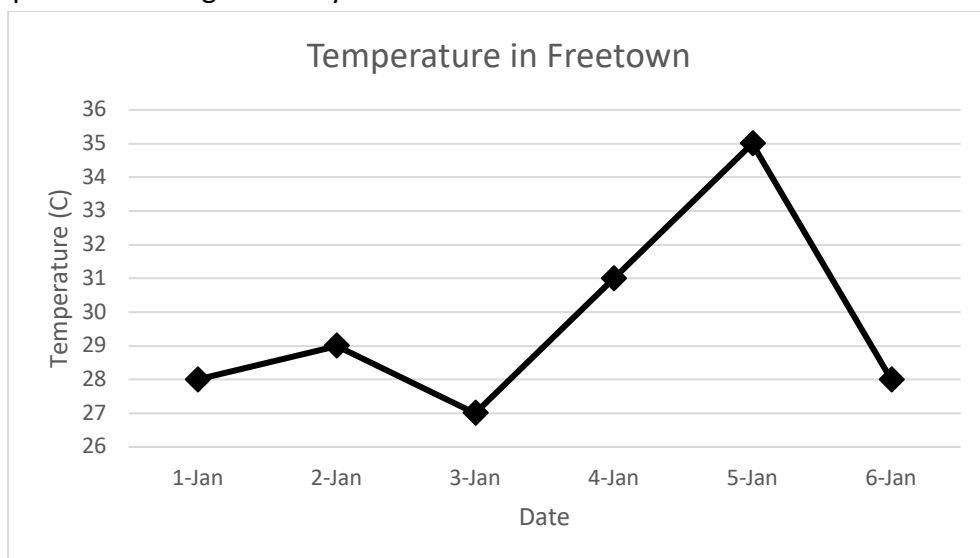
The mean amount of rain was $2\frac{4}{7}$ cm per day.

Practice

1. Agnes received the following scores in examinations in 8 subjects: 67, 75, 80, 56, 77, 68, 98, and 87. Calculate the mean of her scores, correct to the nearest whole number.
2. The shoe sizes of five pupils are 10, 9, 10, 11 and 8. Calculate the mean shoe size, correct to 1 decimal place.
3. The number of goals scored by a team in 9 football matches are as follows: 3, 5, 7, 7, 8, 8, 8, 11, 15. Calculate the mean number of goals scored.
4. Fatu took a survey of her classmates and recorded their favourite fruits in the bar chart below. Find the mean number of pupils who prefer each fruit.



5. The line graph below gives the temperature in Freetown over 6 days. Find the average temperature during the 6 days.



Lesson Title: Median	Theme: Statistics and Probability
Practice Activity: PHM-09-125	Class: JSS 3



Learning Outcomes

By the end of the lesson, you will be able to:

1. Calculate the median of a set of data from a list, chart, or graph.
2. Interpret median.

Overview

The number in the middle when the numbers are listed in ascending or descending order is called the **median**. When there is an even number of items in the list, there are two numbers in the middle. The median is found by calculating the mean of these two numbers.

To find the median from a chart or graph, identify the bar or point with its height in the middle.

Solved Examples

1. Mustapha has 5 children aged 10, 6, 7, 4, and 13. Find the median of their ages.

Solution

List the numbers in ascending order: 4, 6, 7, 10, 13

Identify the middle of the list: 7

The median age of Mustapha's children is 7 years old.

2. Ten pupils received the following scores on their maths exam: 87, 100, 76, 92, 90, 95, 85, 67, 99 and 95. Find the median of the scores.

Solution

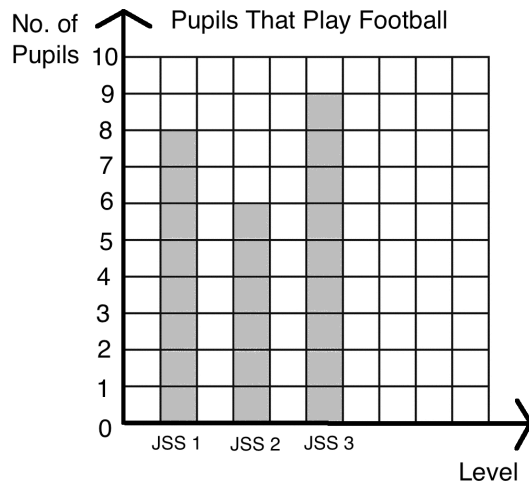
List the numbers in ascending order: 67, 76, 85, 87, 90, 92, 95, 95, 99, 100

Identify the middle of the list: 90, 92

Since there is not one number in the middle, find the mean of the 2 numbers in the

middle. Add them together and divide by 2: median = $\frac{90+92}{2} = 91$

3. The bar chart shows the number of JSS pupils that play football on the school team from each class. Find the median number of players in each class.



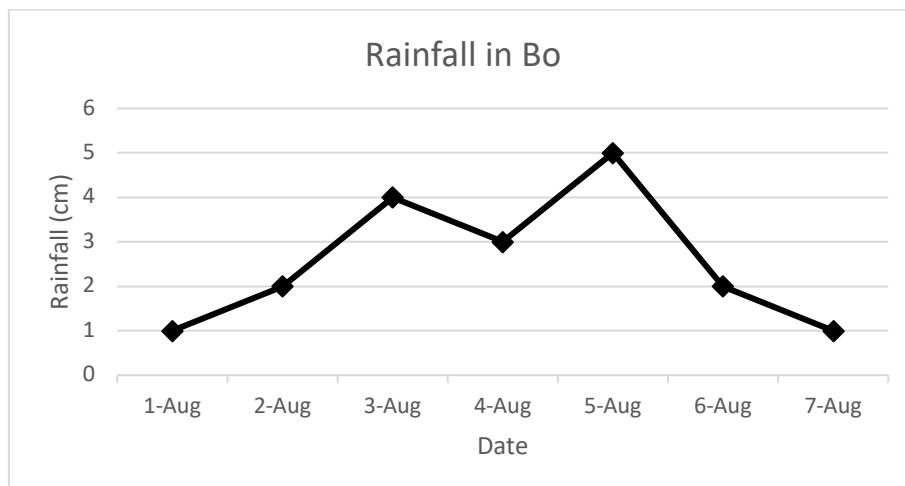
Solution

The heights of the bars give the number of pupils on the football team in each class. You may record them in a list: 8, 6, 9.

Write the numbers in ascending order: 6, 8, 9.

The median is the number in the middle, 8 pupils.

4. The line graph below shows the rainfall in Bo over a 1-week period. What is the median amount of rain fall?



Solution

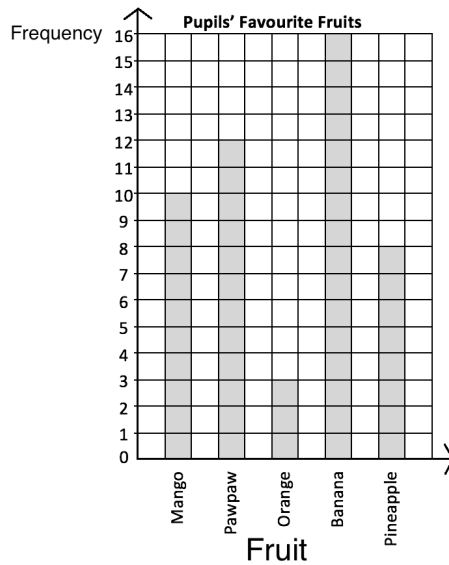
You may record the rainfall for each day in a list: 1, 2, 4, 3, 5, 2, 1

Write the list of numbers in ascending order: 1, 1, 2, 2, 3, 4, 5

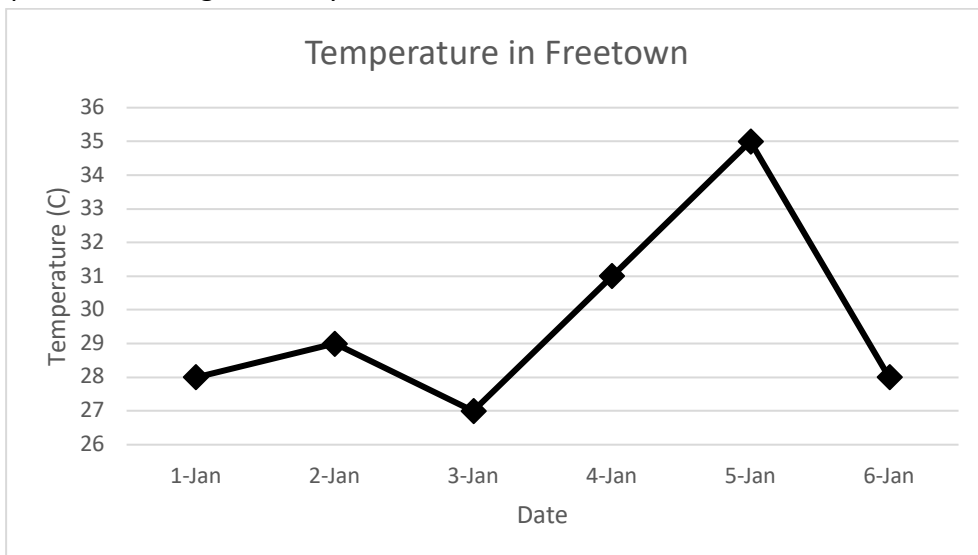
The median rainfall was 2 cm. This is the number in the middle.

Practice


1. Agnes received the following scores in examinations in 8 subjects: 67, 75, 80, 56, 77, 68, 98, and 87. Calculate the median of her scores, correct to the nearest whole number.
2. The shoe sizes of five pupils are 10, 9, 10, 11 and 8. Find the median shoe size.
3. The number of goals scored by a team in 9 football matches are as follows: 3, 5, 7, 7, 8, 8, 8, 11, 15. Calculate the median number of goals scored.
4. Fatu took a survey of her classmates and recorded their favourite fruits in the bar chart below. Find the median number of pupils who prefer each fruit.



5. The line graph below gives the temperature in Freetown over 6 days. Find the median temperature during the 6 days.



Lesson Title: Mode and Range	Theme: Statistics and Probability
Practice Activity: PHM-09-126	Class: JSS 3

	<p>Learning Outcomes</p> <p>By the end of the lesson, you will be able to:</p> <ol style="list-style-type: none"> 1. Calculate the mode and range of a set of data from a list, chart, or graph. 2. Interpret mode and range.
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Overview

In the previous lessons, you calculated the mean and median of a list of numbers. In this lesson, you will calculate the mode and range. These can also be calculated from a list, chart or graph.

The **mode** is the number that appears most often in a list. It can often be easily observed. If no number appears more than once, there is no mode. If multiple numbers appear more than once, there are multiple modes.

The **range** is the difference between the highest and lowest numbers. It tells us how spread apart our numbers are. To find the range, subtract the lowest number from the highest number.

Solved Examples

1. Find the mode and range of the list of numbers: 2, 1, 7, 5, 6, 8, 6, 9, 6, 9

Solution

It is helpful to write the numbers in ascending order. It becomes easy to see the lowest and highest numbers, and the mode.

In ascending order, we have: 1, 2, 5, 6, 6, 6, 7, 8, 9, 9.

Mode: The mode is the number that appears most often. The number 6 appears 3 times, so 6 is the mode.

Range: Subtract the lowest number from the highest number: $9 - 1 = 8$. The range is 8.

2. The ages of 10 pupils are listed below. Find the mode and range of their ages.

10, 11, 13, 10, 10, 12, 11, 13, 11, 12

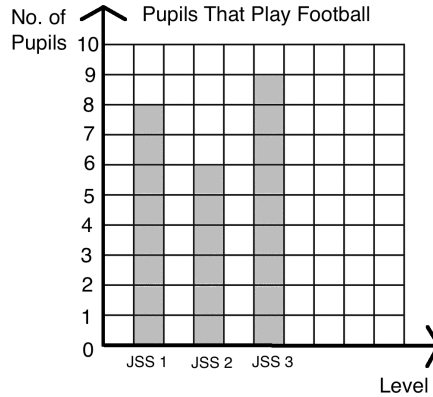
Solution

Write the numbers in ascending order: 10, 10, 10, 11, 11, 11, 12, 12, 13, 13

Mode: The numbers 10 and 11 both appear 3 times. This is the greatest number of times any number appears. There are 2 modes: 10 and 11 years old.

Range: Subtract the lowest number from the highest number: $13 - 10 = 3$. The range is 3 years.

3. The bar chart below shows the number of JSS pupils that play football on the school team from each class. Find the mode and range of the number of players in each class.



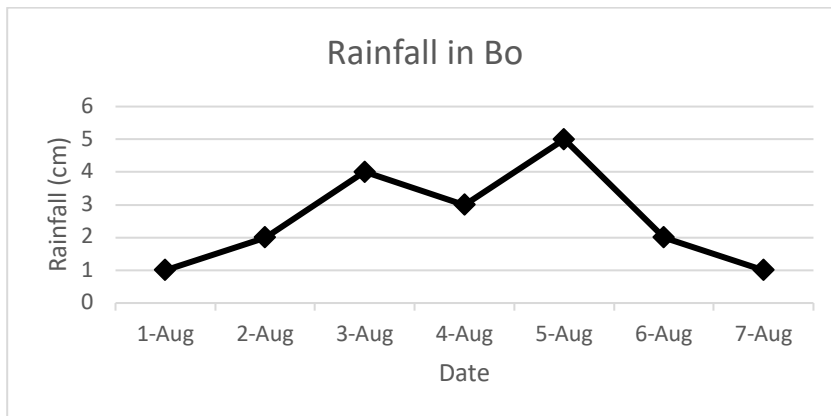
Solution

The number of pupils on the football team in each class are: 8, 6, 9.

Mode: There is **no** mode, because each number appears only once.

Range: Subtract the lowest number from the highest number: $9 - 6 = 3$. The range is 3 pupils.

4. The line graph below shows the rainfall in Bo over a 1-week period. What is the median amount of rain fall?



Solution

The rainfall for each day is: 1, 1, 2, 2, 3, 4, 5

Mode: There are 2 modes, 1 cm and 2 cm. These amounts appear twice.

Range: Subtract the lowest number from the highest number: $5 - 1 = 4$. The range is 4 cm.

Practice

1. The heights of 15 pupils are given in centimetres. Calculate the mode and range of the data:

155, 161, 160, 157, 155, 159, 160, 156, 155, 162, 158, 157, 161, 163, 153

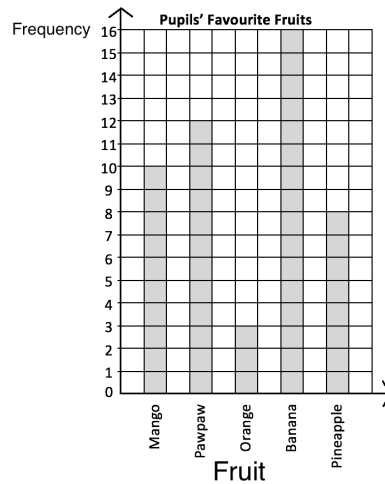
2. Hawa is a doctor. Today, she treated 10 children. She recorded the weight of each child in kilogrammes, listed below. Find the mode and range of their weights.

14, 20, 17, 21, 15, 13, 20, 19, 15, 12

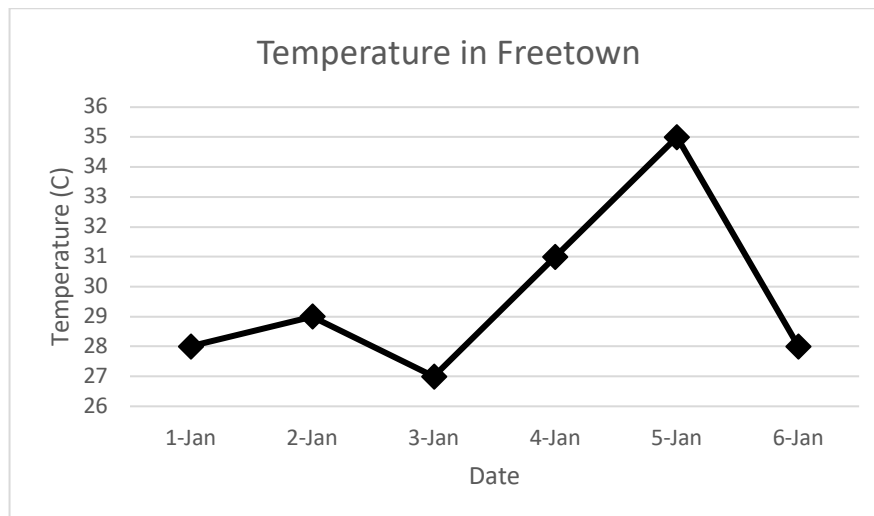
3. Mustapha sells watermelons from his farm. He recorded the number of watermelons that he sold each day for a week. Find the mode and range of the data.

5, 12, 14, 7, 9, 10, 11


4. Fatu took a survey of her classmates and recorded their favourite fruits in the bar chart below. Find the mode and range in number of pupils who prefer each fruit.



5. The line graph below gives the temperature in Freetown over 6 days. Find the mode and range in temperature during the 6 days.



Lesson Title: Introduction to Grouped Data	Theme: Statistics and Probability
Practice Activity: PHM-09-127	Class: JSS 3



Learning Outcomes

By the end of the lesson, you will be able to:

1. Identify that 'grouped data' involves dividing a set of data into groups, or 'class intervals'.
2. Create a frequency table for grouped data.

Overview

Previous statistics lessons on lists and bar charts have handled ungrouped data. This is the first lesson on **grouped data**. Grouped data is divided into **class intervals** that are equal in size, and each cover a range of values.

Grouped data can be arranged in a frequency table. Each class interval is represented on one row.

For example, consider the grouped frequency table:

Pupils' Heights	
Height	Frequency
150 – 159 cm	8
160 – 169 cm	7
170 – 179 cm	5
Total	20

There are 3 class intervals represented in the table: 150-159 cm, 160-169 cm, and 170-179 cm. The table tells us how many pupils fall into each class interval. For example, there are 8 pupils in the class interval 150-159 cm. We do not know their exact heights.

Solved Examples

1. The frequency table below shows the scores that one class of pupils received on an exam. Use the table to answer the questions.

Pupils' Scores on an Exam	
Marks (%)	Frequency
41-50	3
51-60	2
61-70	4
71-80	11
81-90	10
91-100	5

- a. How many pupils are in the class?
- b. If pupils need to score more than 60% to pass, how many pupils passed?
- c. How many pupils failed?

d. How many pupils scored more than 80%?

Solutions

- a. Add the frequencies to find the total number of pupils: $3 + 2 + 4 + 11 + 10 + 5 = 35$ pupils
- b. Add the frequencies for class intervals greater than 60%: $4 + 11 + 10 + 5 = 30$ pupils
- c. Add the frequencies for class intervals 60% or lower: $3 + 2 = 5$ pupils
- d. Add the frequencies for class intervals greater than 80%: $10 + 5 = 15$ pupils

2. The following are the shoe sizes of 25 adults: 32, 35, 38, 41, 43, 42, 45, 38, 39, 39, 36, 34, 37, 42, 36, 41, 34, 33, 42, 36, 35, 40, 44, 37, 35.

- a. Draw a frequency table using class intervals 32-34, 35-37, 38-40, 41-43, 44-46.
- b. Which interval do the greatest number of pupils fall into?
- c. A certain type of shoe is only available in size 40 or smaller. How many people could wear that shoe?

Solutions

a. First, write the shoe sizes in ascending order: 32, 33, 34, 34, 35, 35, 35, 36, 36, 36, 37, 37, 38, 38, 39, 39, 40, 41, 41, 42, 42, 42, 43, 44, 45.

Next, draw and fill the frequency table. You may use tally marks to keep count because there are many numbers in the list.

Adults' Shoe Sizes		
Size	Tally	Frequency
32-34		4
35-37		8
38-40		5
41-43		6
44-46		2
Total		25

- b. The greatest number of pupils fall into size 35-37.
- c. Add the frequencies in sizes 40 and smaller: $4 + 8 + 5 = 17$ people

Practice

1. The following table gives the distribution of the ages in years of all persons in a town who were under the ages of 40 years on 30 June, 1990. The frequency represents thousands of people.

Age	Frequency (thousands)
-----	-----------------------

0 – 4	2
5 – 9	3
10 – 14	6
15 – 19	15
20 – 24	12
25 – 29	7
30 – 34	4
35 – 39	1

Using the table, calculate:

- a. The total number of persons between 15 and 29 years.
 - b. The total number of persons under 20 years.
2. The following gives the weights of 50 children in a sports camp.

64	14	17	48	58	60	43	44	15	32
47	21	23	37	51	26	18	36	22	24
43	45	29	33	46	38	19	38	36	30
31	72	57	41	41	44	54	24	26	41
22	25	35	35	37	36	51	52	19	62

Using class intervals 10 – 19, 20 – 29, ... prepare a frequency table and use it to calculate the:


- a. Total number of children with weights less than 30.
 - b. Total number of children weighing between 50 and 79.
3. The table below shows the distribution of times spent by motorists awaiting purchases of petrol at a filling station.

Waiting Time (Min)	No. of Motorists
15 – 19	3
20 – 24	7
25 – 29	10
30 – 34	25
35 – 39	18
40 – 44	2

Calculate:

- a. Total number of motorists who purchased petrol.
- b. Total number of motorists awaiting purchase of petrol at the filling station for 30 to 44 minutes.

Lesson Title: Mean of Grouped Data	Theme: Statistics and Probability
Practice Activity: PHM-09-128	Class: JSS 3

	<p>Learning Outcomes</p> <p>By the end of the lesson, you will be able to estimate the mean of grouped data from a frequency table using the formula: $\bar{x} = \frac{\sum fx}{\sum f}$.</p>
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Overview

When data is divided into groups, we cannot determine the value of each piece of data in the set. Therefore, we cannot determine the exact mean. We can **estimate** the mean of grouped data using a formula.

The formula for estimating mean is $\bar{x} = \frac{\sum fx}{\sum f}$, where f is frequency, and x is the corresponding class mid-point.

The sigma symbol (Σ) tells us to find the sum. The numerator tells us to find the sum of each frequency multiplied by each corresponding class mid-point. The denominator tells us to find the sum of the frequencies.

The class mid-point is the point in exactly the middle of the class interval. For example, if a class interval is 0 – 4, the mid-point is 2. The mid-point can be found by counting, or by finding the mean of the 2 endpoints of the interval: $\frac{0+4}{2} = 2$.

Solved Examples

1. In one village, 15 farmers have harvested their pepper. The table below shows the amount of pepper they harvested in kilogrammes. Estimate how much pepper each farmer harvested on average.

Farmers' Harvests	
Pepper (kg)	Frequency
0 – 4	2
5 – 9	5
10 – 14	4
15 – 19	3
20 – 24	1
Total	15

Solution

First, find each class mid-point. These can be added to the table.

Farmers' Harvests		
Pepper (kg)	Frequency (f)	Mid-point (x)

0 – 4	2	2
5 – 9	5	7
10 – 14	4	12
15 – 19	3	17
20 – 24	1	22
Total	15	

Use the frequencies and mid-points to calculate the estimated mean:

$$\begin{aligned}
 \bar{x} &= \frac{\sum fx}{\sum f} = \frac{(2 \times 2) + (5 \times 7) + (4 \times 12) + (3 \times 17) + (1 \times 22)}{2 + 5 + 4 + 3 + 1} \\
 &= \frac{4 + 35 + 48 + 51 + 22}{15} \\
 &= \frac{160}{15} \\
 &= 10.67 \text{ to 2 d.p.}
 \end{aligned}$$

2. The following are the weights in kilogrammes of 50 children in a sports camp.

64 14 17 48 58 60 43 44 15 32
 47 21 23 37 51 26 10 36 22 24
 43 45 29 33 46 38 19 38 36 30
 31 72 57 41 41 44 54 24 26 41
 22 25 35 35 37 36 51 52 19 62

Using class intervals 10 – 19, 20 – 29, ... prepare a frequency table and use it to calculate the mean weight.

Solution

Another way to do the calculations is to add columns to the frequency table for both the mid-point, and the mid-point multiplied by the frequency. This is shown in the table below. You can find the sums $\sum fx$ and $\sum f$ by simply adding the values in the table. Then, you can do the calculation as shown below the table.

Weight (kg)	Mid-point (x)	Frequency	fx
10 – 19	14.5	6	87
20 – 29	24.5	10	245
30 – 39	34.5	13	448.5
40 – 49	44.5	11	489.5
50 – 59	54.5	6	327
60 – 69	64.5	3	193.5
70 – 79	74.5	1	74.5
Total		$\sum f = 50$	$\sum fx = 1,865$

$$\text{Mean: } \bar{x} = \frac{\sum fx}{\sum f} = \frac{1865}{50} = 37.3$$

Practice

1. The frequency table below gives the weights of 15 players on the football team. Use the table to estimate the mean weight of the football players.

Football Players' Weights	
Weight (kg.)	Frequency
50 – 54	1
55 – 59	3
60 – 64	5
65 – 69	4
70 – 74	2
Total	15

2. The numbers below are the ages of 24 patients in a hospital:

3 2 10 15 12 9 8 20
7 3 14 16 4 6 16 18
5 15 21 19 2 1 8 12


- a. Construct a frequency table, using class intervals 0 – 4, 5 – 9, and so on.
b. Calculate the mean.

3. The marks obtained in a test by 40 pupils are as follows:

78 42 30 33 66 39 72 60
54 44 42 24 45 60 72 30
39 36 66 33 36 84 18 45
27 78 33 42 67 63 30 63
60 27 30 81 27 33 72 18

- a. Construct a frequency table, using class intervals of 10 – 19, 20 – 29, 30 – 39, and so on.
b. Calculate the mean.

Lesson Title: Median and Modal Class of Grouped Data	Theme: Statistics and Probability
Practice Activity: PHM-09-129	Class: JSS 3



Learning Outcomes
By the end of the lesson, you will be able to:

1. Identify the modal class as the class interval with the highest frequency.
2. Estimate the median of grouped data from a frequency table using the formula:

$$L_m + \left[\frac{\frac{n}{2} - F_{m-1}}{f_m} \right] \times c$$

Overview

In this lesson, you will learn how to identify the modal class as the class interval with the highest frequency, and estimate the median of grouped data from a frequency table.

The **modal class** can easily be found by looking at the grouped frequency table, and noting the class interval with the greatest frequency.

The **median class** is the class interval that contains the median, or the value in the middle of the data. We can find the class interval that the median falls into using its position. When data is grouped, we do not know the exact values and we must estimate the median from information that we know. We estimate the median of grouped data using a formula.

The **estimated median** is given by the formula: $L_m + \left[\frac{\frac{n}{2} - F_{m-1}}{f_m} \right] \times c$, where:

- L_m is the lower class boundary of the group containing the median.
- n is the total frequency of the data.
- F_{m-1} is the total frequency for the groups **before** the median group.
- f_m is the frequency of the median group.
- c is the group width.

Solved Examples

1. In one village, 17 farmers have just harvested cassava. The table shows the amount of cassava they harvested in kilogrammes.
 - a. Find the modal class.
 - b. Find the median class.
 - c. Estimate the median amount of cassava that was harvested.

Farmers' Harvests	
Cassava (kg)	Frequency
10 – 14	1
15 – 19	3
20 – 24	6
25 – 29	5
30 – 34	2
Total	17

Solutions

- a. The modal class is 20 – 24, because it has the greatest frequency.

- b. Since there are 17 farmers, the 9th farmer has the median harvest. Eight farmers harvested more, and 8 farmers harvested less. The 9th farmer is in interval 20-24. The median class is 20-24.
- c. Identify the value for each variable in the formula, substitute them, and evaluate.
- L_m is the lower class boundary of 20-24. That is, the lowest possible value in the class boundary. We have $L_m = 20$.
 - n is the total frequency of the data, 17.
 - F_{m-1} is the total frequency before the median class. Add to find it: $F_{m-1} = 1 + 3 = 4$.
 - f_m is the frequency of the median class, which $f_m = 6$.
 - c is the group width. The width of each group here is 5 units. For example, 10-14 contains 10, 11, 12, 13, 14.

Substitute the values into the formula and evaluate:

$$\begin{aligned}
 \text{Median} &= L_m + \left[\frac{\frac{n}{2} - F_{m-1}}{f_m} \right] \times c &= 20 + \left[\frac{\frac{17}{2} - (1+3)}{6} \right] \times 5 && \text{Substitute values} \\
 & &= 20 + \left[\frac{8.5 - 4}{6} \right] \times 5 && \text{Simplify} \\
 & &= 20 + \left[\frac{4.5}{6} \right] \times 5 \\
 & &= 20 + 0.75 \times 5 \\
 & &= 20 + 3.75 \\
 & &= 23.75
 \end{aligned}$$

The estimated median is 23.75 kg of cassava.

2. The data below shows the age distribution of dwellers in a village.
- Identify the modal class.
 - Identify the median class.
 - Estimate the median age of the villagers.

Age	Frequency
20 – 29	12
30 – 39	13
40 – 49	39
50 – 59	28
60 – 69	11
70 – 79	7
80 – 89	10

Solutions

- The modal class is 40 – 49, because it has the greatest frequency.
- Add the frequencies to find the total number of residents: $12 + 13 + 39 + 28 + 11 + 7 + 10 = 120$. The median is the mean of the 60th and 61st positions. These both lie in the interval 40 – 49. The median class is 40 – 49.
- Identify the values of variables that will be used in calculating the median.

$$\begin{aligned}
 \text{Median} &= L_m + \left[\frac{\frac{n}{2} - F_{m-1}}{f_m} \right] \times c = 40 + \left[\frac{\frac{120}{2} - (12+13)}{39} \right] \times 10 && \text{Substitute values} \\
 &= 40 + \left[\frac{60-25}{39} \right] \times 10 && \text{Simplify} \\
 &= 40 + \left[\frac{35}{39} \right] \times 10 \\
 &= 40 + 0.9 \times 10 \\
 &= 40 + 9 \\
 &= 49 \text{ years old}
 \end{aligned}$$

Practice

- The table below shows the ages of teachers from a certain school.
 - Identify the modal class.
 - Identify the median class.
 - Estimate the median age, correct to the nearest whole number.

Age(years)	25-29	30-34	35-39	40-44	45-49	50-54	55-59
Frequency	1	2	4	5	3	3	2


- The marks scored by 20 pupils in an examination are given in the frequency table below.
 - Identify the modal class.
 - Identify the median class.
 - Estimate the median, correct to 1 decimal place.

Marks	0 – 4	5 – 9	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34
No of pupils	2	5	2	7	2	1	1

- The table below shows the height, in millimeters, of a sample of 250 seedlings on an experimental farm.
 - Identify the modal class.
 - Identify the median class.
 - Estimate the median height.

Seedling height (mm)	Frequency
0 – 4	40
5 – 9	45
10 – 14	60
15 – 19	40
20 – 24	30
25 – 29	20
30 – 34	10
35 – 39	5

Lesson Title: Practice with Grouped Data	Theme: Statistics and Probability
Practice Activity: PHM-09-130	Class: JSS 3

	<p>Learning Outcome By the end of the lesson, you will be able to estimate the mean, median, and modal class for grouped data.</p>
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Overview

This lesson is on the practise of estimating the mean, median, and modal class of grouped data. Recall that the mean and median are estimated using the formula:

$$\text{Estimated mean} = \bar{x} = \frac{\sum fx}{\sum f}$$

$$\text{Estimated median} = L_m + \left[\frac{\frac{n}{2} - F_{m-1}}{f_m} \right] \times c$$

The median is estimated using data from the median class, which is the class interval that the median falls into.

The modal class of grouped data is the class with the greatest frequency.

Solved Examples

1. The table below shows the height distribution of 40 children in a village.
 - a. Estimate the mean to 1 decimal place.
 - b. Identify the modal class.
 - c. Identify the median class.
 - d. Estimate the median to 1 decimal place.

Height (cm)	110-118	119-127	128-136	137-145	146-154	155-163	164-172
Frequency	9	3	4	5	2	5	12

Solutions

- a. Estimate the mean using the formula. Note that the class mid-points are 114, 123, 132, 141, 150, 159 and 168.

$$\begin{aligned} \bar{x} = \frac{\sum fx}{\sum f} &= \frac{(9 \times 114) + (3 \times 123) + (4 \times 132) + (5 \times 141) + (2 \times 150) + (5 \times 159) + (12 \times 168)}{9 + 3 + 4 + 5 + 2 + 5 + 12} \\ &= \frac{1026 + 369 + 528 + 705 + 300 + 795 + 2,016}{40} \\ &= \frac{5,739}{40} \\ &= 143.5 \text{ cm to 1 d.p.} \end{aligned}$$

- b. The modal class is the class with the highest frequency, 164 – 172.

- c. The median falls into class interval 137-145. Since there are 40 children, the median height is the average of the 20th and 21st children's height. The 20th and 21st children are in interval 137-145.
- d. Identify the value for each variable in the formula, substitute them, and evaluate.

$$\begin{aligned}
 \text{Median} = L_m + \left[\frac{\frac{n}{2} - F_{m-1}}{f_m} \right] \times c &= 137 + \left[\frac{\frac{40}{2} - (9+3+4)}{5} \right] \times 9 && \text{Substitute values} \\
 &= 137 + \left[\frac{20-16}{5} \right] \times 9 && \text{Simplify} \\
 &= 137 + \left[\frac{4}{5} \right] \times 9 \\
 &= 137 + 0.8 \times 9 \\
 &= 137 + 7.2 \\
 &= 144.2 \text{ cm}
 \end{aligned}$$

2. The table shows the ages of 25 patients in a hospital.
- Estimate the mean age of the patients.
 - Identify the modal class.
 - Identify the median class.
 - Estimate the median age of the patients.

Age (years)	Frequency
1 – 3	5
4 – 6	7
7 – 9	3
10 – 12	5
13 – 15	3
16 – 18	2

Solutions

- a. Use the formula to estimate the mean. Note that the class mid-points are 2, 5, 8, 11, 14 and 17.

$$\begin{aligned}
 \bar{x} = \frac{\sum fx}{\sum f} &= \frac{(5 \times 2) + (7 \times 5) + (3 \times 8) + (5 \times 11) + (3 \times 14) + (2 \times 17)}{5 + 7 + 3 + 5 + 3 + 2} && \text{Substitute frequencies and mid-points} \\
 &= \frac{10 + 35 + 24 + 55 + 42 + 34}{25} && \text{Simplify} \\
 &= \frac{200}{25} \\
 &= 8 \text{ years old}
 \end{aligned}$$

- b. The modal class is 4 – 6, which has the greatest frequency.
- c. There are 25 patients, so the 13th patient is in the middle. The 13th patient falls into interval 7 – 9, which is the median class.
- d. Estimate the median using the formula:

$$\begin{aligned}
 \text{Median} = L_m + \left[\frac{\frac{n}{2} - F_{m-1}}{f_m} \right] \times c &= 7 + \left[\frac{\frac{25}{2} - (5+7)}{3} \right] \times 3 && \text{Substitute values} \\
 &= 7 + \left[\frac{12.5 - 12}{3} \right] \times 3 && \text{Simplify} \\
 &= 7 + \left[\frac{0.5}{3} \right] \times 3 \\
 &= 7 + 0.5 \\
 &= 7.5 \text{ years old}
 \end{aligned}$$

Practice

1. The table below shows the ages of teachers from a certain school. Estimate, correct to the nearest whole number the:
- Mean
 - Median

Age (Years)	Frequency
25 – 29	1
30 – 34	2
35 – 39	4
40 – 44	5
45 – 49	3
50 – 54	3
55 – 59	2


2. The table below shows the frequency distribution of marks scored by some pupils in an examination.
- Estimate the mean, correct to 1 decimal place.
 - Identify the modal class.
 - Identify the median class.
 - Estimate the median, correct to 1 decimal place.

Marks	21 – 30	31 – 40	41 – 50	51 – 60	61 - 70	71 – 80
Frequency	2	5	12	15	10	6

3. The distribution below gives the weights (in kg) of bags in a store. Estimate the mean and median of the distribution to 2 decimal places.

Weight (kg)	10 – 21	22 – 33	34 – 45	46 – 57	58 – 69
No. of bags	15	5	21	10	29

Lesson Title: Probability	Theme: Statistics and Probability
Practice Activity: PHM-09-131	Class: JSS 3

	<p>Learning Outcomes</p> <p>By the end of the lesson, you will be able to:</p> <ol style="list-style-type: none"> 1. Identify that probability describes the chance of something happening. 2. Discuss the probability of an event happening in words.
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Overview

In math, we can study the possibility of something happening. We call this **probability**. In probability, we use numbers and math language to show the chances of something happening. In this lesson, you will describe the probability of different events happening in words.

There are 4 words that you will use in this lesson to describe probability: **Impossible, unlikely, likely** and **certain**.

These are some words we use to talk about the chances of something happening. Impossible means that something will not happen. Unlikely means that it probably won't happen, and likely means that it probably will happen. Certain means that it will definitely happen.

Solved Examples

1. Determine whether each sentence is impossible, unlikely, likely or certain:
 - a. A fish will walk on land.
 - b. If you study hard, you will pass the maths exam.
 - c. If you travel on the highway you will get into a car accident.
 - d. The sun will rise in the morning.

Solutions

- a. **Impossible**. Fish can't walk.
 - b. **Likely**. The more you study, the more likely it is that you will pass the exam.
 - c. **Unlikely**. A car accident could happen, but many people travel on the highway everyday without being in an accident.
 - d. **Certain**. The sun rises each day.
2. What is the probability that next year is 2007?

Solution

It is **impossible**. 2007 is in the past.

3. What is the probability that there will be a Tuesday next week?

Solution

It is **certain**. There is a Tuesday in each week.

4. What is the probability that it will rain tomorrow?


Solution

The answer depends on the season. If it is currently the rainy season, it is likely that it will rain tomorrow. If it is currently the dry season, it is unlikely that it will rain tomorrow.

Practice

1. Determine whether each sentence is impossible, unlikely, likely or certain:
 - a. A mother is younger than her daughter.
 - b. Your hair will become grey someday.
 - c. The capital city of Sierra Leone is Freetown.
 - d. The well is dry in the rainy season.
2. What is the probability that a monkey falls from a tree?
3. What is the probability that there will be a February next year?
4. What is the probability that a dog has 8 legs?
5. Write 1 statement of your own for each of the following probabilities:
 - a. Impossible
 - b. Unlikely
 - c. Likely
 - d. Certain

Lesson Title: Probability Experiments with 1 Event	Theme: Statistics and Probability
Practice Activity: PHM-09-132	Class: JSS 3

	<p>Learning Outcomes By the end of the lesson, you will be able to:</p> <ol style="list-style-type: none"> 1. Conduct simple probability experiments 2. Use probability terms such as 'experiment', 'outcome' and 'event'.
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Overview

In this lesson, you will learn how we use experiments to understand probability in mathematics. Before we can do calculations with probability, we must understand experiments. It is important to learn the correct words for talking about probability.

In probability, an **experiment** is a situation involving chance. An experiment leads to results called **outcomes**.

For example, I can toss a coin as an experiment. There is a chance that the coin will land on heads, and a chance that it will land on tails. This is an experiment involving chance. There are 2 possible outcomes: heads or tails. An outcome is a single result of an experiment. It is something that could possibly happen.



In probability, the outcomes of an experiment can also be called **events**. Events can involve one outcome or more than one outcome.

For example, consider a situation where you randomly select a number between 1 and 5. Selecting the number 3 is an outcome, and it is also an event. Selecting an even number is also an event. We could select 2 or 4. They are two different outcomes, but they are both even numbers, so in this case we can describe this as one event.

Solved Examples

1. Fatu rolls a 6-sided die. What are the possible outcomes?



Solution

A 6-sided die has faces with numbers 1-6. There are 6 possible outcomes when rolling a die: 1, 2, 3, 4, 5 or 6.

2. Foday rolls a 6-sided die. Determine how many possible outcomes there are in each event:
 - a. Foday rolls a 1.
 - b. Foday rolls an even number.
 - c. Foday rolls a number greater than 3.

- d. Foday rolls a 3 or 4.
- e. Foday rolls a 6.

Solutions

- a. Rolling 1 is the only 1 possible outcome.
- b. There are 3 even numbers on a die: 2, 4 and 6. There are 3 possible outcomes.
- c. There are 3 numbers greater than 3 on a die: 4, 5 and 6. There are 3 possible outcomes.
- d. There are 2 possible outcomes: 3 or 4.
- e. Rolling 6 is the only 1 possible outcome.

3. Hawa, Sia and Martin solved a math problem together as a group. The teacher will randomly select one pupil from their group to write the solution on the board. What are the possible outcomes?

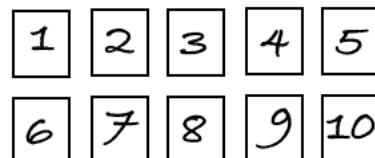
Solution

The experiment is that the teacher will randomly select 1 pupil from the group. There are 3 possible outcomes:

- The teacher selects Hawa.
- The teacher selects Sia.
- The teacher selects Martin.

Practice

1. Determine whether each of the following is an experiment or an event:
 - a. A teacher selects a random pupil from the class.
 - b. A coin is tossed.
 - c. A teacher selects Fatu.
 - d. A die lands on 4.
 - e. A coin lands on tails.
 - f. Michael rolls a die.
2. In one box, there is a Manchester United jersey, a Chelsea jersey, an Arsenal jersey, and a Real Madrid jersey. Mustapha closes his eyes and selects one at random. What are the possible outcomes?
3. The numbers 1 through 10 are written in pieces of paper and placed in a box. Determine how many possible outcomes there are in each event:
 - a. Choosing 3.
 - b. Choosing 4 or 6.
 - c. Choosing an odd number.
 - d. Choosing a number greater than 5.
 - e. Choosing a number less than 4.



Lesson Title: Expressing Probability with Numbers	Theme: Statistics and Probability
Practice Activity: PHM-09-133	Class: JSS 3



Learning Outcomes

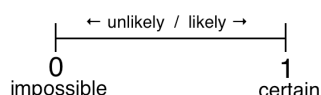
By the end of the lesson, you will be able to:

1. Express the probability of an event happening as a fraction.
2. Express the probability of an event happening as a percentage.

Overview

In this lesson, you will learn how to express the probability of an event happening as a fraction and as a percentage.

If an event is **impossible**, then the probability of it happening is **0**. If an event is **certain**, then the probability of it happening is **1**. If there is a chance of them happening, then probability is between 0 and 1. That is, it is a fraction.



To find the probability of an event occurring as a **fraction**, use the formula:

$$\text{probability} = \frac{\text{the number of ways an event can occur}}{\text{total number of possible outcomes}}$$

For example, consider a group of pupils where 3 of them are girls and 4 of them are boys. There are 7 pupils in total. If a pupil is randomly selected from the group, the probability of choosing a girl is $\frac{3}{7}$. There are 3 possible girls that could be selected, and 7 possible pupils in total. Similarly, the probability of choosing a boy is $\frac{4}{7}$.

We have a special language that we use to discuss probabilities as fractions. For example, if there is a $\frac{4}{7}$ chance of an event occurring, we say, "There is a four in seven chance." For any fraction $\frac{a}{b}$, the probability can be described as "an a in b chance".

You can also represent probability with **percentages**. To find the probability that something will occur as a percentage, follow these steps:

- Find the probability as a fraction.
- Convert the fraction to a percentage.

In this lesson, give your answers as fractions unless you are asked to give them as percentages.

Solved Examples

1. A bag contains 3 red balls, 7 blue balls, and 2 yellow balls. If one ball is selected at random, what is the probability of choosing:
 - a. A red ball
 - b. A blue ball
 - c. A yellow ball

Solutions

First, find the total number of possible outcomes. This will be the denominator of the fractions. The total number of possible outcomes is the total number of balls that you are choosing from: $3 + 7 + 2 = 12$ outcomes.

- a. There are 3 red balls. The probability of choosing one is $\frac{3}{12} = \frac{1}{4}$.
 - b. There are 7 blue balls. The probability of choosing one is $\frac{7}{12}$.
 - c. There are 2 yellow balls. The probability of choosing one is $\frac{2}{12} = \frac{1}{6}$.
2. In one JSS1 classroom, all of the pupils are between 11 and 13 years old. There are five 11-year-old pupils, twenty 12-year-old pupils, and ten 13-year-old pupils. The teacher selects one pupil at random to solve a maths problem on the board. Find the probability that she selects:
 - a. An 11-year-old pupil
 - b. A 12-year-old pupil
 - c. A 13-year-old pupil

Solutions

First, find the total number of possible outcomes. This is the total number of pupils in the class: $5 + 20 + 10 = 35$ pupils.

Write the probability of choosing from each age group as a fraction:

- a. $\frac{5}{35} = \frac{1}{7}$
 - b. $\frac{20}{35} = \frac{4}{7}$
 - c. $\frac{10}{35} = \frac{2}{7}$
3. There are 7 red balls and 3 blue balls in a bag. A ball is selected at random from the bag. Find the following, and express your answers as percentages:
 - a. The probability of choosing a red ball.
 - b. The probability of choosing a blue ball.

Solutions

Write each probability as a fraction, then convert the fraction to a percentage.

Remember that a fraction can be converted to a percentage by multiplying by 100%.

a. Probability of choosing red: $\frac{7}{10} = \frac{7}{10} \times 100\% = \frac{700}{10}\% = 70\%$.

There is a 70% chance of choosing red.

b. Probability of choosing blue: $\frac{3}{10} = \frac{3}{10} \times 100\% = \frac{300}{10}\% = 30\%$.

There is a 30% chance of choosing blue.

4. Foday has 20 oranges in a bag. Four of them are spoiled. If he chooses an orange at random, what is the probability that it will be spoiled? Express your answer as a percentage.

Solution


The probability of choosing a spoiled orange is $\frac{4}{20}$. Convert this to a percentage: $\frac{4}{20} = \frac{4}{20} \times 100\% = \frac{400}{20}\% = 20\%$.

There is a 20% probability that Foday will choose a spoiled orange.

Practice

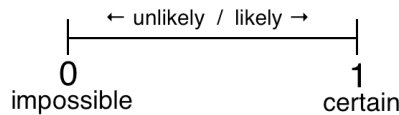
- In one JSS1 classroom, there are 22 boys and 20 girls. If a pupil is selected at random, find the probability that the pupil:
 - Is a girl
 - Is a boy
- A box contains 3 red jerseys, 12 blue jerseys, 8 yellow jerseys, and 1 black jersey. If a football player selects a jersey at random, find the probability that she selects:
 - A red jersey
 - A blue jersey
 - A yellow jersey
 - A black jersey
- There are 9 boys and 11 girls in a class. If a pupil is selected at random, what is the probability that it will be a girl? Give your answer as a percentage.
- In a bag there are 7 green shirts, 8 blue shirts, 9 red shirts and 1 black shirt. A shirt is selected at random. Express the probability of selecting each of the following. Express your answers as percentages:
 - A green shirt
 - A blue shirt
 - A red shirt
 - A black shirt

Lesson Title: Likelihood of Events	Theme: Statistics and Probability
Practice Activity: PHM-09-134	Class: JSS 3

	<p>Learning Outcome By the end of the lesson, you will be able to compare whether events are impossible, unlikely, likely, or certain.</p>
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Overview

If an event is **impossible**, then it definitely won't happen and the probability of it happening is **0**. If an event is **certain**, then it will definitely happen and the probability of it happening is **1**. We have also discussed likely and unlikely events. Those events might happen, or they might not. If there is a chance of them happening, the probability is between 0 and 1. That is, it is a fraction.



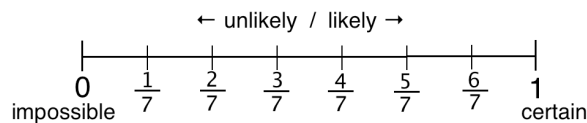
If more than half of the possible outcomes make the event true, then it is a likely event. If less than half of the possible outcomes make the event true, then it is an unlikely event.

In the previous lesson, we had a group of pupils where 3 of them were girls and 4 of them were boys. There were 7 pupils in total. The probability of choosing a girl is $\frac{3}{7}$ and the probability of choosing a boy is $\frac{4}{7}$.

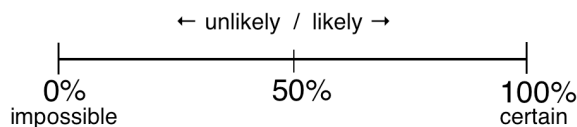
It is likely that we will choose a boy, because $\frac{4}{7} > \frac{1}{2}$. It is unlikely that we will choose a girl, because $\frac{3}{7} < \frac{1}{2}$.

The bigger a fraction is, the more likely the event is to occur. The smaller the fraction, the less likely. In this example, we have $\frac{4}{7} > \frac{3}{7}$, so it is more likely that we will choose a boy.

This can be displayed with a diagram:



With percentages, we have:



0% is impossible, and 100% is certain. Likely events have a more than 50% chance of occurring. Unlikely events have a less than 50% chance of occurring.

Solved Examples

1. There is a family with 5 members. The family has 1 girl, 2 boys, a mother, and a father. A member of the family is selected at random to win a prize. Answer the following questions and give your reasons:
 - a. Is it more likely that an adult or child will be selected?
 - b. Is it more likely that a male or female will be selected?

Solutions

- a. The probability of choosing a child is $\frac{3}{5}$ and the probability of choosing an adult is $\frac{2}{5}$. It is more likely that a child will be selected.
 - b. The probability of choosing a male is $\frac{3}{5}$ and the probability of choosing a female is $\frac{2}{5}$. It is more likely that a male will be selected.
2. Mustapha will randomly choose a day of the week to wash his clothes. Is it more likely that he will wash his clothes during the week or during the weekend?

Solution

There are 5 days during the week (Monday – Friday) and only 2 days during the weekend (Saturday and Sunday). The probability that he will choose a weekday is $\frac{5}{7}$ and the probability that he will choose the weekend is $\frac{2}{7}$.

It is more likely that Mustapha will wash his clothes during the week.

3. Determine whether each statement is impossible, unlikely, likely, or certain:
 - a. There is a 30% chance of rain tomorrow.
 - b. There is a 0% chance of snow tomorrow.
 - c. There is an 80% chance that the building will collapse.
 - d. There is a 100% chance that tomorrow is Tuesday.

Solutions

- a. Unlikely. The percentage is less than 50%.
 - b. Impossible. An event with a probability of 0% will not occur.
 - c. Likely. The percentage is greater than 50%.
 - d. Certain. An event with a probability of 100% will definitely occur.
4. There are 4 blue balls, 3 red balls, and 5 yellow balls in a bag. One ball is randomly selected. Calculate the probability of each of the following events. Then, rank the events from most likely (or certain) to least likely (or impossible).

- a. Selecting a blue ball
- b. Selecting a black ball
- c. Selecting a red ball
- d. Selecting a yellow ball

Solutions

Calculate the probability of selecting each colour. Then, rank them from largest fraction (most likely) to smallest fraction (least likely).

- a. Probability of selecting a blue ball: $\frac{4}{12} = \frac{1}{3}$
- b. Probability of selecting a black ball: $\frac{0}{12} = 0$
- c. Probability of selecting a red ball: $\frac{3}{12} = \frac{1}{4}$
- d. Probability of selecting a yellow ball: $\frac{5}{12}$

Write the fractions in order from greatest to smallest, then rank the events using their letters.


Fractions: $\frac{5}{12}, \frac{1}{3}, \frac{1}{4}, 0$

Ranked events: d, a, c, b

Practice

1. In one room, there are 5 boys, 6 girls, 4 adult women, and 3 adult men. One person is selected at random. Answer the following questions and give your reasons:
 - a. Is it more likely that an adult or child will be selected?
 - b. Is it more likely that a male or female will be selected?
2. There are 14 animals in a yard. There are 3 cats, 1 dog, and the rest are ducks. Hawa wants a new pet, and she selects an animal at random. Rank the following events from most likely (or certain) to least likely (or impossible).
 - a. Hawa selects a cat.
 - b. Hawa selects a dog.
 - c. Hawa selects a duck.
 - d. Hawa selects a chicken.
3. Determine whether each statement is impossible, unlikely, likely, or certain:
 - a. There is a 50% chance of rain tomorrow.
 - b. There is a 20% chance that lessons will be canceled tomorrow.
 - c. There is a 90% chance that she has malaria.
 - d. There is a 0% chance that the car will start.

Lesson Title: Probability Experiments with 2 Independent Events	Theme: Statistics and Probability
Practice Activity: PHM-09-135	Class: JSS 3

	<p>Learning Outcomes</p> <p>By the end of the lesson, you will be able to:</p> <ol style="list-style-type: none"> 1. Conduct simple probability experiments with 2 independent events. 2. Identify that if 2 events are independent, the outcome of 1 does not affect the outcome of the other.
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Overview

In this lesson, you will learn about probability experiments with 2 independent events. In probability, **independent** means that the outcome of 1 event does not affect the outcome of the other event.

For example, consider an experiment. I will roll a die and select a ball from a bag of blue and red balls. The probability of rolling a 6 does not affect the probability of choosing a blue ball. These are independent events, because one does not affect the other.

Solved Examples

1. Determine whether events a. and b. are independent or not:
 - a. A die is rolled and lands on 3.
 - b. The die is rolled again and lands on 4.

Solutions

The events are independent. The outcome of event a. does not affect the outcome of event b. There is an equal chance of rolling a 1, 2, 3, 4, 5 or 6 each time.

2. Determine whether events a. and b. are independent or not:
 - a. Mohamed draws the 2 of hearts from a deck of cards and lays it on the table.
 - b. Mohamed draws the 3 of clubs from the same deck of cards and lays it on the table with the 2 of hearts.



Solutions

The events are **not** independent. Event a. can affect the outcome of event b. During event a., Mohamed was using the full deck of cards, and during event b., one card was missing from the deck.

Note that there are 52 different cards in a deck. Mohamad had a $\frac{1}{52}$ chance of drawing the 2 of hearts in event a. In event b., he had a $\frac{1}{51}$ chance of drawing the 3 of clubs.

3. A maths teacher has a bag of papers with each member of the class written on them. Consider the following events:
- d. The teacher draws a name and asks that pupil to solve a maths problem on the board. She puts the name back into the bag.
 - e. The teacher draws a name again and asks that pupil to solve another maths problem on the board.

Are events a. and b. independent events? Give your reasons.

Solutions

Events a. and b. are independent. The outcome of event a. does not affect the outcome of event b., because the teacher put the name back into the bag. The same pupils' names are in the bag during each event.

4. A science teacher has a bag of papers with each member of the class written on them. Consider the following events:
- a. The teacher draws a name and asks that pupil to solve a science problem on the board. She **does not** put the name back into the bag.
 - b. The teacher draws a name again and asks that pupil to solve another science problem on the board.

Are events a. and b. independent events? Give your reasons.


Solution

Events a. and b. are **not** independent. Event a. can affect the outcome of event b. For example, if Aminata's name is drawn during event a., her name cannot be drawn during event b. The teacher did not place Aminata's name back in the bag.

Practice

1. A bag contains 6 blue balls and 7 red balls. Determine whether events a and b are independent or not:
 - a. Fatu randomly chooses a blue ball. She puts it back in the bag.
 - b. Fatu randomly chooses a red ball.
2. A bag contains 12 yellow balls and 9 orange balls. Determine whether events a. and b. are independent or not:
 - a. Fatu randomly chooses a yellow ball. She throws it away.
 - b. Fatu randomly chooses a yellow ball.
3. Determine whether events a. and b. are independent or not:
 - a. A die is rolled and lands on 6.
 - b. The die is rolled again and lands on 6.
4. Determine whether events a. and b. are independent or not:
 - a. A coin is flipped and lands on heads.
 - b. A coin is flipped and lands on tails.

Lesson Title: Probability of Independent Events I	Theme: Statistics and Probability
Practice Activity: PHM-09-136	Class: JSS 3

	<p>Learning Outcomes</p> <p>By the end of the lesson, you will be able to:</p> <ol style="list-style-type: none"> 1. Solve simple probability problems with 2 independent events. 2. Interpret the word 'and' in probability problems as multiplication.
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Overview

In this lesson, you will learn how to solve simple probability problems with 2 independent events, and interpret the word 'and' in probability problems.

Remember that in probability, **independent** means that the outcome of 1 event does not affect the outcome of the other event

If we have 2 independent events, and we want to calculate the probability of **both** of them happening, we must **multiply** the 2 probabilities. This type of question sometimes has the word 'and'. When you see the word 'and' in a problem, you will know to multiply.

Solved Examples

1. George rolls a die twice. What is the probability that he will roll 5 the first time and 6 the second time?

Solution

The probability that George will roll 5 on a roll is $\frac{1}{6}$. The probability of rolling 6 the second time is also $\frac{1}{6}$. These are independent events.

Multiply to find the probability of rolling 5 then 6: $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.

2. There are 2 classrooms. Classroom A has 10 boys and 14 girls. Classroom B has 16 boys and 12 girls. If the principal randomly selects one pupil from each classroom, find the probability that he selects:
 - a. A girl from classroom A and a boy from classroom B
 - b. Two girls
 - c. Two boys

Solutions

- a. There are 24 pupils in classroom A. The probability that he selects a girl is $\frac{14}{24} = \frac{7}{12}$.
There are 26 pupils in classroom B. The probability that he selects a boy is $\frac{16}{26} = \frac{8}{13}$.


Multiply to find the probability that he chooses a girl and a boy: $\frac{7}{12} \times \frac{8}{13} = \frac{56}{156} = \frac{14}{39}$

- b. The probability that he selects a girl from classroom A is $\frac{14}{24} = \frac{7}{12}$. The probability that he selects a girl from classroom B is $\frac{12}{26} = \frac{6}{13}$. The probability that he will choose 2 girls is $\frac{7}{12} \times \frac{6}{13} = \frac{42}{156} = \frac{7}{26}$.
- c. The probability that he selects a boy from classroom A is $\frac{10}{24} = \frac{5}{12}$. The probability that he selects a boy from classroom B is $\frac{16}{26} = \frac{8}{13}$. The probability that he will choose 2 boys is $\frac{5}{12} \times \frac{8}{13} = \frac{40}{156} = \frac{10}{39}$.

Practice

- Aminata rolls a die twice. What is the probability that she will roll 2 both times?
- There are 2 baskets. Basket A holds 10 oranges and 30 mangos. Basket B holds 15 oranges and 25 avocados. Fatu randomly selects 1 piece of fruit from each basket. Find the probability that:
 - Both fruits are oranges.
 - She chooses a mango from basket A and an avocado from basket B.
- There are 2 bags. Bag A holds 3 maths books and 2 English books. Bag B holds 4 geography books and 1 maths book. If Mr. Kamara randomly chooses 1 book from each bag, what is the probability that they are both maths books?
- Martin has 2 decks of cards. He asks his sister to randomly choose 1 card from each deck. What is the probability that both cards are 2 of hearts?
- There are 2 cups. Cup A holds 2 red pens and 3 blue pens. Cup B holds 1 red pen and 2 blue pens. If 1 pen is selected randomly from each cup, find the probability that:
 - Both pens are red.
 - Both pens are blue.
 - A red pen is selected from cup A and a blue pen is selected from cup B.

Lesson Title: Probability of Independent Events II	Theme: Statistics and Probability
Practice Activity: PHM-09-137	Class: JSS 3

	<p>Learning Outcomes By the end of the lesson, you will be able to:</p> <ol style="list-style-type: none"> 1. Identify whether 2 given events are independent or dependent. 2. Solve more difficult probability problems with 2 independent events.
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Overview

In this lesson, you will identify whether events are independent or dependent, and you will practise solving probability problems.

Remember that **independent events** mean that the outcome of 1 event does not affect the other event. If the outcome of 1 event **does** affect the other, then the events are **dependent events**. Dependent events are the opposite of independent.

For example, consider a pile of fruit. There are 4 bananas and 3 mangos. If you are choosing fruit at random to eat, what is the probability that the first fruit will be a banana? What is the probability that the second fruit will be a mango?

The first event is eating a banana, and the second event is eating a mango. From the beginning, the probability of eating a mango is $\frac{3}{7}$. However, in this problem, you eat a banana first. If you eat the banana, then there will be 3 bananas left and 3 mangos. You will then have a $\frac{3}{6} = \frac{1}{2}$ chance of eating a mango for the second fruit. Eating that first banana changes the probability that you will eat a mango for your second fruit. This means that these are dependent events.

Solved Examples

1. Determine whether each set of events are independent or dependent. Give your reasons:
 - a. Rolling 2 dice and getting 1 on both of them.
 - b. Flipping a coin and getting heads, then flipping the same coin and getting tails.
 - c. Choosing 1 pupil from primary school and 1 pupil from secondary school.
 - d. Choosing 1 pupil from primary school, then a second pupil from primary school.

Solutions

- a. Independent: the outcome on one die does not affect the outcome on the other.
- b. Independent: the outcome of the first coin toss does not affect the outcome of the second coin toss.
- c. Independent: choosing a pupil from primary school does not affect the choice of a pupil from secondary school, because these are different groups.

d. Dependent: After you choose the first pupil, the number of pupils in the primary school is reduced and the probability changes.

2. Consider a pile of fruit. There are 4 bananas and 3 mangos. If you are choosing fruit at random to eat, find the probability that:

- a. The first fruit is a mango.
- b. The second fruit is a banana.

Solutions

- a. The probability of eating a mango for the first fruit is $\frac{3}{7}$.
- b. To find the probability of the second fruit being a banana, find how many fruits are left. There were 7 fruits and you ate 1, so there are 6 fruits left. There are still 4 bananas left. The probability of eating a mango next is $\frac{4}{6} = \frac{2}{3}$.

3. There are 2 red pens, 3 blue pens and 5 black pens in a cup. Sia randomly selects a pen, but it doesn't work and she throws it away. She randomly selects a second pen. What is the probability that:

- a. The first pen is blue.
- b. The second pen is blue.

Solutions

- a. The probability of choosing the first blue pen is $\frac{3}{10}$.
- b. After you choose the first blue pen, there are 9 pens left in the cup and 2 of them are blue. The probability that the second pen will be blue is $\frac{2}{9}$.

4. The principal wants to choose an athlete to represent the school. There are 12 members of the football team and 12 members of the basketball team. She selects 1 player at random from each team. Then she randomly selects a player from between these 2 finalists.

- a. Is this problem describing independent events or dependent events?
- b. What is the probability of being selected as a finalist from 1 of the groups?
- c. What is the probability of being selected as the final winner to represent the school?

Solutions


- a. The events are independent. One event cannot affect the probability of the other.
- b. The probability of being selected as a finalist from 1 of the groups is $\frac{1}{12}$.

- c. To be selected as the final winner, a player needs to be selected from among a team and from among the 2 finalists. Multiply to find the probability of both independent events happening: $\frac{1}{12} \times \frac{1}{2} = \frac{1}{24}$.

Practice

1. Determine whether each set of events are independent or dependent:
 - a. Rolling 2 dice and getting 4 on both of them.
 - b. Choosing a card from a deck of cards, and holding it while choosing a second card.
 - c. Choosing 1 pupil from JSS1, and 1 pupil from JSS2.
 - d. Flipping a coin and getting heads both times.
 - e. Choosing 1 pupil from JSS3, then a second pupil from JSS3.
 - f. Randomly selecting a ball from a bag, and holding it while you select a second ball.
2. There are 3 red balls and 5 blue balls in a bag. Fatu selects 1 ball and gives it to Sia. She then selects a second ball and gives it to Martin. What is the probability that:
 - a. Fatu gave a blue ball to Sia.
 - b. Fatu gave a red ball to Martin.
3. Foday has 3 maths books, 4 English books, and 2 geography books on a shelf. He randomly selects one book and gives it to his brother to read. He randomly selects a second book and reads it himself. Find the probability that:
 - a. Foday gave his brother a maths book.
 - b. Foday is reading an English book.
4. There are 24 girls in a JSS1 class of 40 pupils. There are 18 girls in a JSS2 class of 40 pupils. A teacher selects a pupil from each class at random.
 - a. Is this problem describing independent events or dependent events?
 - b. What is the probability that both selected pupils are girls?
 - c. What is the probability that a girl is selected from JSS1 and a boy is selected from JSS2?

Lesson Title: Sample Space	Theme: Statistics and Probability
Practice Activity: PHM-09-138	Class: JSS 3

	<p>Learning Outcomes</p> <p>By the end of the lesson, you will be able to:</p> <ol style="list-style-type: none"> 1. Identify that the 'sample space' of an experiment is the set of all possible outcomes 2. Record the possible outcomes of an experiment in a sample space diagram.
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Overview

In this lesson, you will learn about sample space and how to record possible outcomes in a sample space.

All the possible outcomes of an experiment are called the sample space. For example, the sample space for rolling a die is {1, 2, 3, 4, 5, 6}. These are all of the possible outcomes.

For experiments with 2 independent events, we record sample space in a diagram. See the examples below.

Solved Examples

1. Find the sample space of selecting 2 balls at random from 2 different bags, each containing 3 blue and 2 red balls. Create a sample space diagram.

Solution

There are 2 independent events in this problem. The sample space for choosing 1 ball from 1 of the bags is {blue, blue, blue, red, red}. However, in this case we are choosing balls from 2 different bags.

We can display all of the possible outcomes in a table. On the left side, we list all the possible outcomes of the first event. On the top, we list all the possible outcomes of the second event:

		Bag B				
		Blue	Blue	Blue	Red	Red
Bag A	Blue					
	Blue					
	Blue					
	Red					
	Red					

In each box, write the outcome of the experiment from Bag A and the experiment from Bag B. Use R for red and B for blue.

		Bag B				
		Blue	Blue	Blue	Red	Red
Bag A	Blue	B, B	B, B	B, B	B, R	B, R
	Blue	B, B	B, B	B, B	B, R	B, R
	Blue	B, B	B, B	B, B	B, R	B, R
	Red	R, B	R, B	R, B	R, R	R, R
	Red	R, B	R, B	R, B	R, R	R, R

There appear to be many possible outcomes. Note that the same outcomes appear more than once. For example, 'B, B' appears 9 times. We can use this information to find the probability of selecting 2 blue balls. In 9 of the outcomes, we select 2 blue balls, and there are 25 outcomes in total. This means that the probability of selecting 2 blue balls is $\frac{9}{25}$.

- Two coins are flipped. Draw a sample space diagram to show the possible outcomes. Use your diagram to find the probability that both coins land on heads.

Solution

A coin has 2 sides: heads and tails. This means there are 2 possible outcomes for each experiment. On the left-hand side, list the outcomes of the first experiment. In the top row, list the outcomes of the second experiment. Use H for heads and T for tails.

		Second coin	
		H	T
First coin	H	HH	HT
	T	TH	TT

There are 4 possible outcomes in the table: HH, TH, HT, TT. One of these represents two heads: HH. This means that the probability that both coins land on heads is $\frac{1}{4}$.

Practice

- What is the sample space of flipping a coin and selecting 1 ball from a bag of 3 red and 2 yellow balls? Draw a sample space diagram.
- Issa rolls a die and flips a coin. Draw a sample space diagram.
- Mary rolls a die and selects a ball from a bag of 2 yellow balls and 1 green ball. Draw a sample space diagram.

Lesson Title: Probability Trees	Theme: Statistics and Probability
Practice Activity: PHM-09-139	Class: JSS 3



Learning Outcome

By the end of the lesson, you will be able to use a probability tree to demonstrate the probability of different outcomes occurring.

Overview

In this lesson, you will learn about probability trees and how to demonstrate the probability of different outcomes occurring. A probability tree is another type of diagram to help us understand the possible outcomes. Every branch of a probability tree represents an event together with its probability of occurring. The probability is written on each branch as a fraction.

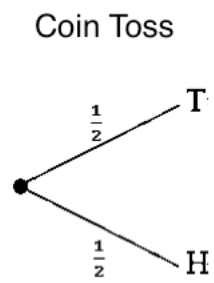
In this lesson, you will represent multiple independent events with a probability tree. To find the probability of an outcome from multiple events, follow the branches and multiply the probabilities that are shown on the branches. See Solved Example 2.

Solved Examples

1. Draw a probability tree to show the possible outcomes from 1 coin toss.

Solution

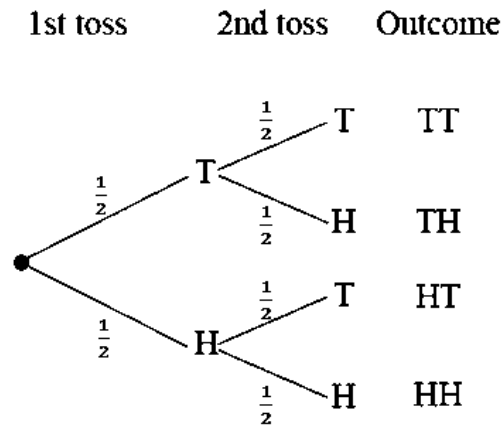
Draw branches to show the 2 possibilities: heads and tails. The branches should be labeled with their probabilities. There is a $\frac{1}{2}$ probability of heads, and $\frac{1}{2}$ probability of tails.



2. A coin is tossed twice. What is the probability of getting 2 heads? Use a probability tree to find the answer.

Solution

Draw a probability tree with 2 experiments. From the probability tree in Solved Example 1 above, you will extend the branches to show the second coin toss.



You can follow any branch to find a possible outcome of the experiment. The 4 possible outcomes are TT, TH, HT and HH. The probability of any one of these outcomes is found by multiplying the fractions on the branches that you follow.

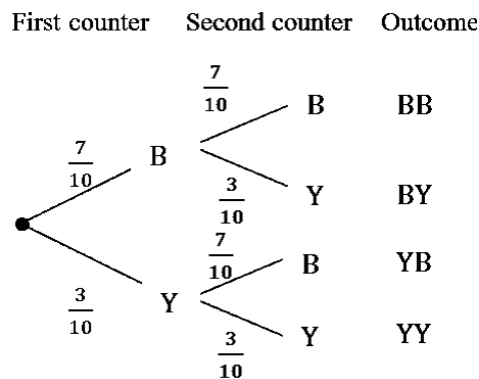
We are asked to find the probability of rolling heads twice. In the diagram, follow the branches marked HH. There are 2 fractions, $\frac{1}{2}$ and $\frac{1}{2}$. Multiply them to find the probability of getting 2 heads: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

3. A bag contains 7 blue counters and 3 yellow counters. A counter is taken at random from the bag, replaced and a second counter taken. What is the probability that both counters are yellow?

Solution

There is a $\frac{7}{10}$ chance of getting a blue counter, and a $\frac{3}{10}$ chance of getting a yellow counter. The events are independent, because the counter is replaced before the second event. The probabilities stay the same throughout the tree for blue and yellow counters.

The possible outcomes are:




To find the probability that both counters are yellow, multiply: $\frac{3}{10} \times \frac{3}{10} = \frac{9}{100}$.

Practice

1. A coin is tossed three times. What is the probability of getting 3 tails? Use a probability tree to find the answer.
2. The likelihood of rain on any day is $\frac{1}{5}$. Use a probability tree to find the probability that it will rain for 2 days in a row.
3. A bag contains 5 red balls and 3 green balls. A ball is chosen from the bag and then replaced. A second ball is chosen.
 - a. Show the possible ways of selecting the balls on a tree diagram.
 - b. Find the probability that both balls are red.
 - c. Find the probability that both balls are green.

Lesson Title: Probability Story Problems	Theme: Statistics and Probability
Practice Activity: PHM-09-140	Class: JSS 3

	<p>Learning Outcome By the end of the lesson, you will be able to solve story problems involving probability of an event happening.</p>
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Overview

In this lesson, you will practise solving story problems involving probability. You will use the information you learned from previous lessons.

Solved Examples

1. Mrs. Bangura has 2 algebra books, 3 geometry books, 4 chemistry books, and 1 biology book in a bag. If she randomly selects a book from her bag, find the probability that:
 - a. It is an algebra book.
 - b. It is a maths book.
 - c. It is either a geometry book or a biology book.
 - d. It is a physics book.

Solutions

First, find the total number of books: $2 + 3 + 4 + 1 = 10$. This will be the denominator.

- a. There are 2 algebra books, so the probability of choosing an algebra book is $\frac{2}{10} = \frac{1}{5}$.
 - b. There are 2 algebra books and 3 geometry books. This makes 5 maths books in total. The probability of choosing a maths book is $\frac{5}{10} = \frac{1}{2}$.
 - c. There are 3 geometry books and 1 biology book. This makes 4 books in total. The probability of choosing one of them is $\frac{4}{10} = \frac{2}{5}$.
 - d. There are no physics books in her bag. The probability of choosing a physics book is 0.
2. Mohamed rolls 2 dice. Draw a sample space diagram to show all of the possible outcomes. Use the sample space to find:
 - a. The probability of rolling two 5s.
 - b. The probability of rolling at least one 5.

Solutions

Draw the sample space to show all the outcomes when 2 dice are rolled:

	Second Roll					
	1	2	3	4	5	6
First Roll 1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6


Note that there are 36 possible outcomes. You can find this by multiplying the number of rows and columns: $6 \times 6 = 36$ outcomes. This will be the denominator.

- To find the probability of both dice showing 5, find all of the outcomes in the table that show (5, 5). There is only 1 outcome. This means that the probability of rolling two 5s is $\frac{1}{36}$.
- To find the probability of rolling at least one 5, count all of the outcomes that include a 5. There are 11 such outcomes. This means that the probability of rolling at least one 5 is $\frac{11}{36}$.

Practice

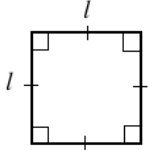
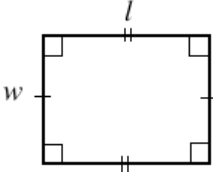
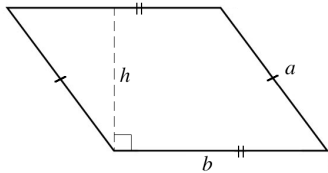
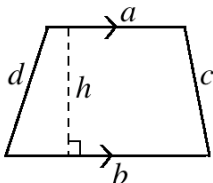
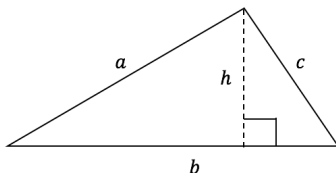
- If a die is rolled 3 times, what is the probability that it lands on 6 all 3 times?
- A coin is tossed 5 times. What is the probability that it lands on heads all 5 times?
- There are 10 pupils sitting in a group. Three are boys from JSS1, and 2 are girls from JSS1. One is a boy from JSS2, and the rest are girls from JSS2. Mr. Kamara chooses 1 pupil at random to help him move a desk. Find the probability that:
 - Mr. Kamara chooses a girl.
 - Mr. Kamara chooses a boy.
 - Mr. Kamara chooses a JSS1 pupil.
 - Mr. Kamara chooses a JSS2 pupil.
 - Mr. Kamara chooses a girl from JSS2.
- Fatu has a bag that contains 2 red balls, 1 blue ball, and 3 green balls. She also has a die. She randomly selects 1 ball from the bag and rolls the die. Draw a sample space diagram to show all of the possible outcomes. Use the sample space to find the probability that she chooses a red ball and the die lands on 3.

Lesson Title: Perimeter of Triangles and Quadrilaterals	Theme: Measurement and Estimation
Practice Activity: PHM-09-141	Class: JSS 3

 **Learning Outcome**
By the end of the lesson, you will be able to find the perimeter of a triangle and quadrilateral.

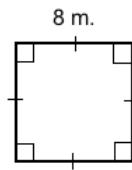
Overview

In maths, “perimeter” is the total length around a shape. To find the **perimeter** of a rectangle or square, add the lengths of all 4 sides together. We have some short cuts for some quadrilaterals, because they have sides of the same length.

Shape	Diagram	Perimeter
Square		$P = l + l + l + l = 4l$
Rectangle		$P = l + l + w + w = 2l + 2w$
Parallelogram		$P = a + a + b + b = 2a + 2b$
Trapezium		$P = a + b + c + d$
Triangle		$P = a + b + c$

Solved Examples

1. Find the perimeter of the square below:



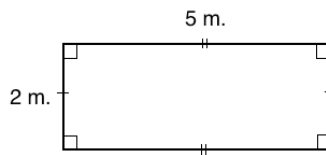
Solution

Find the perimeter by adding the length 4 times, or by multiplying the length of 1 side by 4. Both methods are shown:

$$P = l + l + l + l = 8 + 8 + 8 + 8 = 32 \text{ m}$$

$$P = 4l = 4 \times 8 \text{ m} = 32 \text{ m}$$

2. Find the perimeter of the rectangle below:



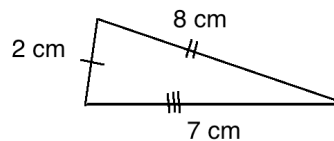
Solution

Find the perimeter by adding the lengths of the 4 sides. You can also find the perimeter by multiplying each length and width by 2, and adding the results. Both methods are shown:

$$P = l + l + w + w = 5 + 5 + 2 + 2 = 14 \text{ m}$$

$$P = 2l + 2w = 2 \times 5 \text{ m} + 2 \times 2 \text{ m} = 10 \text{ m} + 4 \text{ m} = 14 \text{ m}$$

3. Find the perimeter of the triangle below:

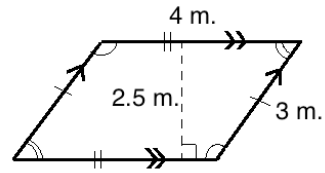


Solution

Add the lengths of the 3 sides: $P = a + b + c = 2 + 8 + 7 = 17 \text{ cm}$

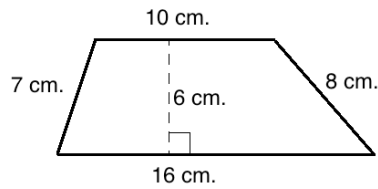
4. Find the perimeter of the parallelogram at right:

Solution



$$\begin{aligned}
 P &= 2a + 2b \\
 &= 2 \times 4 \text{ m} + 2 \times 3 \text{ m} \\
 &= 8 \text{ m} + 6 \text{ m} \\
 &= 14 \text{ m}
 \end{aligned}$$

5. Find the perimeter of the trapezium below:



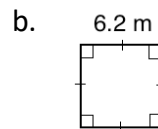
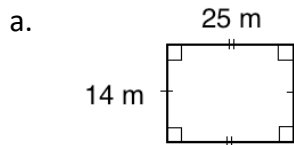
Solution

Add the lengths of the sides to find the perimeter:

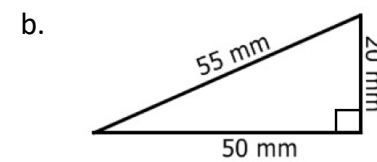
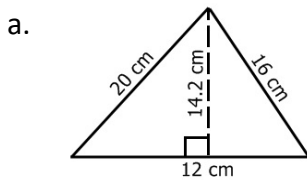
$$\begin{aligned}
 P &= 10 + 16 + 8 + 7 \\
 &= 41 \text{ cm}
 \end{aligned}$$

Practice

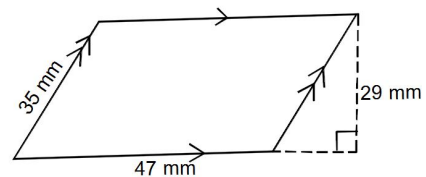
1. Find the perimeter of shapes a. and b.



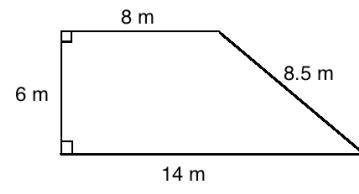
2. Find the perimeter of a square with sides of 14 cm.
 3. Find the perimeter of a rectangle with a length of 5 metres and width of 3 metres.
 4. Find the perimeters of the triangles below:




5. Find the perimeter of the parallelogram at right:



6. Find the perimeter of the trapezium below:



Lesson Title: Area of Triangles	Theme: Measurement and Estimation
Practice Activity: PHM-09-142	Class: JSS 3

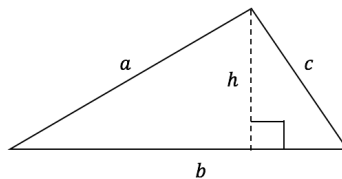
	<p>Learning Outcome By the end of the lesson, you will be able to calculate the area of a triangle.</p>
---	--

Overview

The area of triangles is found by using a formula that is specifically for triangles. Multiply the base and height by $\frac{1}{2}$. The formula is:

Shape	Area
Triangle	$A = \frac{1}{2} b \times h$

For the shape:

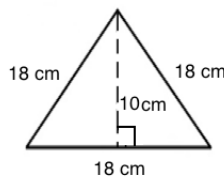


Base and height are always perpendicular to each other. You can take any side of the triangle as its base. Then, you find the height of the triangle from that base. The height is a perpendicular line drawn from the base to the opposite angle of the triangle.

Remember that area is always measured in square units. For example, square metres is given by m^2 .

Solved Examples

- The equilateral triangle shown below has sides of length 18 cm, and a height of 10 cm. Find the area of the triangle.



Solution

In this triangle, the base is 18 cm and the height is 10 cm. Substitute and evaluate:

$$\begin{aligned}
 A &= \frac{1}{2} b \times h \\
 &= \frac{1}{2} \times 18 \times 10 && \text{Substitute values} \\
 &= \frac{1}{2} (180) && \text{Simplify} \\
 &= 90 \text{ cm}^2
 \end{aligned}$$

2. Find the area of the triangle below:



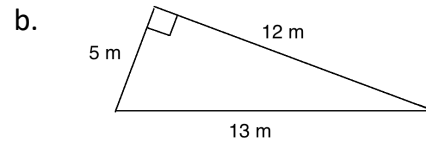
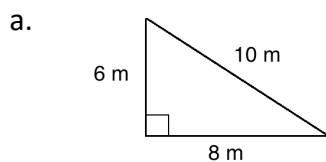
Solution

In this diagram, 2 parts of the base are labelled separately. We need to add to find the length of the base: $14 + 3 = 17 \text{ cm}$

Now, substitute the base and height into the equation and solve:

$$\begin{aligned}
 A &= \frac{1}{2} b \times h \\
 &= \frac{1}{2} \times 17 \times 6 && \text{Substitute values} \\
 &= \frac{1}{2} \times (102) && \text{Simplify} \\
 &= 51 \text{ cm}^2
 \end{aligned}$$

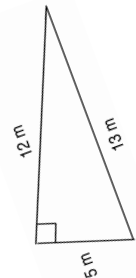
3. Find the area of the triangles below:



Solutions

Remember that any side of the triangle can be taken as the base. It is important that the base and height are perpendicular.

The base and height of triangle a. are 6 m and 8 m. The base and height of triangle b. are 5 m and 12 m. To see why, rotate the triangle. →



a. Substitute $b = 6 \text{ m}$ and $h = 8 \text{ m}$, and solve:

$$\begin{aligned}
 A &= \frac{1}{2} b \times h \\
 &= \frac{1}{2} \times 6 \times 8 && \text{Substitute values} \\
 &= \frac{1}{2} \times 48 && \text{Simplify} \\
 &= 24 \text{ m}^2
 \end{aligned}$$

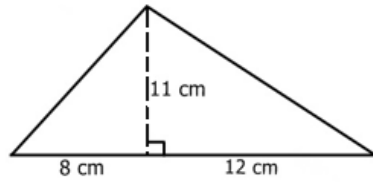
b. Substitute $b = 5 \text{ m}$ and $h = 12 \text{ m}$, and solve:

$$\begin{aligned}
 A &= \frac{1}{2} b \times h \\
 &= \frac{1}{2} \times 5 \times 12 && \text{Substitute values} \\
 &= \frac{1}{2} \times 60 && \text{Simplify} \\
 &= 30 \text{ m}^2
 \end{aligned}$$

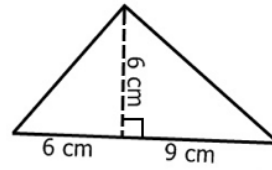
Practice

1. Find the area of the triangles below:

a.

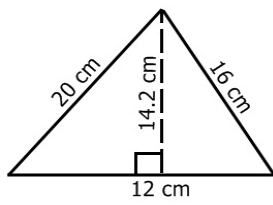


b.

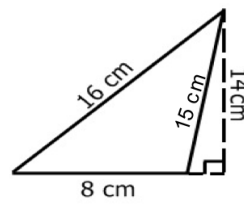


2. Calculate the area of the following triangles:

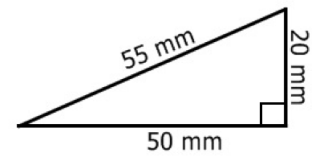
a.




b.



c.



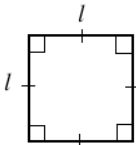
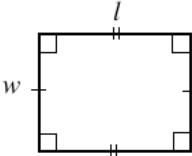
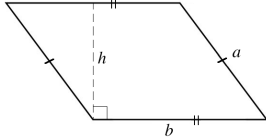
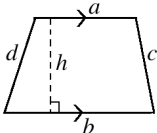
Lesson Title: Area of Quadrilaterals	Theme: Measurement and Estimation
Practice Activity: PHM-09-143	Class: JSS 3

 **Learning Outcome**
By the end of the lesson, you will be able to calculate the area of a square, rectangle, parallelogram, and trapezium.

Overview

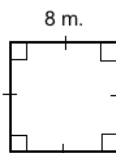
This lesson is on the area of several types of quadrilaterals. Area is the size of the space inside the shape. Remember that area is always measured in square units.

Use the formulae in the table below to solve the problems.

Shape	Diagram	Area
Square		$A = l \times l = l^2$
Rectangle		$A = l \times w$
Parallelogram		$A = b \times h$
Trapezium		$A = \frac{1}{2}(a + b)h$

Solved Examples

1. Find the area of the square at right:

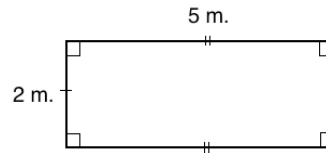


Solution

Substitute the given length in the formula:

$$\begin{aligned}
 A &= l^2 \\
 &= 8 \text{ m} \times 8 \text{ m} \\
 &= 64 \text{ m}^2
 \end{aligned}$$

2. Find the area of the rectangle at right:

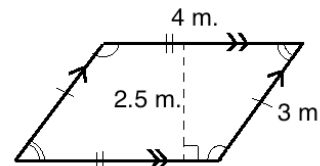


Solution

Substitute the given length and width in the formula:

$$\begin{aligned} A &= l \times w \\ &= 5 \text{ m} \times 2 \text{ m} \\ &= 10 \text{ m}^2 \end{aligned}$$

3. Find the area of the parallelogram at right:



Solution

$$\begin{aligned} A &= b \times h \\ &= 4 \text{ m.} \times 2.5 \text{ m} \\ &= 10 \text{ m}^2 \end{aligned}$$

4. Find the area of a parallelogram with base of 0.5 metres and a height of 25 centimetres. Give your answer in square centimetres.

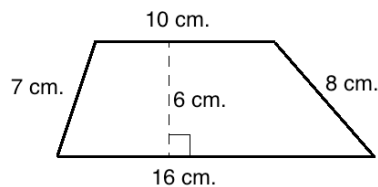
Solution

To solve an area problem, measurements should be in the same units. The answer should be in cm^2 , so convert 0.5 m to cm. Use the fact $100 \text{ cm} = 1 \text{ m}$. Remember to multiply to convert larger units to smaller units: $0.5 \text{ m} = 0.5 \times 100 = 50 \text{ cm}$

Apply the area formula:

$$\begin{aligned} A &= b \times h \\ &= 50 \text{ cm} \times 25 \text{ cm} \\ &= 1,250 \text{ cm}^2 \end{aligned}$$

5. Find the area of the trapezium below:



Solution

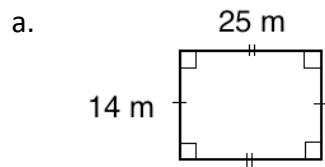
To find the area, first take note of the values needed. The parallel sides are $a = 10$ and $b = 16$. The height is $h = 6$. The other measurements are not needed for the area formula.

Apply the area formula:

$$\begin{aligned}
 A &= \frac{1}{2}(a + b)h \\
 &= \frac{1}{2}(10 + 16)6 \\
 &= \frac{1}{2}(26)6 \\
 &= 78 \text{ cm}^2
 \end{aligned}$$

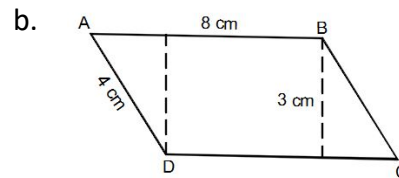
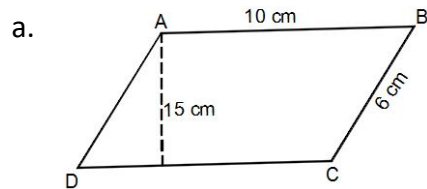
Practice

1. Find the area of the following shapes:

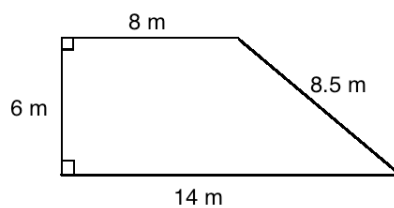


2. Find the area of a square with sides of 14 cm.
3. Find the area of a rectangle with a length of 35 cm and a width of 4 cm.
4. A square has sides of length 4 cm. A rectangle has a length of 5 cm and width of 3 cm. Which shape has a greater area?

5. Find the area of the parallelograms below:




6. Find the area of a parallelogram with a base of 220 cm and height of 2 metres. Give your answer in square metres.
7. Find the area of the trapezium below:



8. The parallel sides PQ and RS of a trapezium are 7.6 and 10 cm. If the distance between them is 6 cm, calculate the area of the trapezium.

Lesson Title: Area and Circumference of Circles	Theme: Measurement and Estimation
Practice Activity: PHM-09-144	Class: JSS 3

	<p>Learning Outcome By the end of the lesson, you will be able to calculate the area and circumference of a circle.</p>
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Overview

To calculate the circumference or area of a circle, we have special formulae. The circumference is the same as the perimeter of the circle. It is the distance around a circle.

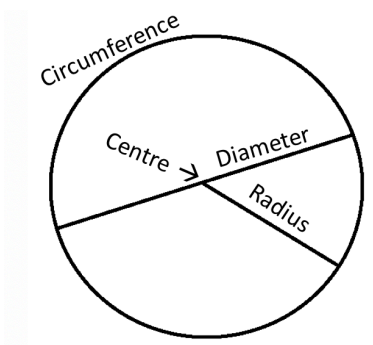
Each formula involves the radius, which is the distance from the centre of the circle to its side. In some problems, the diameter is given. The diameter is the distance across the circle, passing through its centre. The diameter is twice the radius ($d = 2r$), and the radius is half of the diameter ($r = \frac{1}{2}d$).

The formulae also use pi (π). Pi is a decimal number that stretches on forever. It can be estimated with numbers such as 3.14 and $\frac{22}{7}$. We will use these numbers in our calculations.

The formulae for circumference and area of a circle are:

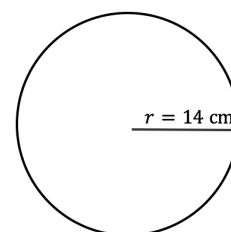
Shape	Circumference	Area
Circle	$C = 2\pi r$	$A = \pi r^2$

For the shape:



Solved Examples

- Find the circumference and area of the circle. Use $\pi = \frac{22}{7}$.



Solution

Apply the circumference formula:

$$\begin{aligned}
 C &= 2\pi r && \text{Formula} \\
 &= 2 \times \frac{22}{7} \times 14 && \text{Substitute values} \\
 &= 2 \times 22 \times 2 && \text{Cancel 7} \\
 &= 88 \text{ cm}
 \end{aligned}$$

Apply the area formula:

$$\begin{aligned}
 A &= \pi r^2 && \text{Formula} \\
 &= \left(\frac{22}{7}\right) 14^2 && \text{Substitute values} \\
 &= \left(\frac{22}{7}\right) 196 && \text{Simplify} \\
 &= 22 \times 28 \\
 &= 616 \text{ cm}^2
 \end{aligned}$$

2. Find the circumference and area of the circle. Use $\pi = 3.14$.

Solution

First find the radius. Divide diameter by 2:

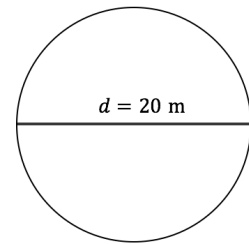
$$r = \frac{1}{2}d = \frac{1}{2}(20 \text{ m}) = 10 \text{ m}$$

Find the circumference:

$$\begin{aligned}
 C &= 2\pi r && \text{Formula} \\
 &= 2(3.14)(10) && \text{Substitute values} \\
 &= 2(31.4) && \text{Multiply 10} \\
 &= 62.8 \text{ m}
 \end{aligned}$$

Find the area:

$$\begin{aligned}
 A &= \pi r^2 && \text{Formula} \\
 &= (3.14)(10^2) && \text{Substitute values} \\
 &= (3.14)(100) && \text{Simplify} \\
 &= 314 \text{ m}^2
 \end{aligned}$$



3. Find the circumference of the circle whose radius is 3.5 mm. Use $\pi = \frac{22}{7}$.

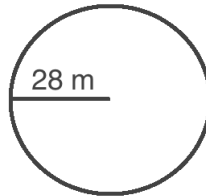
Solution

Apply the circumference formula:

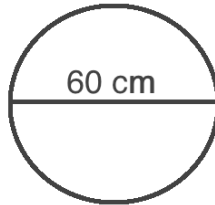
$$\begin{aligned}
 C &= 2\pi r && \text{Formula} \\
 &= 2 \times \frac{22}{7} \times 3.5 && \text{Substitute values} \\
 &= \frac{154}{7} && \text{Simplify} \\
 &= 22 \text{ mm}
 \end{aligned}$$

Practice


1. Find the circumference and area of a circle with a diameter of 14 cm, using $\pi = \frac{22}{7}$.
2. The radius of a circle is 100 cm. Find its circumference using $\pi = 3.14$.
3. Find the circumference of a circle whose radius is 35 cm. Use $\pi = \frac{22}{7}$.
4. Find the area of a circle with a radius of 3 cm. Give your answer to 1 decimal place. (Use $\pi = 3.14$)
5. Find the area of a circle with a diameter of 8 m. Give your answer to 2 decimal places. (Use $\pi = \frac{22}{7}$)
6. Find the circumference of a circle whose diameter is 140 cm. Use $\pi = \frac{22}{7}$.
7. Find the circumference and area of the circle below, using $\pi = \frac{22}{7}$:



8. Find the circumference and area of the circle below, using $\pi = 3.14$:



Lesson Title: Practical Problems with Area and Perimeter	Theme: Measurement and Estimation
Practice Activity: PHM-09-145	Class: JSS 3

	<p>Learning Outcomes</p> <p>By the end of the lesson, you will be able to:</p> <ol style="list-style-type: none"> 1. Find the perimeter and area of composite shapes. 2. Solve multi-step word problems on perimeter and area.
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Overview

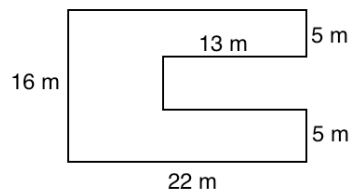
In this lesson, you will solve perimeter and area problems from stories, and for composite shapes. Remember that perimeter is the distance around a shape, and area is the space inside a shape. It can be helpful to draw a diagram before solving story problems.

Composite shapes are shapes made up of one or more different types of shapes. They can be made up of a combination of circles, triangles, rectangles and other polygons.

To find the perimeter and area of composite shapes, first divide the shape into its individual parts and find any missing lengths of sides. To find the **perimeter** of a composite shape, add all of its sides. To find the **area** of a composite shape, find the individual areas of its parts using the appropriate formulae. Then, add the areas together.

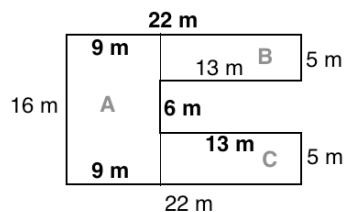
Solved Examples

1. Find the perimeter and area of the shape at right:



Solution

Divide the shape into its individual parts and find the lengths of the missing sides:



In this diagram, the shape is divided into 3 rectangles: A, B and C. All of the sides needed to find the perimeter and area are labelled. Subtraction is used to find each of these missing sides:

- To find the missing side of A, subtract $22 - 13 = 9$ m
- To find the length between B and C, subtract $16 - 5 - 5 = 6$ m

Find the perimeter by adding all of the outside edges of the shape:

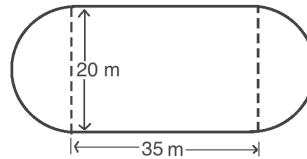
$$P = 16 + 22 + 5 + 13 + 6 + 13 + 5 + 22 = 102 \text{ m}$$

Find the area of each shape and add them to find the total area:

- Area of A: $16 \times 9 = 144 \text{ m}^2$
- Area of B: $13 \times 5 = 65 \text{ m}^2$
- Area of C: $13 \times 5 = 65 \text{ m}^2$

Total area: $A = 144 + 65 + 65 = 274 \text{ m}^2$

2. A farmer's field is in the shape of the figure below. Using $\pi = 3.14$, find the area of the field.



Solution

Note that this shape is a rectangle with a semi-circle on each side. Together, 2 semicircles make 1 full circle. To find the area of the shape, find the area of the circle and the area of the rectangle. Add them together to find the total area.

The diameter of this circle is 20 m, which means the radius is $r = \frac{20}{2} = 10 \text{ m}$.

Area of circle: $A = \pi r^2 = 3.14 \times 10^2 = 314 \text{ m}^2$

Area of the rectangle: $A = l \times w = 35 \times 20 = 700 \text{ m}^2$

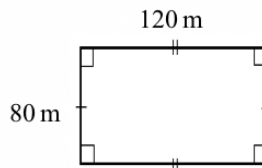
Total area: $A = 314 + 700 = 1,014 \text{ m}^2$

Answer: $1,014 \text{ m}^2$

3. Bright Secondary School has a football field that measures 120 metres on one side and 80 metres on the other side. A gardener is hired to plant carpet grass on the field.
- Calculate the area of the field.
 - If the cost of carpet grass is Le 200.00 per square metre, how much will it cost to cover the field?

Solution

First, draw a diagram. →



a. Calculate the area: $A = l \times w = 120 \text{ m} \times 80 \text{ m} = 9,600 \text{ m}^2$

b. Find the cost. Multiply the cost per square metre by the number of square metres.
 Cost = $9,600 \times \text{Le } 200 = \text{Le } 1,920,000.00$

4. The dimensions of the floor of a room are 20 ft by 15 ft. If rectangular tiles of dimensions 1.0 ft by 1.5 ft are used to tile the room, find the number of tiles required.

Solution

Find the area of the room, then the area of each tile. Divide the area of the room by the area of 1 tile to find the total number of times required.

Step 1. Area of the room: $A = l \times w = 20 \times 15 = 300 \text{ ft}^2$

Step 2. Area of 1 tile: $A = l \times w = 1.5 \times 1.0 = 1.5 \text{ ft}^2$

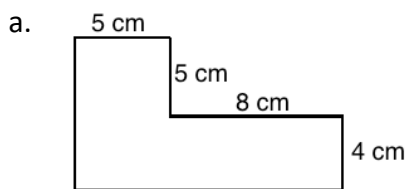
Step 3. Divide the area of the room by the area of 1 tile:

$$300 \text{ ft}^2 \div 1.5 \text{ ft}^2 = 3000 \div 15 = 200$$

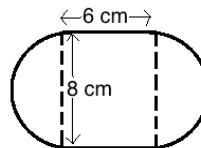
Answer: 200 tiles are needed.

Practice

1. Find the area and perimeter of the shapes:




b. Use $\pi = 3.14$



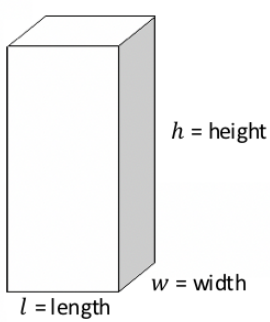
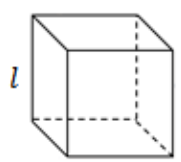
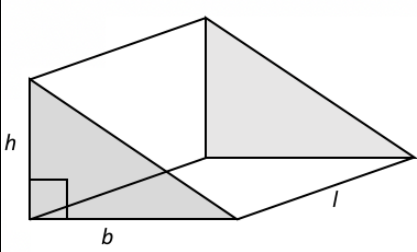
- A farmer wants to build a fence around his rectangular field. The field is 30 metres long by 25 metres wide.
 - What is the perimeter of the field?
 - If the cost of the fence is Le 4,000.00 per metre, how much will the fence cost?
- A goat is tied to a pole with a rope 15 metres in length. He can eat all the grass around him, the area of which forms a perfect circle. For the questions below, use $\pi = \frac{22}{7}$ and give your answers to the nearest tenth.
 - What is the area of the grass which the goat can eat?
 - What is the circumference of the area he can eat?
- The area of a square is 36 m^2 . Find the perimeter of the square.
- The dimensions of the floor of a rectangular room are 10 metres by 12 metres.
 - Find the area of the floor of the room.
 - How many tiles would you need to cover the floor if the tiles are 20 cm by 20 cm?

Lesson Title: Volume of Prisms	Theme: Measurement and Estimation
Practice Activity: PHM-09-146	Class: JSS 3

	<p>Learning Outcome By the end of the lesson, you will be able to find the volume of cubes, rectangular prisms, and triangular prisms.</p>
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Overview

In this lesson, you will calculate the volume of prisms, a type of 3-dimensional solid. Volume is the measurement of space taken up by a 3-dimensional solid. To find the volume of prisms, we use the area of one of its faces. This is called the **cross-section**. Multiply the cross-section by the solid's height or length. For a rectangular prism or cube, you may take any face as the cross-section. For a triangular prism, the triangular face is the cross-section.

	<p>Rectangular Prism</p> <p>Rectangular prisms (or cuboids) are solids with 6 rectangular faces. The cross-section of a rectangular prism is the area of one of its faces, $A = l \times w$.</p> <p>To calculate the volume of the prism, multiply the cross-section by height: $V = l \times w \times h$ or $V = A \times h$</p>
	<p>Cube</p> <p>A cube is a special type of rectangular prism. The height, length and width are all equal.</p> <p>Volume of a cube is $V = l \times l \times l = l^3$</p>
	<p>Triangular Prism</p> <p>In the diagram, the shaded faces are parallel. These are the cross-sections of the solid. The cross-section of a triangular prism is the area of a triangle, $A = \frac{1}{2}bh$</p> <p>To calculate the volume of the prism, multiply the cross-section by height: $V = \frac{1}{2} \times b \times h \times l = \frac{1}{2}bhl$</p>

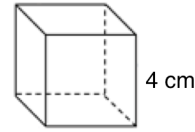
Volume is measured in cubic units. For example, if the size of a rectangular prism is measured in centimetres, its volume is given in cm^3 . This is read as “cubic centimetres” or “centimetres cubed”. To find volume, it is important that the measurements of the solid are all given in the same units.

Solved Examples

1. Find the volume of a cube of side length $l = 4$ cm.

Solution

You may sketch the cube to help solve the problem. →



Apply the formula for volume of a cube:

$$\begin{aligned} V &= l^3 && \text{Formula} \\ &= 4^3 && \text{Substitute } l = 4 \\ &= 4 \times 4 \times 4 && \text{Calculate } 4^3 \\ &= 64 \text{ cm}^3 \end{aligned}$$

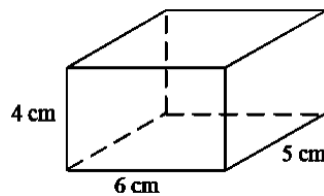
2. Determine the length of the side of a cube if its volume is 27 cm^3 .

Solution

This problem asks us to do the opposite of the problems above. You are given the volume and asked to find the sides of the cube. Since $V = l^3$, we have $27 = l^3$.

Note that $3^3 = 3 \times 3 \times 3 = 27$. The sides of the cube are 3 cm.

3. Find the volume of the given cuboid:



Solution

Note that the measurements of the cuboid are $l = 6$ cm, $w = 5$ cm, $h = 4$ cm.

Apply the volume formula:

$$\begin{aligned} V &= lwh && \text{Formula} \\ &= 6 \times 5 \times 4 && \text{Substitute values} \\ &= 120 \text{ cm}^3 && \text{Multiply} \end{aligned}$$

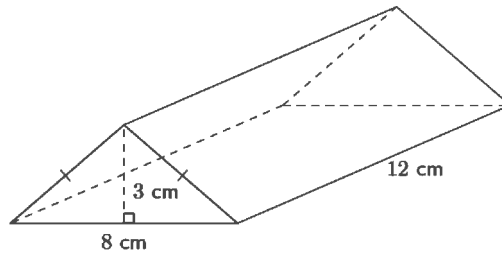
4. The length of a cuboid is 1 metre; its breadth is 15 cm and its height is 5 cm. Find its volume in cm^3 .

Solution

Note that the measurements are given in m, cm, and cm. You are asked to find the answer in cm^3 . This means that you must convert all units to cm first. Convert 1 m to cm:
 $1 \text{ m} = 100 \text{ cm}$.

$$\begin{aligned} V &= lwh && \text{Formula} \\ &= 100 \times 15 \times 5 && \text{Substitute values} \\ &= 7,500 \text{ cm}^3 && \text{Multiply} \end{aligned}$$

5. Find the volume of the figure below.



Solution

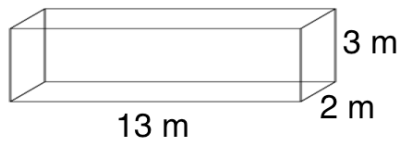
Identify the base, height, and length of the prism: $b = 8 \text{ cm}$, $h = 3 \text{ cm}$, $l = 12 \text{ cm}$

Apply the volume formula:

$V = \frac{1}{2}bhl$	Formula
$= \frac{1}{2} \times 8 \times 3 \times 12$	Substitute values
$= 144 \text{ cm}^3$	Multiply

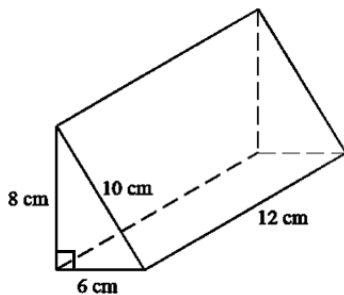
Practice

- Find the volume of a cube with sides of 7 cm.
- Find the volume of a cube with sides of 2.5 metres. Give your answer to 1 decimal place.
- If the volume of a cube is 8 ft^3 , what is the length of its sides?
- Find the volume of the cuboid shown:

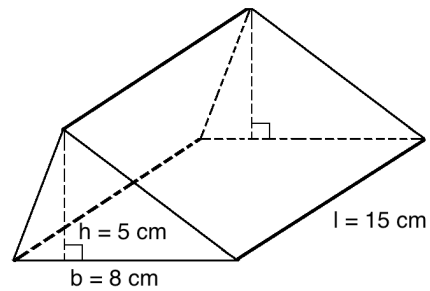


- The length of a cuboid is 2 m; its breadth is 30 cm and its height is 20 cm. Find its volume in cm^3 .
- The dimensions of a cuboid are 4 m, 3 m, and 50 cm. Find its volume in m^3 .
- Find the volume of the triangular prisms:


a.



b.



Lesson Title: Volume of Cylinders	Theme: Measurement and Estimation
Practice Activity: PHM-09-147	Class: JSS 3

	<p>Learning Outcome By the end of the lesson, you will be able to find the volume of a cylinder.</p>
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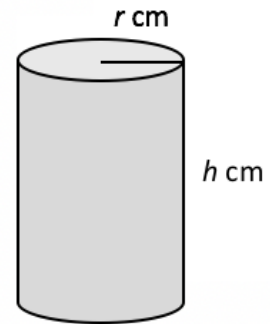
Overview

A cylinder is another 3-dimensional solid. It has 2 parallel, circular faces that are equal in size. It also has 1 rectangular face that curves around it.

The cross-section of a cylinder is the area of one of its circular faces, $A = \pi r^2$.

To find the volume of a cylinder, we multiply the cross-section by its height: $V = \pi r^2 h$ or $V = A \times h$.

You will use an estimated value for pi, $\pi = 3.14$ or $\pi = \frac{22}{7}$.

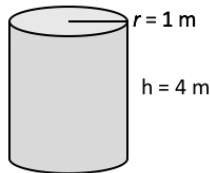


Solved Examples

1. Find the volume of a cylinder with a radius of 1 m and height of 4 m. Take $\pi = 3.14$.

Solution

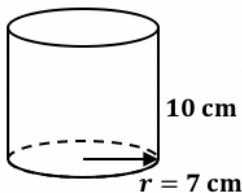
First, draw a diagram:



Apply the volume formula:

$V = \pi r^2 h$	Formula
$= 3.14 \times (1)^2 \times 4$	Substitute values
$= 3.14 \times 4$	Multiply
$= 12.56 \text{ m}^3$	

2. Find the volume of the figure below. Use $\pi = \frac{22}{7}$.



Solution

Identify the value of the radius and height: $r = 7 \text{ cm}$, $h = 10 \text{ cm}$

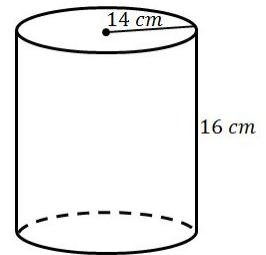
Apply the volume formula:

$$\begin{aligned}
 V &= \pi r^2 h && \text{Formula} \\
 &= \frac{22}{7} \times (7)^2 \times 10 && \text{Substitute values} \\
 &= 22 \times 7 \times 10 && \text{Multiply} \\
 &= 154 \times 10 \\
 &= 1,540 \text{ cm}^3
 \end{aligned}$$

3. Find the volume of a cylinder with a radius of 14 cm and height of 16 cm. Use $\pi = 3.14$. Give your answer to the nearest whole number.

Solution

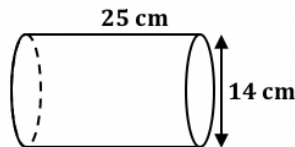
First, draw a diagram. →



Apply the volume formula:

$$\begin{aligned}
 V &= \pi r^2 h && \text{Formula} \\
 &= 3.14 \times (14)^2 \times 16 && \text{Substitute values} \\
 &= 3.14 \times 196 \times 16 && \text{Multiply} \\
 &= 9847.04 \\
 &= 9,847 \text{ cm}^3
 \end{aligned}$$

4. Find the volume of the figure below. Use $\pi = \frac{22}{7}$.

**Solution**

Note that the diameter is $d = 14 \text{ cm}$. Find the radius: $r = \frac{d}{2} = \frac{14}{2} = 7 \text{ cm}$

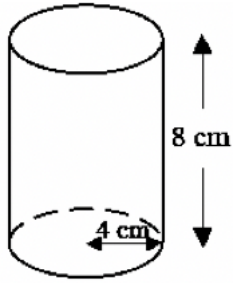
Apply the formula:

$$\begin{aligned}
 V &= \pi r^2 h && \text{Formula} \\
 &= \frac{22}{7} \times (7)^2 \times 25 && \text{Substitute values} \\
 &= 22 \times 7 \times 25 && \text{Multiply} \\
 &= 3,850 \text{ cm}^3
 \end{aligned}$$

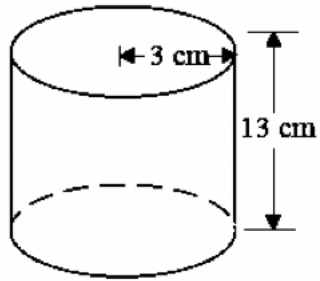
Practice

1. Find the volume of each figure to the nearest whole number. Use $\pi = 3.14$.

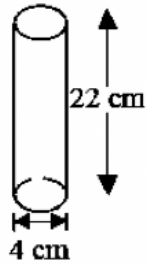
a.



b.




2. Find the volume of a cylinder with a radius of 14 mm and height of 10 mm. Use $\pi = \frac{22}{7}$.
3. Find the volume of the cylinder below. Give your answer to 1 decimal place. Use $\pi = 3.14$.



4. Calculate the volume of a cylindrical pipe with a radius of 1.4 cm and height of 30 cm. Give your answer correct to 1 decimal place. Use $\pi = \frac{22}{7}$.
5. A trader filled a cylindrical container with a radius of 6 cm and height of 7 cm with cooking oil. Find the volume of oil in the container. Use $\pi = \frac{22}{7}$.

Lesson Title: Surface Area of Prisms	Theme: Measurement and Estimation
Practice Activity: PHM-09-148	Class: JSS 3

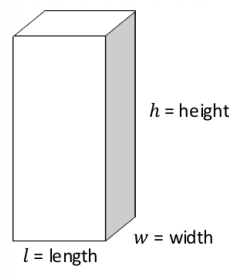
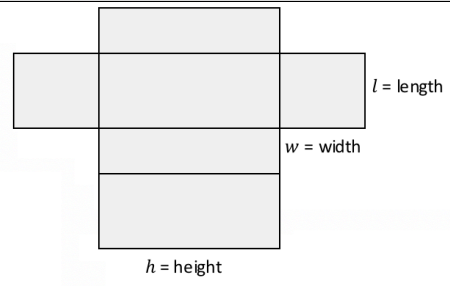
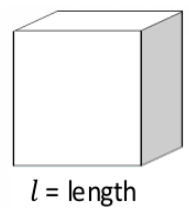
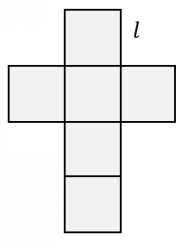
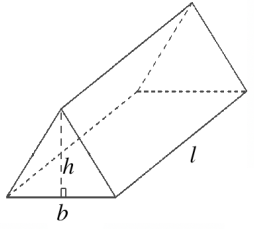
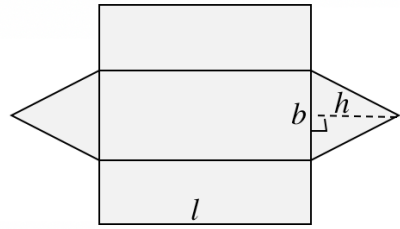
	<p>Learning Outcome By the end of the lesson, you will be able to calculate the surface area of cubes, rectangular prisms, and triangular prisms.</p>
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Overview

The outside layer of a solid is called the **surface area**. For example, imagine you have a box in the shape of a rectangular prism. You want to cover the outside of your box with paper. If you calculate the surface area of your box, you will know the size of paper that you need to completely cover the box. Surface area is measured in units squared, for example, cm^2 .

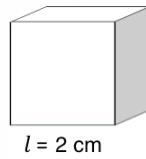
Using a **net** can help us understand surface area. A net is like a paper version of a solid that can be opened up and laid flat. A net is made of plane shapes such as rectangles and circles.

The surface area of a shape is the sum of the areas of the shapes in its net. The diagrams below are some common solids and their nets. You can find the surface area of any solid by finding the area of each face and adding them. For the solids below, we have special formulae that can also be used.

Solid	Net	Formula
		$SA = 2lw + 2wh + 2lh$
		$SA = 6l^2$
		$SA = bh + (a + b + c)l$

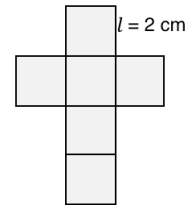
Solved Examples

1. Draw the net of the cube shown. Then, calculate its surface area.



Solution

Draw the net of the cube with each edge at 2 cm. →



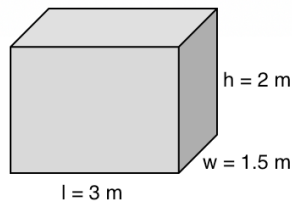
Calculate the surface area of the cube with the formula:

$SA = 6l^2$	Formula
$= 6(2 \text{ cm})^2$	Substitute $l = 2 \text{ cm}$
$= 6 \times 4 \text{ cm}^2$	
$= 24 \text{ cm}^2$	

2. Calculate the surface area of a rectangular prism with a length of 3 m, width of 1.5 m, and height of 2 m.

Solution

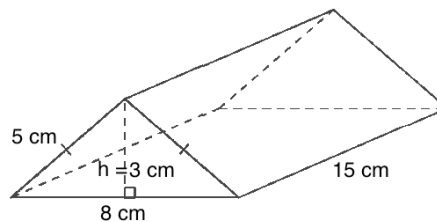
First, draw a diagram:



Calculate the surface area of the prism with the formula:

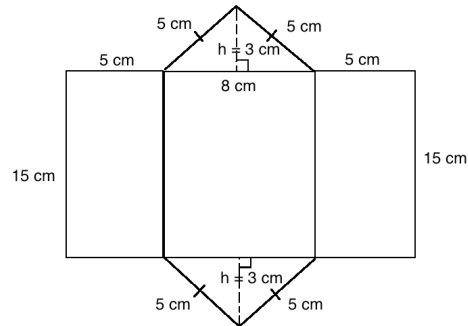
$SA = 2lw + 2wh + 2lh$	Formula
$= 2(3)(1.5) + 2(1.5)(2) + 2(3)(2)$	Substitute values
$= 9 + 6 + 12$	Multiply
$= 27 \text{ m}^2$	

3. Draw a net for the triangular prism below, then find its surface area.



Solution

First, draw the net:



Lengths needed for the formula: $a = 5$ cm, $b = 8$ cm, $c = 5$ cm, $h = 3$ cm, $l = 15$ cm

Calculate the surface area with the formula:

$$\begin{aligned} SA &= bh + (a + b + c)l \\ &= (8)(3) + (5 + 8 + 5)(15) \\ &= 24 + (18)(15) \\ &= 24 + 270 \\ &= 294 \text{ cm}^2 \end{aligned}$$

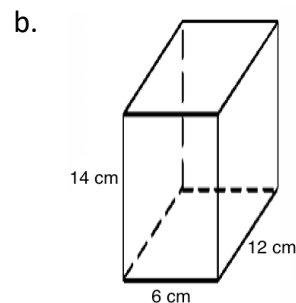
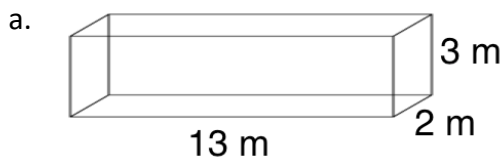
Formula

Substitute values

Simplify

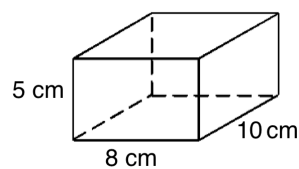
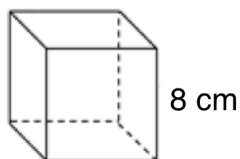
Practice

1. Calculate the surface area for each rectangular prism:



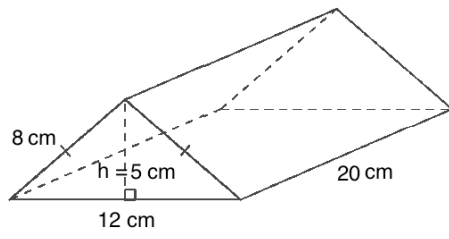
2. Calculate the surface area of a cube with sides of 12 mm.

3. Which has a greater surface area, the cube or the rectangular prism shown:

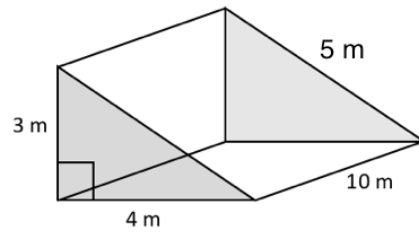


4. Find the surface area of the rectangular prisms:


a.



b.



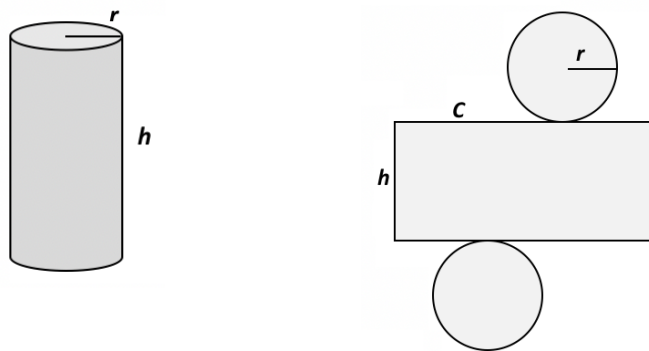
Lesson Title: Surface Area of Cylinders	Theme: Measurement and Estimation
Practice Activity: PHM-09-149	Class: JSS 3

	<p>Learning Outcome By the end of the lesson, you will be able to calculate the surface area of a cylinder.</p>
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Overview

We calculate the surface area of the cylinder by adding the areas of each of the shapes in the net. We can also use a formula.

This is the net of a cylinder:



Remember that the formula for the area of a circle is $A = \pi r^2$. Two of the faces of a cylinder are circles. They are the same size. Together, their area is $2\pi r^2$. This will be part of our surface area formula.

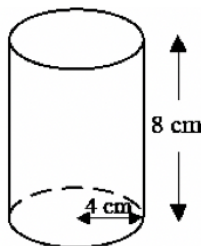
The other face of the cylinder is a rectangle. Notice that one side is the height of the cylinder h . The other side of the rectangle is equal to the circumference of the circular faces.

Remember that the circumference of a circle is $C = 2\pi r$. The area of the rectangular face is the circumference times the height: $Ch = 2\pi rh$. This is another part of our surface area formula.

The formula for the surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$.

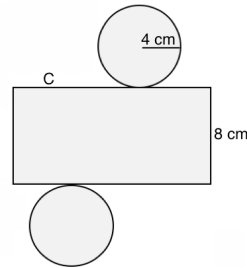
Solved Examples

1. Draw a net for the cylinder below, then find its surface area. Give your answer to the nearest whole number. Use $\pi = 3.14$.



Solution

First, draw the net:



Lengths needed for the formula: $r = 4$ cm, $h = 8$ cm

Calculate the surface area with the formula:

$$\begin{aligned} SA &= 2\pi r^2 + 2\pi r h && \text{Formula} \\ &= 2(3.14)(4)^2 + 2(3.14)(4)(8) && \text{Substitute values} \\ &= 100.48 + 200.96 && \text{Simplify} \\ &= 301.44 \\ &= 301 \text{ cm}^2 \end{aligned}$$

2. Find the surface area of the cylinder below to 1 decimal place. Use $\pi = 3.14$.



Solution

Note that the diameter is given in the diagram, 4 cm. Find the radius: $r = \frac{d}{2} = \frac{4}{2} = 2$ cm.

The height is 22 cm.

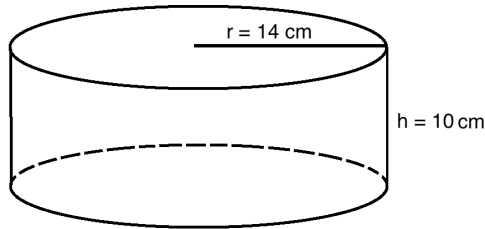
Calculate the surface area with the formula:

$$\begin{aligned} SA &= 2\pi r^2 + 2\pi r h && \text{Formula} \\ &= 2(3.14)(2)^2 + 2(3.14)(2)(22) && \text{Substitute values} \\ &= 25.12 + 276.32 && \text{Simplify} \\ &= 301.44 \\ &= 301.4 \text{ cm}^2 \end{aligned}$$

3. Find the surface area of a cylinder with a radius of 14 and height of 10. Use $\pi = \frac{22}{7}$.

Solution

First, draw a diagram:

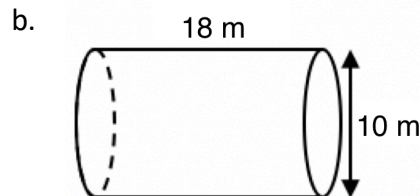
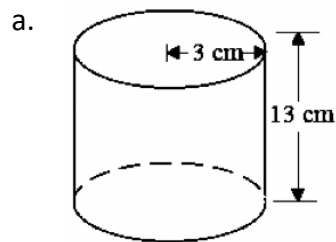


Calculate the surface area with the formula:

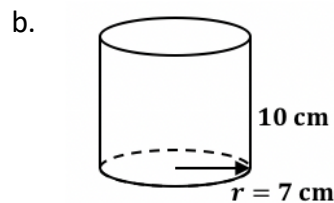
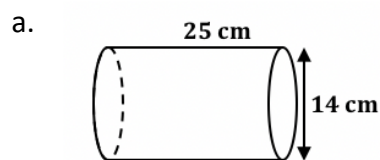
$SA = 2\pi r^2 + 2\pi rh$	Formula
$= 2\left(\frac{22}{7}\right)(14)^2 + 2\left(\frac{22}{7}\right)(14)(10)$	Substitute values
$= 1,232 + 880$	Simplify
$= 2,112 \text{ cm}^2$	

Practice


1. Find the surface area of each cylinder shown below. Use $\pi = 3.14$ and give your answers to the nearest whole number.



2. Find the surface area of each cylinder shown below. Use $\pi = \frac{22}{7}$ and give your answers to the nearest whole number.

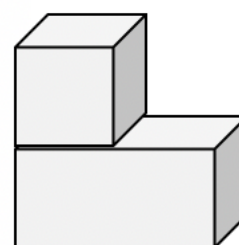


Lesson Title: Practical Problems with Volume and Surface Area	Theme: Measurement and Estimation
Practice Activity: PHM-09-150	Class: JSS 3

	<p>Learning Outcomes</p> <p>By the end of the lesson, you will be able to:</p> <ol style="list-style-type: none"> 1. Find the volume and surface area of composite shapes. 2. Solve multi-step word problems on volume and surface area.
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Overview

In this lesson, you will find the volume of composite solids. Composite solids can be divided into one or more basic solids, like the one on the right. This solid is made up of a cube and a rectangular prism.



To find the **volume** of composite solids, you use the formula for the volume of the solids that make up the composite solid. Add the volumes of the different solids together to find the total volume of the composite solid. In the example figure, the volume would be:

$$\text{Volume of composite solid} = \text{Volume of cube} + \text{Volume of rectangular prism}$$

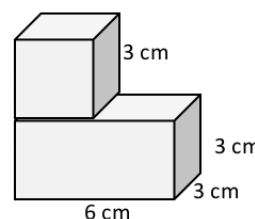
You will also find the **surface area** of composite solids. To find the surface area of composite solids, use the formula for surface area of the solids that make up the composite solid. First, add the surface areas of the different solids together. Then, **subtract the overlapping area**. This step is very important. The overlapping area is the part of each shape that is touching the other. For example, in the figure above, the overlap is one face of the cube and an equal surface area of the rectangular prism. You will subtract the area of the one face of the cube **twice**.

In the example figure above, the surface area would be:

$$\text{SA of composite solid} = \text{SA of cube} + \text{SA of rectangular prism} - 2 \times \text{area of 1 face of the cube}$$

Solved Examples

1. Find the volume and surface area of the solid shown:



Solution

Volume: Find the volume of the cube (V_1) and the volume of the rectangular prism (V_2) separately, then add them to find the total volume (V).

Volume of cube:

$$\begin{aligned}V_1 &= l^3 \\ &= (3 \text{ cm})^3 \\ &= 27 \text{ cm}^3\end{aligned}$$

Volume of rectangular prism:

$$\begin{aligned}V_2 &= lwh \\ &= 6 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm} \\ &= 54 \text{ cm}^3\end{aligned}$$

$$\text{Total volume: } V = V_1 + V_2 = 27 \text{ cm}^3 + 54 \text{ cm}^3 = 81 \text{ cm}^3$$

Surface area: Add the surface areas of the cube and rectangular prism, and subtract twice the area of one face of the cube.

Surface area of the cube:

$$\begin{aligned}SA_1 &= 6l^2 \\ &= 6(3 \text{ cm})^2 \\ &= 54 \text{ cm}^2\end{aligned}$$

Surface area of the rectangular prism:

$$\begin{aligned}SA_2 &= 2lw + 2wh + 2lh \\ &= 2(6)(3) + 2(3)(3) + 2(6)(3) \\ &= 90 \text{ cm}^2\end{aligned}$$

$$\text{Area of 1 face of the cube: } A = l^2 = (3 \text{ cm})^2 = 9 \text{ cm}^2$$

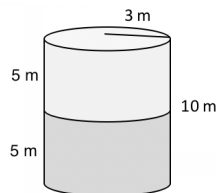
Surface area of the composite solid:

$$\begin{aligned}SA &= SA_1 + SA_2 - 2A \\ &= 54 \text{ cm}^2 + 90 \text{ cm}^2 - 2(9 \text{ cm}^2) \\ &= 126 \text{ cm}^2\end{aligned}$$

2. A water tank with a height of 10 m is in the shape of a cylinder with a radius of 3 m. The tank is half full of water. What is the volume of the water in the tank? Use $\pi = 3.14$.

Solution

First, draw a diagram:



Find the volume of the water tank and divide it in 2, since it is only half full.

Volume of the water tank:

$$\begin{aligned}V &= \pi r^2 h && \text{Formula} \\ &= 3.14 \times (3 \text{ m})^2 \times 10 \text{ m} && \text{Substitute values} \\ &= 282.6 \text{ m}^3\end{aligned}$$

$$\text{Divide by 2 to find the volume of half of the tank: } 282.6 \text{ m}^3 \div 2 = 141.3 \text{ m}^3$$

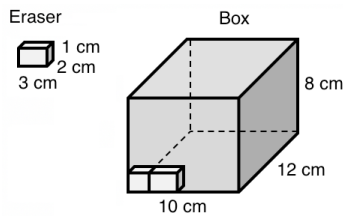
The volume of water in the tank is 141.3 m^3 .

3. Fatu sells pencil erasers from her shop. She bought a large box of erasers to sell. Each eraser is 1 cm by 2 cm by 3 cm. The box of erasers is 10 cm wide, 8 cm tall, and 12 cm long. How many erasers are there in the box?

Solution

To find how many erasers are in the box, divide the total volume of the box by the volume of each eraser.

Draw a diagram. →



Volume of 1 eraser:

$$\begin{aligned}V_1 &= lwh \\ &= 3 \text{ cm} \times 2 \text{ cm} \times 1 \text{ cm} \\ &= 6 \text{ cm}^3\end{aligned}$$

Volume of the box:

$$\begin{aligned}V_2 &= lwh \\ &= 12 \text{ cm} \times 10 \text{ cm} \times 8 \text{ cm} \\ &= 960 \text{ cm}^3\end{aligned}$$

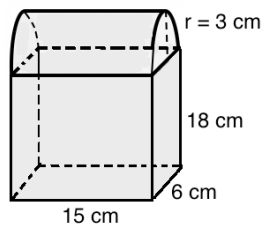
$$\text{Divide to find the number of erasers: } 960 \text{ cm}^3 \div 6 \text{ cm}^3 = 160 \text{ erasers}$$

There are 160 erasers in the box.

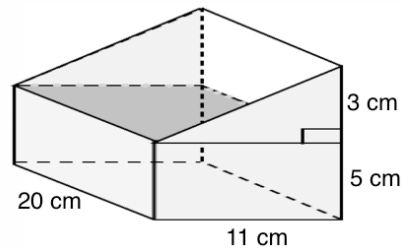
Practice

1. Find the volume and surface area of each solid below. Give your answers to the nearest whole number.

a. Use $\pi = 3.14$:



b.

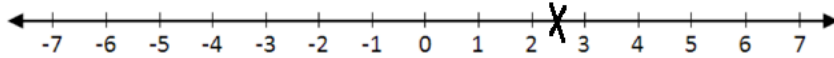


2. A large wooden beam is 10 metres long. Its end face is a rectangle 40 cm by 60 cm.
 - a. Calculate the volume of the beam in cubic metres.
 - b. Find the mass of the beam if 1 cubic metre of wood has a mass of 200 kg.
3. Martin sells packs of biscuits in his shop. Each pack of biscuits is 10 cm long, 5 cm wide and 4 cm high. How many packs of biscuits can fit stacked inside a carton that is 80 cm long, 50 cm wide and 25 cm high?
4. Cooking oil costs Le 100.00 for 20 cm^3 . How much would it cost to fill a container in the shape of a cylinder with a radius of 7 cm and height of 20 cm? Use $\pi = \frac{22}{7}$.
5. A bottle of cola has a label on it. The label is 8 cm tall, and the diameter of the bottle is 10 cm. What is the area covered by the label? (Take $\pi = 3.14$)
6. Hawa is wrapping a gift for her friend's birthday. The box she is wrapping is 15 cm tall, 20 cm long and 10 cm wide. What is the area of paper that she will need to completely cover the box?

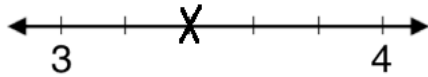
Answer Key – JSS 3 Term 3

Lesson Title: Review of the Number Line
Practice Activity: PHM-09-106

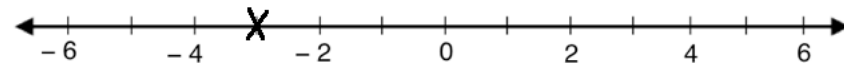
1. a.



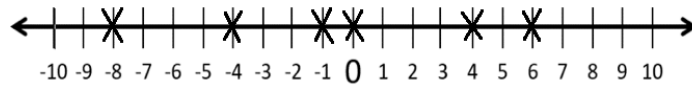
b.



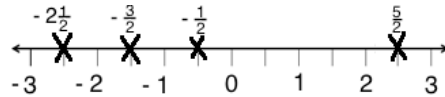
c.



2. Number line:



3. Number line:

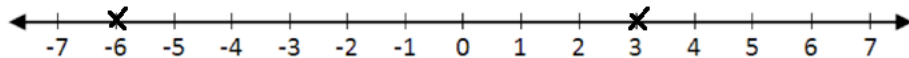


4. Circled: a. 4, b. -1, c. 4

5. a. -3, -2, -1, 1, 2, 3; b. $-1\frac{1}{2}$, $-\frac{1}{2}$, 0, $\frac{1}{2}$, $1\frac{1}{2}$

Lesson Title: Introduction to Inequality
Practice Activity: PHM-09-107

1. $-6 < 3$; number line:



2. a. $9 > -2$; b. $0.8 > -\frac{1}{5}$; c. $-1 > -12$

3. a. $-13 < 2$; b. $8 > -3$; c. $-4 > -7$

4. a. $11 > -8$; b. $-8 < -1$; c. $5 = 5$; d. $9 > 5$

Lesson Title: Linear Inequality

Practice Activity: PHM-09-108

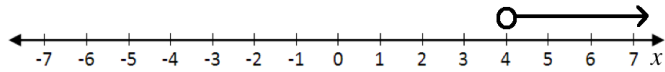
1. $x > -2$

2. $y < 4$

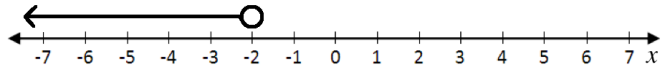
3. $x > 1$

4. $x < -3$

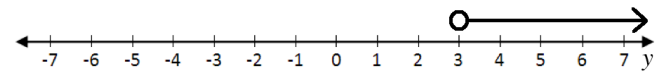
5. Number line:



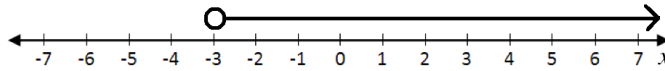
6. Number line:



7. Number line:



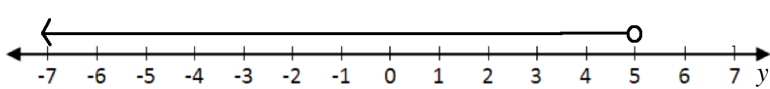
8. Number line:



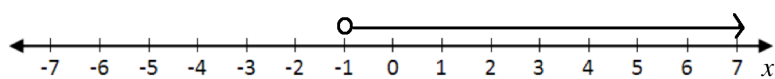
Lesson Title: Solving Linear Inequalities in One Variable I

Practice Activity: PHM-09-109

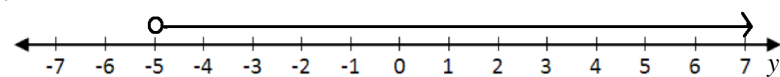
1. $y < 5$



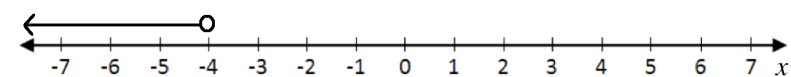
2. $x > -1$



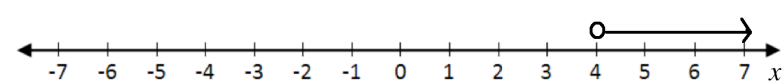
3. $y > -5$



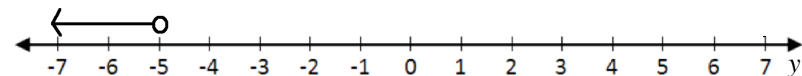
4. $x < -4$



5. $x > 4$



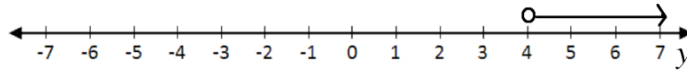
6. $y < -5$



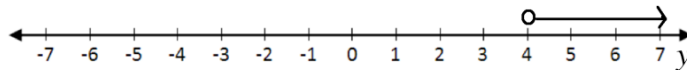
Lesson Title: Solving Linear Inequalities in One Variable II

Practice Activity: PHM-09-110

- a. $3 < y$; b. $y > 10$; c. $y < 3$
- a. $x > 20$; b. $x < -10$; c. $45 > x$ or $x < 45$.
- $y > 4$



- $y > 4$

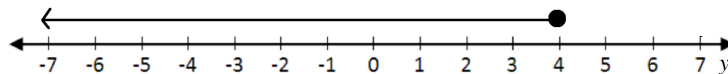


Lesson Title: Solving Linear Inequalities in One Variable III

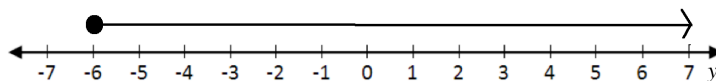
Practice Activity: PHM-09-111

- a. $x \geq 5$; b. $x \leq 2$
- See below:

- a. $y \leq 4$

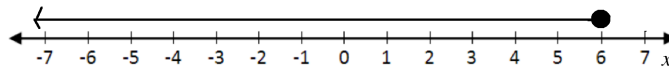


- b. $y \geq -6$

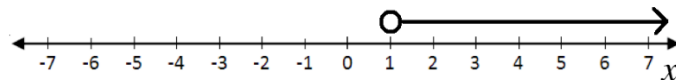


- See below:

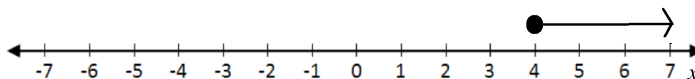
- a. $x \leq 6$



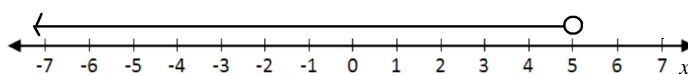
- b. $x > 1$



- c. $x \geq 4$



- d. $x < 5$



Lesson Title: Creating Inequalities from Story Problems

Practice Activity: PHM-09-112

Note that you may use any variable, but you must use the symbol and number given below.

- $p > 70$
- $x < 20$
- $r \geq 5$
- $c \leq 8$

5. $p > 10$
6. $h > 120$
7. $p \geq 40$
8. $s > 20$
9. $m \leq 40$.
10. $v \geq 18$
11. $c \geq 1$
12. $c \leq 0$

Lesson Title: Solving Inequality Story Problems I

Practice Activity: PHM-09-113

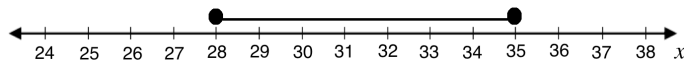
Note that you may use any variable, but you must use the symbol and number given below.

1. $m \geq 8$
2. $m > 5$
3. $p \geq 12$; she needs to sell 12 pumpkins
4. $w \leq 20$

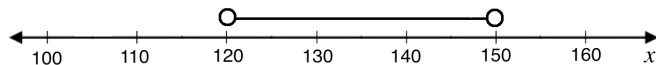
Lesson Title: Solving Inequality Story Problems II

Practice Activity: PHM-09-114

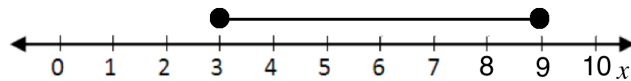
1. $28 \leq x \leq 35$



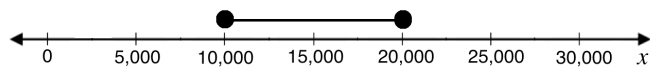
2. $120 < x < 150$



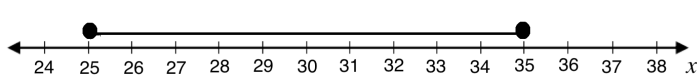
3. $3 \leq x \leq 9$



4. $10,000 \leq x \leq 20,000$



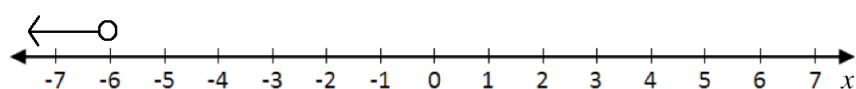
5. $25 \leq x \leq 35$



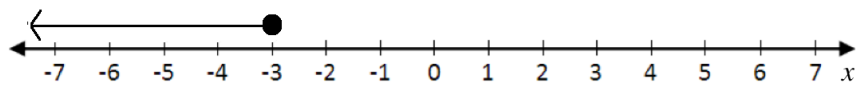
Lesson Title: Practice with Inequalities

Practice Activity: PHM-09-115

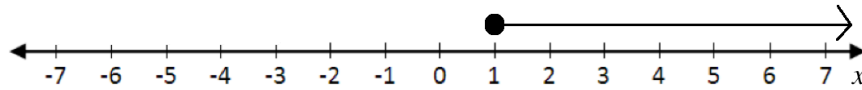
1. $x < -6$



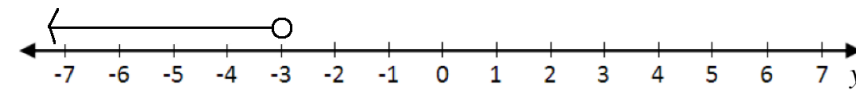
2. $x \leq -3$



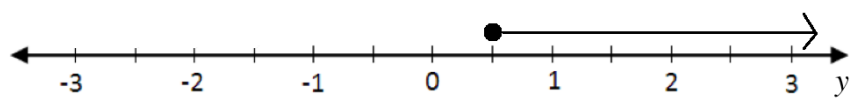
3. $x \geq 1$



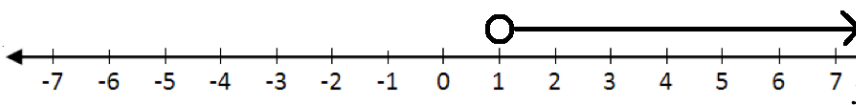
4. $y < -3$



5. $y \geq \frac{1}{2}$

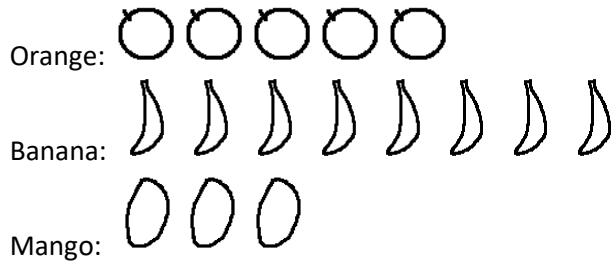


6. $x > 1$



Lesson Title: Data Collection
Practice Activity: PHM-09-116

1. Pictogram of Martin's fruit:



- a. 16 players; b. 2 players; c. 7 players; d. No
- a. 167, 168, 169, 170, 170, 170, 172, 172, 172, 172, 173, 173, 173, 177, 177, 178; b. 178 cm; c. 167 cm; d. 3 large uniforms

Lesson Title: Frequency Tables

Practice Activity: PHM-09-117

1. See table below; a. 19 babies; b. 7 kg; c. 4 babies; d. 5 kg

WEIGHT (KG)	TALLY MARKS	NUMBER OF BABIES
3		4
4		4
5	/	6
6		3
7		2
Total	19	19

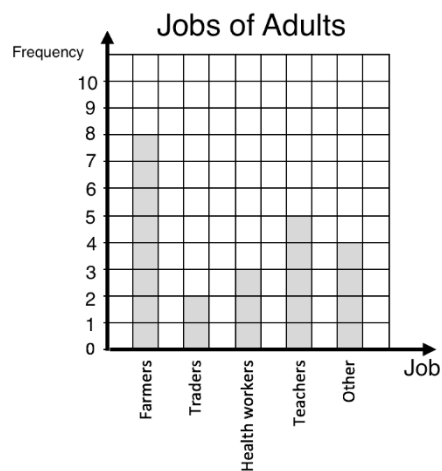
2. See table below; a. 12 pupils; b. 170 cm; c. 2 pupils

HEIGHT (CM)	TALLY MARKS	NUMBER OF PUPILS
150		1
155		4
160		2
165		3
170		2
Total	12	12

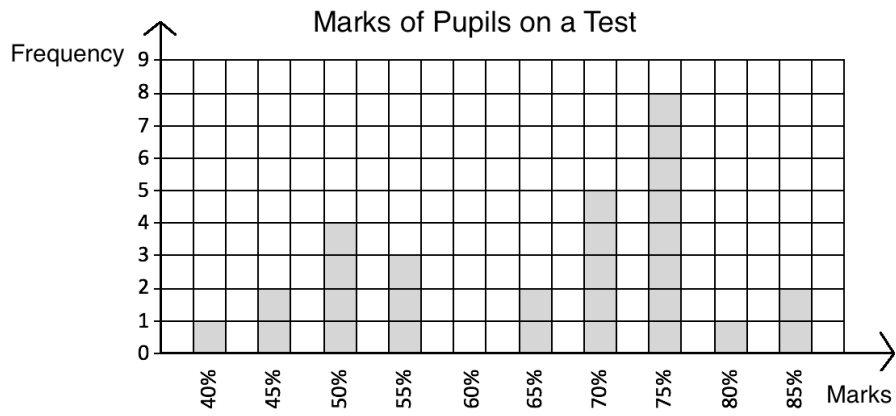
Lesson Title: Bar Charts

Practice Activity: PHM-09-118

1. See below:

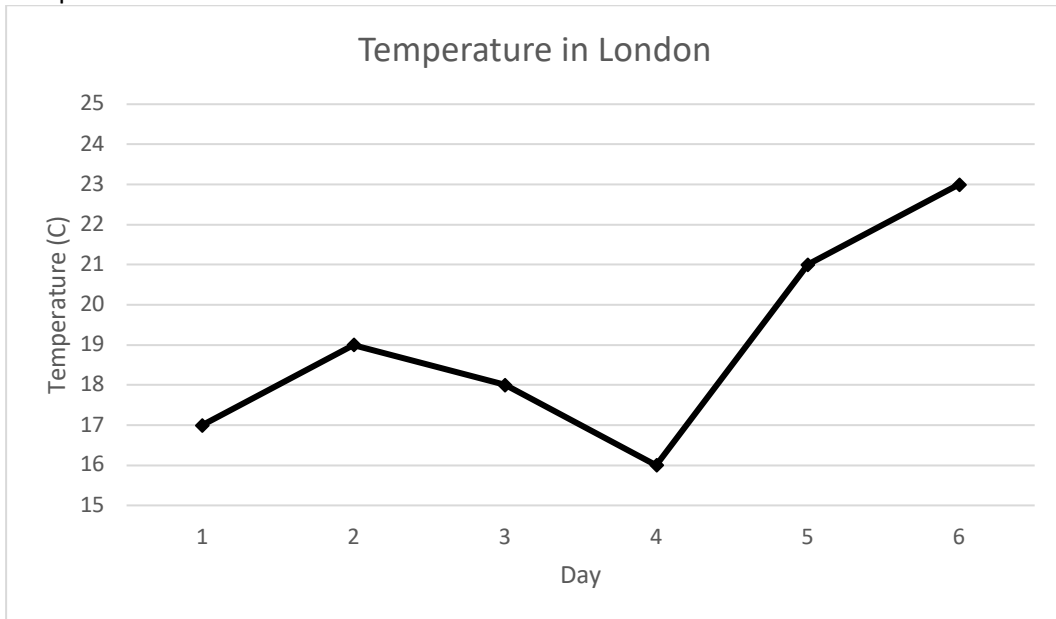


2. See below:

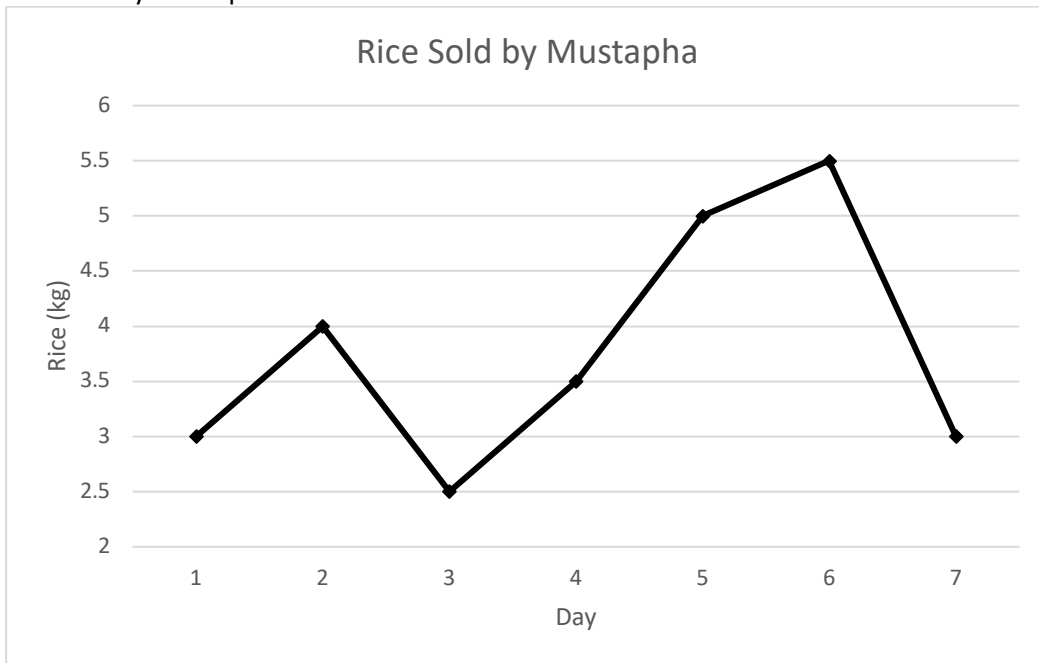


Lesson Title: Line Graphs
Practice Activity: PHM-09-119

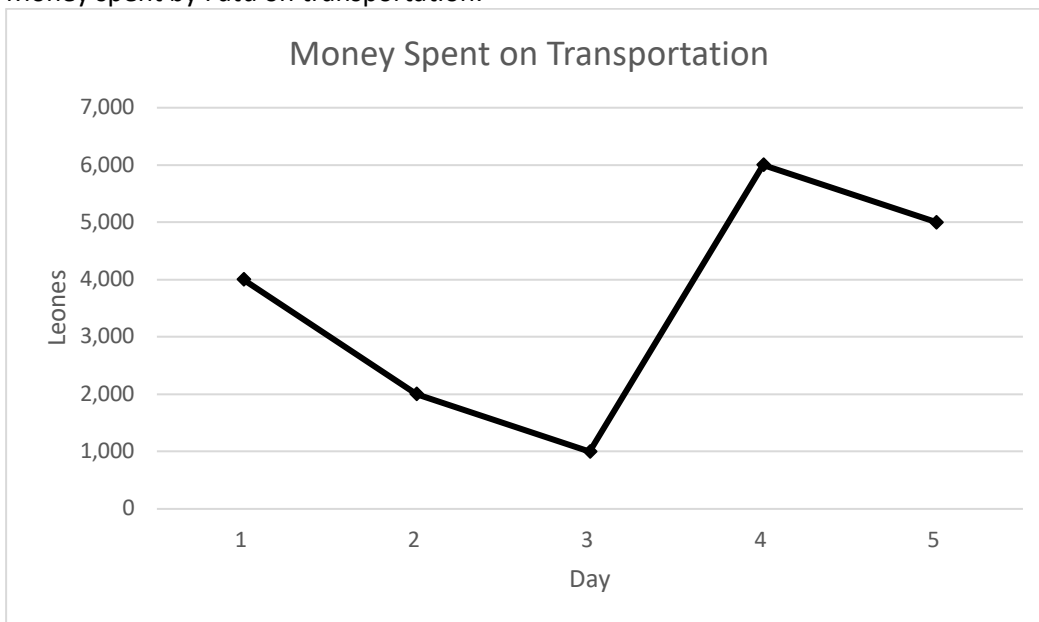
1. Temperatures in London:



2. Rice sold by Mustapha:



3. Money spent by Fatu on transportation:



Lesson Title: Interpreting Charts and Graphs

Practice Activity: PHM-09-120

- a. 41 pupils; b. 1 mark; c. 8 pupils; d. 28 pupils
b. a. 2 cm; b. 5 August; c. 1 August **and** 7 August; d. 2 cm; e. 4 cm

Lesson Title: Interpreting Pie Charts

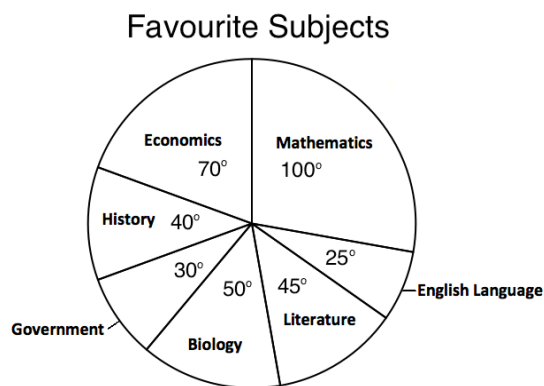
Practice Activity: PHM-09-121

1. a. mobile phone; b. television; c. 105 people; d. 150 people
2. Radio: 126° ; Mobile phone: 180° ; Television: 18° ; Other: 36°
3. a. wages; b. Le 400,000.00; c. Le 600,000.00; d. Le 600,000.00

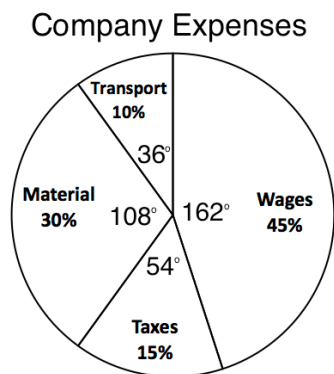
Lesson Title: Creating Pie Charts

Practice Activity: PHM-09-122

1. Pie chart:

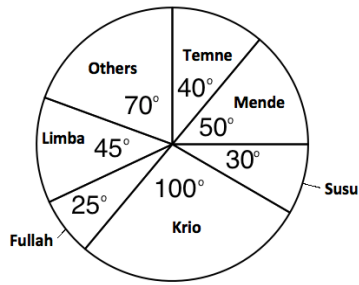


2. Pie chart:



3. Pie chart:

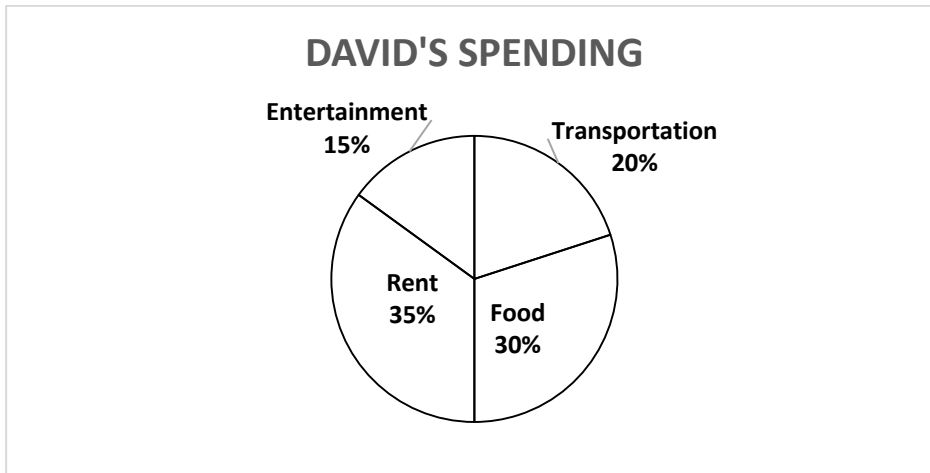
Pupils' Ethnic Groups



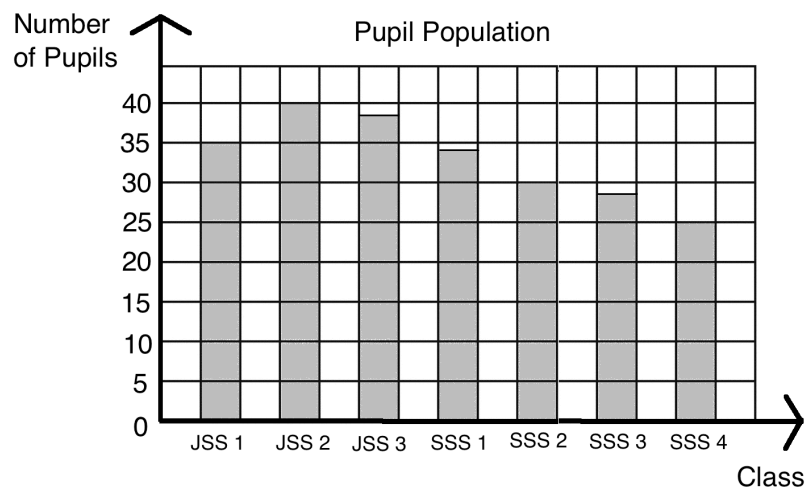
Lesson Title: Choosing a Graph or Chart

Practice Activity: PHM-09-123

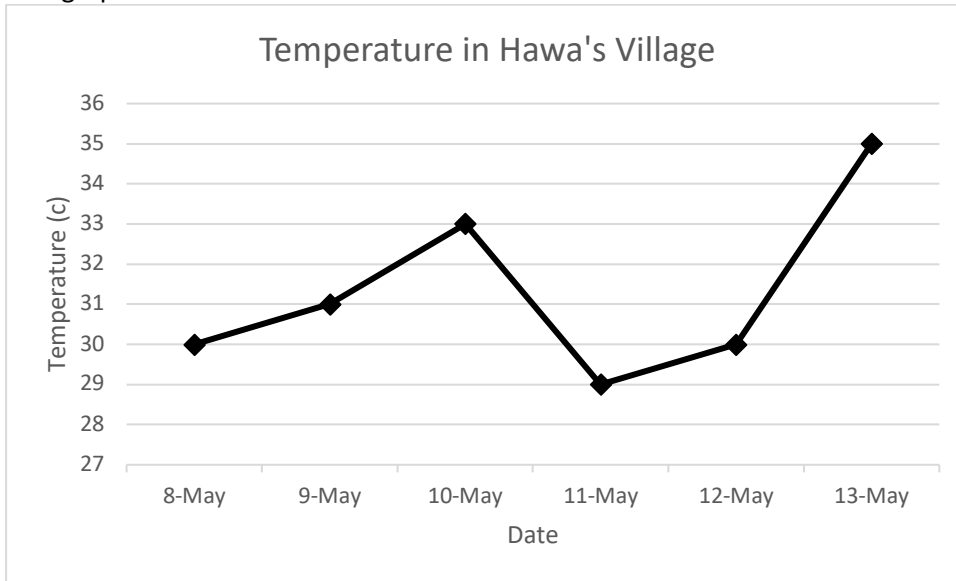
1. Pie chart:



2. Bar chart:



3. Line graph:



Lesson Title: Mean

Practice Activity: PHM-09-124

1. 76 marks
2. Size 9.6
3. 8 goals
4. 9.8 or 10 pupils
5. 29.7°C or $29\frac{2}{3}^{\circ}\text{C}$

Lesson Title: Median

Practice Activity: PHM-09-125

1. 76 marks
2. Size 10
3. 8 goals
4. 10 pupils
5. 28.5°C

Lesson Title: Mode and Range

Practice Activity: PHM-09-126

1. Mode: 155 cm; Range: 10 cm
2. Mode: 15 kg and 20 kg; Range: 9 kg
3. Mode: none; Range: 9 watermelons
4. The number of people who prefer each fruit is 10, 12, 3, 16 and 8. Mode: none; Range: 13 pupils
5. Mode: 28°C; Range: 8°C

Lesson Title: Introduction to Grouped Data

Practice Activity: PHM-09-127

1. a. 34,000 people; b. 26,000 people
2. Frequency table:

Weight (kg)	Tally	Frequency
10 – 19		6
20 – 29		10
30 – 39		13
40 – 49		11
50 – 59		6
60 – 69		3
70 – 79		1

- a. 16; b. 10
3. a. 65 motorists; b. 45 motorists

Lesson Title: Mean of Grouped Data

Practice Activity: PHM-09-128

- 63 kg
- a. See frequency table below; b. $10\frac{1}{3}$ or 10.3 years old

Patients' Ages	
Age (years)	Frequency
0 – 4	6
5 – 9	6
10 – 14	4
15 – 19	6
20 – 24	2
Total	24

- a. See frequency table below; b. Mean: 48 marks

Class Interval	Frequency
10 – 19	2
20 – 29	4
30 – 39	12
40 – 49	6
50 – 59	1
60 – 69	8
70 – 79	5
80 – 89	2
Total	40

Lesson Title: Median and Modal Class of Grouped Data

Practice Activity: PHM-09-129

- a. 40-44; b. 40-44; c. 43 years
- a. 15-19; b. 15-19; c. 15.7 marks
- a. 10-14; b. 10-14; c. 13.3 mm

Lesson Title: Practice with Grouped Data

Practice Activity: PHM-09-130

- a. 43 years; b. 43 years.
- a. 54.3; b. 51 – 60; c. 51 – 60; d. 55.0;
- Mean: 44.45 kg.; Median: 45.43 kg.

Lesson Title: Probability

Practice Activity: PHM-09-131

1. a. Impossible; b. Likely; c. Certain; d. Unlikely
2. Unlikely
3. Certain
4. Impossible
5. You will write statements of your own. Example statements are:
 - a. Impossible: A cat has 3 heads.
 - b. Unlikely: A pupil gets lost on her way home from school.
 - c. Likely: You will pass to JSS2.
 - d. Certain: Your mother gave birth to you.

Lesson Title: Probability Experiments with 1 Event

Practice Activity: PHM-09-132

1. a. Experiment; b. Experiment; c. Event; d. Event; e. Event; f. Experiment
2. There are 4 possible outcomes: choosing Manchester United, choosing Chelsea, choosing Arsenal, or choosing Real Madrid.
3. a. 1; b. 2; c. 5; d. 5; e. 3

Lesson Title: Expressing Probability with Numbers

Practice Activity: PHM-09-133

1. a. $\frac{10}{21}$; b. $\frac{11}{21}$
2. a. $\frac{1}{8}$; b. $\frac{1}{2}$; c. $\frac{1}{3}$; d. $\frac{1}{24}$
3. 55%
4. a. 28%; b. 32%; c. 36%; d. 4%

Lesson Title: Likelihood of Events

Practice Activity: PHM-09-134

1. a. It is more likely that a child will be selected. The probability of selecting a child ($\frac{11}{18}$) is greater than the probability of selecting an adult ($\frac{7}{18}$).
b. It is more likely that a female will be selected. The probability of selecting a female ($\frac{5}{9}$) is greater than the probability of selecting a male ($\frac{4}{9}$).
2. c, a, b, d
3. a. Neither unlikely or likely, there is an equal chance that it will rain or not rain;
b. unlikely; c. likely; d. impossible

Lesson Title: Probability Experiments with 2 Independent Events

Practice Activity: PHM-09-135

1. The events are independent.
2. The events are not independent.
3. The events are independent.
4. The events are independent.

Lesson Title: Probability Experiments with Independent Events I

Practice Activity: PHM-09-136

1. $\frac{1}{36}$
2. a. $\frac{3}{32}$; b. $\frac{15}{32}$
3. $\frac{3}{25}$
4. $\frac{1}{2704}$
5. a. $\frac{2}{15}$; b. $\frac{2}{5}$; c. $\frac{4}{15}$

Lesson Title: Probability Experiments with Independent Events II

Practice Activity: PHM-09-137

1. a. Independent; b. Dependent; c. Independent; d. Independent; e. Dependent; Dependent
2. a. $\frac{5}{8}$; b. $\frac{3}{7}$
3. a. $\frac{1}{3}$; b. $\frac{1}{2}$
4. a. Independent; b. $\frac{27}{100}$; c. $\frac{33}{100}$

Lesson Title: Sample Space

Practice Activity: PHM-09-138

1. Sample Space:

		Draw a Ball				
		Red	Red	Red	Yellow	Yellow
Coin Flip	Heads	H, R	H, R	H, R	H, Y	H, Y
	Tails	T, R	T, R	T, R	T, Y	T, Y

2. Sample Space:

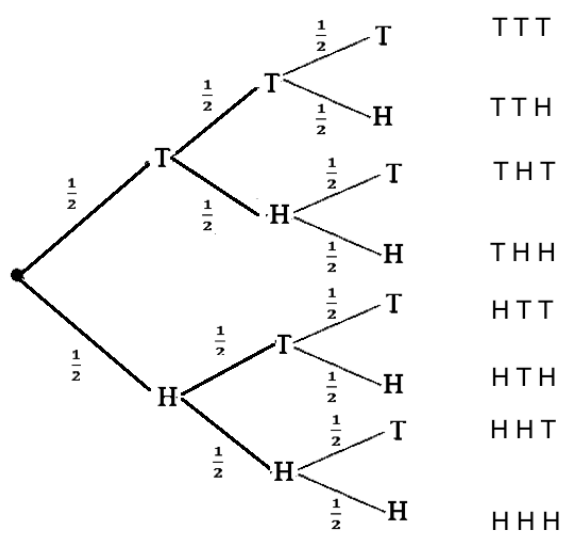
		Coin Flip	
		Heads	Tails
Die Roll	1	1, H	1, T
	2	2, H	2, T
	3	3, H	3, T
	4	4, H	4, T
	5	5, H	5, T
	6	6, H	6, T

3. Sample Space:

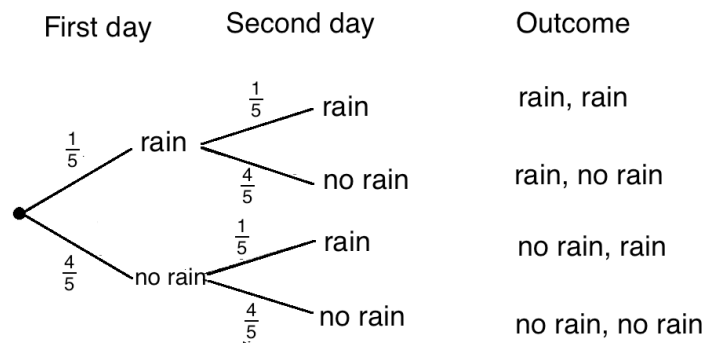
		Draw a Ball		
		Yellow	Yellow	Green
Die Roll	1	1, Y	1, Y	1, G
	2	2, Y	2, Y	2, G
	3	3, Y	3, Y	3, G
	4	4, Y	4, Y	4, G
	5	5, Y	5, Y	5, G
	6	6, Y	6, Y	6, G

Lesson Title: Probability Trees
Practice Activity: PHM-09-139

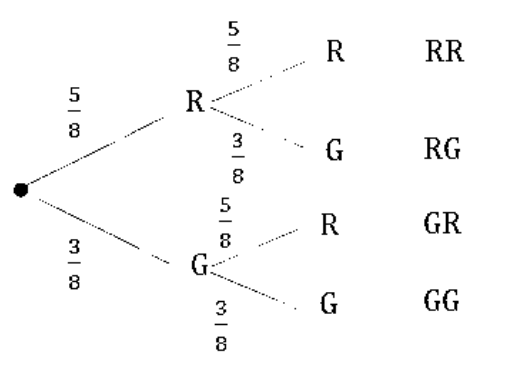
1. See probability tree below; $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$



2. See probability tree below; $\frac{1}{25}$



3. a. See probability tree below; b. $\frac{25}{64}$; c. $\frac{9}{64}$



Lesson Title: Probability Story Problems

Practice Activity: PHM-09-140

1. $\frac{1}{216}$
2. $\frac{1}{32}$
3. a. $\frac{3}{5}$; b. $\frac{2}{5}$; c. $\frac{1}{2}$; d. $\frac{1}{2}$; e. $\frac{2}{5}$
4. See sample space below; $\frac{2}{36} = \frac{1}{18}$

	Draw a ball						
		Red	Red	Blue	Green	Green	Green
Roll a Die	1	1, R	1, R	1, B	1, G	1, G	1, G
	2	2, R	2, R	2, B	2, G	2, G	2, G
	3	3, R	3, R	3, B	3, G	3, G	3, G
	4	4, R	4, R	4, B	4, G	4, G	4, G
	5	5, R	5, R	5, B	5, G	5, G	5, G
	6	6, R	6, R	6, B	6, G	6, G	6, G

Lesson Title: Perimeter of Triangles and Quadrilaterals

Practice Activity: PHM-09-141

1. a. 78 m; b. 24.8 m
2. 56 cm
3. 16 m
4. a. 48 cm; b. 125 mm
5. 164 mm
6. 36.5 m

Lesson Title: Area of Triangles

Practice Activity: PHM-09-142

1. a. 110 cm^2 ; b. 45 cm^2
2. a. 85.2 cm^2 ; b. 56 cm^2 ; c. 500 mm^2

Lesson Title: Area of Quadrilaterals

Practice Activity: PHM-09-143

1. a. 350 m^2 ; b. 38.44 m^2

2. 196 cm^2
3. 140 cm^2
4. The square has greater area
5. a. 150 cm^2 ; b. 24 cm^2
6. 4.4 m^2
7. 66 m^2
8. 52.8 cm^2

Lesson Title: Area and Circumference of Circles
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Practice Activity: PHM-09-144

1. $C = 44 \text{ cm}$, $A = 154 \text{ cm}^2$
2. $C = 628 \text{ cm}$
3. $C = 220 \text{ cm}$
4. $A = 28.3 \text{ cm}^2$
5. $A = 50.29 \text{ m}^2$
6. $C = 440 \text{ cm}$
7. $C = 176 \text{ m}$; $A = 2,464 \text{ m}^2$
8. $C = 188.4 \text{ cm}$; $A = 2,826 \text{ cm}^2$

Lesson Title: Practical Problems with Area and Perimeter

Practice Activity: PHM-09-145

1. a. $A = 77 \text{ cm}^2$; $P = 44 \text{ cm}$; b. $A = 98.24 \text{ cm}^2$; $P = 37.12 \text{ cm}$
2. a. $P = 110 \text{ m}$; b. Le 440,000.00
3. a. $A = 707.1 \text{ m}^2$; b. $P = 94.3 \text{ m}$
4. $P = 24 \text{ m}$
5. a. $A = 120 \text{ m}^2$; b. 3000 tiles

Lesson Title: Volume of Prisms

Practice Activity: PHM-09-146

1. 343 cm^3
2. 15.6 m^3
3. 2 ft
4. 78 m^3
5. $120,000 \text{ cm}^3$
6. 6 m^3
7. a. 288 cm^3 ; b. 300 cm^3

Lesson Title: Volume of Cylinders
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Practice Activity: PHM-09-147

1. a. 402 cm^3 ; b. 367 cm^3
2. $6,160 \text{ mm}^3$
3. 276.3 cm^3
4. 184.8 cm^3
5. 792 cm^3

Lesson Title: Surface Area of Prisms

Practice Activity: PHM-09-148

1. a. 142 m^2 ; b. 648 cm^2
2. 864 mm^2
3. Cube
4. a. 620 cm^2 ; b. 132 m^2

Lesson Title: Surface Area of Cylinders
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Practice Activity: PHM-09-149

1. a. 301 cm^2 ; b. 722 m^2
2. a. 1408 cm^2 ; b. 748 cm^2

Lesson Title: Practical Problems with Volume and Surface Area
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Practice Activity: PHM-09-150

1. a. $\text{SA} = 926 \text{ cm}^2$, $V = 1,832 \text{ cm}^3$; b. $\text{SA} = 851 \text{ cm}^2$; $V = 1,430 \text{ cm}^3$
2. a. 2.4 m^3 ; b. 480 kg
3. 500 packs of biscuits
4. Le 15,400.00
5. 251.2 cm^2
6. $1,300 \text{ cm}^2$

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