Free Quality School Education

Ministry of Basic and Senior Secondary Education

Lesson Plans for

Senior Secondary *Mathematics*

SSS III

Term III

Foreword

These Lesson Plans and the accompanying Pupils' Handbooks are essential educational resources for the promotion of quality education in senior secondary schools in Sierra Leone. As Minister of Basic and Senior Secondary Education, I am pleased with the professional competencies demonstrated by the writers of these educational materials in English Language and Mathematics.

The Lesson Plans give teachers the support they need to cover each element of the national curriculum, as well as prepare pupils for the West African Examinations Council's (WAEC) examinations. The practice activities in the Pupils' Handbooks are designed to support self-study by pupils, and to give them additional opportunities to learn independently. In total, we have produced 516 lesson plans and 516 practice activities – one for each lesson, in each term, in each year, for each class. The production of these materials in a matter of months is a remarkable achievement.

These plans have been written by experienced Sierra Leoneans together with international educators. They have been reviewed by officials of my Ministry to ensure that they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognized techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make the best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new lesson plans. It is really important that the Lesson Plans and Pupils' Handbooks are used, together with any other materials they may have.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank our partners for their continued support. Finally, I also thank the teachers of our country for their hard work in securing our future.

Timbo

Mr. Alpha Osman Timbo

Minister of Basic and Senior Secondary Education

The policy of the Ministry of Basic and Senior Secondary Education, Sierra Leone, on textbooks stipulates that every printed book should have a lifespan of three years.

To achieve thus, <u>DO NOT WRITE IN THE BOOKS</u>.

Table of Contents

Introduction

to the Lesson Plans

These lesson plans are based on the National Curriculum and the West Africa Examination Council syllabus guidelines, and meet the requirements established by the Ministry of Basic and Senior Secondary Education.

1. The lesson plans will not take the whole term, so use spare time to review material or prepare for examinations.
2. Teachers can use other textbooks alongside or instead of these lesson plans.
3. Read the lesson plan before you start the lesson. Look ahead to the next lesson, and see if you need to tell pupils to bring materials for next time.
4. Make sure you understand the learning outcomes, and have teaching aids and other preparation ready – each lesson plan shows these using the symbols on the right.
5. If there is time, quickly review what you taught last time before starting each lesson.
6. Follow the suggested time allocations for each part of the lesson. If time permits, extend practice with additional work.
7. Lesson plans have a mix of activities for the whole class and for individuals or in pairs.
8. Use the board and other visual aids as you teach.
9. Interact with all pupils in the class – including the quiet ones.
10. Congratulate pupils when they get questions right! Offer solutions when they don't, and thank them for trying.

Learning outcomes

Preparation

KEY TAKEAWAYS FROM SIERRA LEONE'S PERFORMANCE IN WEST AFRICAN SENIOR SCHOOL CERTIFICATE EXAMINATION – GENERAL MATHEMATICS[1]

This section, seeks to outline key takeaways from assessing Sierra Leonean pupils' responses on the West African Senior School Certificate Examination. The common errors pupils make are highlighted below with the intention of giving teachers an insight into areas to focus on, to improve pupil performance on the examination. Suggestions are provided for addressing these issues.

Common errors

1. Errors in applying principles of BODMAS
2. Mistakes in simplifying fractions
3. Errors in application of Maths learned in class to real-life situations, and vis-a-versa.
4. Errors in solving geometric constructions.
5. Mistakes in solving problems on circle theorems.
6. Proofs are often left out from solutions, derivations are often missing from quadratic equations.

Suggested solutions

1. Practice answering questions to the detail requested
2. Practice re-reading questions to make sure all the components are answered.
3. If possible, procure as many geometry sets to practice geometry construction.
4. Check that depth and level of the lesson taught is appropriate for the grade level.

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[1] This information is derived from an evaluation of WAEC Examiners' Reports, as well as input from their examiners and Sierra Leonean teachers.

FACILITATION STRATEGIES

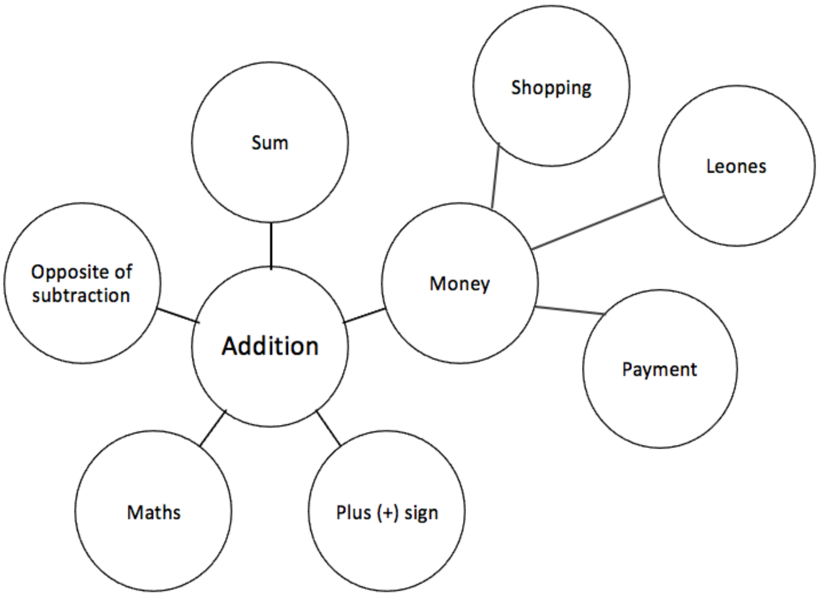
This section includes a list of suggested strategies for facilitating specific classroom and evaluation activities. These strategies were developed with input from national experts and international consultants during the materials development process for the Lesson Plans and Pupils' Handbooks for Senior Secondary Schools in Sierra Leone.

Strategies for introducing a new concept

- **Unpack prior knowledge:** Find out what pupils know about the topic before introducing new concepts, through questions and discussion. This will activate the relevant information in pupils' minds and give the teacher a good starting point for teaching, based on pupils' knowledge of the topic.
- **Relate to real-life experiences:** Ask questions or discuss real-life situations where the topic of the lesson can be applied. This will make the lesson relevant for pupils.
- **K-W-L:** Briefly tell pupils about the topic of the lesson, and ask them to discuss 'What I know' and 'What I want to know' about the topic. At the end of the lesson have pupils share 'What I learned' about the topic. This strategy activates prior knowledge, gives the teacher a sense of what pupils already know and gets pupils to think about how the lesson is relevant to what they want to learn.
- **Use teaching aids from the environment:** Use everyday objects available in the classroom or home as examples or tools to explain a concept. Being able to relate concepts to tangible examples will aid pupils' understanding and retention.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, can be used to activate prior knowledge and engage pupils in the content which is going to be taught in the lesson.

Strategies for reviewing a concept in 3-5 minutes

- **Mind-mapping:** Write the name of the topic on the board. Ask pupils to identify words or phrases related to the topic. Draw lines from the topic to other related words. This will create a 'mind-map', showing pupils how the topic of the lesson can be mapped out to relate to other themes. Example below:



- **Ask questions:** Ask short questions to review key concepts. Questions that ask pupils to summarise the main idea or recall what was taught is an effective way to review a concept quickly. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, is an effective way to review concepts as a whole group.
- **Matching:** Write the main concepts in one column and a word or a phrase related to each concept in the second column, in a jumbled order. Ask pupils to match the concept in the first column with the words or phrases that relate to in the second column.

Strategies for assessing learning without writing

- **Raise your hand:** Ask a question with multiple-choice answers. Give pupils time to think about the answer and then go through the multiple-choice options one by one, asking pupils to raise their hand if they agree with the option being presented. Then give the correct answer and explain why the other answers are incorrect.
- **Ask questions:** Ask short questions about the core concepts. Questions which require pupils to recall concepts and key information from the lesson are an effective way to assess understanding. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Think-pair-share:** Give pupils a question or topic and ask them to turn to seatmates to discuss it. Then, have pupils volunteer to share their ideas with the rest of the class.
- **Oral evaluation:** Invite volunteers to share their answers with the class to assess their work.

Strategies for assessing learning with writing

- **Exit ticket:** At the end of the lesson, assign a short 2-3 minute task to assess how much pupils have understood from the lesson. Pupils must hand in their answers on a sheet of paper before the end of the lesson.
- **Answer on the board:** Ask pupils to volunteer to come up to the board and answer a question. In order to keep all pupils engaged, the rest of the class can also answer the question in their exercise books. Check the answers together. If needed, correct the answer on the board and ask pupils to correct their own work.
- **Continuous assessment of written work:** Collect a set number of exercise books per day/per week to review pupils' written work in order to get a sense of their level of understanding. This is a useful way to review all the exercise books in a class which may have a large number of pupils.
- **Write and share:** Have pupils answer a question in their exercise books and then invite volunteers to read their answers aloud. Answer the question on the board at the end for the benefit of all pupils.
- **Paired check:** After pupils have completed a given activity, ask them to exchange their exercise books with someone sitting near them. Provide the answers, and ask pupils to check their partner's work.
- **Move around:** If there is enough space, move around the classroom and check pupils' work as they are working on a given task or after they have completed a given task and are working on a different activity.

Strategies for engaging different kinds of learners

- For pupils who progress faster than others:
  - Plan extension activities in the lesson.
  - Plan a small writing project which they can work on independently.
  - Plan more challenging tasks than the ones assigned to the rest of the class.
  - Pair them with pupils who need more support.

- For pupils who need more time or support:
  - Pair them with pupils who are progressing faster, and have the latter support the former.
  - Set aside time to revise previously taught concepts while other pupils are working independently.
  - Organise extra lessons or private meetings to learn more about their progress and provide support.
  - Plan revision activities to be completed in the class or for homework.
  - Pay special attention to them in class, to observe their participation and engagement.

Lesson Title: Introduction to probability – Part 1	**Theme:** Statistics and Probability	
Lesson Number: M3-L097	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Define, use, and give examples of terms used in probability. 2. Use the language of probability to describe events in real life.	**Preparation** 1. Write the table giving PROBABILITY TERMS AND DEFINITIONS found at the end of this lesson plan on the board. Leave the Examples column blank. 2. Write the questions also found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to think about something they are likely to do when they go home today.
2. Invite volunteers to answer. (Example answers: do their homework; have a meal; listen to music; watch a film; play football with friends)
3. Tell pupils that after today's lesson, they will be able to define, use, and give examples of terms used in probability. They will also be able to use the language of probability to describe events in real life.

Teaching and Learning *(20 minutes)*

1. Explain:
   - Probability is used to describe how likely or unlikely it is for something to happen.
   - It is used in many different fields such as statistics, commerce, gambling, insurance, science and technology,
   - Before we can study probability, we need to be able to define and use the terms used in probability.
2. Point to the list of words for question a. on the board and explain: Because probability is based on uncertainty and chance, in everyday life we use words such as those shown.
3. Ask pupils to work with seatmates to answer question a.
4. Invite volunteers to give answers and note some of the answers on the board. The answers will very likely be different depending on the time of year, who is answering the question, and so on.

   Solution:

a. **Statement**	**Example answers**
i. It will rain tomorrow	likely / unlikely depending on the time of year
ii. The sun will rise in the east tomorrow	certain
iii. You are late for school tomorrow	likely / unlikely
iv. You complete all your Maths practice correctly	likely / unlikely

v.	A human being grows to 20 feet tall	impossible
vi.	Your favourite football team wins its next match.	likely / unlikely

- ii. is a certainty – the sun always rises in the east.
- v. is impossible. No human being will grow to be 20 feet tall.
- The total opposite of a certain event is an impossible event.
- The only impossible statement is v. All the others have a degree of uncertainty depending on time of year, the person answering and other similar factors.

5. Explain:
   - We have different terms to explain and describe probability.
   - Give each term and definition in the table of PROBABILITY TERMS AND DEFINITION.
   - Give some of the examples in the example column.
   - Ask pupils to discuss with seatmates to come up with more examples.
   - Invite volunteers to give their examples.
6. Go through all the terms in the table making sure pupils are clear on each term.
7. Explain: New probability terms will be defined as we encounter them during the topic.
8. Invite a volunteer to fully assess question b. on the board and extract the given information. (Example answer: give all the possible outcomes of given experiments)

   Solution:

   b. Given: give all the possible outcomes of given experiments
      i. guessing the gender of a new-born child: $S = \{\text{girl}, \text{boy}\}$
      ii. guessing the prime numbers less than 20: $S = \{2, 3, 5, 7, 11, 13, 17, 19\}$
9. Ask pupils to work with seatmates to answer question b. iii.
10. Invite a volunteer to give their answer. The rest of the class should check and correct their answer.
      iii. guessing a vowel $S = \{\text{a}, \text{e}, \text{i}, \text{o}, \text{u}\}$
11. Ask pupils to work with seatmates to answer question c.
12. Invite volunteers to give their examples of each type of probability outcome. Examples are given below in the solution. Pupils will give their own examples.

   Solution:

   c. Given: describe two events for each given probability outcome

      certain
      It will rain in August
      Independence Day will fall on 27 April next year
      likely
      You eat rice for lunch today
      You score 100% in your next Maths test
      unlikely
      People will be able to live on Mars
      You win first prize in the state lottery
      Impossible

A human being will live to be 200 years old
The sun will rise in the west tomorrow

Practice *(15 minutes)*

1. Ask pupils to work independently to answer question d.
2. Walk around, if possible, to check the answers and clear misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solutions:

   d. Given: give all the possible outcomes of given experiments

guessing a month ending in "ber"	$S = \{\text{September, October, November, December}\}$
guessing a multiple of 10 under 100	$S = \{10, 20, 30, 40, 50, 60, 70, 80, 90\}$
arranging the letters of the word cat	$S = \{\text{cat, cta, atc, act, tac, tca}\}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L097 in the Pupil Handbook.

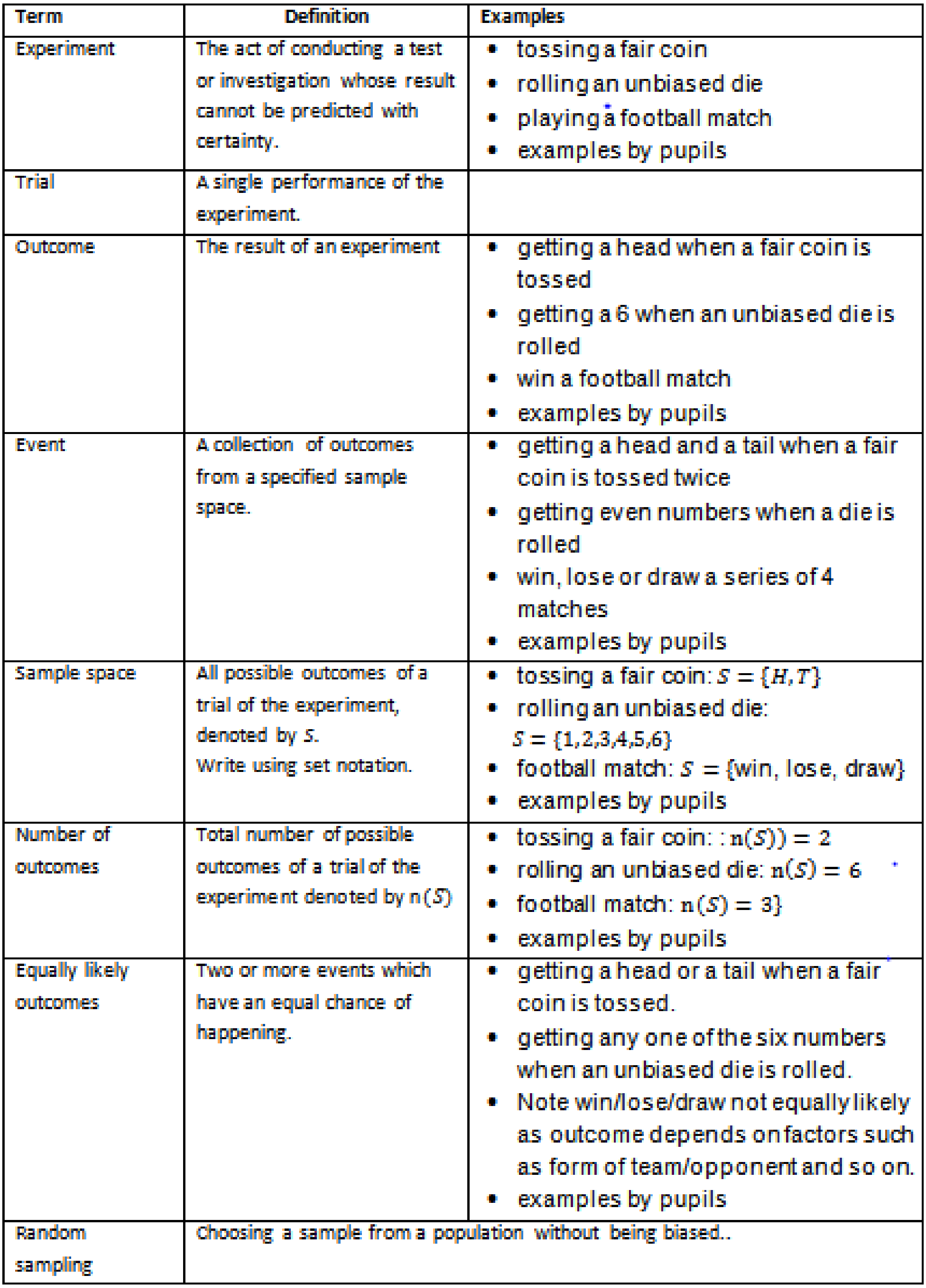
[QUESTIONS]

a. Use one of the following words to describe the given statements:
   certain likely unlikely impossible
   i. It will rain tomorrow.
   ii. The sun will rise in the east tomorrow.
   iii. You are late for school tomorrow.
   iv. You complete all your Maths practice correctly.
   v. A human being grows to 20 feet tall.
   vii. Your favourite football team win its next match.

b. Give all the possible outcomes for the following experiment.
   Write the sample space using set notation.
   i. Guessing the gender of a new-born child.
   ii. Guessing a prime number less than 20.
   iii. Guessing a vowel.

c. Describe two events that are:
   i. certain ii. likely iii. unlikely iv. impossible

d. Give all the possible outcomes for the following experiments (i.e. write the sample space using set notation)
   i. Guessing a month ending in "ber".
   ii. Guessing a multiple of 10 under 100.
   iii. Arranging the letters of the word cat.



Term	Definition	Examples
Experiment	The act of conducting a test or investigation whose result cannot be predicted with certainty.	• tossing a fair coin • rolling an unbiased die • playing a football match • examples by pupils
Trial	A single performance of the experiment.	
Outcome	The result of an experiment	• getting a head when a fair coin is tossed • getting a 6 when an unbiased die is rolled • win a football match • examples by pupils
Event	A collection of outcomes from a specified sample space.	• getting a head and a tail when a fair coin is tossed twice • getting even numbers when a die is rolled • win, lose or draw a series of 4 matches • examples by pupils
Sample space	All possible outcomes of a trial of the experiment, denoted by S. Write using set notation.	• tossing a fair coin: $S = \{H, T\}$ • rolling an unbiased die: $S = \{1,2,3,4,5,6\}$ • football match: $S = \{\text{win, lose, draw}\}$ • examples by pupils
Number of outcomes	Total number of possible outcomes of a trial of the experiment denoted by $n(S)$	• tossing a fair coin: : $n(S)) = 2$ • rolling an unbiased die: $n(S) = 6$ • football match: $n(S) = 3\}$ • examples by pupils
Equally likely outcomes	Two or more events which have an equal chance of happening.	• getting a head or a tail when a fair coin is tossed. • getting any one of the six numbers when an unbiased die is rolled. • Note win/lose/draw not equally likely as outcome depends on factors such as form of team/opponent and so on. • examples by pupils
Random sampling	Choosing a sample from a population without being biased..	

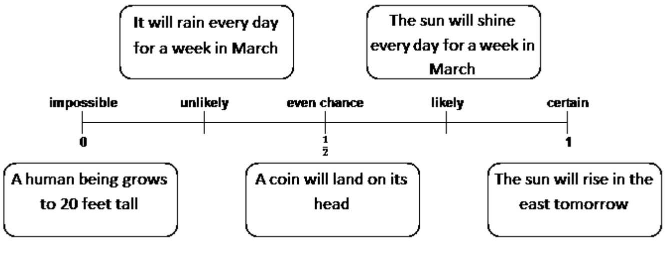
Lesson Title: Introduction to probability – Part 2	**Theme:** Statistics and Probability	
Lesson Number: M3-L098	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use probability notation to describe simple events.	**Preparation** 1. Draw the probability scale shown in Figure 1 under [DIAGRAMS] on the board. 2. Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Invite volunteers to give some of the words used to describe the chance of something happening. (Answer: certain, likely, unlikely, impossible)
2. Tell pupils that after today's lesson, they will be able to use probability notation to describe basic events.

Teaching and Learning *(25 minutes)*

1. Explain:
   - We use a probability scale shown in Figure 1 to indicate the likelihood of something happening. Probabilities are given values between 0 and 1.
   - A probability of 0 means that the event is impossible.
     Mark this on the probability scale – Figure 1. (The completed scale is shown in Figure 2)
   - A probability of 1 means that it is certain. Mark this on the probability scale.
   - The closer the probability of an event is to 1, the more likely it is to happen. The closer the probability is to 0, the less likely it is to happen.
   - Probabilities cannot be greater than 1. (1.0 as decimal, 100% as percentage)
2. Invite a volunteer to mark on the probability scale (Figure 1) where they think likely and unlikely will fall. Correct any mistakes and draw Figure 2.
3. Explain:
   - When we toss a coin, it is equally likely for it to fall on its head as on its tail.
   - The event of getting a head has an even chance of happening and we mark that in the middle of the probability scale. Its probability is $\frac{1}{2}, 0.5$ or 50%.
   - Mark these on the scale.



4. Ask pupils to work with seatmates to answer question a.
5. Invite volunteers to give answers and note some of the answers on the probability scale on the board. An example is shown on the right.
6. Explain:

   - Experiments have to be fairly conducted to ensure that the outcomes are indeed equally likely to happen.
   - In doing experiments, we talk about tossing fair coins, rolling unbiased dice, picking cards or numbers at random, even choosing people at random.

7. Invite a volunteer to give the sample space for tossing a coin. (Answer: $\{\text{head}, \text{tail}\}$)
8. Explain:
   - There are two outcomes, each of which is equally likely to occur.

     $\therefore$ probability of obtaining a head: $P(\text{head}) = \frac{1}{2}$

     probability of obtaining a tail: $P(\text{tail}) = \frac{1}{2}$
   - For equally likely outcomes, the probability that an event, E, will happen is:

     $$P(E) = \frac{\text{number of ways of obtaining event } E}{\text{total number of possible outcomes}}$$

     In set notation, this is given as: $= \frac{n(E)}{n(S)}$

     where $n(E)$ is the number of elements in event E
     $n(S)$ is the number of elements in the sample space, S
   - Probabilities can be expressed **as a fraction, a decimal, or a percentage**.

	fraction	decimal	percentage
$\therefore P(\text{head}) =$	$\frac{1}{2}$	0.5	50%
   - From the outcomes for tossing a coin, we can clearly see that:

     $$P(\text{head}) + P(\text{tail}) = 1 \qquad (1)$$
   - **Since this gives all the possible outcomes for the experiment, we can conclude that the sum of all probabilities in an experiment is equal to 1.**
   - From equation (1), obtaining a tail is the same as not obtaining a head.

     $$\therefore P(\text{head}) + P(\text{not head}) = 1$$

     $$P(\text{not head}) = 1 - P(\text{head}) \qquad (2)$$

     This is called the **complement of the event.** If the probability of obtaining a head is denoted as $P(A)$, the complement is written as $P(\overline{A})$.
9. Invite a volunteer to fully assess question b. on the board. (Example answer: An unbiased die is rolled, find required probabilities.)

   Solution:

   b. Given: An unbiased die is rolled, find required probabilities

   the sample space for the experiment is given by $S = \{1, 2, 3, 4, 5, 6\}$

   the number of elements in the sample space gives the total number of possible outcomes $n(S) = 6$

   probability of an event occurring $P(E) = \frac{n(E)}{n(S)}$

   i. the event of obtaining a 6, $E = \{6\}$ number of elements in the event $n(E) = 1$

   probability of obtaining a 6 $P(6) = \frac{1}{6}$

   ii. the event of obtaining a 5, $E = \{5\}$ number of elements in the event $n(E) = 1$

probability of obtaining a 5 $P(5) = \frac{1}{6}$

Since the probability of obtaining any of the numbers is equally likely,

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$$

iii. the event of obtaining even numbers $E = \{2, 4, 6\}$

number of elements in the event $n(E) = 3$

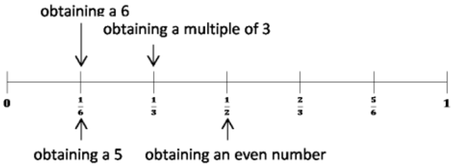
probability of obtaining an even number $P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$

iv. the event of obtaining multiples of 3 $E = \{3, 6\}$

number of elements in the event $n(E) = 2$

probability of obtaining a multiple of 3 $P(\text{multiple of 3}) = \frac{2}{6} = \frac{1}{3}$

Since there are 6 elements in the sample space, we divide the probability scale into 6.



10. Invite a volunteer to fully assess question c. on the board and extract the given information. (Example answer: A card is taken at random from a full pack of cards (no jokers); find the required probabilities)

Solutions:

a. Given: A card is taken at random from a full pack of cards (no jokers);
Explain the sample space and ensure pupils understand the suits and cards

There are 4 suits: clubs, spades, diamonds and hearts.

Clubs and spades are black. Diamonds and hearts are red. Each suit has 13 cards. Ace (A) is taken as 1. Jack (J), queen (Q) and king (K) are called picture cards. The remaining cards are numbered 2 to 10. There are 52 cards in total.

the total number of possible outcomes $n(S) = 52$

probability of an event E occurring $P(E) = \frac{n(E)}{n(S)}$

For this question, let us identify each set by a capital letter.

i. Event ace, $A = \{\text{ace of clubs, ace of spades, ace of diamonds, ace of hearts}\}$

$n(A) = 4$ $P(A) = \frac{4}{52} = \frac{1}{13}$

ii. Event black, $B = \{2, 3, 4, 5, \ldots \text{J, Q. K, A of clubs}, 2, 3, 4, 5, \ldots \text{J, Q. K, A of spades}\}$

$n(B) = 26$ $P(B) = \frac{26}{52} = \frac{1}{2}$

iii. Event hearts, $H = \{2, 3, 4, 5, 6, 7, 8, 9, 10, \text{J, Q. K, A of hearts}\}$

$n(H) = 13$ $P(H) = \frac{13}{52} = \frac{1}{4}$

iv. Event even number, $E = \{2, 4, 6, 8{,}10 \text{ of clubs}, \ldots \text{ spades}, \ldots \text{ hearts}, \ldots \text{ diamonds}\}$

$n(E) = 20$ $P(E) = \frac{20}{52} = \frac{5}{13}$

11. Ask pupils to work with seatmates to answer questions c. v. and c. vi.
12. Invite volunteers to give their answers. The rest of the class should check their answers and correct any mistakes.

Solutions:

v. Event Ace of diamonds, $D = \{\text{A of diamonds}\}$

$n(D) = 1$ $\quad$ $P(D) = \frac{1}{52}$

vi. Event not hearts, N $= \{2, 3, 4, 5, \ldots \text{J, Q. K, A of clubs}, \ldots \text{spades}, \ldots \text{diamonds}\}$

= the complement of Hearts

$= 1 - P(H)$

$= 1 - \frac{1}{4} \quad = \frac{3}{4}$

Practice *(10 minutes)*

1. Ask pupils to work independently to answer question d.
2. Walk around, if possible, to check the answers and clear misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solution:

   d. Given: number is chosen at random from the set $S = \{1, 2, 4, 7, 11, 16, 22\}$.

   the total number of possible outcomes $\quad n(S) = 7$

   probability of an event E occurring $\quad P(E) = \frac{n(E)}{n(S)}$

   i. Event is odd, $O = \{1, 7, 11\}$

   $n(O) = 3$ $\quad$ $P(O) = \frac{3}{7}$

   ii. Event is greater than 5, $G = \{7, 11, 16, 22\}$

   $n(G) = 4$ $\quad$ $P(G) = \frac{4}{7}$

   iv. Event is a multiple of 3, M $\quad \{\,\}$ $\quad$ $P(M) = 0$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L098 in the Pupil Handbook.

[DIAGRAMS FOR TEACHING AND LEARNING]

Figure 1

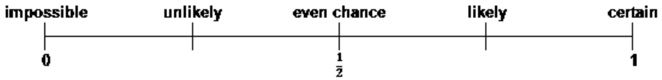


Figure 2

[QUESTIONS]

a. Draw the probability scale (Figure 2) shown on the board.
   Mark on the scale an example from last lesson for each of the probability terms.
b. An unbiased die is rolled. What is the probability of obtaining the following:
   - i. A six
   - ii. A five
   - iii. An even number
   - iv. A multiple of 3

   Mark each probability on a copy of the probability scale in Figure 1.
c. A card is taken at random from a full pack of cards (no jokers).
   What is the probability that the card:

i. Is an ace
ii. Is black
iii. Is a heart
iv. Has an even number on it
v. Is the ace of diamonds
vi. Is not a heart

a. A number is chosen at random from the set $S = \{1, 2, 4, 7, 11, 16, 22\}$.
What is the probability that the number:
   i. Is odd
   ii. Is greater than 5
   iii. Is a multiple of 3

Lesson Title: Addition law for mutually exclusive events – Part 1	**Theme:** Statistics and Probability	
Lesson Number: M3-L099	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply the addition law to find the probability of mutually exclusive events.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

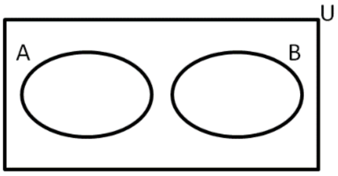
1. Ask pupils to answer question a. on the board.
2. Invite a volunteer to give their answer. (Answer: $1 - P(\text{red}) = 1 - \frac{4}{9} = \frac{5}{9}$).
3. Tell pupils that after today's lesson, they will be able to apply the addition law to find the probability of mutually exclusive events.

Teaching and Learning *(20 minutes)*

1. Ask pupils to consider and discuss with the following 2 events with seatmates:
   A: Attending school on Monday
   B: Being late for school
   Can they both happen at the same time?
2. Invite volunteers to give their answers and state the reason why. (Example answer: Yes, you can attend school on Monday and be late, early or on time.)
3. Ask pupils now to consider and discuss the following 2 events with seatmates:
   A: Winning a football match
   B: Losing a football match
   Can they both happen at the same time?
4. Invite volunteers to give their answers and state the reason why. (Example answer: No, you cannot win and lose the same football match.)
5. Explain:
   - If two events cannot happen at the same time, then they are called **mutually exclusive events.**
   - In the example of the football match, the event "winning" and the event "losing" cannot both happen at the same time so they are mutually exclusive.
   - Mutually exclusive events are examples of compound or combination events.
   - The events are connected by the word "or".
6. Ask pupils to discuss with seatmates an example of 2 mutually exclusive events.
7. Invite volunteers to answer. (Example answers: rainy all day and sunny all day; a tossed coin lands on head and tail; a rolled die lands on a 6 and an odd number)
8. Explain:
   - If two events A and B are mutually exclusive events, then the probability of A or B is given by:

$$\begin{aligned} P(A \text{ or } B) &= P(A \cup B) \\ &= P(A) + P(B) \end{aligned}$$

- This is the Addition Law for mutually exclusive events.
- In probability, the word "or" or the symbol ∪ indicates addition.
- A and B are disjoint sets as shown by the Venn diagram.
- For two mutually exclusive events which cover all possible outcomes, all the individual probabilities add up to 1.



$$P(A) + P(B) = 1$$

- If there are more than 2 events, then:

$$\begin{aligned} P(A \text{ or } B \text{ or } C \text{ or } D \text{ or } \ldots) &= P(A) + P(B) + P(C) + P(D) + \cdots \\ P(A) + P(B) + P(C) + P(D) + \cdots &= 1 \\ \text{Also} \quad P(\text{not } A) &= 1 - P(A) \end{aligned}$$

9. Invite a volunteer to fully assess question b. on the board. (Example answer: A card taken at random from an ordinary pack of cards; find probability it is an Ace or the 10 of clubs).

   Solution:

   b. **Step 1.** Assess and extract the given information from the problem.

   Given: card taken at random from an ordinary pack of cards; find probability of Ace or 10 of Clubs

   Step 2. Find the individual probabilities.

   the total number of possible outcomes $n(S) = 52$

   probability of an event E occurring $P(E) = \frac{n(E)}{n(S)}$

   Let A be the event of choosing an ace, B the event of 10 of clubs

   $A = \{\text{ace of clubs, ace of spades, ace of diamonds, ace of hearts}\}$

   $n(A) = 4 \qquad P(A) = \frac{4}{52} = \frac{1}{13}$

   $B = \{10 \text{ of clubs}\}$

   $n(B) = 1 \qquad P(B) = \frac{1}{52}$

   Step 3. Find the probability of Ace or 10 of Clubs.

   $$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) \\ &= \frac{1}{13} + \frac{1}{52} \quad = \frac{4}{52} + \frac{1}{52} \\ &= \frac{5}{52} \end{aligned}$$

   Step 4. Write the answer.

   The probability of choosing an ace or the 10 of clubs is $\frac{5}{52}$.

10. Invite a volunteer to fully assess question c.

    (Example answer: P A R A L L E L O G R A M written on identical pieces of paper and put in a bag, find required probabilities)

    Solution:

    c. Given: P A R A L L E L O G R A M written on identical pieces of paper and put in a bag, find required probabilities

the total number of possible outcomes $n(S) = 13$
probability of an event E occurring $P(E) = \frac{n(E)}{n(S)}$

Let the letters represent their respective events.

i. $n(A) = 3$ $P(A) = \frac{3}{13}$
ii. $n(L) = 3$ $P(B) = \frac{3}{13}$
iii. $n(O) = 1$ $P(O) = \frac{1}{13}$
iv. $P(A \text{ or } L) = P(A) + P(L)$
$= \frac{3}{13} + \frac{3}{13} = \frac{6}{13}$
v. $P(A \text{ or } O) = P(A) + P(O)$
$= \frac{3}{13} + \frac{1}{13} = \frac{4}{13}$
iv. $P(A \text{ or } L\ or\ O = P(A) + P(L) + P(O)$
$= \frac{3}{13} + \frac{3}{13} + \frac{1}{13} = \frac{7}{13}$

11. Ask pupils to work with seatmates to answer question d.
12. Invite a volunteer to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

d. Given: Probability table of obtaining 1, 2, 3, 4 on a 4-sided spinner.

Number	1	2	3	4
Probability	0.2	0.35	0.15	0.3

the total number of possible outcomes $n(S) = 4$
probability of an event E occurring $P(E) = \frac{n(E)}{n(S)}$

Let the numbers represent their respective events.

i. $P(1 \text{ or } 4) = P(1) + P(4)$
$= 0.2 + 0.3 = 0.5$
ii. $P(2 \text{ or } 3) = P(2) + P(3)$
$= 0.35 + 0.15 = 0.5$
iii. $P(2 \text{ or } 4) = P(2) + P(4)$
$= 0.35 + 0.3 = 0.65$
iv. $P(1 \text{ or } 2\ or\ 3) = P(1) + P(2) + P(3)$
$= 0.2 + 0.35 + 0.15 = 0.7$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer question e.
2. Walk around, if possible, to check the answers and clear misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solution:

e. Given: card taken at random from an ordinary pack of cards; find required probabilities.

the total number of possible outcomes $n(S) = 52$

probability of an event E occurring $P(E) = \frac{n(E)}{n(S)}$

$$P(A \text{ or } B) = P(A) + P(B)$$

Let events be as shown.

i. $P(3 \text{ of hearts or } 5 \text{ of spades}) = P(3 \text{ of hearts}) + P(5 \text{ of spades})$

$= \frac{1}{52} + \frac{1}{52} = \frac{2}{52} = \frac{1}{26}$

ii. $P(\text{hearts or spades}) = P(\text{hearts}) + P(\text{spades})$

$= \frac{13}{52} + \frac{13}{52} = \frac{26}{52} = \frac{1}{2}$

iii. $P(\text{king of clubs or queen}) = P(\text{king of clubs}) + P(\text{queen})$

$= \frac{1}{52} + \frac{4}{52} = \frac{5}{52}$

iii. $P(\text{diamond or the ace of hearts}) = P(\text{diamond}) + P(\text{the ace of hearts})$

$= \frac{13}{52} + \frac{1}{52} = \frac{14}{52} = \frac{7}{26}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L099 in the Pupil Handbook.

[QUESTIONS]

a. A bag contains 4 red, 2 green, and 3 yellow balls. What is the probability of not picking a red ball?
b. A card is taken at random from an ordinary pack of cards. What is the probability that it will be an Ace or the 10 of Clubs?
c. The word P A R A L L E L O G R A M was written on identical pieces of paper and put in a bag. One of the pieces of paper is selected at random.
   What is the probability of getting:
   i. A ii. L iii. O
   iv. A or L v. A or O vi. A or L or O
d. The table gives the probability of getting 1, 2, 3 or 4 on a biased 4-sided spinner.

Number	1	2	3	4
Probability	0.2	0.35	0.15	0.3

   What is the probability of getting:
   i. 1 or 4 ii. 2 or 3 iii. 2 or 4 iv. 1 or 2 or 3
e. A card is taken at random from an ordinary pack of cards. What is the probability that the card will be:
   i. A 3 of hearts or a 5 of spades ii. A heart or a spade
   iii. A king of clubs or a queen of any suit iv. A diamond or the ace of hearts

Lesson Title: Addition law for mutually exclusive events – Part 2	**Theme:** Statistics and Probability	
Lesson Number: M3-L100	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to further apply the addition law to find the probability of mutually exclusive events.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to write down the addition law for 3 mutually exclusive events.
2. Invite a volunteer to give their answer. (Answer: $P\ (A \text{ or } B \text{ or } C) = P(A) + P(B)\ + P(C)$).
3. Invite a volunteer to say what all probabilities in an experiment should add up to (Answer: 1)
4. Tell pupils that after today's lesson, they will be able to further apply the addition law to find the probability of mutually exclusive events.

Teaching and Learning *(20 minutes)*

1. Invite a volunteer to fully assess question a. on the board and extract the given information. (Example answer: given: probabilities of obtaining a ball of a particular colour in a bag, find required probabilities)

Solution:

a. **Step 1.** Assess and extract the given information from the problem.
Given: probabilities of obtaining a ball of a particular colour in a bag

Colour	Probability
black	$\frac{3}{8}$
white	$\frac{1}{4}$
yellow	$\frac{1}{5}$

Step 2. Find the probabilities

the total number of possible outcomes $\quad n(S) = 3$

probability of an event E occurring $\quad P(E) = \frac{n(E)}{n(S)}$

Let the initials of the colours represent their respective events.

i.
$$P(B \text{or } W) = P(B) + P(W)$$
$$= \frac{3}{8} + \frac{1}{4} \qquad = \frac{5}{8}$$

ii.
$$P(\overline{W \text{ or } Y}) = P(\overline{W} \text{ or } \overline{Y}) \quad = \quad 1 - (P(W) + P(Y))$$

$\overline{W}$ is the complement of W, $\overline{Y}$ is the complement of Y

$$P(W) + P(Y)) = \frac{1}{4} + \frac{1}{5} \qquad = \frac{9}{20}$$
$$P(\overline{W} \text{ or } \overline{Y}) = 1 - \frac{9}{20} \qquad = \frac{11}{20}$$

iii.
$$P(\text{none of colours listed}) = 1 - P(\text{all colours listed})$$
$$= 1 - (P(B) + P(W) + P(Y))$$

$$\begin{aligned} &= 1-\frac{3}{8}+\frac{1}{4}+\frac{1}{5} \\ &= 1-\frac{33}{40} \qquad = \qquad \frac{7}{40} \end{aligned}$$

Step 3. Write the answers.

The probability of black or white is $\frac{5}{8}$.

The probability of not yellow or white is $\frac{11}{20}$.

The probability of none of the colours listed in the table is $\frac{7}{40}$.

2. Ask pupils to work with seatmates to answer question b.
3. Invite a volunteer to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

b. Given: pink, yellow or black cards. Card chosen at random gives probability of obtaining a black or pink card $\frac{5}{7}$, the probability of obtaining a black or yellow card $\frac{3}{5}$. What is the probability of obtaining a card of each colour?

Let the initials of the colours represent their respective events.

$$\begin{aligned}
P(B)+P(P)+P(Y) &= 1 && \text{(1)} \\
P(B\text{or } P) &= \frac{5}{7} \\
\Longrightarrow \qquad P(B)+P(P) &= \frac{5}{7} && \text{(2)} \\
\frac{5}{7}+P(Y) &= 1 && \text{from equation (!)} \\
P(Y) &= 1-\frac{5}{7} \qquad = \frac{2}{7} \\
P(B \text{ or } Y)) &= \frac{3}{5} && \text{given} \\
\Longrightarrow \qquad P(B)+P(Y) &= \frac{3}{5} \\
P(B) &= \frac{3}{5}-P(Y) \\
&= \frac{3}{5}-\frac{2}{7} \qquad = \frac{11}{35} \\
P(B)+P(P) &= \frac{5}{7} && \text{from equation (2)} \\
P(P) &= \frac{5}{7}-P(B) \\
&= \frac{5}{7}-\frac{11}{35} \qquad = \frac{14}{35} \\
&= \frac{2}{5}
\end{aligned}$$

The probability of a yellow card is $\frac{2}{7}$, black card is $\frac{11}{35}$ and pink card is $\frac{2}{5}$.

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions c., d. and e.
2. Walk around, if possible, to check the answers and clear misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. Given: probability that a football team wins a match is 0.4 and the probability they draw is 0.3; find probability they lose.

$$\begin{aligned} P(\text{team loses}) &= 1 - P(\text{team wins or draws}) \\ P(\text{team wins or loses}) &= P(\text{team wins}) + P(\text{team draws}) \\ &= 0.4 + 0.3 \quad = \quad 0.7 \\ \therefore P(\text{team loses}) &= 1 - 0.7 \quad = \quad 0.3 \end{aligned}$$

The probability that the team loses is 0.3 (decimal), $\frac{3}{10}$ (fraction), 30% (percentage).

d. Given: A letter is chosen at random from the word M A G N I T U D E. Find required probabilities.

$$\begin{aligned} \text{the total number of possible outcomes} \qquad n(S) &= 9 \\ \text{probability of an event } E \text{ occurring} \qquad P(E) &= \frac{n(E)}{n(S)} \end{aligned}$$

i. $M = \{M, U, G\}$, $n(M) = 3$ $\qquad I = \{I, D, E, A\}$, $n(I) = 4$

$$\begin{aligned} P(M\text{or } I) &= P(M) + P(I) \\ &= \frac{3}{9} + \frac{4}{9} \quad = \quad \frac{7}{9} \end{aligned}$$

The probability that the letter is either in the word M U G or in the word I D E A is $\frac{7}{9}$.

ii. $A = \{A, G, E, N, T\}$, $n(I) = 5$ $\qquad D = \{M, I, D\}$, $n(D) = 3$

$$P(\overline{A} \text{ or } \overline{D}) = 1 - (P(A) + P(D))$$

$\overline{A}$ is the complement of A, D is the complement of D

$$\begin{aligned} P(A) + P(D)) &= \frac{5}{9} + \frac{3}{9} \quad = \quad \frac{8}{9} \\ P(\overline{A} \text{ or } \overline{D}) &= 1 - \frac{8}{9} \quad = \quad \frac{1}{9} \end{aligned}$$

The probability the letter is neither in the word A G E N T nor in the word M I D is $\frac{1}{9}$.

e. Given: weather classified as very good, good, poor and very poor; probability of good or very good is 0.6, probability of poor is 0.3.

i.

$$P(\text{good}\} + P(\text{very good}) + P(\text{poor}) + P(\text{very poor}) = 1$$

$$P(\text{good or very good}\} = 0.6 \qquad P(\text{ poor}\} = 0.3$$

$$\begin{aligned} \Longrightarrow \quad P(\text{very poor}) &= 1 - P(\text{good}\} + P(\text{very good}) + P(\text{poor}) \\ &= 1 - (0.6 + 0.3) \quad = \quad 1 - 0.9 \\ &= 0.1 \end{aligned}$$

The probability that the weather is very poor is 0.1.

ii.

$$\begin{aligned} P(\text{good}) &= 2 \times P(\text{very good}) \\ 2 \times P(\text{very good}) &= 0.6 - P(\text{very good}) && \text{from part i.} \\ 3 \times P(\text{very good}) &= 0.6 \\ P(\text{very good}) &= \frac{0.6}{3} \quad = \quad 0.2 \\ P(\text{good}) + P(\text{poor}) &= 1 - (P(\text{very good}) + P(\text{very poor})) \\ &= 1 - (0.2 + 0.1) \quad = \quad 1 - 0.3 \\ &= 0.7 \end{aligned}$$

The probability that it will be good or poor is 0.7.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L100 in the Pupil Handbook.

[QUESTIONS]

a. A bag contains a number of balls of different colours. The probability of obtaining a ball of a particular colour is given in the table below.

Colour	Probability
black	$\frac{3}{8}$
white	$\frac{1}{4}$
yellow	$\frac{1}{5}$

What is the probability that a ball taken at random from the bag is:

  i. Black or white
  ii. Not yellow or white
  iii. Not one of the colours listed in the table.

b. A pack contains cards that are coloured pink, yellow or black. When the card is chosen at random, the probability of obtaining a black or pink card is $\frac{5}{7}$, and the probability of obtaining a black or yellow card is $\frac{3}{5}$. Find the probability of obtaining a card of each colour.

c. The probability that a football team wins a match is 0.4 and the probability they draw is 0.3. What is the probability that they lose? Give your answer as a fraction, decimal and percentage.

d. A letter is chosen at random from the word M A G N I T U D E. What is the probability that it is:

  i. Either in the word M U G or in the word I D E A.
  ii. Neither in the word A G E N T nor in the word M I D.

e. Pupils in SS3 classified the weather for a fortnight in October as very good, good, poor and very poor. They found out that probability the weather will be good or very good is 0.6. The probability that it will be poor is 0.3.

  i. What is the probability that the weather will be very poor?
  ii. If the probability that the weather is good is twice the probability that it is very good, what is the probability that it will be good or poor?

Lesson Title: Multiplication law for independent events – Part 1	**Theme:** Statistics and Probability	
Lesson Number: M3-L101	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply the multiplication law to find the probability of independent events occurring.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to apply the multiplication law to find the probability of independent events.

Teaching and Learning *(20 minutes)*

1. Ask pupils to consider and discuss with seatmates the following 2 events:
   A: You travel in a poda-poda to school which breaks down.
   B: You are late for school.
   Does one event have any effect on the other?
2. Invite volunteers to give their answer and state the reason why. (Example answer: Yes, the poda-poda breaking down caused you to be late.)
3. Ask pupils now to consider and discuss with seatmates the 2 events:
   A: Being a girl
   B: Being left-handed
   Does one event have any effect on the other?
4. Invite volunteers to give their answer and state the reason why. (Example answer: No, being a girl does not have any effect on which hand is used to write and being left-handed does not have an effect on being a girl)
5. Explain:
   - If one event happening has no effect on another event happening they are called **independent events**.
   - In the example above, the event "being a girl" and the event "being left-handed" do not affect each other, so they are independent events.
   - Independent events are examples of compound or combination events.
   - The events are connected by the word "and".
6. Ask pupils to discuss with seatmates an example of 2 independent events. You can prompt them by asking them to look at some of the previous probability experiments and think of outcomes that lead to independent events.
7. Invite volunteers to answer. (Example answers: it rains on Monday this week, it rains on Monday next week; a coin tossed twice lands on head then lands on tail; a die rolled twice shows a 6 then an odd number)
8. Explain:

- If two events A and B are independent events, then the probability of A and B is given by:

$$\begin{aligned} P(A \text{ and } B) &= P(A \cap B) \\ &= P(A) \times P(B) \end{aligned}$$

- This is the Multiplication Law for independent events.
- In probability, the word “and” or the symbol ∩ indicates multiplication.
- If there are more than 2 events, then:

$$\begin{aligned} P(A \text{ and } B \text{ and } C \text{ and } D \text{ and } \ldots) &= P(A \cap B \cap C \cap D \ \ldots) \\ &= P(A) \times P(B) \times P(C) \times P(D) + \cdots \end{aligned}$$

- As before:

$$P(\text{not } A) = 1 - P(A)$$

9. Invite a volunteer to fully assess question b. on the board and extract the given information. (Example answer: fair die rolled twice, find probability of 6 in the first roll and odd number in the second)

Solution:

a. **Step 1.** Assess and extract the given information from the problem.

Given: fair die rolled twice, find probability of 6 in the first roll and odd number in the second

Step 2. Calculate the required probability.

$$S = \{1, 2, 3, 4, 5, 6\}$$

the total number of possible outcomes $n(S) = 6$

probability of an event E occurring $P(E) = \frac{n(E)}{n(S)}$

$$P(6) = \frac{1}{6} \qquad P(\text{odd number}) = \frac{3}{6} = \frac{1}{2}$$

$$\begin{aligned} P(6 \text{ and odd number}) &= P(6) \times P(\text{odd number}) \\ &= \frac{1}{6} \times \frac{1}{2} = \frac{1}{12} \end{aligned}$$

Step 3. Write the answer.

The probability of 6 in the first roll and odd number in the second is $\frac{1}{12}$.

10. Invite a volunteer to fully assess question b. on the board. (Example answer: A coin is tossed and a die is rolled; find the probability of getting a tail on the coin and a 3 on the die)

Solution:

b. Given: coin is tossed and a die is rolled; find probability of getting a tail on the coin and a 3 on the die

$$S_{\text{die}} = \{H, T\} \qquad S_{\text{coin}} = \{1, 2, 3, 4, 5, 6\}$$

$$n(S_{\text{die}}) = 2 \qquad n(S_{\text{coin}}) = 6$$

probability of an event E occurring $P(E) = \frac{n(E)}{n(S)}$

$$P(T) = \frac{1}{2} \qquad P(3) = \frac{1}{6}$$

$$\begin{aligned} P(T \text{and } 3) &= P(T) \times P(3) \\ &= \frac{1}{2} \times \frac{1}{6} = \frac{1}{12} \end{aligned}$$

The probability of getting a tail on the coin and a 3 on the die is $\frac{1}{12}$.

11. Invite a volunteer to fully assess question c. (Example answer: spinner with eight sections of equal size, coloured white or black; find the required probabilities)

    Solution:

    c. Given: spinner with eight sections of equal size, coloured white or black; find required probabilities.

    $$S = \{B, B, B, B, B, W, W, W\}$$
    $$n(S) = 8$$

    probability of an event E occurring $\quad P(E) = \frac{n(E)}{n(S)}$

    - Ask pupils to give the individual probability.

    i. $P(B) = \frac{5}{8}$

    ii. $P(W) = 1 - \frac{5}{8} = \frac{3}{8}$

    iii. $P(B \text{ and } B) = P(B) \times P(B)$
    $= \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$

    iv. $P(W \text{ and } W) = P(W) \times P(W)$
    $= \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$

    v. $P(B \text{ and } W) = P(B) \times P(W)$
    $= \frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$

    vi. $P(W \text{ and } B) = P(W) \times P(B)$
    $= \frac{3}{8} \times \frac{5}{8} = \frac{15}{64}$

12. Ask pupils to work with seatmates to answer questions c. vii., viii., and ix.
13. Invite volunteers to show their solutions on the board. The rest of the class should check their solutions and correct any mistakes.

    Solutions:

    vii. $P(\text{White is obtained both times}) = P(W \text{ and } W) = P(W) \times P(W)$
    $= \frac{9}{64}$

    viii. $P(\text{different colour obtained on each spin}) = P(B \text{ and } W) \text{ or } P(W \text{ and } B)$
    $= (P(B) \times P(W)) + (P(W) \times P(B))$
    $= \frac{15}{64} + \frac{15}{64} = \frac{30}{64}$
    $= \frac{15}{32}$

    ix. $P(\text{same colour }) = 1 - P(\text{different colour})$
    $= 1 - \frac{15}{32}$
    $= \frac{17}{32}$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer question d.
2. Walk around, if possible, to check the answers and clear misconceptions.
3. Ask volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solution:

d. Given: probability that Jamil will forget his ruler for his Maths examination is 0.35, probability that he will forget his calculator for the examination is 0.15; find required probabilities.

$$P(\text{Jamil forgets ruler}) = 0.35$$

$$P(\text{Jamil forgets calculator}) = 0.15$$

i. $P(\text{Jamil does not forget ruler}) = 1 - 0.35 = 0.65$

ii. $P(\text{Jamil does not forget calculator}) = 1 - 0.15 = 0.85$

iii. $P(\text{Jamil does not forget calculator and ruler}) = 0.65 \times 0.85$
$= 0.5525$

Closing *(4 minutes)*

1. Ask pupils to tell seatmates one new thing they learned in this lesson.
2. Invite volunteers to answer. (Answers: Various)
3. For homework, have pupils do the practice activity PHM3-L101 in the Pupil Handbook.

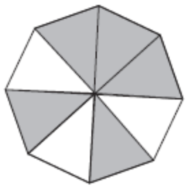
[QUESTIONS]

a. A fair die is rolled twice. What is the probability that it will land on a 6 in the first roll and land on an odd number in the second roll?

b. A coin is tossed and a die is rolled. What is the probability of getting a tail on the coin and a 3 on the die?

c. The spinner shown has eight sections of equal size. Each one is coloured white or black. The events B and W are:

B: the spinner lands on black,
W: the spinner lands on white.



Find the following probabilities:

i. $P(B)$ ii. $P(W)$ iii. $P(B \text{ and } B)$
iv. $P(W \text{ and } W)$ v. $P(B \text{ and } W)$ vi. $P(W \text{ and } B)$

If the spinner is spun twice, find the probabilities of the following outcomes:

vii. White is obtained both times.
viii. A different colour is obtained on each spin.
ix. The same colour is obtained on each spin.

d. The probability that Jamil will forget his ruler for his Maths examination is 0.35. The probability that he will forget his calculator for the examination is 0.15. What is the probability that he will:

i. Not forget his ruler?
ii. Not forget his calculator?
iii. Not forget his ruler and not forget his calculator?

<table>
<tr><td>Lesson Title: Multiplication law for independent events – Part 2</td><td colspan="2">Theme: Statistics and Probability</td></tr>
<tr><td>Lesson Number: M3-L102</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to further apply the multiplication law to find the probability of independent events.</td><td colspan="2">Preparation
Write the questions found at the end of this lesson plan on the board.</td></tr>
</table>

Opening *(4 minutes)*

1. Ask pupils to write down the multiplication law for 3 independent events.
2. Invite a volunteer to give their answer.
   (Answer: $P(A \text{ and } B \text{ and } C) = P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$)
3. Tell pupils that after today's lesson, they will be able to further apply the multiplication law to find the probability of independent events.

Teaching and Learning *(20 minutes)*

1. Invite a volunteer to fully assess question a. on the board. (Example answer: $A = \{2, 4, 6\}$ and $B = \{6, 8\}$ are subsets of the sample space $S = \{2, 4, 6, 8, 10, 12\}$; prove the required equality)

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.
   given: $A = \{2, 4, 6\}$ and $B = \{6, 8\}$ are subsets of the sample space $S = \{2, 4, 6, 8, 10, 12\}$.

   Step 2. Calculate the individual probabilities.
   If A and B are independent, then:

$$\begin{aligned} P(A \cap B) &= P(A) \times P(B) \qquad (1) \\ (A \cap B) &= \{6\} \\ P(A \cap B) &= \tfrac{1}{6} \\ P(A) &= \tfrac{3}{6} = \tfrac{1}{2} \\ P(B) &= \tfrac{2}{6} = \tfrac{1}{3} \end{aligned}$$

   Step 3. Substitute into equation (1)

$$\begin{aligned} \tfrac{1}{6} &= \tfrac{1}{2} \times \tfrac{1}{3} \\ \tfrac{1}{6} &= \tfrac{1}{6} \\ \text{LHS} &= \text{RHS} \end{aligned}$$

   Step 4. Write the answer.
   The events A and B are independent.

2. Invite a volunteer to fully assess question b. on the board and extract the given information. (Example answer: A card is taken at random pack A and pack B; find the required probabilities.)

Solution:

b. Given: A card is taken at random pack A and pack B; find the required probabilities

$$n(S) = 52$$

$$P(A) \text{ and } P(B) = P(A) \times P(B)$$

i. $P(\text{red}) = \frac{26}{52} = \frac{1}{2}$

$$P(r\text{ed from } A) \text{ and } P(\text{red from } B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

ii. $P(\text{diamond}) = P(\text{club}) = \frac{13}{52} = \frac{1}{4}$

$$P(\text{diamond from } A) \text{ and } P(\text{club from } B) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

iii. $P(\text{king}) = \frac{4}{52} = \frac{1}{13}$

$$P(\text{picture card}) = \frac{12}{52} = \frac{3}{13}$$

$$P(\text{king from } A) \text{ and } P(\text{picture card from } B) = \frac{1}{13} \times \frac{3}{13} = \frac{3}{169}$$

iv. $P(10) = \frac{4}{52} = \frac{1}{13}$

$$P(10 \text{ of clubs}) = \frac{1}{52}$$

$$P(10 \text{ from } A) \text{ and } P(10 \text{ of clubs from } B) = \frac{1}{13} \times \frac{1}{52} = \frac{1}{676}$$

v. $P(\text{ace of hearts}) = \frac{1}{52}$

$$P(\text{ace of hearts from } A) \text{ and } P(\text{ace of hearts from } B) = \frac{1}{52} \times \frac{1}{52} = \frac{1}{2{,}704}$$

3. Ask pupils to work with seatmates to answer question c.
4. Invite a volunteer to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

c. Given: probabilities given in table; find required probabilities

Let $A = \text{note Ahmed picks}$ $B = \text{note Mommoh picks}$

i. $P(A \text{ being Le } 5{,}000.00) = \frac{3}{11}$

$$P(B \text{being Le } 5{,}000.00) = \frac{2}{8} = \frac{1}{4}$$

$$P(A \text{ being Le } 5{,}000.00) \text{ and } P(B \text{being Le } 5{,}000.00 = \frac{3}{11} \times \frac{1}{4}$$

$$= \frac{3}{44}$$

ii. $P(A \text{ being Le}1{,}000.00) = \frac{5}{11}$

$$P(B \text{being Le}2{,}000.00) = \frac{1}{8}$$

$$P(A \text{ being Le } 1{,}000.00) \text{ and } P(B \text{being Le } 2{,}000.00 = \frac{5}{11} \times \frac{1}{8}$$

$= \frac{5}{88}$

iii. $P(A \text{ not being Le } 10{,}000.00) = 1 - P(A \text{ being Le } 10{,}000.00)$

$= 1 - \frac{1}{11} = \frac{10}{11}$

$P(B \text{ not being Le } 10{,}000.00) \quad 1 - P(B \text{ being Le } 10{,}000.00)$

$= 1 - \frac{2}{8} = \frac{6}{8} = \frac{3}{4}$

$P(A \text{ not being Le } 10{,}000.00) \text{ and } P(B \text{ not being Le } 10{,}000.00 = \frac{10}{11} \times \frac{3}{4} = \frac{30}{44}$

$= \frac{15}{22}$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d., e. and f.
2. Walk around, if possible, to check the answers and clear misconceptions.
3. Ask volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

For all questions, $P(\ A) \text{ and } P(B) \text{ and} \ldots = PA) \times P(B) \times \ldots$

d. Given: probability of Sally eating sweet bread is 0.3; she eats her bread with either eggs or sardines; probability that she eats eggs is 0.4.

$P(\text{sweet bread}) = 0.3 \Rightarrow P(\text{tapalapa bread}) = 1 - 0.3 = 0.7$

$P(\text{eggs}) = 0.4 \Rightarrow P(\text{sardines}) = 1 - 0.4 = 0.6$

i. $P(\text{tapalapa bread}) \text{ and } P(\text{sardines}) = 0.7 \times 0.6 = 0.42$

ii. $P(\text{eggs on two consecutive days}) = 0.4 \times 0.4 = 0.16$

e. Given: 55% of pupils are girls and 10% of pupils are left-handed; find required probabilities

$P(\text{girl}) = 55\% \Rightarrow P(\text{boys}) = P(\text{not girl}) = 45\%$

$P(\text{left-handed}) = 10\% \Rightarrow P(\text{not left-handed}) = 90\%$

i. $P(\text{not girl}) \text{ and } P(\text{not left-handed}) = 0.45 \times 0.9 = 0.405$

ii. $P(\text{girl}) \text{ and } P(\text{left-handed}) = 0.55 \times 0.1 = 0.055$

iii. $P(\text{left-handed}) \text{ and } P(\text{not girl}) = 0.1 \times 0.45 = 0.045$

f. Given: three fair dice are thrown; find required probabilities

$S = \{1, 2, 3, 4, 5, 6\} \qquad n(S) = 6$

i. $P(18) = P(\ 6) \text{ and } P(6) \text{ and } P(6)$

$= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{256}$

ii. $P(4 \text{ on die 1 and odd numbers on dice 2 and 3}) = P(4) \text{ and } P(\text{odd}) \text{ and } P(\text{odd})$

$= \frac{1}{6} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{24}$

iii. $P(\text{even numbers on all 3 dice}) = P(\text{even})\text{and } P(\text{even}) \text{ and } P(\text{even})$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \quad = \frac{1}{8}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L102 in the Pupil Handbook.

[QUESTIONS]

a. The events $A = \{2, 4, 6\}$ and $B = \{6, 8\}$ are subsets of the sample space $S = \{2, 4, 6, 8, 10, 12\}$. Show that A and B are independent.

b. A card is taken at random from each of two ordinary packs of cards, pack A and pack B. What is the probability of getting:
   i. A red card from pack A and a red card from pack B,
   ii. A diamond from pack A and a club from pack B,
   iii. A king from pack A and a picture card (king, queen, jack) from pack B,
   iv. A 10 from pack A and a 10 of clubs from pack B
   v. An ace of hearts from each pack?

c. The table below show the money Ahmed and Mommoh each have.

	Number of Leone notes			
	1,000	2,000	5,000	10,000
Ahmed	5	2	3	1
Mommoh	3	1	2	2

Ahmed and Mommoh each put their notes in a bag. Each pick one of their own notes from their bag at random. What is the probability that:
   i. Ahmed picks Le 5,000.00 and Mommoh picks Le 5,000.00.
   ii. Ahmed picks Le 1,000.00 and Mommoh picks Le 2,000.00.
   iii. Ahmed does not pick Le 10,000.00 and Mommoh does not pick Le 10,000.00.

d. Sally eats either sweet bread or tapalapa bread for her evening meal. The probability of her eating sweet bread is 0.3. She likes to eat her bread with either eggs or sardines. The probability that she eats eggs is 0.4. What is the probability that Sally eats:
   i. Tapalapa bread and sardines. ii. Eggs on two consecutive days.

e. In a school, 55% of the pupils are girls and 10% of pupils are left-handed. If a pupil is selected at random, what is the probability they are:
   i. Neither a girl or left-handed.
   ii. A girl and left-handed.
   iii. Left-handed but not a gir.

f. Three fair dice are thrown. What is the probability of getting:
   i. 18
   ii. Four on the first die and odd numbers on the other 2 dice.
   iii. Even numbers on all 3 dice.

Lesson Title: Application of the addition and multiplication laws	**Theme:** Statistics and Probability	
Lesson Number: M3-L103	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply the addition and multiplication laws to a variety of probability questions.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to discuss question a. with seatmates.
2. Invite volunteers to give their answers stating the reason why. (Example answers: Events A and B in experiment I are independent because the number on the die after the first throw will not affect what shows up after the second; Events A and B are mutually exclusive in experiment II because they cannot both happen at the same time when a die is rolled once)
3. Tell pupils that after today's lesson, they will be able to apply the addition and multiplication laws to a variety of probability questions.

Teaching and Learning *(20 minutes)*

1. Explain:
    - Consider two events, A and B.
    - If they are mutually exclusive then if A happens B cannot happen.
    - This is the opposite of independent events, if A happens it has no effect on whether B happens.
    - Mutually exclusive events result from the outcomes of one experiment.
    - Independent events arise when considering the outcomes of either the same experiment several times such as rolling one dice twice or a single experiment such as rolling two dice at once.
2. Invite a volunteer to assess question b. on the board and extract the given information. (Example answer: a die is rolled twice; find required probabilities)

   Solution:

   For all questions $P(A) \text{ or } P(B) = P(A) + P(B)$

   $P(A) \text{ and } P(B) = P(A) \times P(B)$

   b. **Step 1.** Assess and extract the given information from the problem.
   Given: a die is rolled twice; find required probabilities

   Step 2. Calculate the required probabilities.

   $S = \{1,2,3,4,5,6\}$ $\quad n(S) = 6$

   Let $A = \text{score on first roll}$ $\quad B = \text{score on second roll}$

   i. $P(A \text{ is } 2) \text{ and } P(B \text{ is } 5) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

   ii. $P(A \text{ is } 1) \text{ and } P(B \text{ is even}) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

iii. $P(A\text{ is 3 or 5}) = \frac{1}{6}+\frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

iv $P(A\text{ is 3 or 5}) \text{ and } P(B\text{ is odd}) = \frac{1}{3}\times\frac{1}{2} = \frac{1}{6}$

v. $P(A\text{ is 3 or 5}) \text{ and } P(B\text{ is 3 or 5}) = \frac{1}{3}\times\frac{1}{3} = \frac{1}{9}$

3. Explain:
   - There are instances when we are required to find the probability of two events either both or neither occurring; or one, or at least one of them occurring.
   - We will use the next example to see how we answer such a problem.
4. Invite a volunteer to fully assess question c. on the board and extract the given information. (Example answer: the probability that Ahmed gains admission into the university is $\frac{4}{5}$ and that of Brima is $\frac{2}{3}$; find required probabilities)

Solution:

c. Given: the probability that Ahmed gains admission into the university is $\frac{4}{5}$ and that of Brima is $\frac{2}{3}$; find the required probabilities

Let $A = \text{Ahmed gains admission}$

$P(A) = \frac{4}{5} \quad P(\overline{A}) = 1-\frac{4}{5} = \frac{1}{5}$

$B = \text{Brima gains admission}$

$P(B) = \frac{2}{3} \quad P(\overline{B}) = 1-\frac{2}{3} = \frac{1}{3}$

i. $P(\text{both gain admission}) = P(A) \text{ and } P(B)$

$= \frac{4}{5}\times\frac{2}{3} = \frac{8}{15}$

ii. $P(\text{none gain admission}) \quad P(\overline{A}) \text{ and } P(\overline{B})$

$= \frac{1}{5}\times\frac{1}{3} = \frac{1}{15}$

iii. $P(\text{one gain admission}) \Longrightarrow \text{Ahmed gains admission and Brima does not}$

or $\text{Ahmed does not gain admission and Brima does}$

$= P(A) \text{ and } P(\overline{B}) \text{ or } P(\overline{A}) \text{ and } P(B)$

$= \left(\frac{4}{5}\times\frac{1}{3}\right)+\left(\frac{1}{5}\times\frac{2}{3}\right) = \frac{4}{15}+\frac{2}{15} = \frac{6}{15}$

$= \frac{2}{5}$

iv. $P(\text{at least one gain admission}) = 1 - P(\text{none gain admission})$

$= 1-\frac{1}{15} = \frac{14}{15}$

5. Ask pupils to work with seatmates to answer question d.
6. Invite a volunteer to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

d. Given: probability that Mariama gets to the top level in her game is $\frac{1}{7}$; find the required probabilities.

Let $A = \text{1st game} \qquad B = \text{2nd game}$

$P(A) = P(B) = \frac{1}{7} \qquad P(\overline{A}) = P(\overline{B}) = 1-\frac{1}{7} = \frac{6}{7}$

i. $P(\text{top level in } A \text{ and } B) = P(A) \text{ and } P(B)$

$= \frac{1}{7} \times \frac{1}{7} \quad = \quad \frac{1}{49}$

ii. $P(\text{not top level in } A \text{ and } B) \quad = \quad P(\overline{A}) \text{ and } P(\overline{B})$

$= \frac{6}{7} \times \frac{6}{7} \quad = \quad \frac{36}{49}$

iii. $P(\text{one of the two games}) \quad \Longrightarrow \quad \text{top level in } A \text{ and not top level in } B$

or $\text{not top level in } A \text{ and top level in } B$

$= \left(P(A) \text{ and } P(\overline{B})\right) \text{ or } \left(P(\overline{A}) \text{ and } PP(B)\right)$

$= \left(\frac{1}{7} \times \frac{6}{7}\right) + \left(\frac{6}{7} \times \frac{1}{7}\right) \quad = \quad \frac{6}{49} + \frac{6}{49} \quad = \quad \frac{12}{49}$

iv. $P(\text{top level in 3 games}) \quad = \quad P(A) \text{ and } P(B) \; and \; P(C)$

$= \frac{1}{7} \times \frac{1}{7} \times \frac{1}{7} \quad = \quad \frac{1}{343}$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer question e.
2. Walk around, if possible, to check the answers and clear misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solution:

   e. Given: probability that Fatu, Mary and Sia will pass Mathematics are 0.4, 0.7 and 0.8 respectively; find required probabilities in percentages

Let $A \quad = \quad \text{Fatu passes Mathematics}$

$P(A) \quad = \quad 0.4 \qquad P(\overline{A}) \quad = \quad 1 - 0.4 \quad = \quad 0.6$

$B \quad = \quad \text{Mary passes Mathematics}$

$P(B) \quad = \quad 0.7 \qquad P(\overline{B}) \quad = \quad 1 - 0.7 \quad = \quad 0.3$

$C \quad = \quad \text{Sia passes Mathematics}$

$P(C) \quad = \quad 0.8 \qquad P(\overline{C}) \quad = \quad 1 - 0.8 \quad = \quad 0.2$

i. $P(\text{all three girls pass}) \quad = \quad P(A) \text{ and } P(B) \text{ and } P(C)$

$= \quad 0.4 \times 0.7 \times 0.8 \quad = \quad 0.224$

$= \quad 22.4\%$

ii. Only one pass means Fatu passes, not Mary or Sia; Mary passes, not Fatu or Sia, Sia passes, not Fatu or Mary.

$\left(P(A) \text{ and } P(\overline{B}) \text{ and } P(\overline{C})\right) \text{ or } \left(P(\overline{A}) \text{ and } P\right.$

$P(\text{only one pass}) \quad = \quad \left.(B) \text{ and } P(\overline{C})\right)$

$\text{or } \left(P(\overline{A}) \text{ and } P(\overline{B}) \text{ and } P(C)\right)$

$= \quad (0.4 \times 0.3 \times 0.2) + (0.6 \times 0.7 \times 0.2) + (0.4 \times 0.3 \times 0.8)$

$= \quad 0.024 + 0.084 + 0.096 \quad = \quad 0.204$

$= \quad 20.4\%$

iii. $P(\text{none pass}) \quad P(\overline{A}) \text{ and } P(\overline{B}) \text{ and } P(\overline{C})$

$= \quad 0.6 \times 0.3 \times 0.2 \quad = \quad 0.036$

$= \quad 3.6\%$

iv $P(\text{at least one pass}) = 1 - P(\text{none pass})$

$= 1 - 0.036 = 0.964$

$= 96.4\%$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L103 in the Pupil Handbook.

[QUESTIONS]

a. For each experiment, decide whether the pair of events are mutually exclusive or independent. Explain your answer.
   - I: A die is rolled twice
     - A: first roll lands on 4
     - B: second roll lands on 6
   - II: A die is rolled once
     - A: it lands on 4
     - B: it lands on 6

b. A die is rolled twice. What is the probability that:
   - i. The score on the first roll is 2 and the second roll is 5.
   - ii. The score on the first roll is 1 and the second roll is even.
   - iii. The score on the first roll is either 3 or 5.
   - iv. The score on the first roll is either 3 or 5 and the second roll is odd.
   - v. The score on the first and second roll is either 3 or 5.

c. The probability that Ahmed gains admission into the university is $\frac{4}{5}$ and that of Brima is $\frac{2}{3}$. What is the probability that:
   - i. Both gain admission.
   - ii. None gain admission
   - iii. One gains admission.
   - iv. At least one gains admission.

d. When Mariama plays a game on her mobile phone, the probability that she gets to the top level is $\frac{1}{7}$. One day, Mariama plays two games on her mobile phone. What is the probability that:
   - i. She gets to the top level in both games.
   - ii. She does not get to the top level in both games.
   - iii. She gets to the top level in one of the two games.

If Mariama plays three games instead of two, what is the probability that:
   - iv. She will get to the top level in all three.

e. Fatu, Mary and Sia are taking their WASSCE examination this year. The probability that they will pass Mathematics are respectively 0.4, 0.7 and 0.8. If the events are independent, what is the probability that:
   - i. All three girls pass.
   - ii. Only one of them pass.
   - iii. None pass.
   - iv. At least one pass.

   Give your answers as percentages.

Lesson Title: Outcome tables	**Theme:** Statistics and Probability	
Lesson Number: M3-L104	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to illustrate probability spaces with outcome tables and use them to solve probability problems.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(4 minutes)*

1. Ask pupils to write down the sample space S for throwing an unbiased die.
2. Invite a volunteer to give their answer. (Answer: $S = \{1, 2, 3, 4, 5, 6\}$)
3. Tell pupils that after today's lesson, they will be able to illustrate probability spaces with outcome tables and use them to solve probability problems.

Teaching and Learning *(20 minutes)*

1. Explain:
   - When dealing with the probability of an event occurring, it is very important to identify all the outcomes of the experiment.
   - One way in which this can be done **for outcomes which are all equally likely** is by systematically listing all of them as we did for throwing a die.
   - However, when we have to identify the outcomes for two equally likely events occurring, listing can result in missing out some of the outcomes.
   - Instead, we use an outcome or 2-way table to identify all the outcomes.
   - Drawing a table means we do not have to calculate the required probabilities.
2. Draw the table for question a. i. making sure pupils understand how to list the outcome in each cell.
3. Draw the table and first row of question a. ii. Ask pupils to work with seatmates to complete the rest of the table.
4. Invite volunteers to come to the board to complete the table of outcomes.

   Solution:

   a. Given: 2 fair coins 2 unbiased dice

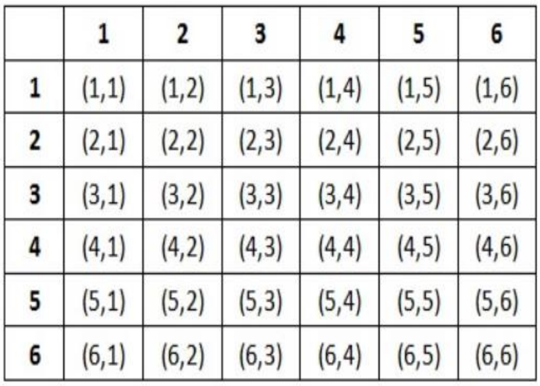
   i. **Second coin**

   First coin

	H	**T**
H	HH	HT
T	TH	TT

   ii. **Second die**

   First die



	1	**2**	**3**	**4**	**5**	**6**
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

5. Invite a volunteer to fully assess question b. on the board and extract the given information (Answer: Find the required probabilities)

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.

   Given: table a. i. find the required probabilities

   Step 2. Calculate the required probabilities.

   From the table: $n(S) = 4$

   i. $P(\text{both coins show a head}) = \frac{1}{4}$

   ii. $P(\text{only one coin shows a tail}) = \frac{2}{4} = \frac{1}{2}$

   iii. $P(\text{both coins land the same way up}) = \frac{2}{4} = \frac{1}{2}$

6. Ask pupils to work with seatmates to answer question c.
7. Invite a volunteer to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

   Solutions:

   c. Given: table a. ii. find the required probabilities

   From the table: $n(S) = 36$

   i. $P(\text{both coins show even number}) = \frac{9}{36} = \frac{1}{4}$

   ii. $P(\text{at least one coin shows 5}) = \frac{11}{36}$

   iii. $P(\text{no coin shows 5}) = 1 - P(\text{at least one coin shows 5})$

   $= 1 - \frac{11}{36} = \frac{25}{36}$

   iv. $P(\text{both coins show the same number}) = \frac{6}{36} = \frac{1}{6}$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d., e. and f.
2. Walk around, if possible, to check the answers and clear misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solutions:

   d. Given: outcomes of 2 unbiased dice; find the required probabilities

   i. Table shown to the right.

   From the table: $n(S) = 36$

   ii. $P(\text{a score of 7}) = \frac{6}{36}$

   $= \frac{1}{6}$

   iii. $P(\text{a score of 5}) = \frac{4}{36}$

   $= \frac{1}{9}$

   iv. $P(\text{an even number score}) = \frac{18}{36}$

   $= \frac{1}{2}$

Second die

First die

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

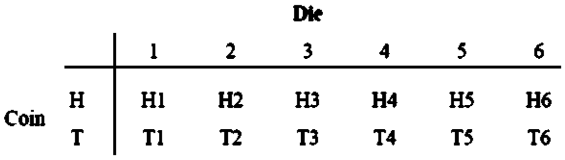
v. $P(\text{score of more than 8}) = \frac{10}{36} = \frac{5}{18}$

vi. $P(\text{score of less than 6}) = \frac{10}{36} = \frac{5}{18}$

From the table, a score of 7 has the highest chance of occurring when the two dice are rolled. The probability is $\frac{1}{6}$ from question ii.

e. Given: an unbiased die and a fair coin are thrown together: find the required probabilities

From the table: $n(S) = 12$



		Die					
		1	2	3	4	5	6
Coin	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

i. $P(\text{a head and a 6}) = \frac{1}{12}$

ii. $P(\text{a tail and an odd numbe}) = \frac{3}{12}$

$= \frac{1}{4}$

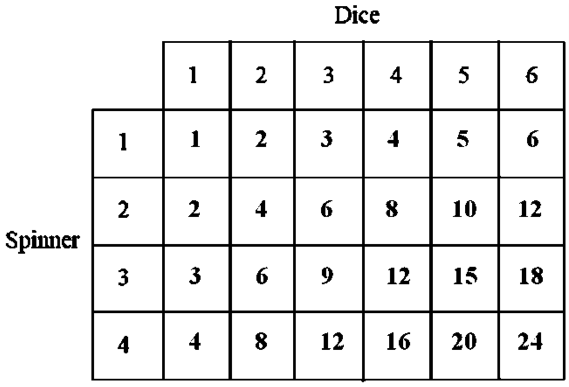
iii. $P(\text{a head and an even no.}) = \frac{3}{12} = \frac{1}{4}$

iv. $P(\text{a head and a no.} > \text{ than 2 }) = \frac{4}{12} = \frac{1}{3}$

v. $P(\text{an even number}) = \frac{6}{12} = \frac{1}{2}$

f. Given: a four-sided spinner is spun and a die is rolled find the required probabilities

From the table: $n(S) = 24$



		Dice					
		1	2	3	4	5	6
Spinner	1	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24

i. $P(\text{a score of 12}) = \frac{3}{24}$

$= \frac{1}{8}$

ii. $P(\text{a score of more than 15}) = \frac{4}{24}$

$= \frac{1}{6}$

iii. $P(\text{a score of less than 2}) = \frac{3}{24}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM3-L104 in the Pupil Handbook.

[QUESTIONS]

a. Use 2-way tables to list all the outcomes for:
   i. Tossing 2 fair coins.
   ii. Rolling 2 unbiased dice.
b. Using the table obtained in question a. i., find the probability that:
   i. Both coins show heads.
   ii. Only one coin shows a tail.
   iii. Both coins land the same way up.
c. Using the table obtained in question a. ii., find the probability that:

i. Both coins show an even number.
ii. At least one coin shows a 5.
iii. No coin shows a 5.
iv. Both coins show the same number.

d. Using the table obtained in question a. ii.,
   i. Create an outcome table for when 2 unbiased dice are rolled, and the outcomes are added together.

   Using the table obtained, find the probability of getting:
   ii. A score of 7.
   iii. A score of 5.
   iv. A score that is an even number.
   v. A score of more than 8.
   vi. A score of less than 6.

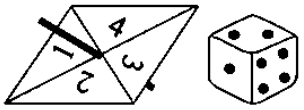
   What score has the highest chance of occurring when the two dice are rolled?

e. An unbiased die and a fair coin are thrown together.
   Copy and complete the table below to show the possible outcomes.

		Die					
		1	2	3	4	5	6
Coin	**H**	**H1**					
	T	T1					

   What is the probability of getting:
   i. A head and a 6.
   ii. A tail and an odd number.
   iii. A head and an even number.
   iv. A head and a number greater than 2.
   v. An even number.

f. A four-sided spinner is spun and a die is rolled. The two results are then multiplied to give a score.
   i. Draw a 2-way outcome table to show all the possible outcomes.



   What is the probability of getting:
   ii. A score of 12.
   iii. A score of more than 20.
   iv. A score of less than 2.

<table>
<tr><td>Lesson Title: Tree diagrams – Part 1</td><td colspan="2">Theme: Statistics and Probability</td></tr>
<tr><td>Lesson Number: M3-L105</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to illustrate probability spaces with tree diagrams and use them to solve probability problems.</td><td colspan="2">Preparation
Write the questions found at the end of this lesson plan on the board.</td></tr>
</table>

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to illustrate probability spaces with tree diagrams and use them to solve probability problems.

Teaching and Learning *(20 minutes)*

1. Explain:
   - Using 2-way outcome tables is a systematic method of listing all the outcomes from two equally likely events.
   - However, they cannot be used when the events are not equally likely to occur or when we have more than 2 events.
   - In such situations, we use a tree diagram where every branch represents an event together with its probability of occurring.
   - Let us look at a simple example of how to draw a tree diagram and use it to solve for required probabilities.
2. Invite a volunteer to assess question a. and say what we are asked to do. (Answer: toss a fair coin twice and find required probabilities)
3. Work through the solution taking care to explain how the tree diagram is created.

Solution:

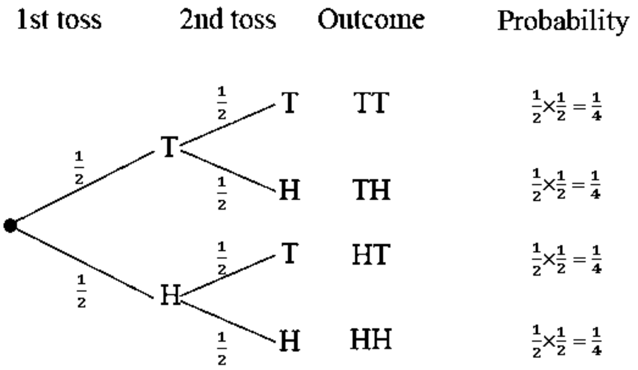
a. **Step 1.** Assess and extract the given information from the problem.

Given: toss a fair coin twice and find the required probabilities.

Step 2. Draw the tree diagram showing all the outcomes.

Step 3. Find the probability of each outcome.

The probability of each outcome is obtained by multiplying the probabilities of the branches leading to that outcome.



Step 4. Find and write the required probabilities.

i. The probability of 2 heads is given by the bottom branch and is $\frac{1}{4}$.

ii. The probability of no heads is given by the top branch and is $\frac{1}{4}$.

iii. The probability of only head is given by the 2 middle branches – The combined probability is: $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$.

The probabilities should all add up to 1.

$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4} \quad = \quad 1$ as expected

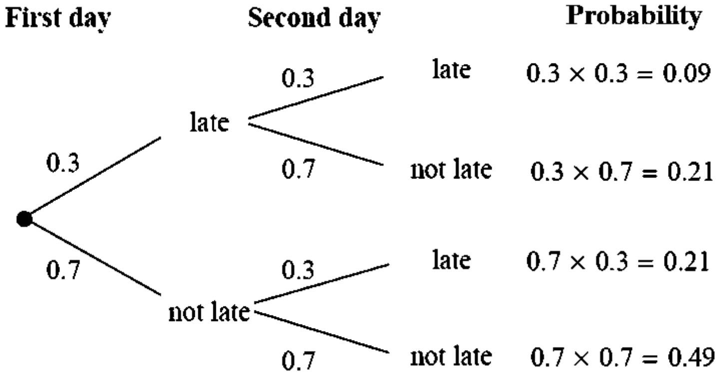
4. Invite a volunteer to assess question b. on the board and extract the given information. (Example answer: probability Jane is late for school $= 0.3$)
5. Invite another volunteer to say what the probability of not being late is. (Answer: $1 - 0.3 = 0.7$)
6. Invite a volunteer to say what is different in this question to question a.? (Answer: the outcomes are not equally likely to occur.)

Solution:

b. Given: probability of being late $= 0.3$

$$P(\text{not being late}) \quad = \quad 1 - 0.3 \quad = \quad 0.7$$

The probability of each outcome is obtained by multiplying the probabilities of the branches leading to that outcome.



From the tree diagram,

i. $P(\text{Jane is never late}) \quad = \quad 0.49$

ii. Let L = late, N = not late

$$\begin{aligned} P(\text{Jane is late only once}) &= P(LN) + P(NL) \\ &= 0.21 + 0.21 \\ &= 0.42 \end{aligned}$$

The probabilities should all add up to 1.

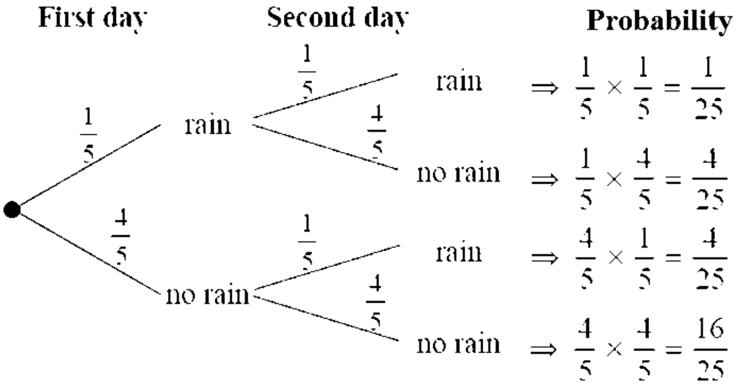
$0.09 + 0.21 + 0.21 + 0.49 \quad = \quad 1$ as expected

7. Ask pupils to work with seatmates to answer question c.
8. Invite a volunteer to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

c. Given: likelihood of rain on any day is $\frac{1}{5}$.

$$P(\text{no rain}) \quad = \quad 1 - \frac{1}{5} \quad = \quad \frac{4}{5}$$



From the tree diagram:

i. $P(\text{rain on 2 consecutive days}) = \frac{1}{25}$

ii. Let R = rain, N = no rain

$$P(\text{rain on only one of two consecutive days}) = P(RN) + P(NR)$$
$$= \frac{4}{25} + \frac{4}{25}$$
$$P(\text{rain on only one of two consecutive days}) = \frac{8}{25}$$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d. and e.
2. Walk around, if possible, to check the answers and clear misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

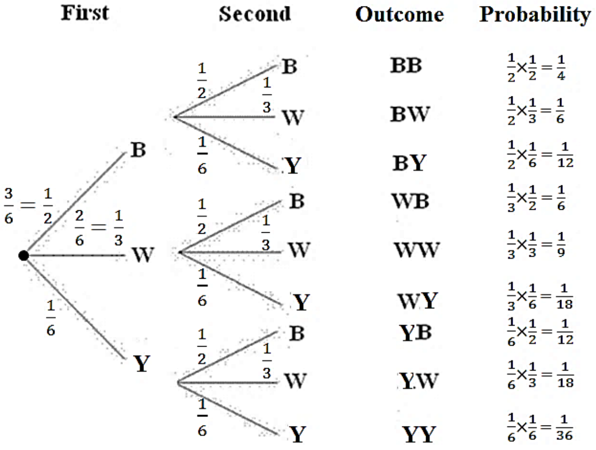
Solutions:

d. Given: die with 6 faces, 3 black, 2 white, 1 yellow; die rolled twice

i. $P(\text{both yellow}) = \frac{1}{36}$

ii. $P(\text{same colour})$
$$= P(BB) + P(WW) + P(YY)$$
$$= \frac{1}{4} + \frac{1}{9} + \frac{1}{36}$$
$$= \frac{7}{18}$$

$P(\text{neither is black})$
$$= P(WW) + P(WY) + P(YW) + P(YY)$$
$$= \frac{1}{9} + \frac{1}{18} + \frac{1}{18} + \frac{1}{36} = \frac{9}{36}$$
$$= \frac{1}{4}$$



e. Given: 3 fair coins tossed at the same time

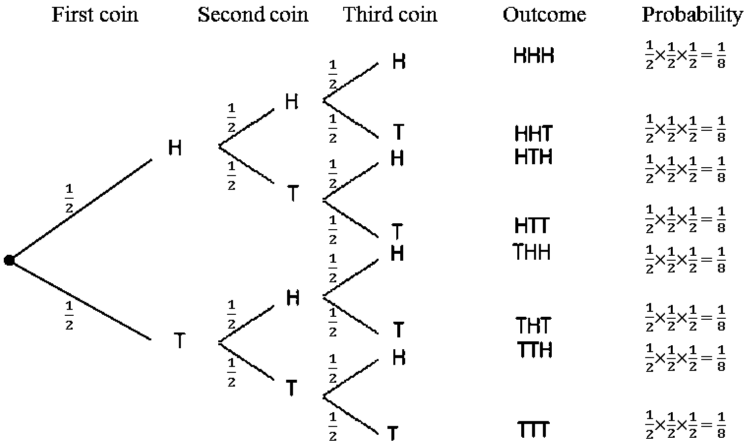
i. $P(\text{three heads}) = P(HHH) = \frac{1}{8}$

ii. $P(\text{at least 2 heads}) = P(HHH) + P(HHT) + P(HTH) + P(THH)$
$$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8}$$
$$= \frac{1}{2}$$

iii. $P(\text{exactly 1 tail}) = P(HHT) + P(HTH) + P(THH) +$

$$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$= \frac{3}{8}$$



Closing *(4 minutes)*

1. Invite volunteers to say one new thing they have learned in this lesson. (Answer: Various)
2. For homework, have pupils do the practice activity PHM3-L105 in the Pupil Handbook.

[QUESTIONS]

For each of the questions below, list all the possible outcomes in a tree diagram. Include the probability on each branch.

a. A fair coin is tossed twice. What is the probability of getting:
   i. Two heads ii. No heads iii. Only one head
b. The probability that Jane is late for school is 0.3. What is the probability that on two consecutive days, she is:
   i. Never late ii. Late only once
c. The likelihood of rain on any day is $\frac{1}{5}$. What is the probability that it rains on:
   i. On two consecutive days, ii. On only one of two consecutive days?
d. A die has 6 faces of which 3 are black, 2 white and 1 yellow. If the die is rolled twice, what is the probability of getting:
   i. Both faces are yellow.
   ii. Both faces the same colour.
   iii. Neither face is black.
e. Three fair coins are tossed at the same time. What is the probability of getting:
   i. Three heads ii. At least 2 heads iii. Exactly one tail

Lesson Title: Tree diagrams – Part 2	**Theme:** Statistics and Probability	
Lesson Number: M3-L106	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use tree diagrams to further solve probability problems.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to use tree diagrams to further solve probability problems.

Teaching and Learning *(20 minutes)*

1. Explain: We will use the tree diagrams from the previous lesson to further solve probability problems including conditional probability.
2. Invite a volunteer to fully assess question a. on the board and extract the given information. (Example answer: bag with 7 blue counters and 3 yellow counters, 1st counter taken randomly, replaced, 2nd counter taken randomly, find required probability)

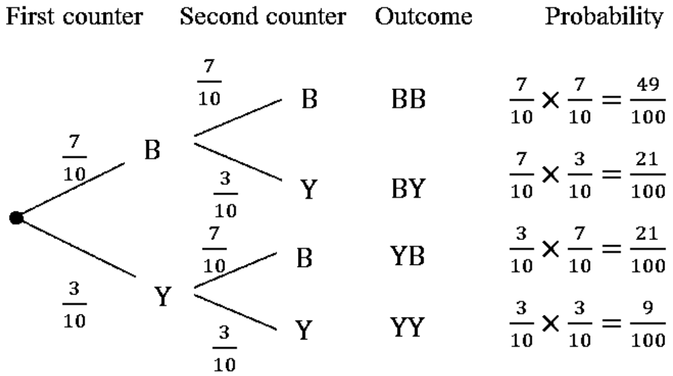
   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.

      Given: bag with 7 blue counters and 3 yellow counters, 1st counter taken randomly, replaced, 2nd counter taken randomly, find required probability

      Step 2. Draw the tree diagram showing all the outcomes.

      total number of counters $=$ 10



      Step 3. Find the probability of each outcome.

      From the tree diagram:

      $$P(\text{both counters are yellow}) = \frac{9}{100}$$

      Step 4. Write the answer.

      The probability that both counters are yellow is $\frac{9}{100}$.

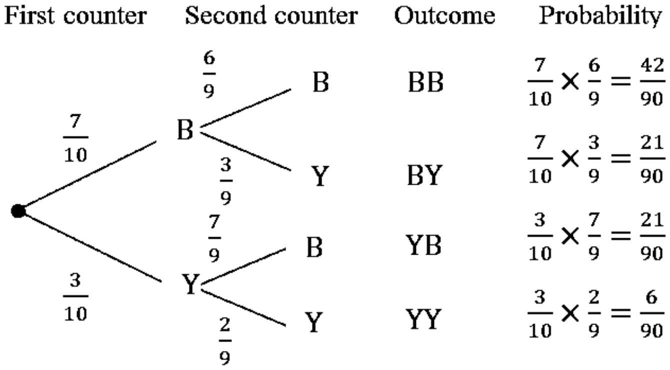
3. Ask pupils to assess question b. with seatmates and identify any differences with question a.

4. Invite a volunteer to give the differences between the two questions. (Example answer: question a. the counters are replaced, in b. the counters are not replaced)
5. Solve question b. explaining clearly what happens after the first counter is taken out to change the probabilities for the second counter.

Solution:

b. Given: bag with 7 blue counters and 3 yellow counters, counter taken randomly, not replaced, find required probability

total number of counters $=$ 10



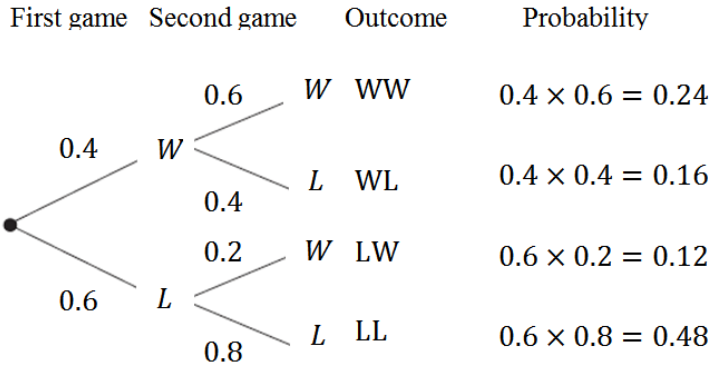
From the tree diagram: $P(\text{both counters are yellow}) = \frac{6}{90}$

6. Explain:
   - As we can see from question b. the probabilities of certain events may change as a result of earlier events.
   - This is called **conditional probability**.
   - Tree diagrams greatly help in solving conditional probability problems as we can adjust the probabilities as we move along the branches.
7. Ask pupils to work with seatmates to answer question c.
8. Invite a volunteer to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

c. Given: game with 2 outcomes, win or lose is played twice; probability of winning the first game is 0.4.

Let W = win, L = lose 1st game: $P(\text{W}) = 0.4$ $P(L) = 1 - 0.4 = 0.6$



From the tree diagram:

i. $P(\text{winning both games}) = P(WW) = 0.24$

ii. $P(\text{winning at least 1}) = P(WW) + P(WL) + P(LW)$

$$= 0.24 + 0.16 + 0.12$$
$$= 0.52$$

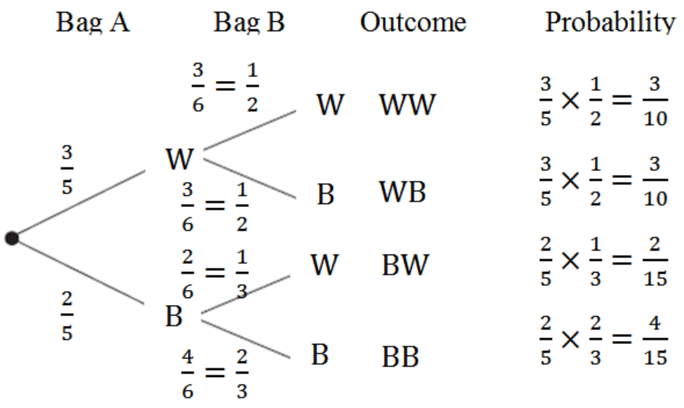
iii. $P(\text{losing both games}) = P(LL)$ this is also $P(1 - \text{winning at least } 1)$
$$= 0.48$$

Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d. and e.
2. Walk around, if possible, to check the answers and clear misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

   Solution:

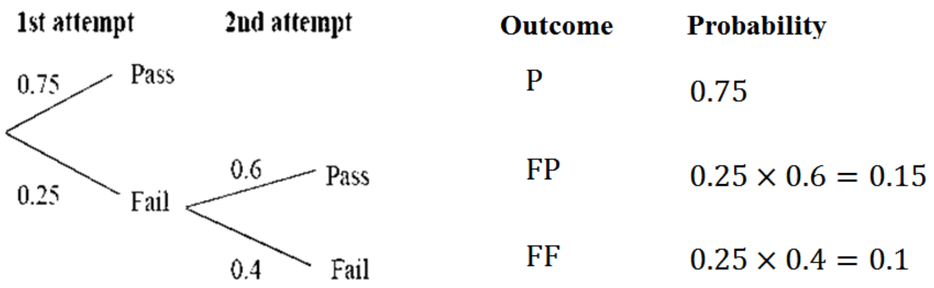
   d. Given: Bag A contains 3 white counters and 2 black counters. Bag B contains 2 white and 3 black counters.



From the tree diagram:

$$P(\text{white counter}) = P(WW) + P(BW)$$
$$= \frac{3}{10} + \frac{2}{15}$$
$$= \frac{13}{30}$$

   e. Given: probability of passing an exam at the first attempt is 0.75 and probability of passing the re-sit is 0.6.



From the tree diagram:

i. $P(a \text{ pupil fails}) = 0.1$

ii. Since the probability of any pupil passing does not depend on any other pupil passing:

$$
\begin{aligned}
P(\text{all 3 pass}) &= P(\text{pupil 1 passes}) \times P(\text{pupil 2 passes}) \times \\
&\quad P(\text{pupil 3 passes}) \\
P(\text{pupil 1 passes}) &= 1 - P(\text{pupil 1 fails}) \\
&= 1 - 0.1 \qquad \text{from i. above} \qquad \text{(or } 0.75 + 0.15 \text{ from} \\
&= 0.9 \qquad\qquad\qquad\qquad\qquad\quad \text{tree diagram)} \\
\therefore P(\text{all 3 pass}) &= 0.9 \times 0.9 \times 0.9 \qquad\qquad\quad \text{3 independent events} \\
&= 0.729
\end{aligned}
$$

Closing *(4 minutes)*

1. Invite volunteers to say one new thing they have learned in this lesson. (Answers: Various)
2. For homework, have pupils do the practice activity PHM3-L106 in the Pupil Handbook.

[QUESTIONS]

Draw a tree diagram with the probability for each branch for the questions below.

a. A bag contains 7 blue counters and 3 yellow counters. A counter is taken at random from the bag, replaced and a second counter taken. What is the probability that both counters are yellow?
b. A bag contains 7 blue counters and 3 yellow counters. Two counters are taken at random from the bag without replacement. What is the probability that both counters are yellow?
c. A game with 2 outcomes, win or lose is played twice. The probability of a team winning the first game is 0.4. If the team wins the first game the probability of them winning the second game is 0.6. If the team loses the first game, the probability of losing the second is 0.8. What is the probability of the team:
    i. Winning both games.
    ii. Winning at least one game.
    iii. Losing both games.
d. Bag A contains 3 white counters and 2 black counters. Bag B contains 2 white and 3 black counters. One counter is removed from bag A and placed in bag B without its colour being seen. What is the probability that the counter removed from bag B will be white?
e. At the end of a training programme, pupils have to pass an exam to gain a certificate. The probability of passing an exam at the first attempt is 0.75.
   Those who fail are allowed to re-sit. The probability of passing the re-sit is 0.6. No further attempts are allowed.
    i. What is the probability that a pupil fails to get a certificate?
    ii. Three pupils take the exam. What is the probability that all 3 pass?

Lesson Title: Venn diagrams	**Theme:** Statistics and Probability	
Lesson Number: M3-L107	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to illustrate probability spaces with Venn diagrams and use them to solve probability problems.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(3 minutes)*

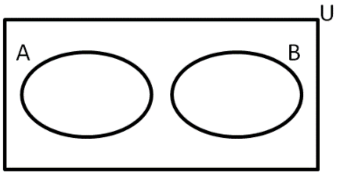
1. Ask pupils to write a simple description of mutually exclusive events.
2. Invite volunteers to answer. (Example answer: Mutually exclusive events cannot occur at the same time.)
3. Tell pupils that after today's lesson, they will be able to illustrate probability spaces with Venn diagrams and use them to solve probability problems.

Teaching and Learning *(20 minutes)*

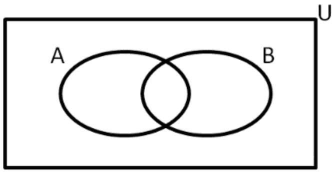
1. Explain:
   - Consider two events – A and B.
   - We know that if they are mutually exclusive then they cannot both occur at the same time. The probability of either one of them occurring is given by:

$$\begin{aligned} P(A \text{ or } B) &= P(A \cup B) \\ &= P(A) + P(B) \end{aligned}$$

   - The first Venn diagram shows the two mutually exclusive events as disjoint sets with no common elements.
   - The complete set is denoted by U which is the Universal set.



   - Now consider events which cannot be classified as mutually exclusive events.
   - The second Venn diagram shows such events.
   - It is clear from the diagram that if the events are not mutually exclusive then:



$$P(A \text{ or } B) = P(A \cup B)$$

Since $A \cup B = A + B - A \cap B$ where $A \cap B$ gives the common elements for both A and B

We can write:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

   - This is the general Addition Law of Probability.
     The law for mutually exclusive events is the special case for when $P(A \cap B) = 0$.

2. Invite a volunteer to fully assess question a. on the board and extract the given information. (Example answer: in a class of 60 SS3 pupils, 40 enjoy listening to music, 50 enjoy watching films, every pupil enjoys at least one of these)
3. Work through the solution taking care to explain how the Venn diagram is constructed from the information given.

   Solution:

   a. **Step 1.** Assess and extract the given information from the problem.

   Given: in a class of 60 SS3 pupils, 40 enjoy listening to music, 50 enjoy watching films, every pupil enjoys at least one of these

   Step 2. Explain how to draw the Venn diagram.

   Let $A = \text{enjoy listening to music}$, $B = \text{enjoy watching films}$, $n(U) = 60$

   - Draw an empty Venn diagram on the board. Label U, A and B
   - Since every pupil enjoys one of the activities, a 0 can be placed outside the ovals representing the events.

   We know that:

   $$A \cup B = A + B - A \cap B$$

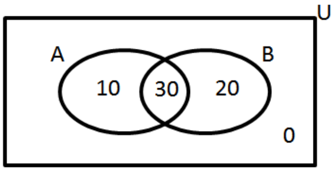
   $$A \cup B = 60 \qquad A = 40 \qquad B = 50$$

   $$\therefore \quad 60 = 40 + 50 - A \cap B$$

   $$= 90 - A \cap B$$

   $$A \cap B = 90 - 60$$

   $$= 30$$

   - Complete the diagram as shown.
   - Explain that we subtract the pupils who enjoy both activities from those who enjoy just one.
   - The total number will always add up to the universal set.

   Step 3. Find the required probabilities.

   i. Let $C = \text{enjoy listening to music and watching films}$

   $$P(C) = P(A \cap B) = \frac{30}{60} = \frac{1}{2}$$

   ii. Let $D = \text{does not enjoy listening to music but enjoys watching films}$

   $$P(D) = \frac{20}{60} = \frac{1}{3}$$

   iii. Let $E = \text{enjoy watching films but does not enjoy listening to music}$

   $$P(E) = \frac{10}{60} = \frac{1}{6}$$

   Check that the probabilities add up to 1: $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$ as expected.
4. Ask pupils to work with seatmates to answer question b.
5. Invite a volunteer to show their answer on the board.

   The rest of the class should check their solution and correct any mistakes.

   Solution:

   b. Given: Venn diagrams I, II and III show the ways two events A and B can take place; find required probabilities.

Diagram I $n(U) = 20$

i. $P(A \text{ and } B) = \frac{7}{20}$ $P(A \cap B)$

ii. $P(A \text{ but not } B) = \frac{4}{20}$

iii. $P(A \text{ or } B) = 1$ $P(A \cup B)$

iv. $P(B \text{ but not } A) = \frac{9}{20}$

Diagram I

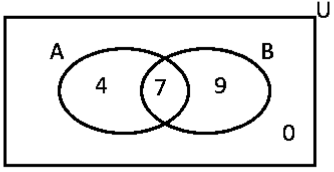


Diagram II $n(U) = 50$

i. $P(A \text{ and } B) = 0$ $P(A \cap B)$

ii. $P(A \text{ but not } B) = \frac{10}{50} = \frac{1}{5}$

iii. $P(A \text{ or } B) = \frac{45}{50} = \frac{9}{10}$ $P(A \cup B)$

iv. $P(B \text{ but not } A) = \frac{35}{50} = \frac{7}{10}$

Diagram II

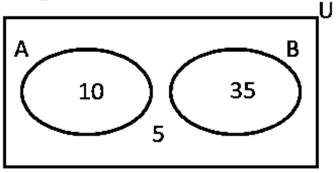


Diagram III $n(U) = 40$

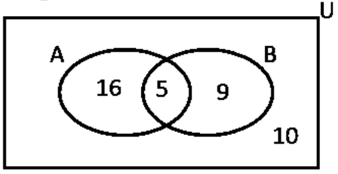
i. $P(A \text{ and } B) = \frac{5}{40} = \frac{1}{8}$ $P(A \cap B)$

ii. $P(A \text{ but not } B) = \frac{16}{40} = \frac{2}{5}$

iii. $P(A \text{ or } B) = \frac{30}{40} = \frac{3}{4}$ $P(A \cup B)$

iv. $P(B \text{ but not } A) = \frac{9}{40}$

Diagram III



Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions d. and e.
2. Walk around, if possible, to check the answers and clear misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

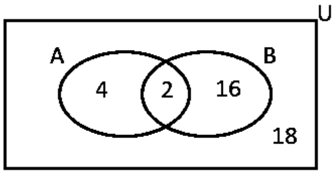
Solutions:

c. Given: 40 people, 6 of them are left-handed, 18 have size 7 feet and 2 are left-handed with size 7 feet; find required probabilities.

Let $A = \text{left} - \text{handed}$, $B = \text{size 7 feet}$, $n(U) = 40$

$A = 6$ $B = 18$ $A \cap B = 2$

Subtract $A \cap B$ (2) from A and from B to leave just left-handed or size 7 feet. (see Venn diagram)



$$\begin{aligned} A \cup B &= A + B - A \cap B \\ &= 6 + 18 - 2 \\ &= 22 \end{aligned}$$

Number of people who are not left-handed and do not have size 7 feet:

$$\begin{aligned} &= n(U) - A \cup B \\ &= 40 - 22 \quad = \quad 18 \end{aligned}$$

i. Let $C = \text{left} - \text{handed or has size 7 feet}$

$P(C) = P(A \cup B) = \frac{22}{40} = \frac{11}{20}$

ii. Let $D = \text{not left} - \text{handed and does not have size 7 feet}$

$P(D) = \frac{18}{40} = \frac{9}{20}$ this is the same as $1 - P(C)$

iii. Let $E = \text{not left} - \text{handed and has size 7 feet}$

$$P(E) \quad = \quad \frac{16}{20} \quad = \quad \frac{4}{5}$$

d. Given: probability a randomly selected person is over 6 feet is 0.08, female is 0.52 and female and over 6 feet is 0.042; find required probability.

Let $A = \text{over 6 feet tall}$, $\quad B = \text{female}$

$$P(A) \;=\; 0.08 \qquad P(B) \;=\; 0.52 \qquad P(A \cap B) \;=\; 0.042$$

$$\begin{aligned} P(A \text{ or } B) = P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.08 + 0.52 - 0.042 \\ &= 0.558 \end{aligned}$$

Closing *(2 minutes)*

1. Invite a volunteer to say what the difference is between the general addition law of probability and that for mutually exclusive events. (Answer: For mutually exclusive events $P(A \cap B) = 0$.)
2. For homework, have pupils do the practice activity PHM3-L107 in the Pupil Handbook.

[QUESTIONS]

a. In a class of 60 SS3 pupils, there are 40 who enjoy listening to music and 50 who enjoy watching films. Every pupil enjoys at least one of these activities.
Draw a Venn diagram and find the probability that a pupil selected at random:
   i. Enjoys listening to music and watching films.
   ii. Does not enjoy listening to music but enjoys watching films.
   iii. Enjoys watching films but does not enjoy listening to music.

b. The Venn diagrams I, II and III show the ways two events A and B can take place.
For each case find:
   i. $P(A \text{ and } B)$ ii. $P(A \text{ but not } B)$ iii. $P(A \text{ or } B)$ iv. $P(B \text{ but not } A)$

Diagram I

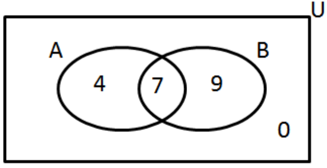


Diagram II

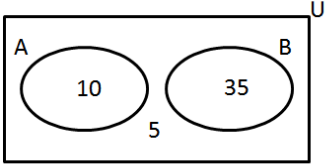
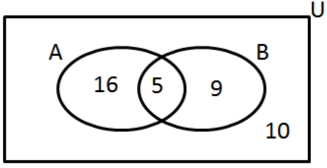


Diagram III



c. In a group of 40 people, 6 of them are left-handed, 18 have size 7 feet and 2 are left-handed with size 7 feet.
Draw a Venn diagram and find the probability that someone selected at random:
   i. Is left-handed or has size 7 feet.
   ii. Is not left-handed and does not have size 7 feet.
   iii. Is not left-handed and has size 7 feet.

d. When a person is selected at random, the probability that they are over 6 feet is 0.08 and the probability that they are female is 0.52. If the probability that a female is over 6 feet is 0.042, find the probability that a person selected at random is female or over 6 feet.

Lesson Title: Solve probability problems	**Theme:** Statistics and Probability	
Lesson Number: M3-L108	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve a variety of probability problems.	**Preparation** Write the questions found at the end of this lesson plan on the board.	

Opening *(1 minute)*

1. Tell pupils that after today's lesson, they will be able to solve a variety of probability problems.

Teaching and Learning *(20 minutes)*

1. Explain: Problems will cover all the concepts learned during our study of probability.
2. Invite a volunteer to fully assess question a. on the board and extract the given information. (Example answer: probability Kelfala pumps tyres $\frac{1}{20}$, probability that he adds oil is $\frac{1}{10}$ and the probability he adds water is $\frac{1}{5}$, find required probabilities)

Solution:

a. **Step 1.** Assess and extract the given information from the problem.

Given: probability Kelfala pumps tyre is $\frac{1}{20}$, probability that he adds oil is $\frac{1}{10}$ and the probability he adds water is $\frac{1}{5}$, find required probabilities.

$$\begin{array}{llllll}
\text{Let } A = \text{Kelfala pumps tyre} & P(A) & = \frac{1}{20} & P(\overline{A}) & = \frac{19}{20} \\
B = \text{Kelfala adds oil} & P(B) & = \frac{1}{10} & P(\overline{B}) & = \frac{9}{10} \\
C = \text{Kelfala adds water} & P(C) & = \frac{1}{5} & P(\overline{C}) & = \frac{4}{5}
\end{array}$$

Step 2. Calculate the required probabilities.

i. $P(\text{Kelfala does not do anything to his car}) = P(\overline{A}) \times P(\overline{B}) \times P(\overline{C})$

$$\begin{aligned}
&= \frac{19}{20} \times \frac{9}{10} \times \frac{4}{5} \\
&= \frac{684}{1000} \\
&= \frac{171}{250}
\end{aligned}$$

ii. $P(\text{Kelfala does one thing to his car})$

$$\begin{aligned}
&= P(A) \times P(\overline{B}) \times P(\overline{C}) + P(\overline{A}) \times P(B) \times P(\overline{C}) \\
&\quad + P(\overline{A}) \times P(\overline{B}) \times P(C) \\
&= \left(\frac{1}{20} \times \frac{9}{10} \times \frac{4}{5}\right) + \left(\frac{19}{20} \times \frac{1}{10} \times \frac{4}{5}\right) + \left(\frac{19}{20} \times \frac{9}{10} \times \frac{1}{5}\right) \\
&= \frac{36}{1{,}000} + \frac{76}{1{,}000} + \frac{171}{1{,}000} \\
&= \frac{283}{1{,}000}
\end{aligned}$$

iii. $P(\text{Kelfala does at least one thing to his car})$

$$= 1 - P(\text{Kelfala does not do anything to his car})$$
$$= 1 - \frac{171}{250}$$
$$= \frac{79}{250}$$

3. Ask pupils to work with seatmates to answer question b.
4. Invite a volunteer to show their answer on the board. The rest of the class should check their solution and correct any mistakes.

Solution:

b. Given: Venn diagram showing the ways in which the events A, B and C can take place; find the required probabilities

$n(U) = 40$

i. $P(A \text{ and } B \text{ and } C) = \frac{3}{40}$ $\quad P(A \cap B \cap C)$

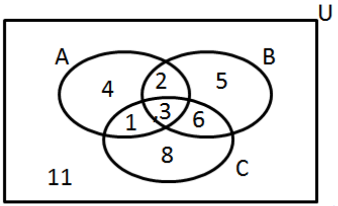
ii. $P(A \text{ or } B \text{ or } C) = \frac{29}{40}$ $\quad P(A \cup B \cup C)$

iii. $P(A \text{ or } B) = \frac{21}{40}$ $\quad P(A \cup B)$

iv. $P(A \text{ and } B) = \frac{5}{40} = \frac{1}{8}$ $\quad P(A \cap B)$

v. $P(A \text{ or } C) = \frac{24}{40} = \frac{3}{5}$ $\quad P(A \cup C)$

vi. $P(A) = \frac{10}{40} = \frac{1}{4}$



Practice *(15 minutes)*

1. Ask pupils to work independently to answer questions c. d. and e.
2. Walk around, if possible, to check the answers and clear misconceptions.
3. Invite volunteers to come to the board to show their solutions. The rest of the class should check their solutions and correct any mistakes.

Solutions:

c. Given: a white die, a blue die and a yellow die are rolled; find required probabilities.

Let $A = \text{score on white die}$
$B = \text{score on blue die}$
$C = \text{score on yellow die}$

For each die
$S = \{1, 2, 3, 4, 5, 6\}$ $\quad n(S) = 6$

i. $P(\text{the score on all 3 dice is odd}) = P(A \text{ odd}) \times P(B \text{ odd}) \times P(C \text{ odd})$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{8}$$

ii. $P(A \text{ is 2 or 3 and } B \text{ is greater than 3})$

$$= P(A \text{ is 2 or 3}) \times P(B > 3)$$
$$= \left(\frac{1}{6} + \frac{1}{6}\right) \times \frac{1}{2}$$
$$= \frac{1}{6}$$

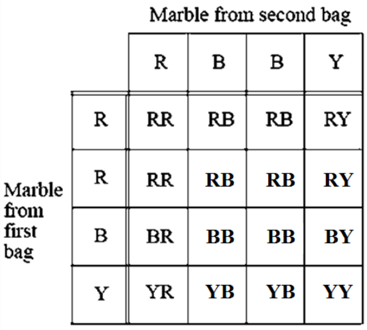
iii. $P(A \text{ is prime number and B is 1 or 4 and C is a multiple of 3})$

$$= \frac{1}{2} \times \left(\frac{1}{6} + \frac{1}{6}\right) \times \frac{1}{3}$$

$$= \frac{1}{18}$$

d. Given: A bag contains 2 red marbles, 1 blue marble and 1 yellow marble; second bag contains 1 red, 2 blue and 1 yellow marble.

i. Completed table shown right



		Marble from second bag			
		R	B	B	Y
Marble from first bag	R	RR	RB	RB	RY
	R	RR	RB	RB	RY
	B	BR	BB	BB	BY
	Y	YR	YB	YB	YY

ii. $P(\text{both marbles are same colour}) = \frac{5}{16}$

iii. $P(\text{at least one marble is yellow}) = \frac{7}{16}$

iv. $P(\text{no marble is yellow})$

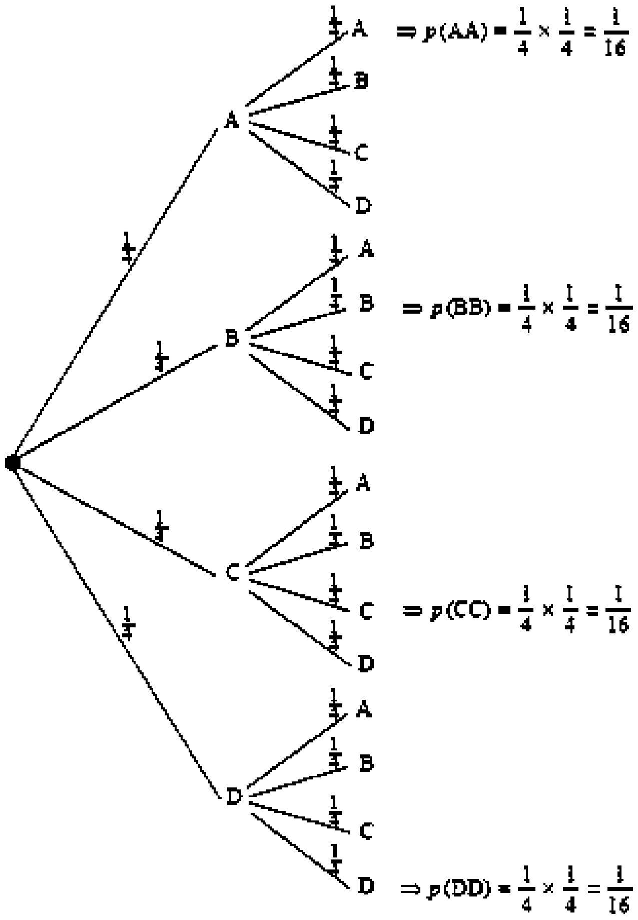
$$= 1 - P(\text{at least one marble is yellow})$$

$$= 1 - \frac{7}{16}$$

$$= \frac{9}{16}$$

e. Given: child's toy can point to one of 4 regions A, B, C or D when spun

i. **1st spin** **2nd spin** **Probability**



ii. $P(\text{same letter}) = \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{4}{16}$

$$= \frac{1}{4}$$

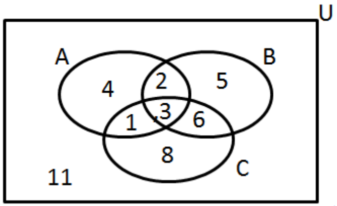
Closing *(4 minutes)*

1. Ask pupils to discuss with seatmates two new things they learned in this lesson.
2. Invite volunteers to share with the class. (Answer: Various)
3. For homework, have pupils do the practice activity PHM3-L108 in the Pupil Handbook.

[QUESTIONS]

a. Kelfala checks his car once a week. The probability that he needs to pump up a tyre is $\frac{1}{20}$. The probability that he has to add oil is $\frac{1}{10}$ and the probability he has to add water is $\frac{1}{5}$. If the events are independent, what is the probability that Kelfala:
   i. Does not need to do anything to his car.
   ii. Has to do one thing to his car.
   iii. Has to do at least one thing to his car.

b. The Venn diagram shows the ways in which the events A, B and C can take place. Find:
   i. $P(A \text{ and } B \text{ and } C)$
   ii. $P(A \text{ or } B \text{ or } C)$
   iii. $P(A \text{ or } B)$
   iv. $P(A \text{ and } B)$
   v. $P(A \text{ or } C)$
   vi. $P(A)$



c. A white die, a blue die and a yellow die are rolled. What is the probability that:
   i. The score on all 3 dice is odd.
   ii. The score on the white die is 2 or 3, and the yellow die is greater than 3.
   iii. The score on the white die is a prime number, the blue die is 1 or 4 and the yellow die is a multiple of 3.

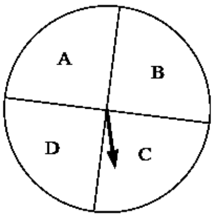
d. A bag contains 2 red marbles, 1 blue marble and 1 yellow marble. A second bag contains 1 red marble, 2 blue marbles and 1 yellow marble. A marble is drawn from each bag.
   i. Complete the table showing all the possible pairs of colours.

   What is the probability:
   ii. That both marbles are the same colour.
   iii. At least one marble is yellow.
   iv. No marble is yellow.

Marble from second bag

Marble from first bag		R	B	B	Y
	R	RR	RB	RB	RY
	R	RR			
	B	BR			
	Y	YR			

e. A child's toy shown can point to one of 4 regions A, B, C or D when spun.
   i. Draw a tree diagram to show all the possible outcomes.
   ii. What is the probability that when it is spin twice, it points to the same letter?



Lesson Title: Review of cumulative frequency curve	**Theme:** Statistics and Probability	
Lesson Number: M3-L109	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Construct a cumulative frequency curve and estimate quartiles. 2. Calculate inter-quartile range and semi interquartile range.	**Preparation** Write the frequency table in Opening on the board, with the "Cumulative Frequency" column empty.	

Opening *(3 minutes)*

1. Draw the frequency table below on the board, but leave the "Cumulative Frequency" column empty.
2. Invite volunteers to come to the board and find the cumulative frequency of each row. Remind them to add each frequency to the total frequency of the rows above it.
3. Explain that this lesson is a review of drawing and interpreting a cumulative frequency curve.

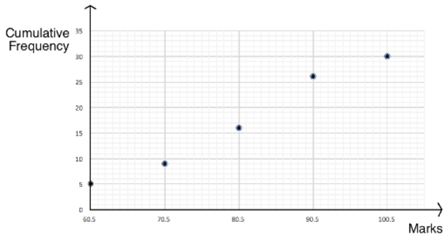
Pupils' Scores on a Maths Test		
Marks	**Frequency**	**Cumulative Frequency**
51 – 60	5	5
61 – 70	4	5+4=9
71 – 80	7	9+7=16
81 – 90	10	16+10=26
91 – 100	4	26+4=30
Total	30	

Teaching and Learning *(18 minutes)*

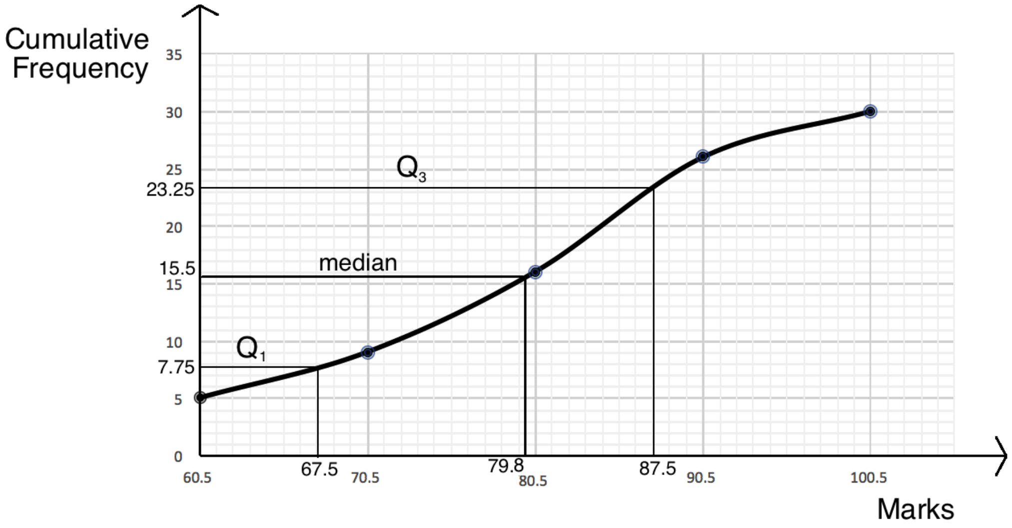
1. Explain: A **cumulative frequency curve** (or "**ogive**") can be graphed in a similar way to line graphs and frequency polygons.
2. Explain:
   - For the x-values, plot the upper class boundary of each class interval. This is the highest data point in each class interval.
   - Where there is a space of 1 unit between each interval, take the middle (for example, the upper class boundary of 51-60 is 60.5, covering the gap to the next interval, which starts at 61.)
   - For the y-value, plot the cumulative frequency from the table.
3. Draw another column in the table on the board, and write the upper class boundary for each class interval. Make sure pupils understand.

Pupils' Scores on a Maths Test			
Marks	**Frequency**	**Cumulative Frequency**	**Upper Class Boundary**
51 – 60	5	5	60.5
61 – 70	4	5 + 4 = 9	70.5
71 – 80	7	7 + 9 = 16	80.5
81 – 90	10	10 + 16 = 26	90.5

91 – 100	4	4 + 26 = 30	100.5
Total	30		



4. Draw the axes on the board, and plot each point from the table (see larger graph below). → Note that for the sake of time, it is not necessary to draw each minor gridline on the graph.
5. Connect the points with a smooth curve.
6. Explain **quartiles:**
   - We can estimate the quartiles using the cumulative frequency curve.
   - Quartiles divide a data set into 4 equal parts. The lower quartile (Q_1) is one-quarter of the way from the bottom of the data. The upper quartile (Q_3) is one-quarter of the way from the top of the data set. The second quartile (Q_2) is the median, or the middle quartile.
   - We estimate quartiles by finding their placement. We then locate their placement on the x-axis, and find the corresponding value on the y-axis.
7. Write on the board: Positions of quartiles: $Q_1: \frac{1}{4}(n+1)$, $Q_2: \frac{1}{2}(n+1)$ and $Q_3: \frac{3}{4}(n+1)$,
8. Use the formulae to find the position of each quartile as a class on the board. (Answers: $Q_1: \frac{1}{4}(n+1) = \frac{1}{4}(31) = 7.75$; $Q_2: \frac{1}{2}(31) = 15.5$; $Q_3: = \frac{3}{4}(31) = 23.25$)
9. Draw each horizontal line from the y-axis, then draw a vertical line connecting this point on the curve to the x-axis. Identify the marks at the corresponding point.



10. Explain: The lower quartile (Q_1) is 67.5, the median (Q_2) is 79.8, and the upper quartile (Q_3) is 87.5. Remember that these are estimates.
11. Explain **interquartile range:**
    - Just as we can calculate the range of a data set, we can calculate the interquartile range. Subtract the lower quartile from the upper quartile.
    - This represents how spread out the middle half of the data is.
12. Calculate interquartile range on the board: $Q_3 - Q_1 = 87.5 - 67.5 = 20$ marks

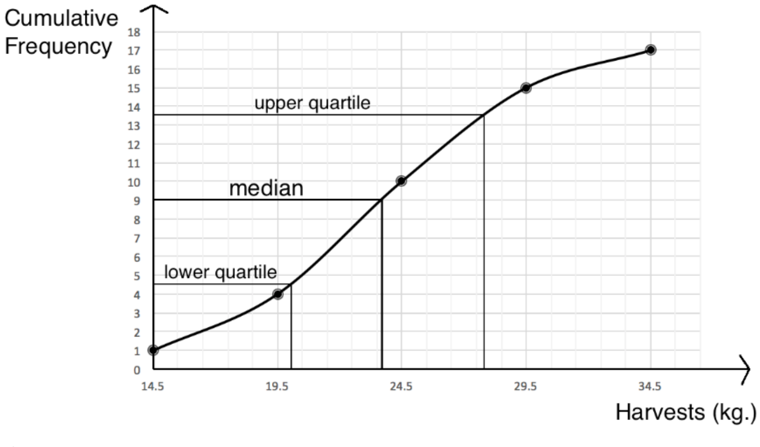
13. Explain semi-interquartile range: The semi-interquartile range tells us about one quarter of the data set ("semi" means half, so it is half of the interquartile range).
14. Calculate semi-interquartile range: $Q = \frac{Q_3 - Q_1}{2} = \frac{87.5-67.5}{2} = \frac{20}{2} = 10$ marks
15. Explain: This tells us that about half of the pupils scored within 10 marks of the median score.

Practice *(18 minutes)*

1. Write the following problem on the board: The table below gives the cassava harvests of 17 farmers.
    a. Fill the empty columns.
    b. Draw the cumulative frequency curve.
    c. Use the curve to estimate each quartile of the distribution.
    d. Calculate the interquartile range and semi-interquartile range.

Farmers' Harvests			
Cassava (kg)	**Frequency**	**Upper Class Boundary**	**Cumulative Frequency**
10 – 14	1		
15 – 19	3		
20 – 24	6		
25 – 29	5		
30 – 34	2		
Total	17		

2. Ask pupils to work independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain:
    a. For the table, upper class intervals: 14.5, 19.5, 24.5, 29.5, 34.5; Cumulative frequencies: 1, 4, 10, 15, 17
    b. Curve →
    c. Accept approximate answers: $Q_1 = 20.1$ kg; $Q_2 = 23.6$ kg; $Q_3 = 27.7$ kg
    d. Interquartile range: $Q_3 - Q_1 = 27.7 - 20.1 = 6.6$ kg



Semi-interquartile range: $Q = \frac{Q_3 - Q_1}{2} = \frac{27.7-20.1}{2} = \frac{6.6}{2} = 3.3$ kg

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L109 in the Pupil Handbook.

Lesson Title: Percentiles	**Theme:** Statistics and Probability	
Lesson Number: M3-L110	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to estimate percentiles of data from the cumulative frequency curve.	**Preparation** Write the table and c.f. curve at the start of Teaching and Learning on the board.	

Opening *(3 minutes)*

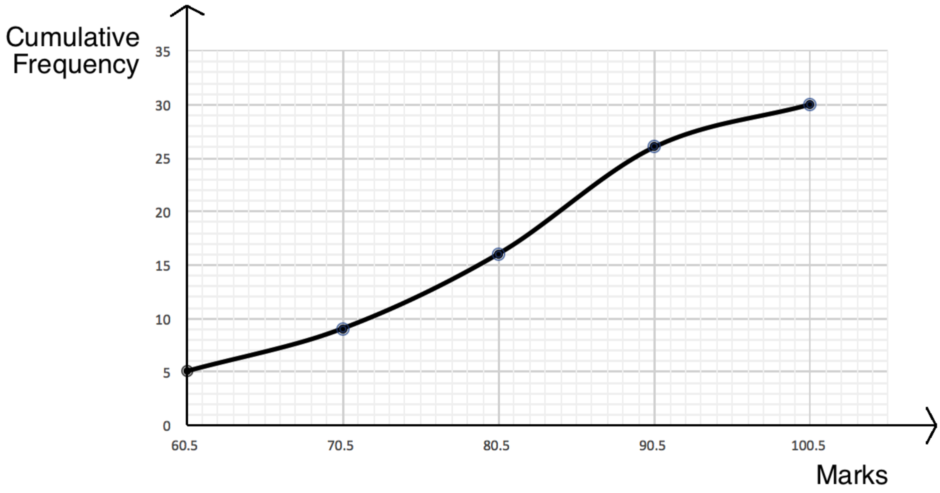
1. Discuss:
    - What are quartiles? (Answer: They are values that divide a data set into 4 equal parts.)
    - How do we estimate quartiles? (Answer: Use the formulae to find their positions in the data, then estimate the value of each quartile using the cumulative frequency curve.)
2. Explain that this lesson is on estimating percentiles. This is similar to estimating quartiles.

Teaching and Learning *(16 minutes)*

1. Write the table and c.f. curve on the board (from the previous lesson):

Pupils' Scores on a Maths Test

Marks	Frequency	Cumulative Frequency	Upper Class Interval
51 – 60	5	5	60.5
61 – 70	4	5 + 4 = 9	70.5
71 – 80	7	7 + 9 = 16	80.5
81 – 90	10	10 + 16 = 26	90.5
91 – 100	4	4 + 26 = 30	100.5
Total	30		



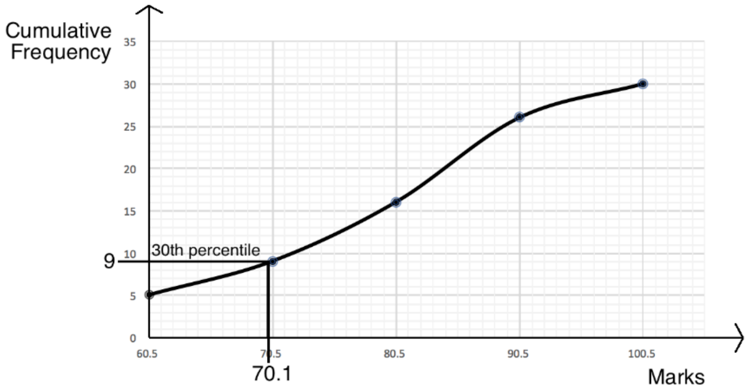
2. Explain:

- In the previous lesson, we used this curve to estimate the quartiles of the data set.
- Today we will use the same curve to estimate the percentiles.
- Percentiles divide the data set into 100 equal parts.
- For example, the 30th percentile divides off the lowest 30% of the data.

3. Write on the board: The nth percentile is the mark at $\frac{n}{100}\sum f$.
4. Explain:
   - This formula tells us the position of the percentile.
   - After using the formula, find the position on the y-axis of the curve. Draw horizontal and vertical lines to estimate the corresponding value on the x-axis. This is the estimated percentile.
5. Write on the board: Using the curve, estimate the:
   a. 30th percentile
   b. 75th percentile
   c. 90th percentile
6. Use the formula to find the position of the 30th percentile on the board:

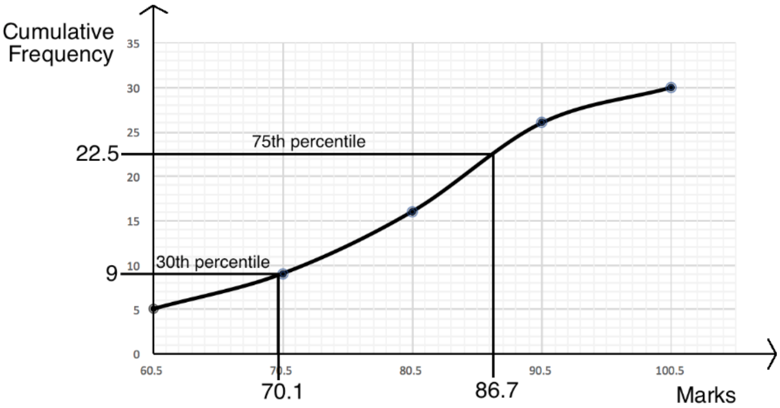
$$\frac{n}{100}\sum f = \frac{30}{100}(30) = \frac{900}{100} = 9$$

7. Estimate the 30th percentile using the curve on the board, as shown:



8. Write on the board: 30th percentile = 70.1 marks
9. Remind pupils that this is an estimated value.
10. Ask pupils to explain how to find b., the 75th percentile. As they give the steps, estimate it on the board:

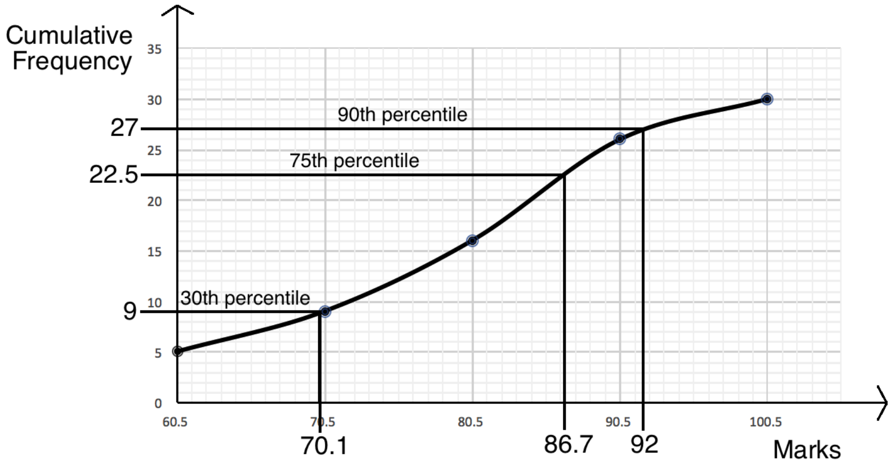
75th percentile: $\frac{n}{100}\sum f = \frac{75}{100}(30) = \frac{225}{10} = 22.5$



75th percentile $= 86.7$ marks

11. Ask pupils to work with seatmates to complete part c.
12. Walk around to check for understanding and clear misconceptions.
13. Invite volunteers to write the solution on the board.

    90th percentile: $\frac{n}{100}\sum f = \frac{90}{100}(30) = \frac{2700}{100} = 27$



90th percentile $= 92$ marks

Practice *(20 minutes)*

1. Write the following problem on the board: The table below gives the age distribution of all of teachers in a certain town.

Age (years)	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64
Frequency	1	9	16	26	32	17	14	5

    a. Construct a cumulative frequency table.
    b. Use the table to draw the cumulative frequency curve.
    c. Use the curve to estimate the following:
        i. 20th percentile
        ii. 45th percentile
        iii. 70th percentile
        iv. 95th percentile
2. Ask pupils to work independently or with seatmates to complete the problem.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain:
    a. Cumulative frequency table:

Teachers' ages			
Age (years)	**Frequency**	**Upper Class Boundary**	**Cumulative Frequency**
25 – 29	1	29.5	1
30 – 34	9	34.5	1+9=10
35 – 39	16	39.5	10+16=26
40 – 44	26	44.5	26+26=52

45 – 49	32	49.5	52+32=84
50 – 54	17	54.5	84+17=101
55 – 59	14	59.5	101+14=115
60 – 64	5	64.5	115+5=120
Total	120		

b. Cumulative frequency curve: See below

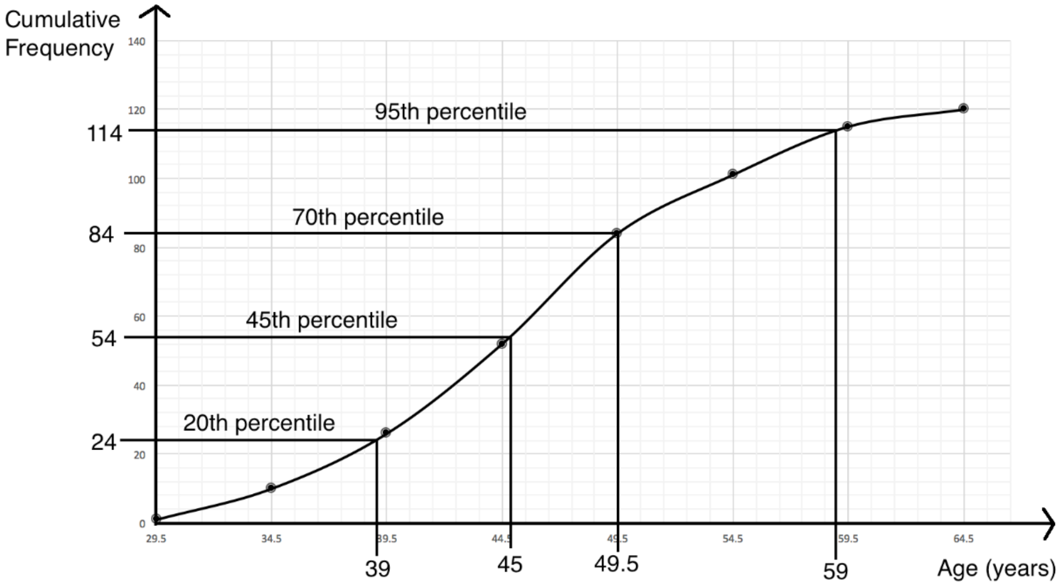
c. Find the position of each percentile, then find its value on the cumulative frequency curve (see curve below).

   i. 20th percentile position: $\frac{n}{100}\sum f = \frac{20}{100}(120) = 24$;
      20th percentile $= 39$ years

   ii. 45th percentile position: $\frac{n}{100}\sum f = \frac{45}{100}(120) = 54$;
      45th percentile $= 45$ years

   iii. 70th percentile position: $\frac{n}{100}\sum f = \frac{70}{100}(120) = 84$;
      70th percentile $= 49.5$ years

   iv. 95th percentile position: $\frac{n}{100}\sum f = \frac{95}{100}(120) = 114$;
      95th percentile $= 59$ years



5. Remind pupils that the percentiles are only estimates.
6. Discuss and interpret each percentile as a class:
   i. 20% of teachers are 39 years or younger
   ii. 45% of teachers are 45 years or younger
   iii. 70% of teachers are 49.5 years or younger
   iv. 95% of teachers are 59 years or younger

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L110 in the Pupil Handbook.

Lesson Title: Applications of percentiles	**Theme:** Statistics and Probability	
Lesson Number: M3-L111	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply percentiles to real-life problems.	**Preparation** Write the problem at the start of Teaching and Learning on the board.	

Opening *(2 minutes)*

1. Discuss:
   - What are percentiles? (Answer: They divide a data set into 100 equal parts.)
   - How do we estimate percentiles? (Answer: Use the formula to find their positions in the data, then estimate the value of each percentile using the cumulative frequency curve.)
2. Explain that this lesson is on solving practical problems related to percentiles.

Teaching and Learning *(18 minutes)*

1. Write the problem on the board: The scores of 50 pupils on a test are shown in the frequency table.

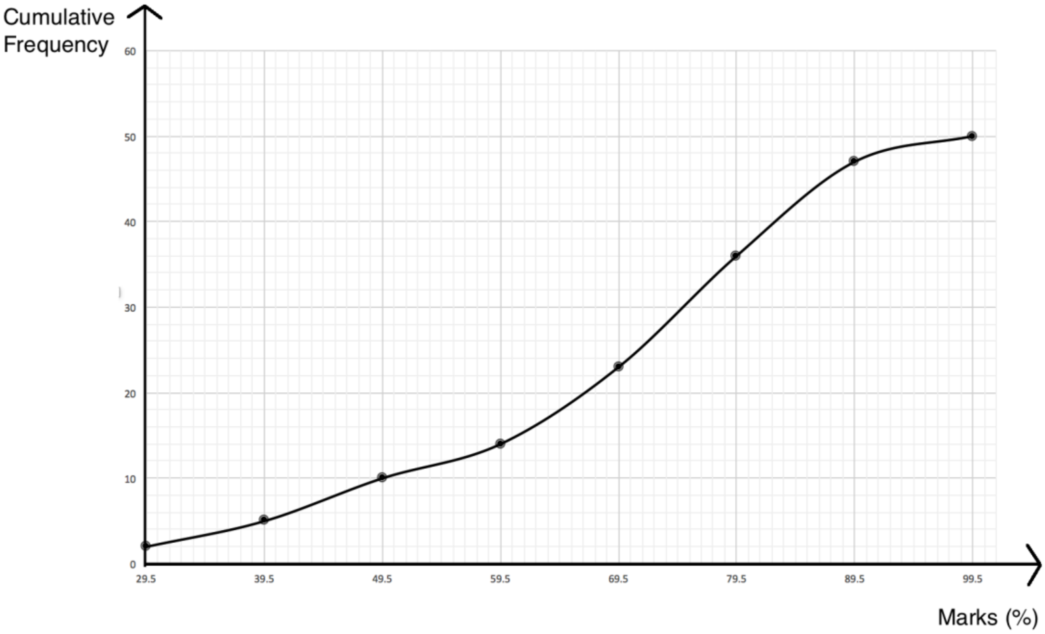
Marks (%)	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	2	3	5	4	9	13	11	3

   a. Draw a cumulative frequency table.
   b. Use the table to draw the cumulative frequency curve.
   c. If 70% of pupils passed, find the pass mark.
   d. If 8% of candidates were awarded distinction, estimate the lowest mark for distinction.
   e. How many pupils were awarded distinction?
2. Complete parts a. and b. on the board as a class. Ask volunteers to draw the cumulative frequency table and curve.

Answers:

Pupils' Scores on a Test

Marks	**Frequency**	**Upper Class Boundary**	**Cumulative Frequency**
20-29	2	29.5	2
30-39	3	39.5	2+3=5
40-49	5	49.5	5+5=10
50-59	4	59.5	10+4=14
60-69	9	69.5	14+9=23
70-79	13	79.5	23+13=36
80-89	11	89.5	36+11=47
90-99	3	99.5	47+3=50
Total	50		



3. Explain part c. of the problem:
    - If 70% of pupils passed, then 30% of pupils failed.
    - If 30% of pupils scored below the passing mark, we can find the 30th percentile to find the passing mark.
    - This question is just another way of asking you to find the 30th percentile.
4. Ask pupils to work with seatmates to find the 30th percentile.
5. Ask a volunteer to write the solution on the board.

   Solution:

   Step 1. Find the position of the 30th percentile: $\frac{n}{100}\sum f = \frac{30}{100}(50) = \frac{1500}{100} = 15$

   Step 2. Find the percentile on the c.f. curve (see below).

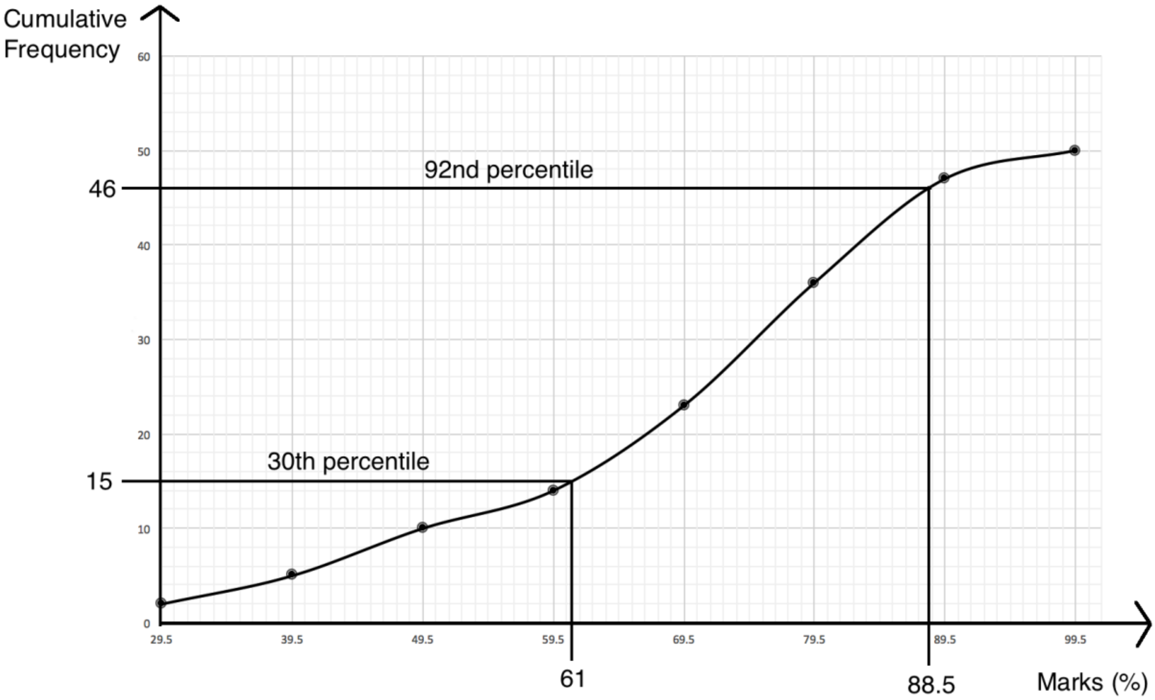
   Answer: The 30th percentile is 61, which is the pass mark.
6. Discuss part d.: How do you think we will solve this? (Answer: If 8% were awarded distinction, then 92% were not. Thus, 92% of pupils had scores under the mark for distinction, and we should find the 92nd percentile.)
7. Ask pupils to work with seatmates to solve part d.
8. Invite a volunteer to write the solution on the board.

   Solution:

   Step 1. Find the position of the 92nd percentile: $\frac{n}{100}\sum f = \frac{92}{100}(50) = \frac{4{,}600}{100} = 46$

   Step 2. Find the percentile on the c.f. curve (see below).

   Answer: The 92nd percentile is 88.5. This gives the lowest mark for distinction.



9. Ask pupils to find the answer to part e. with seatmates.
10. Ask a volunteer to share the answer with the class and explain. (Answer: 4 pupils were awarded distinction. The position of the 92nd percentile is 46, which means there are 4 pupils above distinction ($50 - 46 = 4$).)

Practice *(19 minutes)*

1. Write the following problem on the board: The table below gives the age distribution of all of the people in the village who were under 35 years old on 1 January 2017, when a survey was conducted.

Age (years)	0-4	5-9	10-14	15-19	20-24	25-29	30-34
Frequency	8	6	7	6	5	5	3

   a. Construct a cumulative frequency table.
   b. Use the table to draw the cumulative frequency curve.
   c. Use the curve to estimate the 70th percentile.
   d. If the youngest 40% of the village is eligible for an education program, estimate how many people are eligible.
   e. Estimate the age of the oldest villager eligible for the program.
2. Ask pupils to work with independently or with seatmates to complete the problem.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain:
   a. Cumulative frequency table:

Villagers' ages			
Age (years)	**Frequency**	**Upper Class Boundary**	**Cumulative Frequency**
0 – 4	8	4.5	8
5 – 9	6	9.5	8+6=14
10 – 14	7	14.5	14+7=21
15 – 19	6	19.5	21+6=27
20 – 24	5	24.5	27+5=32
25 – 29	5	29.5	32+5=37
30 – 34	3	34.5	37+3=40
Total	40		

b. Cumulative frequency curve: See below

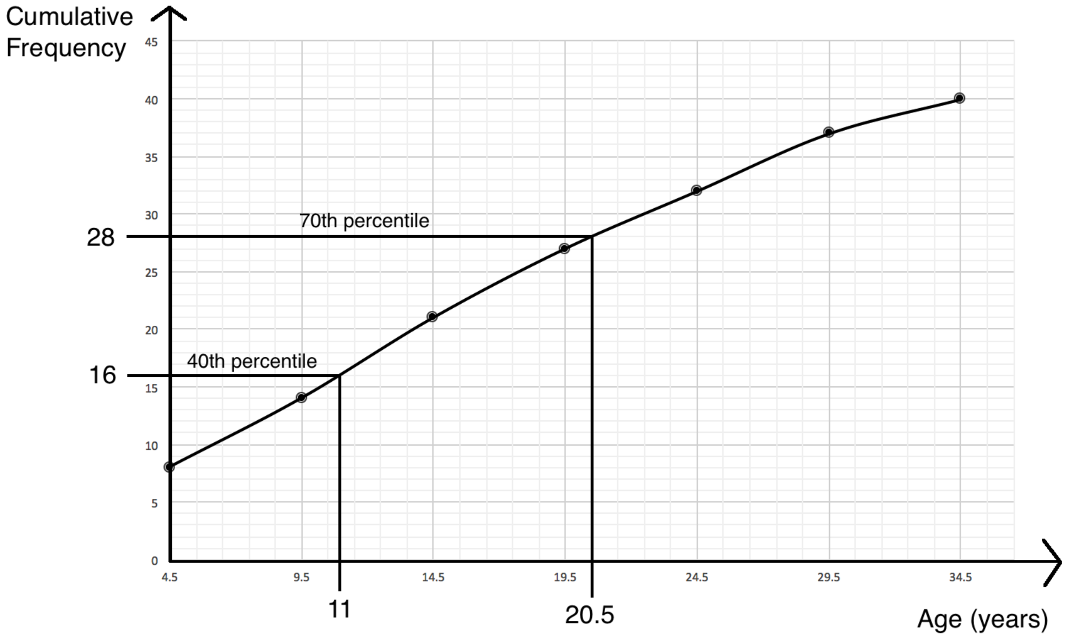
c. Find the position of the 70th percentile: $\frac{n}{100}\sum f = \frac{70}{100}(40) = \frac{2,800}{100} = 28$

Estimate the 70th percentile using the curve: 20.5 years old (see below)

d. Find the position of the 40th percentile: $\frac{n}{100}\sum f = \frac{40}{100}(40) = \frac{1,600}{100} = 16$

An estimated 16 villagers qualify for the program.

e. Using the curve, the 40th percentile is 11 years. Thus, the oldest villager eligible for the program is 11 years old.



Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L111 in the Pupil Handbook.

Lesson Title: Measures of dispersion	**Theme:** Statistics and Probability	
Lesson Number: M3-L112	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Describe and interpret the dispersion or spread of values in a data set. 2. Calculate the range and variance of a set of ungrouped values.	**Preparation** None	

Opening *(2 minutes)*

1. Discuss and allow pupils to share their ideas:
    - What is the range of a data set? (Answer: The difference between the greatest and least values.)
    - How can we calculate the range of a data set? (Answer: Subtract the least value in the data set from the greatest value.)
2. Explain that this lesson is on dispersion. Dispersion is related to how spread out the data is.

Teaching and Learning *(20 minutes)*

1. Explain:
    - Range is a **measure of dispersion**. The dispersion of a set of data tells us how spread out the data is.
    - If a measure of dispersion has greater value, the data is more spread out. In other words, it has greater variation.
2. Write a problem on the board: The ages of 15 university pupils are 18, 18, 18, 19, 19, 19, 19, 20, 20, 20, 21, 21, 22, 22, 24. Calculate the range and mean of the data.
3. Ask pupils to work with seatmates to calculate the range and mean.
4. Walk around to check for understanding and clear misconceptions.
5. Invite volunteers to write the solutions on the board and explain.
   Solutions:
   Range = greatest value – least value $= 24 - 18 = 6$ years
   Mean $= \frac{\text{sum of ages}}{\text{number of pupils}} = \frac{3(18)+4(19)+3(20)+2(21)+2(22)+24}{15} = \frac{300}{15} = 20$ years
6. Discuss: How can you interpret the range of this data? (Example answer: It tells us that the pupils' ages are spread out by 6 years. The oldest pupil is 6 years older than the youngest pupil.)
7. Explain **deviation:**
    - Deviation is another measure of dispersion.
    - If the mean of a distribution is subtracted from any value in the distribution, the result is called the deviation of the value from the mean.
8. Write on the board:

    a. Calculate the deviation of an 18-year-old pupil.
    b. Calculate the deviation of a 20-year-old pupil.
    c. Calculate the deviation of a 24-year-old pupil.

9. Solve the problems on the board and explain:
    a. Deviation $= 18 - \text{mean} = 18 - 20 = -2$
    b. Deviation $= 20 - \text{mean} = 20 - 20 = 0$
    c. Deviation $= 24 - \text{mean} = 24 - 20 = +4$
10. Explain:
    - Deviation can be positive or negative.
    - For any distribution, the sum of all of the deviations from the mean is always zero. In other words, if we calculated the deviation for all 15 pupils in the data set, their sum would be 0.
11. Ask pupils to work with seatmates to calculate the deviation for each of the 15 values on the board, and write them in a list.
12. Invite volunteers to write the list on the board. (Answer: $-2, -2, -2, -1, -1, -1, -1, 0, 0, 0, 1, 1, 2, 2, 4$)
13. Explain **variance:**
    - Variance is another measure of dispersion.
    - Variance is calculated using the deviations of the data.
    - The deviations of the data tell you information about each piece of data, and this information can be used to obtain a single number which indicates the overall dispersion of the data.
    - To calculate variance, find the sum of the square of each deviation from the mean. Divide by the frequency (in this case, 15).
14. Calculate the variance of the set of data on the board:

$$\begin{aligned}\text{ariance} &= \frac{(-2)^2+(-2)^2+(-2)^2+(-1)^2+(-1)^2+(-1)^2+(-1)^2+(0)^2+(0)^2+(0)^2+(1)^2+(1)^2+(2)^2+(2)^2+(4)^2}{15} \\ &= \frac{4+4+4+1+1+1+1+0+0+0+1+1+4+4+16}{15} \\ &= \frac{42}{15} \\ &= 2.8\end{aligned}$$

15. Explain:
    - The value 2.8 by itself doesn’t tell us a lot of information.
    - If we had another group of pupils, we could find the variation of that data set too.
    - A larger variation means that a data set is more spread out. For example, if we found that another group of pupils had a variance of 3, then their ages would be more spread out then the ages of the pupils in our data set.
16. Explain dispersion:
    - Range and variance are 2 important measures of dispersion, but there are others as well.
    - Interquartile range and semi-interquartile range are other measures of dispersion that you have calculated before. They are better measures of dispersion and range, because they are not affected by any extremely low

or high values. Remember that they are based on where the middle half of the data is.

Practice *(16 minutes)*

1. Write the following problem on the board: Bentu runs an education centre that children attend while their parents are working. The ages of the children that attend are: 1, 1, 2, 2, 2, 3, 4, 4, 5, 5, 5, 6, 6, 7, 7. Calculate:
    a. The range in ages of the children.
    b. The mean age of the children.
    c. The variance in the children's ages.
2. Ask pupils to work with independently or with seatmates to complete the problem.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain:
    a. Range $= 7 - 1 = 6$ years
    b. Mean $= \frac{\text{sum of ages}}{\text{number of pupils}} = \frac{2(1)+3(2)+3+2(4)+3(5)+2(6)+2(7)}{15} = \frac{60}{15} = 4$ years
    c. **Step 1.** Calculate the deviation of each value from the mean:

$$-3, -3, -2, -2, -2, -1, 0, 0, 1, 1, 1, 2, 2, 3, 3$$

Step 2. Calculate the variance:

$$\begin{aligned}\text{Variance} &= \frac{(-3)^2+(-3)^2+(-2)^2+(-2)^2+(-2)^2+(-1)^2+(0)^2+(0)^2+(1)^2+(1)^2+(1)^2+(2)^2+(2)^2+(3)^2+(3)^2}{15} \\ &= \frac{9+9+4+4+4+1+0+1+1+1+4+4+9+9}{15} \\ &= \frac{60}{15} \\ &= 4\end{aligned}$$

Closing *(2 minutes)*

1. Discuss: Which set of data is more spread out, the data of university pupils or young children? (Answer: The ranges are the same; however, the set of young children have a greater variance, so they are generally more spread out in age.)
2. For homework, have pupils do the practice activity of PHM-L112 in the Pupil Handbook.

Lesson Title: Standard deviation of ungrouped data	**Theme:** Statistics and Probability	
Lesson Number: M3-L113	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the standard deviation of a set of ungrouped values.	**Preparation** Bring a calculator to class, or a phone with a calculator. Ask pupils to bring calculators if they have access to them.	

Opening *(2 minutes)*

1. Discuss:
    - What is the meaning of "dispersion"? (Answer: Dispersion is related to how spread out a set of data is.)
    - What are some measures of dispersion that you know? (Example answers: range, interquartile range, semi-interquartile range, variance.)
2. Explain that this lesson is on standard deviation. This is another measure of dispersion that is very common and useful.

Teaching and Learning *(25 minutes)*

1. Explain: Standard deviation is derived from variance. It is the square root of the variance of a set of data.
2. Write on the board: $\text{standard deviation} = \sqrt{\text{variance}}$
3. Write the data set from the previous lesson on the board as a standard deviation problem: The ages of 15 university pupils are 18, 18, 18, 19, 19, 19, 19, 20, 20, 20, 21, 21, 22, 22, 24. Calculate the standard deviation of the data.
4. Ask pupils to give the variance of the data that was calculated in the previous lesson. (Answer: 2.8)
5. Write on the board: $\text{standard deviation} = \sqrt{\text{variance}} = \sqrt{2.8} = 1.67$
6. Show pupils how to use a calculator to calculate square root if needed. You may allow them to use calculators on their mobile phones if they do not have other calculators.
7. Explain:
    - We can calculate standard deviation with a table.
    - Recall that in the previous lesson, we calculated variance using multiple steps, including finding the deviation of each value.
    - Using a table will keep the steps organised.
8. Write a problem on the board: The ages of 7 children living in one house are 3, 5, 8, 12, 8, 4, and 9. Find the standard deviation of their ages.
9. Ask pupils to work with seatmates to find the mean of the values. Explain that the mean will be used in the calculation of standard deviation.
10. Invite a volunteer to write the solution on the board. (Answer: mean $= \frac{3+5+8+12+8+4+9}{7} = \frac{49}{7} = 7$ years old)

11. Calculate standard deviation on the board a table, following the steps below. Note the meaning of each variable:
    - x represents the numbers in the given data set
    - m represents the mean
    - d represents the deviation from the mean

    Step 1. Write the values from the problem in the x column.

    Step 2. Calculate the deviation of each value from the mean and fill the second column.

    Step 3. Find the square of each deviation and fill the third column.

    Step 4. Find the total in the column for d^2. Note that the variance is this value divided by the number of values in the set (7).

    Step 5. Calculate standard deviation:

x	$x - m = d$	d^2
3	$3 - 7 = -4$	16
5	$5 - 7 = -2$	4
8	$8 - 7 = +1$	1
12	$12 - 7 = +5$	25
8	$8 - 7 = +1$	1
4	$4 - 7 = -3$	9
9	$9 - 7 = +2$	4
	Total	60

$$\text{standard deviation} = \sqrt{\text{variance}} = \sqrt{\frac{60}{7}} \approx 2.93$$

12. Write on the board: $s = \sqrt{\frac{1}{n}\sum(x - \bar{x})^2}$, where n is the frequency, x is each value in the set, and $\bar{x}$ is the mean.
13. Explain the **formula:**
    - This is the general formula for standard deviation. It gives the same method that we have just used, but the notation is different.
    - Mean is sometimes given by the letter x with a bar over it.
    - Note that $x - \bar{x}$ is the deviation of each value. Thus, $\frac{1}{n}\sum(x - \bar{x})^2$ is the variance. The square root of the variance gives us this general form for standard deviation.
14. Explain how to **interpret standard deviation:**
    - Standard deviation is a measure of dispersion that tells us how close data points generally are to the mean.
    - A low standard deviation indicates that points are generally close to the mean (the deviation is low).
    - A high standard deviation indicates that points are spread out over a wide range of values, farther from the mean (the deviation is high).
15. Write the following problem on the board: 10 pupils achieved the following scores on a Maths exam: 80, 82, 88, 89, 84, 79, 81, 82, 85, 80. Calculate:
    a. The mean of the distribution.
    b. The standard deviation of the distribution.
16. Ask pupils to work with seatmates to complete part a.
17. Invite a volunteer to write the solution on the board. (Answer: mean $= \frac{80+82+88+89+84+79+81+82+85+80}{10} = \frac{830}{10} = 83$)
18. Write the table on the board with the first column filled (see below).

19. Ask pupils to work with seatmates to fill the other 2 columns, $x - \bar{x}$ and $(x - \bar{x})^2$, and find the total of the $(x - \bar{x})^2$ column. Remind them that $x - \bar{x}$ is the same as deviance (d).
20. Invite volunteers to fill the columns on the board.
    Answers:

$\boldsymbol{x}$	$\boldsymbol{x - \bar{x}}$	$\boldsymbol{(x - \bar{x})^2}$
80	$80 - 83 = -3$	9
82	$82 - 83 = -1$	1
88	$88 - 83 = +5$	25
89	$89 - 83 = +6$	36
84	$84 - 83 = +1$	1
79	$79 - 83 = -4$	16
81	$81 - 83 = -2$	4
82	$82 - 83 = -1$	1
85	$85 - 83 = +2$	4
80	$80 - 83 = -3$	9
Total	$= \sum(x - \bar{x})^2 =$	106

21. Ask pupils to explain in their own words how to calculate standard deviation from this table. Make sure everyone understands. (Example answer: Divide the total of the $(x - \bar{x})^2$ column by the number of pupils, 10. Find the square root of the result.)
22. Ask pupils to work with seatmates to calculate the standard deviation.
23. Invite a volunteer to write the solution on the board.

$$s = \sqrt{\frac{1}{n}\sum(x - \bar{x})^2} = \sqrt{\frac{106}{10}} \approx 3.26$$

Practice *(12 minutes)*

1. Write the following problem on the board: Six boys weighed themselves and found their weights in kilogrammes to be 43, 48, 45, 50, 47, and 43. Calculate:
    a. The range of their weights.
    b. The mean of their weights.
    c. The standard deviation of their weights.
2. Ask pupils to work with independently or with seatmates to complete the problem.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain:
    a. Range = $50 - 43 = 7$ kg
    b. Mean = $\frac{43+48+45+50+47+43}{6} = \frac{276}{6} = 46$ kg
    c. Standard deviation:

x	$x-\bar{x}$	$(x-\bar{x})^2$
43	$43-46=-3$	9
48	$48-46=+2$	4
45	$45-46=-1$	1
50	$50-46=+4$	16
47	$47-46=+1$	1
43	$43-46=-3$	9
Total $=\sum(x-\bar{x})^2=$		40

$$\sqrt{\frac{1}{n}\sum(x-\bar{x})^2}=\sqrt{\frac{40}{6}}\approx 2.58$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L113 in the Pupil Handbook.

Lesson Title: Standard deviation of grouped data – Part 1	**Theme:** Statistics and Probability	
Lesson Number: M3-L114	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the standard deviation of a set of grouped values *without* class intervals.	**Preparation** Bring a calculator to class, or a phone with a calculator. Ask pupils to bring calculators if they have access to them.	

Opening *(2 minutes)*

1. Discuss and allow pupils to share ideas: What is standard deviation? (Answer: Standard deviation is a measure of dispersion that tells us how spread out a set of data is, by telling us how far from the mean the values generally are.)
2. Explain that this lesson is on calculating the standard deviation of grouped data without class intervals.

Teaching and Learning *(18 minutes)*

1. Write the following problem on the board: The ages of 20 children are given in the table below. Calculate the mean and standard deviation of their ages.

Age (years)						
Frequency (f)						

2. Explain: This is grouped data. Each age in the table appears more than once in the data.
3. Revise calculating the mean of grouped data. Discuss: How can we calculate the mean of grouped data? (Answer: Find the sum of the products of each age and frequency, then divide by the total frequency.)
4. Calculate mean on the board. Involve pupils by asking them to describe the steps:

$$\begin{aligned} \bar{x} &= \frac{\Sigma fx}{\Sigma f} \\ &= \frac{3(1)+4(2)+2(3)+3(4)+6(5)+2(6)}{20} \\ &= \frac{3+8+6+12+30+12}{20} \\ &= \frac{71}{20} = 3.55 \text{ years old} \end{aligned}$$

5. Explain:
   - To calculate standard deviation, we will not use the same formula and table that we used in the previous lesson. If we used that table, it would have 20 rows.
   - We will use a different formula, and a table with a row for each of the 6 ages in the data set.
6. Write on the board: Standard deviation of grouped data: $s = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$, where f is frequency, x is each data point, and $\bar{x}$ is the mean.
7. Draw the empty table on the board:

x	f	fx	fx^2
1			
2			
3			
4			
5			
6			
Totals	$\sum f =$	$\sum fx =$	$\sum fx^2 =$

8. Explain: Each part of the formula is in this table except for the mean. We have already calculated the mean.
9. Invite volunteers to fill the second column of the table, frequency. These values are given in the problem. (Answers: see table below)
10. Invite volunteers to fill the third column of the table. This can be done by multiplying the first and second columns to calculate each fx value. (Answers: see table below)
11. Explain: To find the values in the last column, we simply multiply the first and third columns.
12. Write on the board and make sure this is clear: $fx^2 = x(fx)$
13. Ask volunteers to calculate the values of fx^2 and call them out. Write them in the table as they call them out.
14. Ask pupils to work with seatmates to find the total of each column.
15. Invite volunteers to write the answers on the board.

    Answer:

x	f	fx	fx^2
1	3	$1 \times 3 = 3$	$1 \times 3 = 3$
2	4	$2 \times 4 = 8$	$2 \times 8 = 16$
3	2	$3 \times 2 = 6$	$3 \times 6 = 18$
4	3	$4 \times 3 = 12$	$4 \times 12 = 48$
5	6	$5 \times 6 = 30$	$5 \times 30 = 150$
6	2	$6 \times 2 = 12$	$6 \times 12 = 72$
Totals	$\sum f = 20$	$\sum fx = 71$	$\sum fx^2 = 307$

16. Calculate the standard deviation on the board, explaining each step:

$$\begin{aligned} s &= \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} \\ &= \sqrt{\frac{307}{20} - (3.55)^2} \\ &= \sqrt{15.35 - 12.60} \\ &= \sqrt{2.75} \\ &\approx 1.66 \end{aligned}$$

Practice *(19 minutes)*

1. Write the following problem on the board: The table below shows the amount of cassava sold in one week by 25 saleswomen. Use the table to calculate the mean and standard deviation.

Sales (kg)	1	2	3	4	5	6
Frequency (f)	3	5	4	6	3	4

2. Ask pupils to work with independently or with seatmates to complete the problem.
3. Encourage pupils to complete the table before solving for both mean and standard deviation.
4. Walk around to check for understanding and clear misconceptions. If needed, give the table on the board and fill a few rows as a class.
5. Invite volunteers to write the solutions on the board and explain:

Table:

x	f	fx	fx^2
1	3	$1 \times 3 = 3$	$1 \times 3 = 3$
2	5	$2 \times 5 = 10$	$2 \times 10 = 20$
3	4	$3 \times 4 = 12$	$3 \times 12 = 36$
4	6	$4 \times 6 = 24$	$4 \times 24 = 96$
5	3	$5 \times 3 = 15$	$5 \times 15 = 75$
6	4	$6 \times 4 = 24$	$6 \times 24 = 144$
Totals	$\sum f = 25$	$\sum fx = 88$	$\sum fx^2 = 374$

Mean:

$$\begin{aligned} \bar{x} &= \frac{\sum fx}{\sum f} \\ &= \frac{88}{25} \\ &= 3.52 \text{ kg} \end{aligned}$$

Standard deviation:

$$\begin{aligned} s &= \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} \\ &= \sqrt{\frac{374}{25} - (3.52)^2} \\ &= \sqrt{14.96 - 12.39} \\ &= \sqrt{2.57} \\ &\approx 1.60 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L114 in the Pupil Handbook.

<table>
<tr><td>Lesson Title: Standard deviation of grouped data – Part 2</td><td colspan="2">Theme: Statistics and Probability</td></tr>
<tr><td>Lesson Number: M3-L115</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to calculate the standard deviation of a set of grouped values <i>with</i> class intervals.</td><td colspan="2">Preparation
1. Bring a calculator to class, or a phone with a calculator. Ask pupils to bring calculators if they have access to them.
2. Write the table in Opening on the board.</td></tr>
</table>

Opening *(2 minutes)*

1. Write on the board: The weights of 20 children are given in the table:

Weight (kg)	10-14	15-19	20-24	25-29	30-34	35-39
Frequency (f)						

2. Discuss: What do you notice about this table? How is it different from the tables we used in the previous lesson? (Answer: There are class intervals (or ranges) given for the weights.)
3. Explain that this lesson is on calculating the standard deviation of grouped data with class intervals.

Teaching and Learning *(18 minutes)*

1. Discuss:
   - What are class intervals? (Answer: Class intervals are equal intervals that divide a set of data into groups.)
   - How can we find the mid-point of a class interval? (Answer: For small intervals like the example, we can count. For larger intervals, we can calculate the mean of the 2 endpoints to find the middle.)
2. Explain: To calculate the standard deviation of grouped data with class intervals, we will use the same formula from the previous class.
3. Write the formula on the board: $s = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$, where f is frequency, x is each data point, and $\bar{x}$ is the mean.
4. Explain:
   - We do not have individual data points to substitute for x, but class intervals.
   - We will take the mid-point of each class interval and use it for x.
5. Draw the empty table on the board:

Interval	**Mid-point (x)**	f	fx	fx^2
10-14				
15-19				
20-24				

25-29				
30-34				
35-39				
Totals		$\sum f =$	$\sum fx =$	$\sum fx^2 =$

6. Ask volunteers to give the values for the 2nd, 3rd, and 4th columns. Write them on the board as they give them (see complete table below):
    - The 2nd column is the class mid-points, which can be found by counting.
    - The 3rd column is frequencies, which are given in the problem.
    - The 4th column is found by multiplying the 2nd and 3rd columns.
7. Ask volunteers to calculate the values of fx^2 for the last column and call them out. Write them in the table as they call them out.
8. Ask pupils to work with seatmates to find the totals for the last 3 columns.
9. Invite volunteers to write the totals in the table on the board.

    Answer:

Interval	**Mid-point (x)**	$\boldsymbol{f}$	$\boldsymbol{fx}$	$\boldsymbol{fx^2}$
10-14	12	3	36	432
15-19	17	2	34	578
20-24	22	4	88	1,936
25-29	27	5	135	3,645
30-34	32	3	96	3,072
35-39	37	3	111	4,107
Totals		$\sum f = 20$	$\sum fx = 500$	$\sum fx^2 = 13{,}770$

10. Ask pupils to calculate the mean with seatmates.
11. Invite a volunteer to write the solution on the board.

    Solution:

$$\begin{aligned} \bar{x} &= \frac{\sum fx}{\sum f} \\ &= \frac{500}{20} \\ &= 25 \text{ kg.} \end{aligned}$$

12. Ask volunteers to explain how to calculate the standard deviation. As they give the steps, write the solution on the board:

$$\begin{aligned} s &= \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} \\ &= \sqrt{\frac{13770}{20} - 25^2} \\ &= \sqrt{688.5 - 625} \\ &= \sqrt{63.5} \\ &\approx 7.97 \end{aligned}$$

Practice *(19 minutes)*

1. Write the following problem on the board: The table below shows the distribution of marks scored by pupils in an examination. Calculate, correct to 2 decimal places, the:
   a. Mean
   b. Standard deviation

Marks	60-64	65-69	70-74	75-79	80-84	85-89
Frequency (f)	1	2	6	7	3	1

2. Ask pupils to work with independently or with seatmates to complete the problem.
3. Walk around to check for understanding and clear misconceptions. If needed, give the table on the board and fill a few rows as a class.
4. Invite volunteers to write the solutions on the board and explain:

Table:

Interval	**Mid-point (x)**	f	fx	fx^2
60-64	62	1	62	3,844
65-69	67	2	134	8,978
70-74	72	6	432	31,104
75-79	77	7	539	41,503
80-84	82	3	246	20,172
85-89	87	1	87	7,569
Totals	$\sum f = 20$	$\sum fx = 1{,}500$	$\sum fx^2 = 113{,}170$	

Mean:

$$\begin{aligned} \bar{x} &= \frac{\sum fx}{\sum f} \\ &= \frac{1{,}500}{20} \\ &= 75 \text{ marks} \end{aligned}$$

Standard deviation:

$$\begin{aligned} s &= \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} \\ &= \sqrt{\frac{113{,}170}{20} - 75^2} \\ &= \sqrt{5658.5 - 5625} \\ &= \sqrt{33.5} \\ &\approx 5.79 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L115 in the Pupil Handbook.

Lesson Title: Standard deviation practice	**Theme:** Statistics and Probability	
Lesson Number: M3-L116	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve standard deviation problems using the appropriate formulae.	**Preparation** Bring a calculator to class, or a phone with a calculator. Ask pupils to bring calculators if they have access to them.	

Opening *(2 minutes)*

1. Discuss and allow pupils to share their ideas:
   - What types of standard deviation problems have you learned to solve? (Example answer: Those with ungrouped data; those with grouped data, including with and without class intervals.)
   - How would you decide which standard deviation formula to use for a given problem? (Answer: There is 1 formula for ungrouped data, and another formula for grouped data.)
2. Explain that during this lesson, pupils will practice solving standard deviation problems.

Teaching and Learning *(19 minutes)*

1. Ask volunteers to write the 2 formulae for standard deviation on the board:

   Ungrouped data: $s = \sqrt{\frac{1}{n}\sum(x-\bar{x})^2}$

   Grouped data: $s = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$
2. Ask volunteers to give the meaning of each variable. (Answers: n is the total frequency of ungrouped data; x is each piece of data; $\bar{x}$ is mean; f is frequency of grouped data.)
3. Write the following problems on the board:
   - In a secondary school, the number of absentees recorded over a period of 30 days is shown in the frequency distribution table. Calculate the: i. mean; ii. Standard deviation.

Number of absentees	0-4	5-9	10-14	15-19	20-24
Frequency (f)	1	3	12	11	3

   - The goals scored by 8 football players in a season are 5, 1, 4, 7, 3, 10, 8, and 2. Calculate the mean and standard deviation of the data to 2 decimal places.
4. Ask pupils to work with seatmates to solve the problems using the appropriate formulae and tables.

5. Walk around to check for understanding and clear misconceptions. If needed, write the empty tables on the board and remind pupils of how to fill them.
6. Invite volunteers to write the solutions on the board.

Solutions:

- Table:

Interval	**Mid-point (x)**	f	fx	fx^2
0-4	2	1	2	4
5-9	7	3	21	147
10-14	12	12	144	1,728
15-19	17	11	187	3,179
20-24	22	3	66	1,452
	Totals	$\sum f = 30$	$\sum fx = 420$	$\sum fx^2 = 6{,}510$

Mean:

$$\begin{aligned} \bar{x} &= \frac{\sum fx}{\sum f} \\ &= \frac{420}{30} \\ &= 14 \text{ absentees} \end{aligned}$$

Standard deviation:

$$\begin{aligned} s &= \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} \\ &= \sqrt{\frac{6{,}510}{30} - 14^2} \\ &= \sqrt{217 - 196} \\ &= \sqrt{21} \\ &\approx 4.58 \end{aligned}$$

- Mean: $\bar{x} = \frac{5+1+4+7+3+10+8+2}{8} = \frac{40}{8} = 5$ goals

Table:

x	$x - \bar{x}$	$(x - \bar{x})^2$
5	$5 - 5 = 0$	0
1	$1 - 5 = -4$	16
4	$4 - 5 = -1$	1
7	$7 - 5 = +2$	4
3	$3 - 5 = -2$	4
10	$10 - 5 = 5$	25
8	$8 - 5 = 3$	9
2	$2 - 5 = -3$	9
Total	$= \sum(x - \bar{x})^2 =$	68

Standard deviation: $s = \sqrt{\frac{1}{n}\sum(x - \bar{x})^2} = \sqrt{\frac{68}{8}} \approx 2.92$

Practice *(18 minutes)*

1. Write the following problems on the board:
   a. If the ages of 6 children are 6, 8, 9, 10, 13, and 14, calculate: i. Range; ii. Mean; iii. Standard deviation.
   b. The table below shows the age distribution of the members of a club. Calculate, correct to 2 decimal places, the mean and standard deviation.

Age (years)	15-19	20-24	25-29	30-34	35-39
Frequency (f)	1	5	6	3	1

2. Ask pupils to work with independently to complete the problems.
3. Walk around to check for understanding and clear misconceptions. Support them as needed.
4. Invite volunteers to write the solutions on the board and explain:

a. i. Range: $14 - 6 = 8$ years

ii. Mean: $\bar{x} = \frac{6+8+9+10+13+14}{6} = \frac{60}{6} = 10$ years old

iii. Standard deviation (see table below): $s = \sqrt{\frac{1}{n}\sum(x-\bar{x})^2} = \sqrt{\frac{46}{6}} \approx 2.77$

x	$x - \bar{x}$	$(x - \bar{x})^2$
6	$6 - 10 = -4$	16
8	$8 - 10 = -2$	4
9	$9 - 10 = -1$	1
10	$10 - 10 = 0$	0
13	$13 - 10 = +3$	9
14	$14 - 10 = +4$	16
Total	$= \sum(x - \bar{x})^2 =$	46

b. Table:

Interval	Mid-point (x)	f	fx	fx^2
15-19	17	1	17	289
20-24	22	5	110	2,420
25-29	27	6	162	4,374
30-34	32	3	96	3,072
35-39	37	1	37	1,369
	Totals	$\sum f = 16$	$\sum fx = 422$	$\sum fx^2 = 11{,}524$

Mean:

$$\begin{aligned}\bar{x} &= \frac{\sum fx}{\sum f} \\ &= \frac{422}{16} \\ &= 26.38 \text{ years old}\end{aligned}$$

Standard deviation:

$$\begin{aligned} s &= \sqrt{\frac{\sum f x^2}{\sum f} - \bar{x}^2} \\ &= \sqrt{\frac{11{,}524}{16} - 26.38^2} \\ &= \sqrt{720.25 - 695.90} \\ &= \sqrt{24.35} \\ &\approx 4\,.93 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L116 in the Pupil Handbook.

Lesson Title: Mean deviation of ungrouped data	**Theme:** Statistics and Probability	
Lesson Number: M3-L117	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the mean deviation of ungrouped data.	**Preparation** Bring a calculator to class, or a phone with a calculator. Ask pupils to bring calculators if they have access to them.	

Opening *(2 minutes)*

1. Discuss: What are some measures of dispersion that you know? (Example answers: range, interquartile range, semi-interquartile range, variance, standard deviation.)
2. Explain that this lesson is on mean deviation of ungrouped data. This is another measure of dispersion.

Teaching and Learning *(22 minutes)*

1. Explain:
   - Mean deviation is similar to standard deviation.
   - Mean and standard deviation measures are positive.
   - When we calculate standard deviation, we square the deviations, which makes them positive.
   - For mean deviation, we use absolute value to make the deviations positive.
   - Mean deviation is another measure of the spread (dispersion) of data.
2. Write on the board: Mean deviation $= MD = \frac{\sum|x-\bar{x}|}{n}$, where x is each piece of data, $\bar{x}$ is the mean, and n is the total frequency.
3. Write the following problem on the board: The goals scored by 10 football players in a season are 3, 1, 4, 7, 3, 8, 12, 15, 8 and 9. Calculate the mean and mean deviation of the data to 2 decimal places.
4. Ask pupils to work with seatmates to calculate the mean of the data.
5. Invite a volunteer to write the solution on the board. (Answer: $\bar{x} = \frac{3+1+4+7+3+8+12+15+8+9}{10} = \frac{70}{10} = 7$ goals)
6. Draw the empty table on the board:

$\boldsymbol{x}$	$\boldsymbol{x-\bar{x}}$	$\boldsymbol{\lvert x-\bar{x}\rvert}$
3		
1		
4		
7		
3		
8		
12		

15		
8		
9		
Total $= \sum \lvert \boldsymbol{x} - \bar{\boldsymbol{x}} \rvert =$		

7. Explain:
    a. This is the table that we need to fill in order to find the standard deviation. It has a column for each term in the formula.
    b. This is similar to the table for calculating standard deviation of ungrouped data. The difference is that the last column is $|x - \bar{x}|$ instead of $(x - \bar{x})^2$.
8. Invite volunteers to come to the board and fill the second column (see below).
9. Invite volunteers to come to the board to fill the third column (see below).
10. Ask pupils to work with seatmates to find the sum, $\sum |\boldsymbol{x} - \bar{\boldsymbol{x}}|$.
11. Invite a volunteer to write the sum in the table.

Answer:

$\boldsymbol{x}$	$\boldsymbol{x} - \bar{\boldsymbol{x}}$	$\lvert \boldsymbol{x} - \bar{\boldsymbol{x}} \rvert$
3	$3 - 7 = -4$	4
1	$1 - 7 = -6$	7
4	$4 - 7 = -3$	3
7	$7 - 7 = 0$	0
3	$3 - 7 = -4$	4
8	$8 - 7 = +1$	1
12	$12 - 7 = +5$	5
15	$15 - 7 = +8$	8
8	$8 - 8 = +1$	1
9	$9 - 7 = +2$	2
Total $= \sum \lvert \boldsymbol{x} - \bar{\boldsymbol{x}} \rvert =$		35

12. Calculate the mean deviation on the board, explaining each step:

$$\begin{aligned} MD &= \frac{\sum|x-\bar{x}|}{n} \\ &= \frac{35}{10} \\ &= 3.5 \end{aligned}$$

13. Write the following problem on the board: The heights of 8 pupils in centimetres are 152, 150, 157, 155, 159, 151, 150 and 158. Calculate the mean and mean deviation of their heights.
14. Ask pupils to work with seatmates to solve the problem.
15. Walk around to check for understanding and clear misconceptions.
16. Invite volunteers to write the solution on the board.

Solution:

Mean: $\bar{x} = \frac{152+150+157+155+159+151+150+158}{8} = \frac{1232}{8} = 154 \text{ cm}$

Table:

x	$x - \bar{x}$	$\lvert x - \bar{x} \rvert$
152	$152 - 154 = -2$	2
150	$150 - 154 = -4$	4
157	$157 - 154 = +3$	3
155	$155 - 154 = +1$	1
159	$159 - 154 = +5$	5
151	$151 - 154 = -3$	3
150	$150 - 154 = -4$	4
158	$158 - 154 = +4$	4
	Total $= \sum \lvert x - \bar{x} \rvert =$	26

Mean deviation: $\frac{\sum |x-\bar{x}|}{n} = \frac{26}{8} = 3.25$

Practice *(15 minutes)*

1. Write the following problem on the board: If the ages of 10 children are 2, 3, 8, 4, 9, 10, 12, 7, 6 and 9, calculate: i. Range; ii. Mean; iii. Mean deviation.
2. Ask pupils to work with independently to complete the problem.
3. Walk around to check for understanding and clear misconceptions. Support them as needed.
4. Invite volunteers to write the solutions on the board and explain:
    a. i. Range: $12 - 2 = 10$ years
    ii. Mean: $\bar{x} = \frac{2+3+8+4+9+10+12+7+6+9}{10} = \frac{70}{10} = 7$ years old
    iii. Mean deviation (see table below): $\text{MD} = \frac{\sum |x-\bar{x}|}{n} = \frac{26}{10} = 2.6$

x	$x - \bar{x}$	$\lvert x - \bar{x} \rvert$
2	$2 - 7 = -5$	5
3	$3 - 7 = -4$	4
8	$8 - 7 = +1$	1
4	$4 - 7 = -3$	3
9	$9 - 7 = +2$	2
10	$10 - 7 = +3$	3
12	$12 - 7 = +5$	5
7	$7 - 7 = 0$	0
6	$6 - 7 = -1$	1
9	$9 - 7 = +2$	2
	Total $= \sum \lvert x - \bar{x} \rvert =$	26

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L117 in the Pupil Handbook.

Lesson Title: Mean deviation of grouped data – Part 1	**Theme:** Statistics and Probability	
Lesson Number: M3-L118	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the mean deviation of grouped data **without** class intervals.	**Preparation** Bring a calculator to class, or a phone with a calculator. Ask pupils to bring calculators if they have access to them.	

Opening *(2 minutes)*

1. Discuss: What is mean deviation? (Example answers: Mean deviation is another measure of dispersion that tells us how spread out a set of data is, by telling us how far from the mean the values generally are. It is similar to standard deviation but uses absolute value.)
2. Explain that this lesson is on mean deviation of grouped data without class intervals.

Teaching and Learning *(18 minutes)*

1. Write the following problem on the board: The ages of young children in one community are given in the table below. Calculate the mean and mean deviation of their ages.

Age (years)						
Frequency (f)						

2. Explain:
   - We use a different formula for calculating mean deviation of grouped data.
   - It is similar to the formula from the previous class, but includes frequency as well.
3. Write on the board: Mean deviation of grouped data: $MD = \frac{\sum f|x-\bar{x}|}{\sum f}$, where x is each piece of data, $\bar{x}$ is the mean, and f is frequency.
4. Invite a volunteer to write the formula for mean on the board. (Answer: $\bar{x} = \frac{\sum fx}{\sum f}$)
5. Discuss: We need a table to organise the solution of this problem. What columns will it have? (Answer: x, f, fx, $x - \bar{x}$, $|x - \bar{x}|$, $f|x - \bar{x}|$)
6. Draw the empty table on the board:

$\boldsymbol{x}$	$\boldsymbol{f}$		$\boldsymbol{x - \bar{x}}$	$\boldsymbol{\lvert x - \bar{x} \rvert}$	$\boldsymbol{f\lvert x - \bar{x} \rvert}$
1					
2					
3					
4					
5					
6					
Totals:	$\sum f =$	$\sum fx =$			$\sum f\lvert x - \bar{x}\rvert =$

7. Invite volunteers to come to the board and fill the second column, frequency. This is given in the problem (see answers below).
8. Invite volunteers to come to the board to fill the third column, fx (see answers below).
9. Ask pupils to work with seatmates to find the sums of the second and third columns. Invite volunteers to write them on the board.
10. Ask pupils to work with seatmates to calculate the mean.
11. Invite a volunteer to write the solution on the board: $\bar{x} = \frac{\sum fx}{\sum f} = \frac{90}{30} = 3$
12. Invite volunteers to come to the board and complete the table (see below):
    a. Fill the 4th row by subtracting the mean from each x.
    b. Fill the 5th row by finding the absolute values of the 4th row.
    c. Find the 6th row by multiplying the 2nd and 5th rows.
13. Ask pupils to work with seatmates to find the sum $\sum f|x - \bar{x}|$.
14. Invite a volunteer to write the sum in the table.

Answer:

$\boldsymbol{x}$	$\boldsymbol{f}$	$\boldsymbol{fx}$	$\boldsymbol{x - \bar{x}}$	$\boldsymbol{|x - \bar{x}|}$	$\boldsymbol{f|x - \bar{x}|}$
1	7	7	$1 - 3 = -2$	2	14
2	6	12	$2 - 3 = -1$	1	6
3	7	21	$3 - 3 = 0$	0	0
4	3	12	$4 - 3 = +1$	1	3
5	4	20	$5 - 3 = +2$	2	8
6	3	18	$6 - 3 = +3$	3	9
Totals:	$\sum f = 30$	$\sum fx = 90$			$\sum f|x - \bar{x}| = 40$

15. Calculate the mean deviation on the board, explaining each step:

$$\begin{aligned} MD &= \frac{\sum f|x-\bar{x}|}{\sum f} \\ &= \frac{40}{30} \\ &= 1.33 \end{aligned}$$

Practice *(19 minutes)*

1. Write the following problem on the board: A test worth 10 marks was given to 40 pupils. The results are in the table below. Calculate: a. Range; b. Mean; c. Mean deviation; Standard deviation.

Marks	1	2	3	4	5	6	7	8	9	10
Frequency (f)	1	2	4	3	5	7	8	6	1	3

2. Ask a pupil to write the formula for standard deviation of grouped data on the board. (Answer: $s = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$)

3. Ask volunteers to give the table columns that are needed to calculate all parts of the problem. (Answer: x, f, fx, $x-\bar{x}$, $|x-\bar{x}|$, $f|x-\bar{x}|$, x^2, fx^2)
4. Draw the empty table on the board (see table below).
5. Ask pupils to work with independently or with seatmates to complete the problem.
6. Walk around to check for understanding and clear misconceptions. Support them as needed.
7. Invite volunteers to write the solutions on the board and explain:
    a. Range: $10-1=9$ marks
    b. Mean: $\bar{x}=\frac{\sum fx}{\sum f}=\frac{240}{40}=6$ marks
    c. Mean deviation: $\text{MD}=\frac{\sum f|x-\bar{x}|}{\sum f}=\frac{72}{40}=1.8$
    d. Standard deviation: $s=\sqrt{\frac{\sum fx^2}{\sum f}-\bar{x}^2}=\sqrt{\frac{1640}{40}-6^2}=\sqrt{41-36}=\sqrt{5}\approx 2.24$

$\boldsymbol{x}$	$\boldsymbol{f}$	$\boldsymbol{fx}$	$\boldsymbol{x-\bar{x}}$	$\boldsymbol{\lvert x-\bar{x}\rvert}$	$\boldsymbol{f\lvert x-\bar{x}\rvert}$	x^2	fx^2
1	1	1	$1-6=-5$	5	5	1	1
2	2	4	$2-6=-4$	4	8	4	8
3	4	12	$3-6=-3$	3	12	9	36
4	3	12	$4-6=-2$	2	6	16	48
5	5	25	$5-6=-1$	1	5	25	125
6	6	36	$6-6=0$	0	0	36	216
7	9	63	$7-6=+1$	1	9	49	441
8	6	48	$8-6=+2$	2	12	64	384
9	1	9	$9-6=+3$	3	3	81	81
10	3	30	$10-6=+4$	4	12	100	300
Totals:	$\sum f=40$	$\sum fx=240$			$\sum f\lvert x-\bar{x}\rvert=72$		$\sum fx^2=1640$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L118 in the Pupil Handbook.

Lesson Title: Mean deviation of grouped data – Part 2	**Theme:** Statistics and Probability	
Lesson Number: M3-L119	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the mean deviation of grouped data **with** class intervals.	**Preparation** Bring a calculator to class, or a phone with a calculator. Ask pupils to bring calculators if they have access to them. Write the table in Opening on the board.	

Opening *(2 minutes)*

1. Write on the board: The weights of 20 children are given in the table:

Weight (kg)	10-14	15-19	20-24	25-29	30-34	35-39
Frequency (f)	3	2	4	5	3	3

2. Discuss and allow pupils to share ideas:
   - How do you think we could calculate mean deviation from this data?
   - What values would we use for x?
3. Remind pupils that when calculating standard deviation of grouped data with class intervals, we used the class mid-points as values of x. A similar process will be used in this lesson.
4. Explain that this lesson is on mean deviation of grouped data with class intervals.

Teaching and Learning *(18 minutes)*

1. Explain: To calculate the standard deviation of grouped data with class intervals, we will use the same formula from the previous class.
2. Write the formula on the board: $MD = \frac{\sum f|x-\bar{x}|}{\sum f}$, where x is each piece of data, $\bar{x}$ is the mean, and f is frequency.
3. Explain:
   - We do not have individual data points to substitute for x, but class intervals.
   - We will take the mid-point of each class interval and use it for x.
4. Draw the empty table on the board:

Interval	**Mid-point (x)**	$\boldsymbol{f}$	$\boldsymbol{fx}$	$\boldsymbol{x-\bar{x}}$	$\boldsymbol{\lvert x-\bar{x}\rvert}$	$\boldsymbol{f\lvert x-\bar{x}\rvert}$
10-14						
15-19						
20-24						
25-29						
30-34						
35-39						
	Totals:	$\sum f =$	$\sum fx =$			$\sum f\lvert x-\bar{x}\rvert =$

5. Invite volunteers to find the mid-points and write them in the table (see table below).
6. Invite volunteers to fill the 3rd and 4th columns.
7. Invite a volunteer to write the formula for the mean on the board. (Answer: $\bar{x} = \frac{\sum fx}{\sum f}$)
8. Ask pupils to work with seatmates to calculate the mean.
9. Invite volunteers to write the sums ($\sum f$ and $\sum fx$) in the table on the board, and write the solution for the mean. (Answer: $\bar{x} = \frac{\sum fx}{\sum f} = \frac{500}{20} = 25$ kg.)
10. Invite volunteers to fill the last 3 columns of the table.
11. Ask pupils to work with seatmates to find the sum of the last column, $\sum f|x - \bar{x}|$.
12. Invite a volunteer to write the answer in the table on the board.

Complete table:

Interval	**Mid-point (x)**	f	fx	$x - \bar{x}$	$\lvert x - \bar{x}\rvert$	$f\lvert x - \bar{x}\rvert$
10-14	12	3	36	$12 - 25 = -13$	13	39
15-19	17	2	34	$17 - 25 = -8$	8	16
20-24	22	4	88	$22 - 25 = -3$	3	12
25-29	27	5	135	$27 - 25 = +2$	2	10
30-34	32	3	96	$32 - 25 = +7$	7	21
35-39	37	3	111	$37 - 25 = +12$	12	36
	Totals:	$\sum f = 20$	$\sum fx = 500$			$\sum f\lvert x - \bar{x}\rvert = 134$

13. Calculate the mean deviation on the board, explaining each step:

$$\begin{aligned} MD &= \frac{\sum f|x-\bar{x}|}{\sum f} \\ &= \frac{134}{20} \\ &= 6.7 \end{aligned}$$

Practice *(19 minutes)*

1. Write the following problem on the board: The scores of 20 pupils on a Mathematics exam are given in the table below. Calculate: a. Mean; b. Mean deviation.

Marks (%)	40-49	50-59	60-69	70-79	80-89	90-99
Frequency (f)	1	2	6	7	3	1

2. Ask volunteers to give the table columns that are needed to calculate all parts of the problem. (Answer: interval, mid-point (x), f, fx, $x - \bar{x}$, $|x - \bar{x}|$, $f|x - \bar{x}|$)
3. Draw the empty table on the board (see table below).
4. Ask pupils to work with independently or with seatmates to complete the problem.
5. Walk around to check for understanding and clear misconceptions. Support pupils as needed.

6. Invite volunteers to fill the table and write the solutions on the board:
    a. Mean: $\bar{x} = \frac{\sum fx}{\sum f} = \frac{1440}{20} = 72$ marks
    b. Mean deviation: $\text{MD} = \frac{\sum f|x-\bar{x}|}{\sum f} = \frac{187}{20} = 9.35$

Interval	**Mid-point (x)**	$\boldsymbol{f}$	$\boldsymbol{fx}$	$\boldsymbol{x - \bar{x}}$	$\boldsymbol{\lvert x - \bar{x}\rvert}$	$\boldsymbol{f\lvert x - \bar{x}\rvert}$
40-49	45.5	1	45.5	$45.5 - 72$ $= -26.5$	26.5	26.5
50-59	55.5	2	111	$55.5 - 72$ $= -16.5$	16.5	33
60-69	65.5	6	393	$65.5 - 72$ $= -6.5$	6.5	39
70-79	75.5	7	528.5	$75.5 - 72$ $= +3.5$	3.5	24.5
80-89	85.5	3	256.5	$85.5 - 72$ $= +13.5$	13.5	40.5
90-99	95.5	1	95.5	$95.5 - 72$ $= +23.5$	23.5	23.5
	Totals:	$\sum f =$ 20	$\sum fx =$ 1,430			$\sum f\lvert x - \bar{x}\rvert =$ 187

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L119 in the Pupil Handbook.

Lesson Title: Statistics and probability	**Theme:** Statistics and Probability	
Lesson Number: M3-L120	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve WASSCE-style statistics problems that include probability questions.	**Preparation** 1. Bring a calculator to class, or a phone with a calculator. Ask pupils to bring calculators if they have access to them. 2. Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write on the board: The heights of 20 football players are given in the table below. Calculate the: a. Mean height; b. Mean deviation; c. Probability that a player chosen at random will be at least 170 cm tall.

Height (cm)	50-154	55-159	160-164	165-169	170-174	175-179	180-184
Frequency	1	1	2	4	6	5	1

2. Discuss and allow pupils to share ideas:
   a. How would you solve this problem? (Example answers: Create a table to organise calculations for mean and mean deviation, then apply the formulae; calculate probability as a fraction where the numerator is the number of players 170 cm or taller.)
3. Explain that this lesson is on solving problems that have both statistics and probability questions. Such questions often appear on the WASSCE exam. It is important to be able to answer various questions concerning one set of data.

Teaching and Learning *(12 minutes)*

1. Ask pupils to work with seatmates to solve the problem. Remind them to look at the relevant lessons in their Pupil Handbook if needed.
2. Walk around to check for understanding and clear misconceptions.
3. Invite volunteers to write the solution on the board. Make sure everyone understands.

   Solutions:

   a. Organise a table (see below) and apply the formula:
   $\bar{x} = \frac{\sum fx}{\sum f} = \frac{3400}{20} = 170$ cm

   b. Complete the relevant columns of the table (see below) and apply the formula: $MD = \frac{\sum f|x-\bar{x}|}{\sum f} = \frac{118}{20} = 5.9$

   c. $\text{Probability } = \frac{\text{players taller than 170 cm}}{\text{all players}} = \frac{6+5+1}{20} = \frac{12}{20} = \frac{3}{5} = 0.6$

Complete table:

Interval	Mid-point (x)	f	fx	$x-\bar{x}$	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
150-154	152	1	152	$152-170=-18$	18	18
155-159	157	1	157	$157-170=-13$	13	13
160-164	162	2	324	$162-170=-8$	8	16
165-169	167	4	668	$167-170=-3$	3	12
170-174	172	6	1,032	$172-170=+2$	2	12
175-179	177	5	885	$177-170=+7$	7	35
180-184	182	1	182	$182-170=+12$	12	12
	Totals:	$\sum f = 20$	$\sum fx = 3{,}400$			$\sum f\lvert x-\bar{x}\rvert = 118$

Practice *(24 minutes)*

1. Write the following problems on the board:
   a. The frequency distribution shows the marks scored by 50 pupils on a Maths exam.

Marks (%)	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	3	4	7	13	11	8	4

      i. Draw a cumulative frequency curve for the distribution.
      ii. Use the graph to find the 60th percentile.
      iii. If pupils must score more than 65% to pass, use the graph to find the probability that a pupil chosen at random passed the test.

   b. The ages of 10 children are 5, 4, 3, 7, 10, 2, 3, 8, 6, and 2. Calculate:
      i. Mean.
      ii. Standard deviation.
      iii. The probability that a child chosen at random will be at least 5 years old.
2. Ask pupils to work with independently or with seatmates to complete the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board.

   Solutions:

   a. i. Before drawing the cumulative frequency curve, complete a cumulative frequency table:

Pupils' Marks

Marks	**Frequency**	**Upper Class Boundary**	**Cumulative Frequency**
30 – 39	3	39.5	3
40 – 49	4	49.5	3+4=7
50 – 59	7	59.5	7+7=14
60 – 69	13	69.5	14+13=27
70 – 79	11	79.5	27+11=38
80 – 89	8	89.5	38+8=46
90 – 99	4	99.5	46+4=50
Total	50		

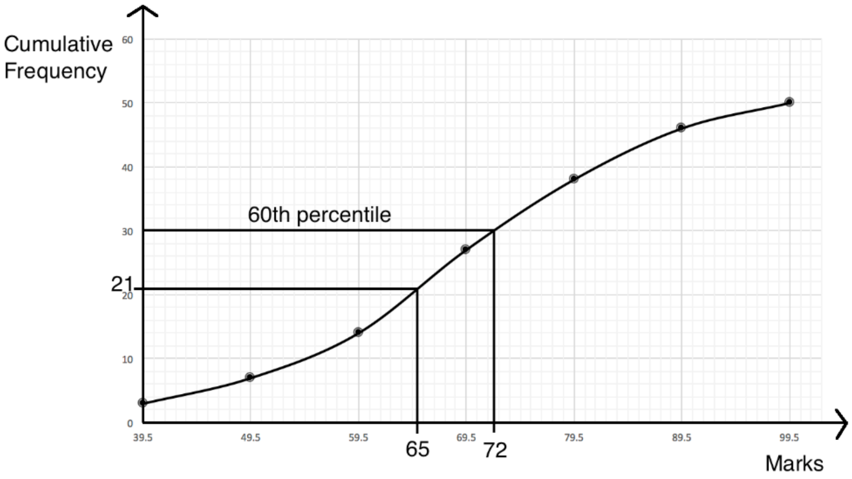
See table below.

i. Find the position of the 60th percentile: $\frac{n}{100}\sum f = \frac{60}{100}(50) = \frac{3000}{100} = 30$

Identify the 60th percentile on the curve as 72 marks. (see curve below).

ii. To identify the number of pupils scoring above 65%, first identify 65 marks on the c.f. curve. 65 marks corresponds to a cumulative frequency of 21. If 21 pupils scored 65 or lower, then the number that passed is $50 - 21 = 29$.

Probability that a pupil passed $= \frac{\text{passing pupils}}{\text{all pupils}} = \frac{29}{50} = 0.58$



b.

i. Mean: $\bar{x} = \frac{5+4+3+7+10+2+3+8+6+2}{10} = \frac{50}{10} = 5$ years old

ii. Standard deviation (see table below): $s = \sqrt{\frac{1}{n}\sum(x-\bar{x})^2} = \sqrt{\frac{66}{10}} = \sqrt{6.6} = 2.57$

$\boldsymbol{x}$	$\boldsymbol{x-\bar{x}}$	$\boldsymbol{(x-\bar{x})^2}$
5	$5-5=0$	0
4	$4-5=-1$	1
3	$3-5=-2$	4
7	$7-5=+2$	4
10	$10-5=+5$	25
2	$2-5=-3$	9
3	$3-5=-2$	4
8	$8-5=+3$	9
6	$6-5=+1$	1
2	$2-5=-3$	9
Total $=\sum(x-\bar{x})^2 =$		66

iii. Probability $= \frac{\text{children 5 and older}}{\text{all children}} = \frac{5}{10} = 0.5$

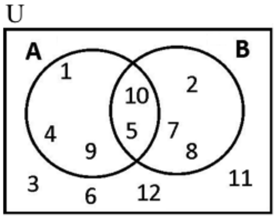
Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L120 in the Pupil Handbook.

Lesson Title: Sets	**Theme:** Review	
Lesson Number: M3-L121	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems on sets.	**Preparation** Draw the Venn diagram in Opening on the board.	

Opening *(4 minutes)*

1. Draw the Venn diagram at right on the board:



2. Discuss and allow pupils to share ideas until arriving at the correct answers.
    a. What are the sets in the diagram? (Answers: A, B, and the universal set U).
    b. What are the elements of A? (Answer: 1, 4, 5, 9, 10)
        b. What are the elements of B? (Answer: 2, 5, 7, 8, 10)
        c. What elements are in the intersection of A and B? (Answer: 5 and 10)
3. Write on the board using set notation, and make sure pupils understand:

$A = \{1, 4, 5, 9, 10\}$

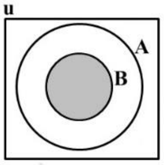
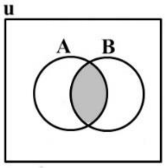
$B = \{2, 5, 7, 8, 10\}$

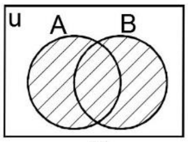
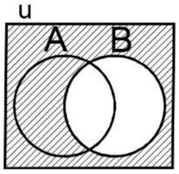
$A \cap B = \{5, 10\}$

4. Explain that this lesson is on solving problems that involve sets.

Teaching and Learning *(17 minutes)*

1. The table shown below is in the Pupil Handbook. Review it with pupils. Write the notation and diagrams on the board as needed. It is not necessary to copy the entire table on the board.

	Explanation	**Notation**	**Reading**	**Diagram**
Subset	If every element in set B is present in set A, then B is a subset of A.	$B \subset A$	"set B is a subset of set A"	
Intersection	The intersection of 2 sets A and B is the element(s) that is/are common to both sets A and B.	$A \cap B$	"A intersection B"	

Union	The union of A and B is the set formed by combining the elements in both sets.	$A \cup B$	“A union B”	
Complement	If B is a subset of the universal set U, then the complement of B is the set of members which belong to U but not to B.	B’	“B prime”	

2. Write a problem on the board: If $A = \{11, 12, 13, 14\}$ and $B = \{13, 14, 15, 16, 17\}$ are subsets of $U = \{10, 11, 12, 13, 14, 15, 16, 17, 18\}$, then list the elements of:

   a. A' b. B' c. $A' \cap B'$

3. Solve the problem as a class. Involve pupils by having them give answers or write them on the board.

   Solutions:

   a. A' is the elements in U that are not in A. $A' = \{10, 15, 16, 17, 18\}$
   b. B' is the elements in U that are not in B. $B' = \{10, 11, 12, 18\}$
   c. $A' \cap B'$ is the common elements in A' and B'. $A' \cap B' = \{10, 18\}$

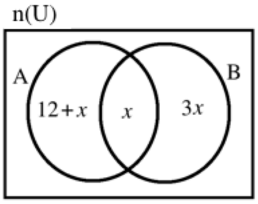
4. Write another problem on the board: If A and B are two finite sets such that $n(A) = 20$, $n(B) = 28$ and $n(A \cap B) = 12$, find $n(A \cup B)$.
5. Discuss:
   - What does the letter n denote? (Answer: It gives the cardinality, which is the number of elements in a set.)
   - What is this problem asking us to find? (Answer: The number of elements in the intersection $A \cap B$)
6. Explain:
   - A formula is given in the Pupil Handbook which can be used to solve a problem like this.
   - This problem can also be solved logically. The number of elements in the union can be found by adding the total number of elements in both A and B, and subtracting the number of elements in their intersection. Recall that elements are only listed once in the union. What is left after subtracting the intersection is the union.
7. Write the formula on the board: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
8. Ask pupils to solve the problem with seatmates.
9. Walk around to check for understanding and clear misconceptions.
10. Ask volunteers to write the solution on the board. Make sure that everyone understands.

Solution:

$$\begin{aligned} n(\mathrm{A} \cup \mathrm{B}) &= n(\mathrm{A}) + n(\mathrm{B}) - n(\mathrm{A} \cap \mathrm{B}) \\ &= 20 + 28 - 12 \\ &= 48 - 12 \\ &= 36 \end{aligned}$$

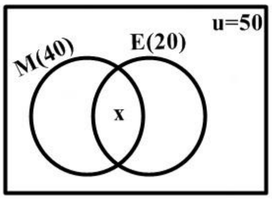
Practice *(18 minutes)*

1. Write the following problems on the board. Give 2 or 3 problems depending on the time:
    a. If $\mathrm{U} = \{1,2,3,6,9,18\}$, $\mathrm{X} = \{2,6,18\}$ and $\mathrm{Y} = \{1,3,6\}$, list the elements of: i. $X' \cap \mathrm{Y}$; ii. $X \cap \mathrm{Y}'$; iii. $(X \cup \mathrm{Y})'$
    b. In a class of 50 pupils, 40 read Mathematics and 20 read English. Each pupil reads at least one subject. How many pupils read both Mathematics and English?
    c. A and B are two sets and the number of elements are shown in the Venn diagram below. Given that $n(\mathrm{A}) = n(\mathrm{B})$, find: i. x ii. $n(\mathrm{A} \cup \mathrm{B})$

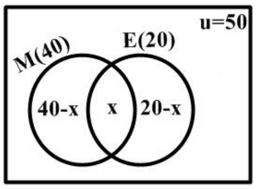


2. Ask pupils to work with independently or with seatmates to complete the problems.
3. Walk around to check for understanding and clear misconceptions. Remind pupils to look at Solved Examples in the Pupil Handbook for support.
4. Invite volunteers to write the solutions on the board.

   Solutions:
    a. Note that $\mathrm{X}' = \{1,3,9\}$ and $\mathrm{Y}' = \{2,9,18\}$.
        i. $\mathrm{X}' \cap \mathrm{Y} = \{1,3,9\} \cap \{1,3,6\} = \{1,3\}$
        ii. $\mathrm{X} \cup \mathrm{Y}' = \{2,6,18\} \cup \{2,9,18\} = \{2,6,9,18\}$
        iii. $(\mathrm{X} \cup \mathrm{Y})'$ is the complement of $\mathrm{X} \cup \mathrm{Y}$.
        $\mathrm{X} \cup \mathrm{Y} = \{1,2,3,6,18\}$, therefore $(\mathrm{X} \cup \mathrm{Y})' = \{9\}$.
    b. 50 pupils give the universal set $n(\mathrm{U}) = 50$. Let M represent pupils who read Mathematics, and E represent pupils who read English. Then $n(\mathrm{M}) = 40, n(\mathrm{E}) = 20$. This can be represented with a Venn diagram, where x is the intersection, $n(\mathrm{M} \cap \mathrm{E}) = x$:



Set up an equation with variable x using the given information. To identify the number of pupils who read only Maths or only English, subtract x from the number who read each subject. This gives the Venn diagram:



From the Venn diagram, we have the equation $(40 - x) + x + (20 - x) = 50$.
Solve the equation for x:

$$\begin{aligned}(40 - x) + x + (20 - x) &= 50\\ 20 - x + x + 20 - x &= 50\\ 40 + 20 - x &= 50\\ x &= - 50\\ x &= \end{aligned}$$

Therefore, 10 pupils read both Mathematics and English.

c. i. We have $n(\mathrm{A}) = n(\mathrm{B})$. Set the number of elements in the 2 sets equal and solve for x.

$$\begin{aligned}12 + x + x &= 3x + x\\ 12 + 2x &= 4x\\ 12 &= 4x - 2x\\ 12 &= 2x\\ 6 &= x\end{aligned}$$

ii. To find $n(\mathrm{A} \cup \mathrm{B})$, find the sum of all of the parts inside A and B. Substitute $x = 6$ and evaluate:

$$\begin{aligned}n(\mathrm{A} \cup \mathrm{B}) &= 12 + x + x + 3x\\ &= 12 + 5x\\ &= 12 + 5(6)\\ &= + 30 = 42\end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L121 in the Pupil Handbook.

Lesson Title: Indices and logarithms	**Theme:** Review	
Lesson Number: M3-L122	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems on indices and logarithms.	**Preparation** Write the laws of indices and logarithms (in the Opening) on the board.	

Opening *(3 minutes)*

1. Write the following on the board:
   Laws of indices:
    1. $a^m \times a^n = a^{m+n}$
    2. $a^m \div a^n = a^{m-n}$
    3. $a^0 = 1$
    4. $(a^x)^y = a^{xy}$

   Laws of logarithms:
    1. $\log_{10} pq = \log_{10} p + \log_{10} q$
    2. $\log_{10}\left(\frac{p}{q}\right) = \log_{10} p - \log_{10} q$
    3. $\log_{10}(P)^n = n \log_{10} P$
2. Explain: These laws will be used throughout this lesson to simplify expressions involving indices and logarithms.
3. Ask volunteers to make observations and explain what they notice about the laws on the board. (Example: Multiplication is associated with addition in both sets of laws; division is associated with subtraction.)
4. Explain that this lesson is on solving problems on indices and logarithms.

Teaching and Learning *(18 minutes)*

1. Ask pupils what else they know about **indices**. Encourage them to look at the Overview in the Pupil Handbook. Write their ideas on the board as they list them.
2. Write the following on the board if pupils do not identify them. Make sure pupils understand each.
   - $a^{-n} = \frac{1}{a^n}$
   - $\sqrt[n]{x} = x^{\frac{1}{n}}$
   - $\left(\frac{\mathrm{a}}{\mathrm{b}}\right)^{\mathrm{n}} = \frac{a^n}{b^n}$
   - $\left(\frac{\mathrm{a}}{\mathrm{b}}\right)^{-\mathrm{n}} = \left(\frac{\mathrm{b}}{\mathrm{a}}\right)^{\mathrm{n}}$
3. Ask pupils what else they know about **logarithms**. Encourage them to look at the Overview in the Pupil Handbook. Write their ideas on the board as they list them.
4. Write the following on the board if pupils do not identify them. Make sure pupils understand each.
   - $\log_a a = 1$
   - $\log_a 1 = 0$

5. Ask pupils to describe the relationship between indices and logarithms in their own words.
6. Write on the board and make sure pupils understand:

$$y = \underset{\substack{\uparrow \\ \text{base}}}{b}^{x} \longleftarrow \text{power} \qquad\qquad \log_{\underset{\substack{\uparrow \\ \text{base}}}{b}} y = x \longleftarrow \text{power}$$

7. Explain: We will now use these various rules to solve problems. Refer to the rules on the board or in your Pupil Handbook any time you need assistance.
8. Write the following problems on the board:
   a. $(4b^2)^3 \div 8b^2a^3$
   b. Solve for x: $2^{x-1} = 16$
9. Ask volunteers to give the steps to solve each. Solve the problems on the board as they give them.

   Solutions:

   a.
$$\begin{aligned}
(4b^2)^3 \div 8b^2a^3 &= 4^3(b^2)^3 \div 8b^2a^3 && \text{Distribute the power} \\
&= 64b^{2\times 3} \div 8b^2a^3 && \text{Multiply powers on } b \\
&= 64b^6 \div 8b^2a^3 && \\
&= (64 \div 8)b^{6-2}a^{0-3} && \text{Divide (subtract powers)} \\
&= 8b^4a^{-3} && \text{Simplify} \\
&= \frac{8b^4}{a^3} && \text{Move negative powers to the denominator}
\end{aligned}$$

   b.
$$\begin{aligned}
2^{x-1} &= 16 && \\
2^{x-1} &= 2^4 && \text{Write 16 with base 2} \\
x-1 &= 4 && \text{Set powers equal} \\
x &= 4+1 && \text{Solve for } x \\
x &= 5 &&
\end{aligned}$$

10. Write the following problems on the board:
    - Solve for x: $x = \log_4 16$
    - Simplify $\frac{\log_2 6^2}{\log_2 6}$
11. Ask volunteers to give the steps to solve each. Solve the problems on the board as they give them.

    Solutions:

    a.
$$\begin{aligned}
x &= \log_4 16 && \\
16 &= 4^x && \text{Write in index form} \\
4^2 &= 4^x && \text{Make bases equal} \\
2 &= x && \text{Set powers equal}
\end{aligned}$$

b. $\frac{\log_2 6^2}{\log_2 6} = \frac{2(\log_2 6)}{(\log_2 6)}$ Rewrite the numerator

$= 2$ $(\log_2 6)$ cancels

Practice *(18 minutes)*

1. Write the following problems on the board:
   Simplify:
   a. $(x^{-2})^3 \times x^{10}$
   b. $\log_2 8 + \log_2 6 - \log_2 2$
   c. $\frac{\log_a 8 + \log_a 6 - \log_a 3}{\log_a 4}$

   Solve for the variable in the following:
   d. $y = \log_3 81$
   e. $10^{2x-1} = \frac{1}{1{,}000}$
2. Ask pupils to work with independently or with seatmates to complete the problems.
3. Walk around to check for understanding and clear misconceptions. Remind pupils to look at Solved Examples in the Pupil Handbook for support.
4. Invite volunteers to write the solutions on the board.

   Solutions:

a.
$$\begin{aligned} (x^{-2})^3 \times x^{10} &= x^{-2\times 3} \times x^{10} \\ &= x^{-6} \times x^{10} \\ &= x^{-6+10} \\ &= x^4 \end{aligned}$$

b.
$$\begin{aligned} \log_2 8 + \log_2 4 - \log_2 2 &= \log_2\left(\frac{8\times 4}{2}\right) \\ &= \log_2 16 \\ &= \log_2 2^4 \\ &= 4(\log_2 2) \\ &= 4 \times 1 = 4 \end{aligned}$$

c.
$$\begin{aligned} \frac{\log_a 8 + \log_a 6 - \log_a 3}{\log_a 4} &= \frac{\log_a\left(\frac{8\times 6}{3}\right)}{\log_a 4} \\ &= \frac{\log_a 16}{\log_a 4} \\ &= \frac{\log_a 2^4}{\log_a 2^2} \\ &= \frac{4(\log_a 2)}{2(\log_a 2)} \\ &= \frac{4}{2} = 2 \end{aligned}$$

d.
$$\begin{aligned} y &= \log_3 81 \\ 81 &= 3^y \\ 3^4 &= 3^y \\ 4 &= y \end{aligned}$$

e.
$$\begin{aligned} 10^{2x-1} &= \frac{1}{1{,}000} \\ 10^{2x-1} &= \frac{1}{10^3} \\ 10^{2x-1} &= 10^{-3} \\ 2x-1 &= -3 \\ 2x &= -2 \\ x &= \frac{-2}{2} = -1 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L122 in the Pupil Handbook.

Lesson Title: Sequences and series	**Theme:** Review	
Lesson Number: M3-L123	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems on sequences and series.	**Preparation** Draw the Venn diagram in Opening on the board.	

Opening *(2 minutes)*

1. Discuss and allow pupils to share ideas:
    - What is a sequence? Explain in your own words. (Example answer: A list of numbers that increases or decreases; a sequence of numbers that follows a certain rule.)
    - What types of sequences do you know about? (Example answers: Arithmetic and geometric sequences.)
    - What is a series? Explain in your own words. (Example answer: A series is like a sequence, but the numbers are added to find a sum.)
2. Explain that this lesson is on various sequence and series problems.

Teaching and Learning *(22 minutes)*

1. Write on the board:
    a. 5, 8, 11, 14, …
    b. 4, 8, 16, 32, …
2. Ask volunteers to identify whether each sequence is an arithmetic or geometric sequence. Ask them to explain.
   Answers:
    a. An arithmetic sequence, because it increases by a common difference (3 is added to each term).
    b. A geometric sequence, because it increases by a common ratio (each term is multiplied by 2).
3. Review **arithmetic sequence:**
    - A sequence in which the terms either increase or decrease by a common difference is an arithmetic sequence, or arithmetic progression. It can be abbreviated to AP.
    - Arithmetic progressions have a **common difference** that is the same between each term and the next term.
4. Write on the board: $U_n = a + (n-1)d$, where U_n is the nth term of the AP, a is the first term, and d is the common difference.
5. Review **geometric sequence:**
    - A sequence in which the terms either increase or decrease by a common ratio is a geometric sequence, or geometric progression. It can be abbreviated to GP.

- Geometric progressions have a **common ratio** that is multiplied by each term to get the next term.

6. Write on the board: $U_n = ar^{n-1}$, where U_n is the nth term of the GP, a is the first term, and r is the common ratio.
7. Write on the board:
   a. $4 + 8 + 16 + 32 + \cdots$
   b. $5 + 8 + 11 + 14$
8. Explain: These have the same values as the sequences given earlier, but these are series.
9. Explain:
   a. Series a. has 3 points after it ($\cdots$), because it continues on forever. This is an **infinite series**. It is often impossible to find the sum of infinite series, but we can find the sum of the first n terms.
   b. Series b stops at 14. It is a finite series, which ends at a certain point.
10. Write the formulae on the board:

    Sum of the first n terms of an AP: $S_n = \frac{1}{2}n[2a + (n-1)d]$

    Sum of the first n terms of a GP if $|r| > 1$: $S_n = \frac{a(r^n-1)}{r-1}$

    Where a is the first term, d is the common difference, and r is the common ratio.
11. Write the problems on the board:
    a. Find the 10th term of 4, 8, 16, 32, …
    b. Find the 10th term of 5, 8, 11, 14, …
    c. Find the sum of the first 16 terms of $3 + 7 + 11 + 15 + 19 + \cdots$.
    a. Find the sum of $2 + 8 + 32 + 128 + 512$.
12. Ask pupils to work with seatmates to solve the problems. Identify the variables and appropriate formulae for each problem as a class if needed.
13. Walk around to check for understanding and clear misconceptions. Remind pupils to refer to the formulae and Solved Examples in the Pupil Handbook if needed.
14. Invite volunteers to write the solutions on the board.

Solutions:

a.

$$\begin{aligned}
U_n &= ar^{n-1} \\
U_{10} &= 4 \times 2^{10-1} && \text{Substitute } a = 4, n = 10, r = 2 \\
&= 4 \times 2^9 && \text{Simplify} \\
&= 4 \times 512 \\
&= 2{,}048
\end{aligned}$$

b.

$$\begin{aligned}
U_n &= a + (n-1)d \\
U_{10} &= 5 + (10-1)3 && \text{Substitute } a = 5, d = 3 \text{ and } n = 10 \\
&= 5 + 9 \times 3 && \text{Clear the brackets} \\
&= 5 + 27 && \text{Simplify} \\
&= 32
\end{aligned}$$

c.
$$S_n = \frac{1}{2}n[2a + (n-1)d]$$
$$S_{16} = \frac{1}{2}(16)[2(3) + (16-1)4]$$ Substitute n, a, and d
$$= (8)[6+60]$$ Simplify
$$= 8(66) = 528$$

d.
$$S_n = \frac{a(r^n-1)}{r-1}$$
$$S_5 = \frac{2(4^5-1)}{4-1}$$ Substitute n, a, and r
$$= \frac{2(1024-1)}{3}$$ Simplify
$$= \frac{2046}{3}$$
$$= 682$$

Practice *(15 minutes)*

1. Write the following problems on the board:
    a. Find the number of terms in the arithmetic sequence $4, 8, 12, 16, \ldots, 64$.
    b. Find the 9th term of the sequence $4, -8, 16, -32, \ldots$
    c. Find the number of terms in the geometric sequence $3, 9, 27, \ldots, 6561$
    d. Find the sum of the first 15 terms of the AP 10, 8, 7, 6, ….
2. Ask pupils to work with independently to complete the problems. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board.
   Solutions:
    a.

$$U_n = a + (n-1)d$$
$$64 = 4 + (n-1)4$$ Substitute $U_n = 64, a = 4, d = 4$
$$64 - 4 = (n-1)4$$ Solve for n
$$60 = 4n - 4$$
$$60 + 4 = 4n$$
$$\frac{64}{4} = n$$
$$n = 21$$

There are 21 terms in the progression.

b.

$$U_n = ar^{n-1}$$
$$U_9 = 4 \times (-2)^{9-1}$$ Substitute $a = 4, n = 9, r = -2$

$= 4 \times (-2)^8$ Simplify

$= 4 \times 256$

$= 1{,}024$

The ninth term is $1{,}024$.

c.

$U_n = ar^{n-1}$

$6{,}561 = 3 \times 3^{n-1}$ Substitute $U_n = 6{,}561, a = 3, r = 3$

$3^8 = 3^n$ Substitute $6{,}561 = 3^8$

$n = 8$

There are 8 terms.

d.

$S_n = \frac{1}{2}n[2a + (n-1)d]$

$7 = \frac{1}{2}(14)[2(20) + (14-1)d]$ Substitute n, a, and d

$7 = 7[40 + 13d]$ Simplify

$1 = 40 + 13d$ Divide throughout by 7

$1 - 40 = 13d$ Transpose 40

$-39 = 13d$

$-3 = d$ Divide throughout by 13

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L123 in the Pupil Handbook.

Lesson Title: Ratio/Proportion/Rate/ Percentages	**Theme:** Review	
Lesson Number: M3-L124	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems on ratio, proportion, rate, and percentages.	**Preparation** Write the problems at the start of Teaching and Learning on the board.	

Opening *(2 minutes)*

1. Ask volunteers to write examples of ratios, rates, and percentages on the board. Example answers:
   - Ratios: 4 : 5   3 girls : 2 boys   130 : 61
   - Rates: 35 m/hr   16 litres/minute   3 hr/day
   - Percentages: 25% 2.67% $31\frac{1}{2}\%$ 135%
2. Explain that this lesson is on various problems related to ratio, rate, and percentage.

Teaching and Learning *(18 minutes)*

1. Write on the board:
   a. Decrease Le 200,000.00 by the ratio 5 : 4.
   b. There are 550 pupils in a school. If 48% of them are boys, how many are girls?
   c. Mamadou has a bag of 60 oranges. She shares them between 3 children in the ratio 5 : 4 : 3. How many oranges does each child receive?
   d. Last year, there were 256 pupils enrolled in a certain school. This year, enrolment increased by 12.5%. How many pupils are enrolled now?
   e. Fatu invested Le 500,000.00 at 2.5% interest. After how many years will she earn Le 75,000.00 interest?
2. Solve the first 2 problems as a class. Ask volunteers to give the steps as you solve on the board.

   Solutions:

   a. Remind pupils that to decrease an amount by a given ratio, multiply by the ratio as a fraction with the smaller number in the numerator.

   New amount $= \frac{4}{5} \times 200{,}000 = \text{Le } 160{,}000.00$

   b. If 48% are boys, then the percentage who are girls is $100 - 48 = 52\%$.

   Number of girls: $550 \times \frac{52}{100} = 286$ girls
3. Ask pupils to work with seatmates to solve problems c., d., and e. Remind them to look at the information and solved examples in the Pupil Handbook if needed.
4. Walk around to check for understanding and clear misconceptions.

5. Invite 3 volunteers to write the solutions on the board and explain. Other pupils should check their work.

   Solutions:

   c. **Step 1.** Find the total number of parts to the ratio: $5 + 4 + 3 = 12$

      Note that the ratio 5 : 4 : 3 means that for every 12 oranges shared, 5 will go to Child 1, 4 will go to Child 2 and 3 will go to Child 3.

      Step 2. Find what proportion (fraction) of the total is given to each child.

      Child 1 receives: $\frac{5}{12} \times 60 = 25$ oranges

      Child 2 receives: $\frac{4}{12} \times 60 = 20$ oranges

      Child 3 receives: $\frac{3}{12} \times 60 = 15$ oranges

   d. To increase a quantity by x%, take the original quantity as 100%, and increase or decrease it by x. The new quantity is $(100 + x)$% of the original.

      New quantity $= 256 \times \frac{112.5}{100} = 288$ pupils

   e. Apply the simple interest formula $I = \frac{PRT}{100}$**, where I is the interest earned, P is the principal (M), R is the rate (x), and T is the time.**

$$\begin{aligned} I &= \frac{PRT}{100} \\ 75{,}000 &= \frac{500{,}000(2.5)T}{100} \\ 75{,}000 &= \frac{1{,}250{,}000T}{100} \\ 75{,}000 &= 12{,}500T \\ 6 &= T \end{aligned}$$

   The interest will be Le 75,000.00 in 6 years.

Practice *(19 minutes)*

1. Write the following problems on the board:
   a. There are 200 pupils in SS3 at a certain school. 28% play football and 15% play basketball. The remainder do not play any sports. How many pupils do not play sports?
   b. A man has Le 800,000.00 that he wants to share between his 3 children in the ratio 5 : 4 : 7. Find how much each child will receive.
   c. Mr. Bah pays Le 75,000.00 for a Maths textbook priced at Le 90,000.00. Find the discount allowed.
   d. Foday earns a commission for each sale he makes. He sold a television for \$620.00, and earned a commission of \$31.00. Find his rate of commission.
2. Ask pupils to work with independently to complete the problems. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.

4. Invite volunteers to write the solutions on the board.

   Solutions:

   a. Find the percentage who do not play sports: $100 - 28 - 15 = 57\%$

   Number of pupils who do not play sports: $200 \times \frac{57}{100} = 114$ pupils

   b. Find the number of parts to the ratio: $5 + 4 + 7 = 16$

   Find what portion is given to each child:

   Child 1 receives: $\frac{5}{16} \times \text{Le } 800{,}000.00 = \text{Le } 250{,}000.00$

   Child 2 receives: $\frac{4}{16} \times \text{Le } 800{,}000.00 = \text{Le } 200{,}000.00$

   Child 3 receives: $\frac{7}{16} \times \text{Le } 800{,}000.00 = \text{Le } 350{,}000.00$

   c. Reduction in price: $90{,}000 - 75{,}000 = 15{,}000$

   $\text{Discount allowed} = \frac{\textit{Reduction in price}}{\textit{Original price}} \times 100 = \frac{15{,}000}{90{,}000} \times 100 = 16.67\%$

   d. Use the formula for commission:

$$\begin{aligned} \text{Commission} &= \frac{x}{100} \times \text{Price} \qquad \text{where } x \text{ is the percent commission} \\ 31 &= \frac{x}{100} \times 620 \\ 3{,}100 &= x \times 620 \\ \frac{3{,}100}{620} &= x \\ 5 &= x \end{aligned}$$

   His rate of commission is 5%.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L124 in the Pupil Handbook.

Lesson Title: Linear equations	**Theme:** Review	
Lesson Number: M3-L125	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to graph and solve problems on linear equations.	**Preparation** None	

Opening *(2 minutes)*

1. Write on the board: Graph $6x - 2y = 4$ on a Cartesian plane.
2. Discuss: How would you solve this problem? (Example answer: Solve for y, then use the equation to fill a table of values and graph the line on the plane.)
3. Explain that this lesson is on graphing linear functions and solving related problems.

Teaching and Learning *(17 minutes)*

1. Ask pupils to solve the problem on the board for y with seatmates.
2. Invite a volunteer to write the solution on the board:

$$\begin{aligned} 6x - 2y &= \\ -2y &= -6x + 4 && \text{Transpose } 6x \\ \frac{-2y}{-2} &= \frac{-6x}{-2} + \frac{4}{-2} && \text{Divide throughout by } -2 \\ y &= 3x - 2 \end{aligned}$$

3. Graph the equation on the board as a class. Involve pupils by asking volunteers to fill the table and plot the points.

$$\begin{aligned} y &= 3(-2) - 2 \\ &= -6 - 2 \\ &= -8 \end{aligned}$$

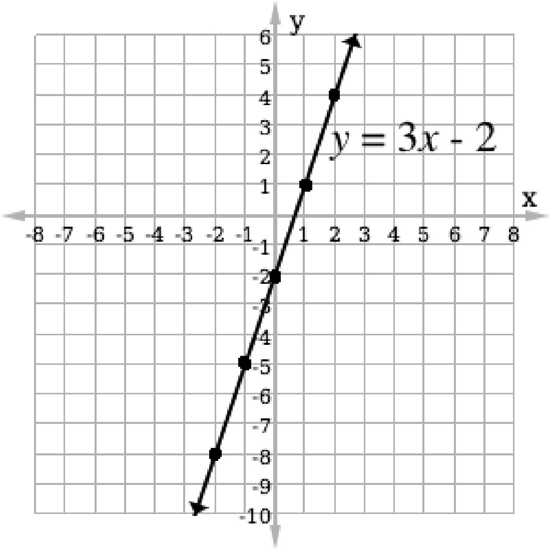
x	-2	-1	0	1	2
y	-8	-5	-2	1	4

$$\begin{aligned} y &= 3(-1) - 2 \\ &= -3 - 2 \\ &= -5 \end{aligned}$$

$$\begin{aligned} y &= 3(0) - 2 \\ &= 0 - 2 \\ &= -2 \end{aligned}$$

$$\begin{aligned} y &= 3(1) - 2 \\ &= 3 - 2 \\ &= 1 \end{aligned}$$

$$\begin{aligned} y &= 3(2) - 2 \\ &= - 2 \\ &= 4 \end{aligned}$$



4. Discuss: What is the gradient of the line? How do you know? (Answer: The gradient is 3.)
5. Write on the board: Slope-intercept form: $y = mx + c$, where m is the gradient and c is the y-intercept of the line.
6. Explain:
    a. When the equation of a line is written in this form, the coefficient of x is the gradient.
    b. Gradient is a number that tells us in which direction a line increases, and how steep it is.
    c. If a line increases as it goes to the right, or in the positive x-direction, the gradient is positive. If a line increases as it goes to the left, or in the negative x-direction, the gradient is negative.
7. Write on the board: Calculate the gradient of the line passing through $(-3, -5)$ and $(5, 11)$.
8. Invite a volunteer to write the formula for calculating gradient on the board. (Answer: $m = \frac{y_2 - y_1}{x_2 - x_1}$)
9. Ask pupils to work with seatmates to solve the problem on the board.
10. Invite a volunteer to write the solution on the board.

Solution:

$$\begin{aligned} m &= \frac{y_2-y_1}{x_2-x_1} \\ &= \frac{11-(-5)}{5-(-3)} && \text{Substitute } x\text{- and } y\text{-values} \\ &= \frac{+5}{3} && \text{Simplify} \\ &= \frac{16}{8} \\ &= 2 \end{aligned}$$

Practice *(20 minutes)*

1. Write the following problems on the board:
    a. Find the gradient of the line passing through $(1, -2)$ and $(-2, 4)$.
    b. Line AB has a gradient of 2, and line CD has a gradient of -4. Which line is steeper?
    c. Graph the linear equation $y + 2x = 3$.
    d. The gradient of the line joining the points $(a, 4)$ and $(3, -2)$ is 2. Find a.
2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board simultaneously to write the solutions.

Solutions:

a. $m = \frac{4-(-2)}{-2-1}$ Substitute x- and y-values

$= \frac{4+2}{-3}$ Simplify

$= -\frac{6}{3}$

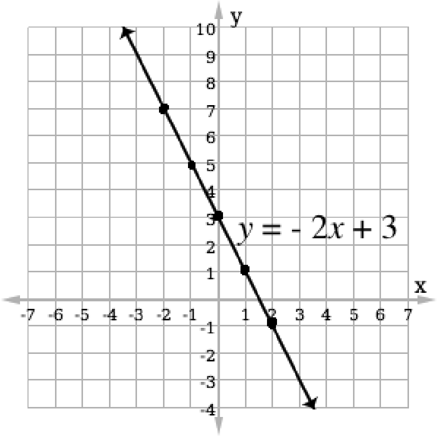
$= -2$

b. Line CD is steeper, because $|CD| > |BC|$.

c. Solve for y: $y = -2x + 3$

Fill a table of values and graph the line:

x	-2	-1	0	1	2
y	7	5	3	1	-1



d. $2 = \frac{-2-4}{3-a}$ Substitute in the gradient formula

$2(3-a) = -6$ Solve for a

$6 - 2a = -6$

$-2a = -6 - 6$

$\frac{-2a}{-2} = \frac{-12}{-2}$ Simplify

$a = 6$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L125 in the Pupil Handbook.

Lesson Title: Quadratic Equations	**Theme:** Review	
Lesson Number: M3-L126	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to graph and solve problems on quadratic equations	**Preparation** None	

Opening *(3 minutes)*

1. Write on the board: Graph the quadratic function $y = x^2 + 2x - 3$ for the interval $-3 \leq x \leq 1$. Use it to answer the following questions:
    a. What is the minimum value of $y = x^2 + 2x - 3$?
    b. What is the solution set of the equation $x^2 + 2x - 3 = 0$?
    c. What is the equation of the line of symmetry?
2. Discuss:
    a. How would you graph this equation? (Example answer: Find the y-values for $-3 \leq x \leq 1$ and plot them on the Cartesian plane.)
    b. What is the "solution set" of a quadratic equation? (Answer: The roots of the equation; where the parabola intersects with the x-axis.)
    c. What is a line of symmetry? (Answer: It is the vertical line about which the parabola is symmetric.)
3. Explain that this lesson is on graphing quadratic functions, and answering questions using the graphs.

Teaching and Learning *(12 minutes)*

1. Draw the table of values on the board:

x	-3	-2	-1	0	1
y					

2. Solve for the first value of y, and write it in the table:

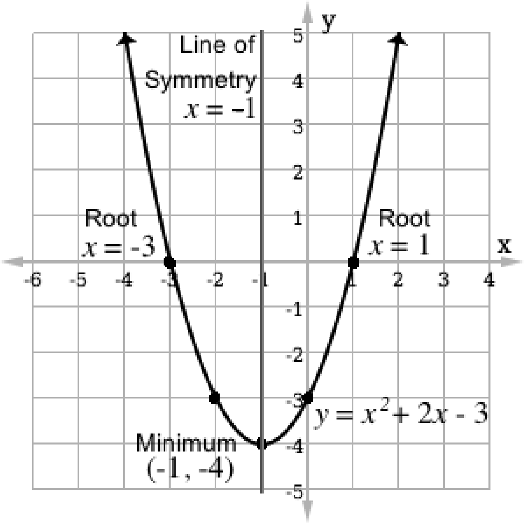
$$\begin{aligned} y &= +2x - 3 \\ &= \cdot 3)^2 + 2(-3) - 3 \\ &= -6 - 3 \\ &= 0 \end{aligned}$$

3. Ask pupils to work with seatmates to complete the table.
4. Ask volunteers to come to the board to fill the table:

x	-3	-2	-1	0	1
y	0	-3	-4	-3	0

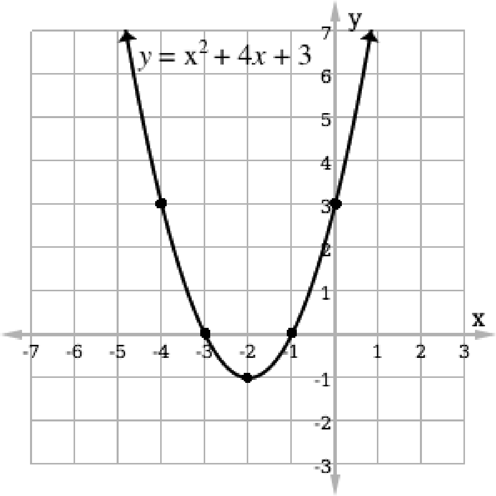
5. Draw an empty Cartesian plane on the board with axes from -5 to 5.
6. Invite volunteers to come to the board and plot the points from the table.
7. Connect the points with a parabola and label it $y = x^2 + 2x - 3$ (see graphed parabola below).

8. Ask a volunteer to answer question a. by identifying the minimum. (Answer: $(-1, -4)$)
9. Ask another volunteer to answer question b. by giving the solution set (roots). (Answer: $x = -3,\ 1$)
10. Ask another volunteer to answer question c. by giving the line of symmetry. (Answer: $x = -1$)
11. Label the parabola as shown:



Practice *(24 minutes)*

1. Write the following problems on the board:
   a. From the graph below, identify for $y = x^2 + 4x + 3$: i. Minimum; ii. Roots; iii. Line of symmetry.



   b. Complete the following:
      i. Complete the table below for the relation $y + 3x^2 + 2x - 9 = 0$.

x	-5	-4	-3	-2	-1	0	1	2	3
y	-56			1			4		

      ii. Using a scale of 2 cm to 1 unit on the x-axis and 2 cm to 10 units on the y-axis, draw the graph of $y = -3x^2 - 2x + 9$.

    iii. Use your graph to find the roots of the equation $-3x^2 - 2x + 9 = 0$.
    iv. Find the maximum value of $y = -3x^2 - 2x + 9$.
    v. Find the equation of the line of symmetry.

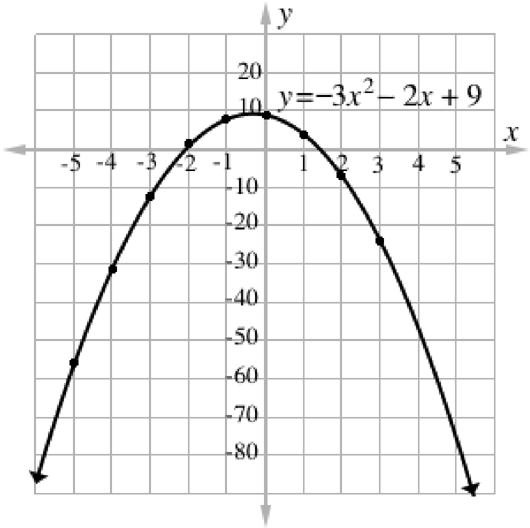
2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board simultaneously to write the solutions.
   Solutions:

   a. i. Minimum: $(-2, -1)$
      ii. Roots: $x = -3, -1$
      iii. Line of symmetry: $x = -2$

   b. i. Complete table:

x	-5	-4	-3	-2	-1	0	1	2	3
y	-56	-31	-12	1	8	9	4	-7	-24

   ii. Graph (not to scale):



   iii. Roots: Accept approximations near $x = -2.1, 1.4$
   iv. Maximum: Accept approximations near $(-0.3, 9.3)$
   v. Line of symmetry: Accept approximations near $x = -0.3$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L126 in the Pupil Handbook.

Lesson Title: Simultaneous equations	**Theme:** Review	
Lesson Number: M3-L127	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve simultaneous equations using graphical and algebraic methods.	**Preparation** None	

Opening *(2 minutes)*

1. Write on the board:

$$\begin{aligned} y &= x+2 \qquad (1) \\ y &= x^2 \qquad (2) \end{aligned}$$

2. Discuss:
   a. What type of equations are these? (Answer: (1) is linear, and (2) is quadratic; together they are a set of simultaneous equations.)
   b. What does the solution to a set of simultaneous equations look like? (Answer: It has two values, x and y, that satisfy both equations.)
   c. How would you solve these simultaneous equations? (Example answers: Pupils may describe solving by elimination, substitution, or graphing.)
3. Explain that this lesson is on solving simultaneous equations.

Teaching and Learning *(17 minutes)*

1. Explain:
   a. In the example on the board, we have simultaneous equations that are linear and quadratic. We can also have simultaneous equations where both equations are linear.
   b. In this lesson we will cover 2 methods of solving simultaneous equations: substitution and graphing. These 2 methods can be used for either type of simultaneous equations.
   - To solve by **substitution:**
     i. To solve using the method of substitution, we must change the subject of one equation.
     ii. We should choose one of the given equations and make one of the variables the subject of the other one.
     iii. After changing the subject, we substitute the expression into the other equation.
   - To solve by **graphing:**
     iv. Solve the set of simultaneous equations by graphing both equations. The solution is the point where the lines/curves intersect.
2. Solve the simultaneous equations on the board using substitution. Involve pupils by asking volunteers to explain each step.

Solution:

$x^2 = x + 2$ (1) Substitute $y = x^2$ for y in equation (1)
$x^2 - x - 2 = 0$ Transpose x and 2
$(x - 2)(x + 1) = 0$ Factorise the quadratic equation

$x - 2 = 0$ or $x + 1 = 0$ Set each binomial equal to 0
$x = 2$ or $x = -1$ Transpose -2 and 1

$y = (2) + 2$ Substitute $x = 2$ into equation (2)
$y = 4$

$y = (-1) + 2$ Substitute $x = -1$ into equation (2)
$y = 1$

4. Draw an empty Cartesian plane and empty tables of values on the board (see below for tables).
5. Ask pupils to work with seatmates to fill the tables of values and graph the line and quadratic function.
6. Invite volunteers to fill the tables of values on the board and graph the parabola and line. Determine the solutions as a class.
Solution:

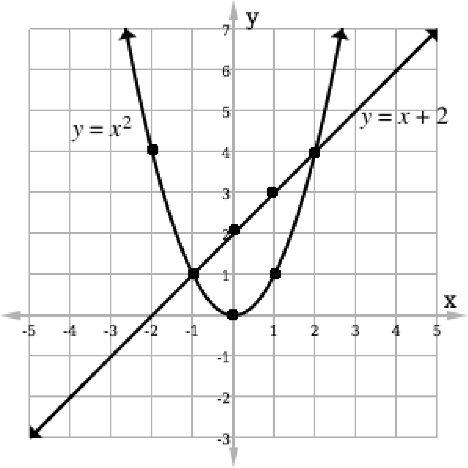
Fill the table of values for $y = x + 2$:

x	-2	-1	0	1	2
y	0	1	2	3	4

Fill the table of values for $y = x^2$:

x	-2	-1	0	1	2
y	0	1	2	3	4

Plot the points from both tables. Draw the line and curve:



Solutions: $(-1, 1)$, $(2, 4)$

7. Explain:
   a. Simultaneous linear and quadratic equations can have 0, 1 or 2 solutions.
   b. The solutions are ordered pairs, (x, y).

c. A system of equations where they are both linear can be solved in a similar way, using either substitution or graphing. Linear equations can only have 1 solution, because 2 straight lines can only intersect at 1 point.

Practice *(20 minutes)*

1. Write the following problems on the board:
    a. Solve by graphing: $y = \frac{1}{2}x + 4$ and $y = -x + 7$.
    b. Solve using substitution: $2a - b = 5$ and $3a + 2b = -24$.
    c. Find the solution(s) to the following equations using substitution: $y = x^2 - 3$ and $x = y - 9$.
2. Ask pupils to solve the problems either independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions at the same time.

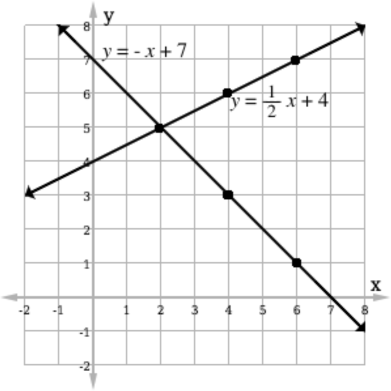
Solutions:

a. Equation (1)

x	2	4	6
y	5	6	7

Equation (2)

x	2	4	6
y	5	3	1



Answer: $(2, 5)$

b.

$$\begin{aligned} 2a - b &= 5 & (1) \\ 2a - 5 &= b & \\ \\ 3a + 2(2a - 5) &= -24 & (2) \\ 3a + 4a - 10 &= -24 & \\ 7a - 10 &= -24 & \\ 7a &= -14 & \\ \frac{7a}{7} &= \frac{-14}{7} & \\ a &= -2 & \end{aligned}$$

Substitute a into the formula for b:

$$\begin{aligned} b &= 2(-2) - 5 \\ b &= -4 - 5 \\ b &= -9 \end{aligned}$$

Answer: $a = -2, b = -9$

c.

$$x = (x^2 - 3) - 9 \quad (2)$$ Substitute equation (1) into equation (2)

$$x = x^2 - 12$$ Simplify

$$0 = x^2 - x - 12$$ Transpose x

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$ Factorise the quadratic equation

$x - 4 = 0$ or $x + 3 = 0$ Set each binomial equal to 0

$x = 4$ or $x = -3$ Transpose -4 and 3

$$y = 4^2 - 3$$ Substitute $x = 4$ into equation (1)

$$y = 16 - 3$$

$$y = 13$$

$$y = (-3)^2 - 3$$ Substitute $x = -3$ into equation (1)

$$y = 9 - 3$$

$$y = 6$$

Solutions: $(4, 13)$ and $(-3, 6)$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L127 in the Pupil Handbook.

<table>
<tr><td>Lesson Title: Variation</td><td colspan="2">Theme: Review</td></tr>
<tr><td>Lesson Number: M3-L128</td><td>Class: SSS 3</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to solve problems on variation.</td><td colspan="2">Preparation
Write the problems in step 2 of Teaching and Learning on the board.</td></tr>
</table>

Opening *(2 minutes)*

1. Discuss:
    - What is variation? (Example answers: A change in the value or quantity of something; a change in one variable that is associated with a change in another variable.)
    - What types of variation do you know? (Example answers: direct, indirect, joint, partial)
2. Explain that this lesson is on solving problems on 4 types of variation: direct, indirect, joint, and partial.

Teaching and Learning *(23 minutes)*

1. Explain briefly:
    - **Direct variation** means that two quantities x and y are related such that a change in one results in a change in the other in the same ratio.
    - **Indirect variation** means that two quantities x and y are related such that an increase in one results in a decrease in the other.
    - **Joint variation** occurs when a variable varies directly or inversely with multiple variables. For example, a variable x could vary directly with y and indirectly with z.
    - **Partial variation** occurs when a variable is related to two or more other variables added together.
2. Write the following problems on the board:
    a. A car travels 330 km in 5 hours with a uniform speed. In how many hours will it travel 4,290 km?
    b. Juliet is traveling to Freetown. If she drives at the rate of 90 kph it will take her 2 hours. How long will it take her to reach Freetown if she drives at the rate of 60 kph?
    c. The cost of making a dress is partly constant and partly varies with time. The fabric has a constant cost of Le 25,000. The tailor charges Le 10,000 per hour of work.
        i. Find the relationship, using C for total cost and t for time.
        ii. If the dress takes 3 hours to make, what is the total cost?
3. Discuss:
    - Problem a. is direct variation, b. is indirect variation, and c. is partial variation.

- In problem a., distance is directly proportional to time. This is because the car is traveling at a uniform speed.

d. In problem b, speed is inversely proportional to time. If Juliet drives faster, it will take less time to reach Freetown. If she drives slower, it will take more time.
e. In problem c, the total cost of a dress varies partly as the amount of time spent in hours.

4. Solve the problems as a class. Ask pupils to give the steps as you solve on the board. Make sure they understand each type of variation.

   Solutions:

   a. A car travels 330 km in 5 hours at a uniform speed. In how many hours will it travel 4,290 km?
   Remind pupils that because distance and time are directly proportional, we have the formula $\mathrm{d} = \mathrm{kt}$, where d and t are time.
   Find the formula:

$$\begin{aligned} d &\propto t \\ d &= kt \\ 330 &= k(5) \\ k &= \frac{330}{5} = 66 \\ d &= 66t \end{aligned}$$

   Solve for t when $d = 4{,}290$ km:

$$\begin{aligned} 4{,}290 &= 66t \\ t &= \frac{4{,}290}{66} \\ t &= 65 \end{aligned}$$

   b. Write the relationship between speed and time, and solve:

$$\begin{aligned} s &\propto \frac{1}{t} && \text{Relationship} \\ s &= \frac{k}{t} && \text{Equation} \\ 90 &= \frac{k}{2} && \text{Substitute known values for } s \text{ and } t \\ k &= 90 \times 2 && \text{Solve for the constant, } k \\ k &= 180 && \\ s &= \frac{180}{t} && \text{Write the formula} \\ 60 &= \frac{180}{t} && \text{Substitute } s = 60 \\ t &= \frac{180}{60} && \\ t &= 3 \text{ hours} && \end{aligned}$$

   c. Apply the partial variation formula $C = k_1 + kt$, where C is total cost, t is time in hours, k_1 is the fixed constant (for material), and k is the constant associated with variation, in this case the amount charged per hour of labor.

i. The relationship is $C = 25{,}000 + 10{,}000t$

ii. Total cost is Le 55,000.00:

$C = 25{,}000 + 10{,}000(3)$ Substitute $t = 3$

$= 25{,}000 + 30{,}000$ Simplify

$= 55{,}000$

Practice *(14 minutes)*

1. Write the following problems on the board:
   a. Three pupils can brush the schoolyard in 4 hours. If it needs to be done in 2 hours, how many pupils are needed in total?
   b. z varies jointly with x and y. When $x = 3$, $y = 8$, and $z = 6$. Find z when $x = 6$ and $y = 4$.
2. Discuss:
   - What type of variation is problem a.? (Answer: Inverse variation)
   - What type of variation is problem b.? (Answer: Joint variation)
3. Ask pupils to work independently or with seatmates to solve the problems. Remind them to refer to the example problems in the Pupil Handbook if needed.
4. Walk around to check for understanding and clear misconceptions.
5. Invite volunteers to come to the board to write the solutions.

Solutions:

i. Let $p =$ pupils and $t =$ time

$p \propto \frac{1}{t}$ Identify the relationship between p and t

$p = \frac{k}{t}$

$3 = \frac{k}{4}$ Substitute known values for p and t

$k = 3 \times 4$ Solve for the constant, k

$k = 12$

$p = \frac{12}{2}$ Write the formula

$p = 6$ pupils

Answer: Six pupils are needed to brush the yard in 2 hours.

ii. When the problem does not say whether joint variation is direct or inverse, assume both variables vary directly.

$z \propto xy$ Identify the relationship between z, x and y

$z = kxy$

$6 = k(3)(8)$ Substitute known values for z, x and y

$6 = 24k$ Solve for the constant, k

$k = \frac{6}{24}$

$k = \frac{1}{4}$

$z = \frac{1}{4}xy$ Write the formula

$z = \frac{1}{4}(6)(4)$ Substitute $x = 6$ and $y = 4$ into the formula

$z = \frac{1}{4}(24)$ Simplify

$z = 6$

Closing *(1 minute)*

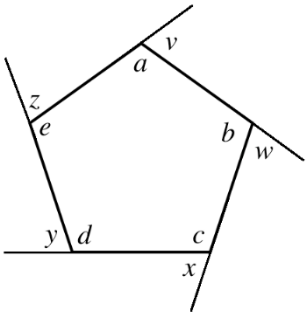
1. For homework, have pupils do the practice activity of PHM3-L128 in the Pupil Handbook.

Lesson Title: Angles of polygons	**Theme:** Review	
Lesson Number: M3-L129	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems on the angles of polygons.	**Preparation** None	

Opening *(2 minutes)*

1. Discuss:
    - What do you know about the angles in a triangle? (Example answer: They add up to 180°.)
    - What do you know about the angles in other shapes? (Example answers: Each shape's interior angles sum to a given amount; shapes have interior and exterior angles.)
2. Explain that this lesson is on solving problems the angles of polygons.

Teaching and Learning *(17 minutes)*

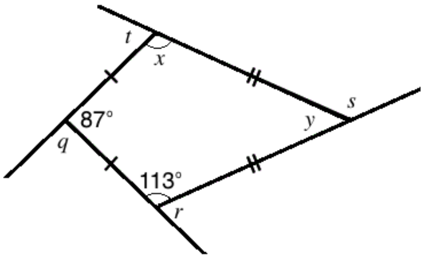


1. Draw the pentagon diagram at right on the board:
2. Explain:
    - Interior angles are inside of a shape, and in this example are a, b, c, d, e.
    - Exterior angles lie outside of the shape, and in this example are v, w, x, y, z.
3. Ask pupils to look at the table in PHM3-T3-W33-L124. This is also given on the right.
4. Explain:
    - The interior angles of the given polygons will always sum to these amounts.
    - These are used to solve various problems. There are also some related equations.
5. Write on the board:
    - Sum of the interior angles in a polygon: $(n-2) \times 180°$ where n is the number of sides.
    - Measure of each interior angle of a **regular** polygon: $\frac{(n-2)\times 180°}{n}$ where n is the number of sides.
6. Explain:
    - A regular polygon has all of its sides equal.

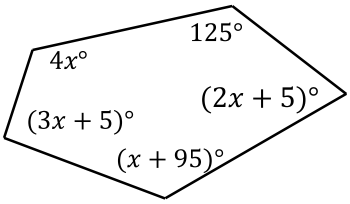
Sides	Name	Sum of Interior Angles
3	Triangle	180°
4	Quadrilateral	360°
5	Pentagon	540°
6	Hexagon	720°
7	Heptagon	900°
8	Octagon	1,080°
9	Nonagon	1,260°
10	Decagon	1,440°

- For polygons that are not regular, missing angles can be found by subtracting known angles from the sum of the angles for that type of polygon.

7. Write the problems on the board:
   a. Calculate the sum of the interior angles of a polygon with 21 sides.
   b. Find the missing interior and exterior angles of the kite:



   c. In the pentagon below, solve for x:



   d. Solve the problems as a class. Ask pupils to give the steps as you solve on the board.

Solutions:

a. Substitute $n = 21$ in the formula and solve:

$$\begin{aligned} \text{Sum of angles} &= (n-2) \times 180° \\ &= (21-2) \times 180° \\ &= 19 \times 180° \\ &= 3{,}420° \end{aligned}$$

b. Find the interior angles first. $x = 113°$ because these 2 opposite angles of the kite are equal. Subtract from $360°$ (the angles of a quadrilateral) to find y:

$y = 360° - 113° - 113° - 87° = 47°$

Find the exterior angles by subtracting the interior angles from $180°$:

$q = 180° - 87° = 93°$

$r = 180° - 113° = 67°$

$s = 180° - 47° = 133°$

$t = 180° - 113° = 67°$

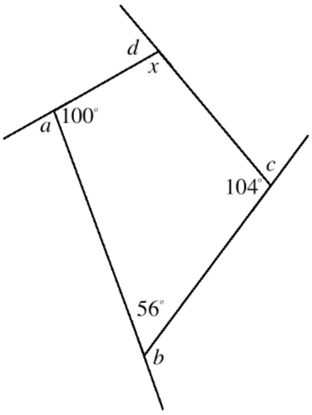
c. Use the fact that the angles of a pentagon add up to 540°.

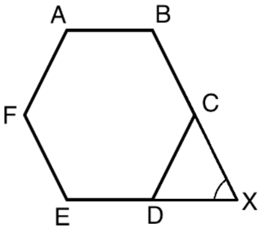
$$\begin{aligned} 540° &= 5° + (2x+5)° + (x+95)° + (3x+5)° + 4x° && \text{Add the angles} \\ &= 25° + 5° + 95° + 5°) + (2x + x + 3x + 4x)° && \text{Combine like terms} \\ &= 0° + 10x° \\ 540° - 230° &= x° \\ 310° &= x° && \text{Transpose } 230° \end{aligned}$$

$31^\circ \;=$ Divide by 10°

Practice *(20 minutes)*

1. Write the following problems on the board:
   a. Find the missing angles a, b, c, d and x in the diagram:
   b. In the diagram, $ABCDEF$ is a regular polygon. When they are extended, sides BC and ED meet at point X. Find the measure of $\angle X$.





   c. A pentagon has one exterior angle of 70°. Two other angles are $(90 - x)^\circ$, while the remaining angles are each $(40 + 2x)^\circ$. Find the value of x.
   d. The interior angle of a regular polygon is 140°. How many sides does it have?
2. Ask pupils to work independently or with seatmates to solve the problems. Remind them to refer to the example problems in the Pupil Handbook if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions.

Solutions:

a. $x = 360^\circ - 100^\circ - 104^\circ - 56^\circ = 100^\circ$; $a = 180^\circ - 100^\circ = 80^\circ$; $b = 180^\circ - 56^\circ = 124^\circ$; $c = 180^\circ - 104^\circ = 76^\circ$; $d = 180^\circ - 100^\circ = 80^\circ$

b. Note that the triangle is made up of 2 external angles of a hexagon, and X.

$$\begin{aligned}\text{Exterior angle of a hexagon} &= \frac{360^\circ}{n}\\ &= \frac{360^\circ}{6}\\ &= 60^\circ\end{aligned}$$

Subtract to find X. $X = 180^\circ - 60^\circ - 60^\circ = 60^\circ$

c. Set the sum of the angles equal to 360°, which is always the sum of the exterior angles. Solve for x.

$$\begin{aligned}360^\circ &= 70^\circ + 2(90 - x)^\circ + 2(40 + 2x)^\circ &&\\ &= 70^\circ + 180^\circ - 2x^\circ + 80^\circ + 4x^\circ && \text{Remove brackets}\\ &= (70^\circ + 180^\circ + 80^\circ) + (-2x^\circ + 4x^\circ) && \text{Combine like terms}\\ &= 330^\circ + 2x^\circ && \text{Simplify}\\ 360^\circ - 330^\circ &= 2x^\circ && \text{Transpose } 330^\circ\\ 30^\circ &= 2x^\circ && \text{Divide by } 2^\circ\\ 15^\circ &= x &&\end{aligned}$$

d. Use the formula for the interior angle to find the number of sides, n:

$$\text{Interior angle} = \frac{(n-2)\times 180^\circ}{n}$$

$$\begin{aligned}
140° &= \frac{(n-2)\times 180°}{n} \\
140°n &= (n-2)\times 180° && \text{Multiply throughout by } n \\
140°n &= 180°n - 360° && \text{Distribute the right-hand side} \\
140°n - 180°n &= -360° && \text{Transpose } 180°n \\
-40°n &= -360° && \text{Divide throughout by } -40° \\
n &= 9
\end{aligned}$$

Answer: The polygon has 9 sides.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L129 in the Pupil Handbook.

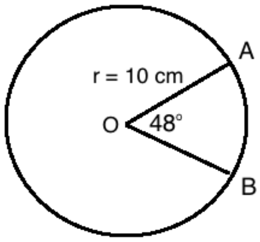
Lesson Title: Circles	**Theme:** Review	
Lesson Number: M3-L130	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems on circumference and area of circles.	**Preparation** None	

Opening *(2 minutes)*

1. Ask volunteers to write the formulae for circumference and area of a circle on the board. (Answers: $C = 2\pi r$; $A = \pi r^2$)
2. Remind pupils that circumference is the perimeter, or distance around a circle.
3. Explain that this lesson is on solving problems related to area and circumference of a circle, including finding the length of an arc and area of a sector of a circle.

Teaching and Learning *(21 minutes)*

1. Write on the board: For a circle with radius 14 metres, find: a. The circumference; b. The area. Use $\pi = \frac{22}{7}$.
2. Ask pupils to give the steps to solve the problem. As they give them, solve on the board:
   a. $C = 2\pi r = 2 \times \frac{22}{7} \times 14 = 2 \times 22 \times 2 = 88$ m
   b. $A = \pi r^2 = \frac{22}{7} \times 14^2 = \frac{22}{7} \times 196 = 616$ m^2
3. Write the following problem on the board: An arc subtends an angle of 48° at the centre of a circle with radius 10 cm. Find the length of the arc. Use $\pi = \frac{22}{7}$.



4. Explain:
   - An **arc** is a part of the circumference of a circle.
   - The length of an arc of a circle is in proportion to the angle it subtends.
   - To find the length of an arc, multiply the circumference by θ as a fraction of $360°$.
5. Write on the board: Length of arc $= \frac{\theta}{360°} \times C = \frac{\theta}{360°} \times 2\pi r$
6. Solve the problem on the board, explaining each step:

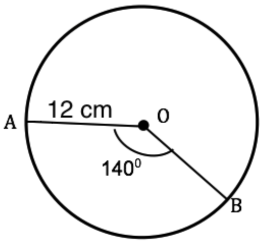
length of arc $= \frac{\theta}{360} \times 2\pi r$ Formula

$= \frac{48}{360} \times 2 \times \frac{22}{7} \times 10$ Substitute values

$\frac{48\times2\times22\times10}{360\times7}$ Simplify

$= 8.38$ cm

7. Write the following problem on the board: The radius of a circle is 12 cm. Find the area of a sector AOB which has an angle of 140^0. Use $\pi = \frac{22}{7}$.



8. Explain:
    - A **sector** is part of the area of a circle.
    - As with an arc, the area of a sector is in proportion to the angle it subtends.
    - To find the area of a sector, multiply the circumference by θ as a fraction of $360°$.
9. Write on the board: Area of sector $= \frac{\theta}{360°} \times A = \frac{\theta}{360°} \times \pi r^2$
10. Solve the problem on the board, explaining each step:

A	$=$	$\frac{140}{360} \times \pi r^2$	Formula
	$=$	$\frac{140}{360} \times \frac{22}{7} \times 12^2$	Substitute values
	$=$	$\frac{140 \times 22 \times 144}{360 \times 7}$	Simplify
A	$=$	176 cm^2	

11. Write the following problem on the board: An arc subtends an angle of $125°$ at the centre of a circle of radius 9 cm. Find, correct to 2 decimal places: a. The length of the arc; b. The area of the sector. [Use $\pi = 3.14$]
12. Ask pupils to work with seatmates to solve.
13. Walk around to check for understanding and clear misconceptions.
14. Invite volunteers to write the solutions on the board.

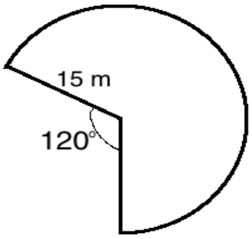
Solutions:

a. $C = \frac{\theta}{360} \times 2\pi r = \frac{125}{360} \times 2 \times 3.14 \times 9 = 19.63$ cm

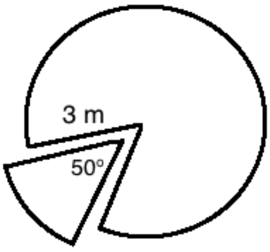
b. $A = \frac{\theta}{360} \times \pi r^2 = \frac{125}{360} \times 3.14 \times 9^2 = 88.31$ cm^2

Practice *(16 minutes)*

1. Write the following problems on the board:
    a. Find the radius of a circle whose area is 66 m^2. Give your answer to 2 decimal places.
    b. A circle with a radius of 7 cm has an arc length of 11 cm. Find its angle. Use $\pi = \frac{22}{7}$
    c. Calculate the arc length of the given shape, which a segment of 120° has been removed from. Give your answer to 3 significant figures. Use $\pi = 3.14$.
    d. The area of a sector is 690 cm^2. If the radius of the circle is 0.45 m, find the angle of the sector to the nearest degree.



   e. A sector of $50°$ was removed from a circle of radius 3 m. What is the area of the circle left? Give your answer to 1 decimal place.



2. Ask pupils to work independently to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions.

   Solutions:

a.

$$A = \pi r^2$$
$$r^2 = \frac{A}{\pi}$$
$$= \frac{66}{3.14} \qquad \text{Take } \pi = 3.14$$
$$= 21.019$$
$$r = \sqrt{21.019} = 4.58$$

b.

$$\text{length of arc} = \frac{\theta}{360} \times 2\pi r$$
$$\theta = \frac{\text{length of arc} \times 360}{2\pi r}$$
$$= \frac{11 \times 360}{2 \times \frac{22}{7} \times 7}$$
$$= \frac{11 \times 360 \times 7}{2 \times 22 \times 7}$$
$$\theta = 90°$$

c.

Subtract to find the angle: $\theta = 360 - 120 = 240°$

$$\text{length of arc} = \frac{\theta}{360} \times 2\pi r$$
$$= \frac{240}{360} \times 2 \times 3.14 \times 15$$
$$= \frac{2}{3} \times 2 \times 3.14 \times 15$$
$$= 62.8 \text{ m}$$

d.

$$A = \frac{\theta}{360} \times \pi r^2$$
$$690 = \frac{\theta}{360} \times 3.14 \times 45^2 \qquad 0.45 \text{ m} = 45 \text{ cm}$$
$$\theta = \frac{690 \times 360}{3.14 \times 45^2}$$
$$\theta = 39°$$

e.

$$\text{From the diagram, } \theta = 360 - 50 = 310°$$
$$\text{Area of the sector} = \frac{\theta}{360} \times \pi r^2$$
$$\text{Area of the sector} = \frac{310}{360} \times 3.14 \times 3^2 = 24.3 \text{ cm}^2$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L130 in the Pupil Handbook.

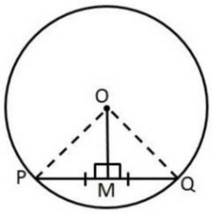
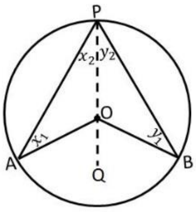
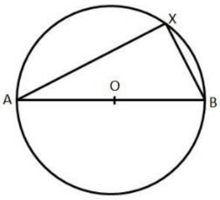
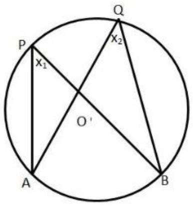
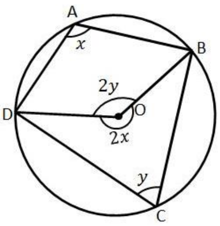
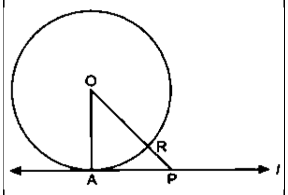
Lesson Title: Circle theorems	**Theme:** Review	
Lesson Number: M3-L131	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems using circle theorems.	**Preparation** None	

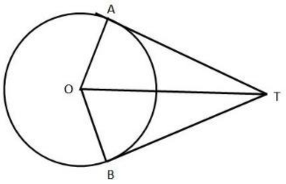
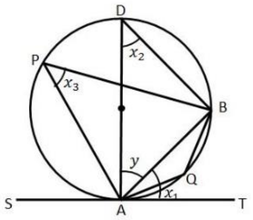
Opening *(2 minutes)*

1. Ask volunteers to describe some circle theorems they can recall from previous lessons. Allow discussion.
2. Explain that this lesson is on solving problems using circle theorems. Such problems may ask you to find the measures of angles or line segments.

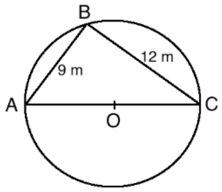
Teaching and Learning *(22 minutes)*

1. Revise each circle theorem with pupils (briefly). The theorems are given in the table below. These are also in the Pupil Handbook, so it is not necessary to write them on the board.

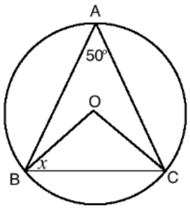
Circle Theorem 1: A straight line from the centre of a circle that bisects a chord is at right angles to the chord. $|\text{PM}| = |\text{QM}|$ and OM ⊥ PQ.		**Circle Theorem 2:** The angle subtended at the centre of a circle is twice that subtended at the circumference. ∠AOB=2×∠APB.	
Circle Theorem 3: The angle in a semi-circle is a right angle. $\angle AXB$=90°		**Circle Theorem 4:** Angles subtended at the circumference by a chord or arc in the same segment of a circle are equal. $\angle APB$=$\angle AQB$.	
Circle Theorem 5: The opposite angles of a cyclic quadrilateral are supplementary. $\angle BAD + \angle BCD =$ 180° and $\angle ABC +$ $\angle ADC = 180°$.		**Circle Theorem 6:** The angle between a tangent and a radius is equal to 90°. OA ⊥ l.	

Circle Theorem 7: The lengths of the two tangents from a point to a circle are equal. |TA| = |TB|. Also, ∠AOT=∠BOT and ∠ATO=∠BTO.		**Circle Theorem 8:** The angle between a chord and a tangent at the end of the chord equals the angle in the alternate segment. ∠TAB = ∠APB and ∠SAB = ∠AQB. This is the alternate segment theorem.	

2. Write the following problems on the board:
   c. In the diagram, AC is a diameter of the circle with centre O. Find |AC|.



   d. O is the centre of the circle in the diagram. Find the value of x.



3. Solve the problems as a class. Ask volunteers to give the steps, and work the problems on the board.
   Solutions:
   a. From circle theorem 3, an angle subtended in a circle by the diameter is a right angle. Thus, ABC is a right-angled triangle. Apply Pythagoras' theorem to find the diameter:

$$AC^2 = AB^2 + BC^2$$
$$AC^2 = 9^2 + 12^2$$
$$AC^2 = +144$$
$$AC^2 = 5$$
$$AC = \overline{25} = 15 \text{ m}$$

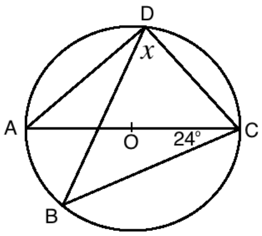
   b. According to circle theorem 2, ∠BOC=2∠BAC. Apply this to find ∠BOC:

$$\angle \text{BOC} = 2 \times 50°$$
$$= 100°$$

Note that triangle BOC is isosceles, because 2 sides are the radius, r=|BO|=|OC|. Therefore, $x = \angle \text{OBC} = \angle \text{OCB}$. Subtract the known angle in the triangle (100°) from 180°, and divide by 2 to find x.

$$x = \frac{180° - 100°}{2}$$
$$= \frac{80°}{2} = 40°$$

4. Write the following problem on the board: Find the measure of x in the diagram at right:



5. Ask pupils to work with seatmates to solve.
6. Walk around to check for understanding and clear misconceptions.
7. Invite volunteers to write the solutions on the board.

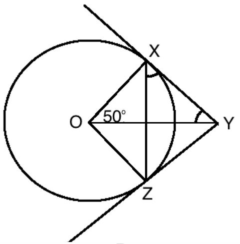
   Solutions:

   Note that $\angle ADB = \angle ACB = 24°$ according to circle theorem 4. Also note that $\angle ADC$ is subtended at the circumference, so $\angle ADC = 90°$ according to circle theorem 3. Find x by subtracting $\angle ADB$ from $\angle ADC$:

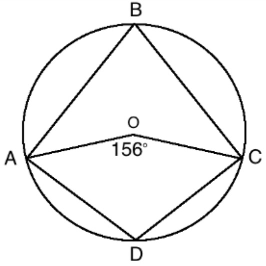
$$\begin{aligned} x &= \angle ADC - \angle ADB \\ &= 90° - 24° \\ &= 66° \end{aligned}$$

Practice *(15 minutes)*

1. Write the following problems on the board:
   a. In the given diagram, O is the centre of the circle. ZY and YZ are tangent lines, and $\angle XOY = 50°$. Calculate the measures of angles $\angle YXZ$ and $\angle XYZ$.



   b. The diagram is a circle with centre O. ABCD are points on the circle. Find the measure of $\angle ADC$.



2. Ask pupils to work independently to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions.

   Solutions:

   a. Note that $\angle OXY = 90°$ according to circle theorem 6. Therefore, we can subtract $\angle OXZ$ from $90°$ to find $\angle YXZ$. Find $\angle OXZ$ using the triangle formed by the chord, which forms a perpendicular angle with YO.

$$\angle OXZ = 180° - 90° - 50° = 40°$$

   Therefore, $\angle YXZ = 90° - \angle OXZ = 90° - 40° = 50°$.

   Note that $\angle XYZ$ can be found using $\triangle\ OXY$. Subtract the known angles from $180°$:

   $\angle XYZ = 180° - 90° - 50° = 40°$

   Answers: $\angle YXZ = 50°$, $\angle XYZ = 40°$

   b. According to circle theorem 2, $\angle \text{AOC}=2\angle \text{ABC}$. Apply this to find $\angle \text{ABC}$:

$$\begin{aligned} \angle \text{ABC} &= \tfrac{1}{2}156° \\ &= 78° \end{aligned}$$

   According to circle theorem 5, $\angle ABC + \angle ADC = 180°$. Apply this to find $\angle ADC$:

$$\begin{aligned} \angle ADC &= 180° - \angle ABC \\ &= 180° - 78° \\ &= 102° \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L131 in the Pupil Handbook.

Lesson Title: Transformations on the Cartesian plane	**Theme:** Review	
Lesson Number: M3-L132	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to transform figures on the Cartesian plane.	**Preparation** None	

Opening *(2 minutes)*

1. Discuss:
    - What is a transformation? (Answer: Transformation changes the position, shape, or size of an object.)
    - What transformations do you know? (Answers: Translation, reflection, rotation, enlargement)
2. Explain that this lesson is on solving problems involving transformations.

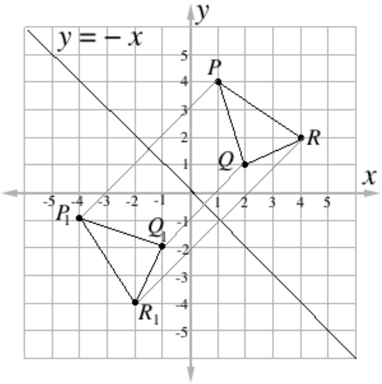
Teaching and Learning *(22 minutes)*

1. Revise each transformation with pupils. Allow discussion of each.
    - **Translation** – moves all the points of an object in the same direction and the same distance without changing its shape or size.
    - **Reflection** – an object is reflected in a line of symmetry. The direction that it faces changes, but not its size.
    - **Rotation** – an object rotates (or turns) around a point, which is called the centre of rotation.
    - **Enlargement** – the object is magnified (made larger) or diminished (made smaller). Its shape does not change, but its size does.
2. Remind pupils that this is review of lessons 87 through 96 of this course. They may refer to information in the appropriate section of the Pupil Handbook.
3. Write the following problems on the board:
    a. Triangle PQR has coordinates $P(1,4), Q(2,1)$ and $R(4,2)$. Find the co-ordinates, P_1, Q_1 and R_1 of the image of the triangle formed under reflection in the line $y = -x$.
    b. Use the appropriate formula to find the co-ordinates of the image point when point $X(-3,-2)$ is rotated $90°$ clockwise about the point $(0,-4)$.
4. Solve the problems as a class. Ask volunteers to give the steps, and work the problems on the board.

    Solutions:

    a. Reflection:

    Step 1. Assess and extract the given information from the problem. Given: points $P(1,4), Q(2,1)$ and $R(4,2)$, line $y = -x$



Step 2. Draw the x- and y- axes Locate the points P, Q and R on the graph. Draw the lines joining the points.

Step 3. Draw the line $y = -x$

Step 4. Draw a line at right angles from P to the mirror line ($y = -x$). Measure this distance.

Step 5. Measure the same distance on the opposite side of the mirror line ($y = x$) to locate the point P_1 on the graph.

Step 6. Write the new coordinates: $P_1(-4,-1)$, $Q_1(-1,-2)$ and $R_1(-2,-4)$

b. Rotation:

Apply the following formula for rotation:

$$\begin{pmatrix} x-a \\ y-b \end{pmatrix} \longrightarrow \begin{pmatrix} (y-b)+a \\ -(x-a)+b \end{pmatrix} = \begin{pmatrix} (y-(-4))+0 \\ -(x-0)+(-4) \end{pmatrix} = \begin{pmatrix} y+4 \\ -x-4 \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ -2 \end{pmatrix} \longrightarrow \left. \begin{matrix} -2+4 \\ (-3)-4 \end{matrix} \right) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$ ply the formula

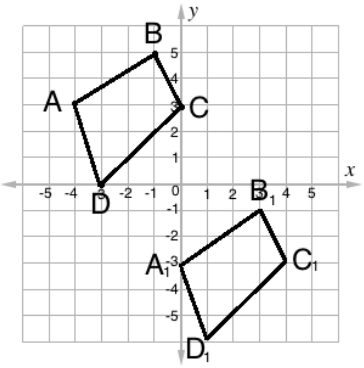
$X(-3,-2)$ rotated 90° clockwise about the point $(0,-4)$ gives $(2,-1)$

5. Write the following problem on the board: Quadrilateral $ABCD$ has co-ordinates $A(-4{,}3)$, $B(-1{,}5)$ and $C(0{,}3)$ and $D(-3{,}0)$.
   a. Draw quadrilateral $ABCD$ on the Cartesian plane.
   b. Draw quadrilateral $A_1B_1C_1D_1$, which is $ABCD$ translated by the vector $\binom{4}{-6}$.
6. Ask pupils to work with seatmates to solve.
7. Walk around to check for understanding and clear misconceptions.
8. Invite volunteers to write the solution on the board.

   Solution:
   a. Identify points A, B, C, and D on the Cartesian plane, and connect them in a quadrilateral as shown below.
   b. Identify point A_1 by translating $A(-4{,}3)$ by the vector $\binom{4}{-6}$:

$$\binom{-4}{3} \longrightarrow \binom{-4}{3} + \binom{4}{-6}$$
$$= \binom{-4+4}{3-6}$$
$$= \binom{0}{-3}$$

Draw the image of $A_1B_1C_1D_1$ using A_1 as a reference point. Note that it has the same shape and size as $ABCD$.



Practice *(15 minutes)*

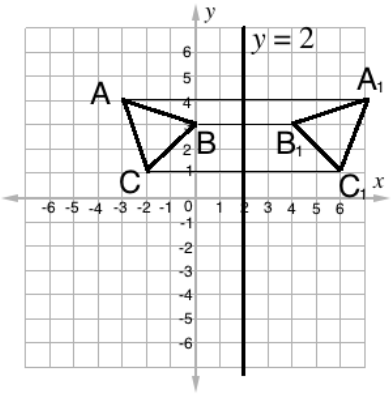
1. Write the following problems on the board:
   a. Triangle ABC has coordinates $A(-3{,}4), B(0{,}3)$ and $C(-2, 1)$. Find the coordinates, A_1, B_1 and C_1 of the image of the triangle formed under reflection in the line $y = 2$.

   b. Find the image of $(3,-2)$ under the enlargement with scale factor of 2 from the point $(2,1)$.

2. Ask pupils to work independently to solve the problems. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board to write the solutions.

   Solutions:

   a. **Step 1.** Draw the x- and y- axes and locate the points $A,\ B$ and C on the graph. Draw the lines joining the points. Draw the line $y = 2$.
   Step 2. Draw a line at right angles from each point (A, B and C) to the mirror line ($y = 2$).
   Step 3. Measure the same distance on the opposite side of the mirror line ($y = 2$) to locate and plot the points A_1, B_1 and C_1 on the graph.
   Step 4. Write the new coordinates: $A_1(7,4)$, $B_1(4,3)$ and $C_1(6,1)$



   b. Apply the formula for enlargement:

$$\begin{pmatrix} x-a \\ y-b \end{pmatrix} \longrightarrow \begin{matrix} -2 \\ 2-1 \end{matrix}\Big) = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$ btract components of $(2,1)$ from $(3,-2)$

$$\begin{pmatrix} 1 \\ -3 \end{pmatrix} \longrightarrow \begin{matrix} 1 \\ -3 \end{matrix}\Big) = \begin{pmatrix} 2 \\ -6 \end{pmatrix}$$ enlarge using given scale factor

$$\begin{pmatrix} 2 \\ -6 \end{pmatrix} \longrightarrow \begin{matrix} +2 \\ 6+1 \end{matrix}\Big) = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$$ add back components of $(2,1)$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L132 in the Pupil Handbook.

Lesson Title: Area and surface area	**Theme:** Review	
Lesson Number: M3-L133	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the area of 2-dimensional figures and the surface area of 3-dimensional figures.	**Preparation** None	

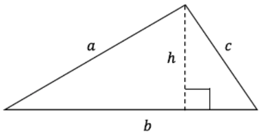
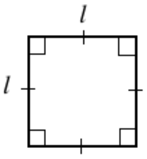
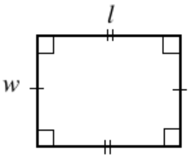
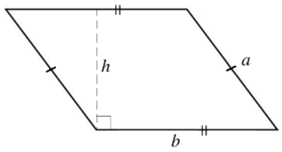
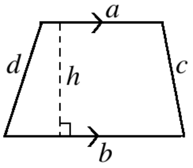
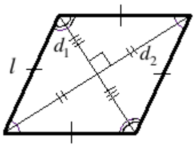
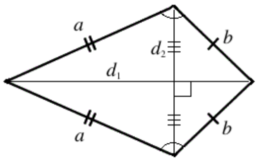
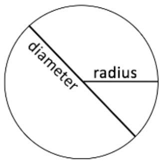
Opening *(1 minute)*

1. Explain that this lesson is on calculating the area of 2-dimensional shapes, and the surface area of 3-dimensional solids.

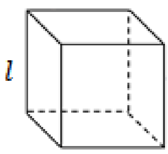
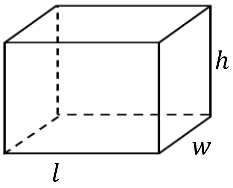
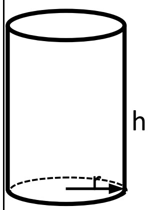
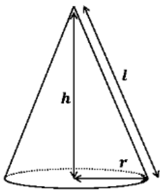
Teaching and Learning *(16 minutes)*

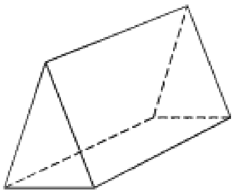
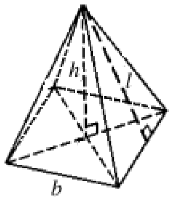
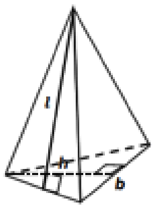
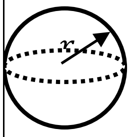
1. Briefly review the area and surface area formulae in the Pupil Handbook (in the table below). For the sake of time, it is not necessary to draw them all on the board. Identify the formulae for each. Make sure pupils understand what the diagrams look like, and what the variables stand for.

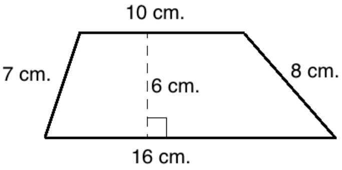
Area Formulae:

Triangle  $A = \frac{1}{2}b \times h$	**Square**  $A = l \times l = l^2$	**Rectangle**  $A = l \times w$	**Parallelogram**  $A = b \times h$
Trapezium  $A = \frac{1}{2}(a+b)h$	**Rhombus**  $A = \frac{1}{2}d_1 \times d_2$	**Kite**  $A = \frac{1}{2}d_1 \times d_2$	**Circle**  $A = \pi r^2$

Surface Area formulae:

Cube  $SA = 6 \times l^2$	**Cuboid**  $SA = 2(lh + hw + lw)$	**Cylinder**  $SA = 2\pi r(r + h)$	**Cone**  $SA = \pi r(l + r)$

Triangular Prism	**Pyramid with a rectangular base**	**Pyramid with a triangular base**	**Sphere**
			
No formula	$SA = b^2 + 2bl$	$SA = \frac{1}{2}b(h + 3l)$	$SA = 4\pi r^2$



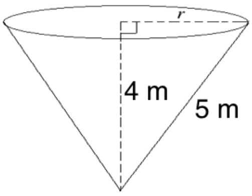
2. Write the following problem on the board: Find the area of the trapezium.
3. Discuss: How would we calculate the area? (Answer: Apply the formula for a trapezium, and substitute the given side lengths.)
4. Ask pupils to work with seatmates to find the area.
5. Invite a volunteer to write the solution on the board.
   Solution:

$$\begin{aligned} A &= \tfrac{1}{2}(a+b)h && \text{Apply the formula} \\ &= \tfrac{1}{2}(10+16)6 && \text{Substitute known values} \\ &= \tfrac{1}{2}(26)6 \\ &= 13 \times 6 \\ &= 78 \text{ cm}^2 \end{aligned}$$

6. Write the following problem on the board: Find the surface area of sphere with a radius of 0.5 metres. (Use $\pi = 3.14$)
7. Ask pupils to work with seatmates to find the surface area.
8. Invite a volunteer to write the solution on the board.
   Solution:

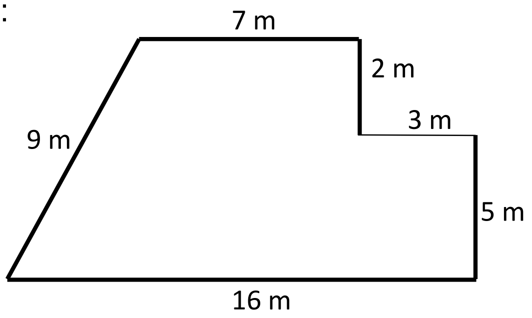
$$\begin{aligned} SA &= 4\pi r^2 && \text{Apply the formula} \\ &= 4(3.14)(0.5)^2 && \text{Substitute known values} \\ &= 4(3.14)(0.25) \\ &= 3.14 \text{ m}^2 \end{aligned}$$

Practice *(22 minutes)*



1. Write the following problems on the board:
   a. The diagram shows a cone with height 4 m and slant height 5 m. Find: i. The base radius, r; ii. The surface area of the cone, correct to 3 significant figures. Use $\pi = \frac{22}{7}$.
   b. The surface area of a cylinder is 440 cm^2. If the radius of its base is 7 cm, find its height. Use $\pi = \frac{22}{7}$.

c. Find the area of the compound shape:



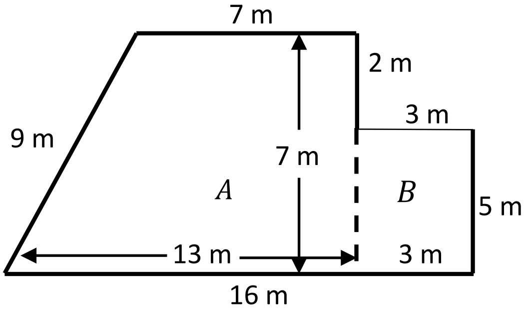
2. Ask pupils to solve the problems either independently or with seatmates. Solve problems as a class if they do not understand.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to come to the board simultaneously to write the solutions.
   Solutions:
   a. i. Use Pythagoras' theorem to find the radius: $r = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3\text{ m}$

   ii. Calculate surface area: $SA = \left(\frac{22}{7}\right)(3)(5+3) = \frac{22}{7}(24) = 75.4\text{ cm}^2$

   b. Apply the formula for surface area of a cylinder. Substitute the given values and solve for h:

$$\begin{aligned} SA &= \left(\tfrac{22}{7}\right)(7)(7+h) && \text{Apply the formula} \\ 440 &= 2(22)(7+h) && \text{Substitute known values} \\ 440 &= 44(7+h) \\ \tfrac{440}{44} &= 7+h \\ 10-7 &= h \\ 3\text{ cm} &= h \end{aligned}$$

   c. Divide the compound shape into shapes from the table of formulae. This gives a trapezium and rectangle. Find the lengths of any unknown sides:



Find the area of each shape, then add them together. Area of the trapezium:

$$\begin{aligned} \text{area of A} &= \tfrac{1}{2}(a+b)h \\ &= \tfrac{1}{2}(7+13)7 \\ &= \tfrac{1}{2}(20)7 \\ &= 10 \times 7 = 70\text{ m}^2 \end{aligned}$$

Area of the rectangle:

$$\begin{aligned} \text{area of } B &= l \times w \\ &= 5 \times 3 \\ &= 15\ \mathrm{m}^2 \end{aligned}$$

Area of the compound shape:

$$\begin{aligned} \text{area of shape} &= \text{area of } A + \text{area of } B \\ &= 70 + 15 \\ &= 85\ \mathrm{m}^2 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L133 in the Pupil Handbook.

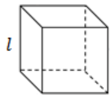
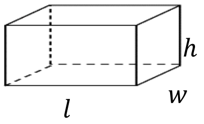
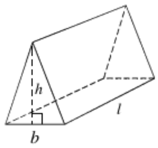
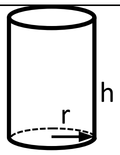
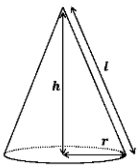
Lesson Title: Volume	**Theme:** Review	
Lesson Number: M3-L134	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate the volume of 3-dimensional figures.	**Preparation** None	

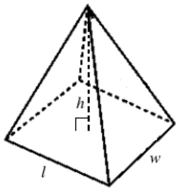
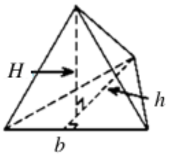
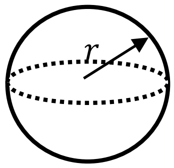
Opening *(1 minute)*

1. Explain: This lesson is on the volume of 8 different types of solids. There is not enough time in this lesson to solve problems on each. However, if you can identify the formula for volume of a solid, you can solve related problems.

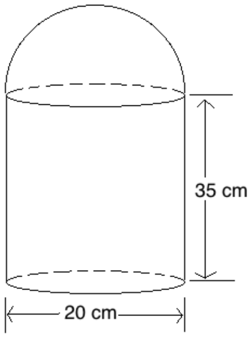
Teaching and Learning *(18 minutes)*

1. Briefly review the volume formulae in the Pupil Handbook (in the table below). For the sake of time, it is not necessary to draw them all on the board. Identify the formulae for each. Make sure pupils understand what the diagrams look like, and what the variables stand for.

Solid	Diagram	Formula
Cube		$V = l^3$, where l is the length of a side.
Cuboid		$V = lwh$, where l and w are the length and width of the base, and h is the height.
Triangular Prism		$V = \frac{1}{2}bhl$, where b and h are the base and height of the triangular face, and l is the length.
Cylinder		$V = \pi r^2 \times h$, where r is the radius of the circular face and h is the height.
Cone		$V = \frac{1}{3}\pi r^2 h,$ where r is the radius and h the height.

Pyramid with a rectangular base		$V = \frac{1}{3}lwh$, where l and w are the length and width of the base rectangle, and h is the height.
Pyramid with a triangular base		$V = \frac{1}{3}AH$, where A is the area of the triangular base, and H is the height.
Sphere		$V = \frac{4}{3}\pi r^3$, where r is the radius.

2. Write the following problem on the board: A container is in the shape of a hemisphere mounted on a cylinder. It is built such that the plane face of the hemisphere fits exactly on a circular face of the cylinder. The diameter of the cylinder is 20 cm, and its height is 35 cm.
    a. Illustrate this information in a diagram
    b. Calculate: i. The volume of the hemisphere; ii. The volume of the entire solid. Use $\pi = \frac{22}{7}$.
3. Discuss:
    - What is a hemisphere? (Answer: Half of a sphere.)
    - What does this solid look like? (Answer: A hemisphere sitting on top of a cylinder.)
4. Ask pupils to work with seatmates to draw the diagram. Remind them to label the lengths.
5. Invite a volunteer to draw the diagram on the board. →
6. Discuss: How would we find the volume of a hemisphere? (Answer: Since it is half of a sphere, find the volume of an entire sphere and multiply it by half.)
7. Solve the problem on the board as a class. Involve pupils.



b. i. Volume of the hemisphere:

$$\begin{aligned} V &= \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3 && \text{Formula for hemisphere} \\ &= \frac{2}{3}\left(\frac{22}{7}\right) 10^3 && \text{Substitute } \pi = \frac{22}{7}, r = \frac{20}{2} = 10 \\ &= \frac{44}{21} 10^3 && \text{Simplify} \\ &= 2{,}095 \text{ cm}^3 \end{aligned}$$

ii. Volume of the solid:

Step 1. Find the volume of the cylinder:

$$\begin{aligned} V &= \pi r^2 h && \text{Formula for hemisphere} \\ &= \left(\frac{22}{7}\right)(10^2)(35) && \text{Substitute } \pi = \frac{22}{7}, r = 7,\ h = 9 \end{aligned}$$

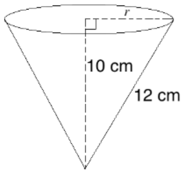
$$= 11{,}000 \text{ cm}^3$$

Step 2. Add the volume of the cylinder and hemisphere:

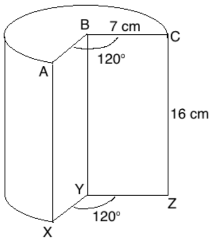
$$V = 11{,}000 + 2{,}095 = 13{,}095 \text{ cm}^3$$

Practice *(20 minutes)*

1. Write the following problems on the board:
   a. A cylindrical container has a base radius of 10 cm and a height 25 cm. How many litres, correct to the nearest litre, of liquid can it hold? Use $\pi = \frac{22}{7}$.
   b. The diagram shows a cone with height 10 cm. and slant height 12 cm. Find: i. the base radius, r; ii. the volume of the cone. Use $\pi = \frac{22}{7}$.



   c. The diagram shows part of a solid cylinder with radius 7 cm and height 16 cm. The missing piece is formed by 2 radii and an angle of 120°. Calculate the volume, correct to 1 decimal place.



2. Explain problem a: A volume of $1{,}000 \text{ cm}^3$ holds 1 litre ($1\ l = 1{,}000\text{cm}^3$). This fact may be needed to solve WASSCE problems.
3. Ask pupils to solve the problems either independently or with seatmates. Solve problems as a class if they do not understand.
4. Walk around to check for understanding and clear misconceptions.
5. Ask volunteers to come to the board simultaneously to write the solutions.

Solutions:

a. **Step 1.** Calculate volume: $V = \pi r^2 \times h = \left(\frac{22}{7}\right) 10^2 \times 25 = 7{,}857 \text{ cm}^3$

   Step 2. Divide by $1{,}000 \text{ cm}^3$ to find litres: $\frac{7{,}857 \text{ cm}^3}{1{,}000 \text{ cm}^3} = 8\ l$ to the nearest litre

b. i. Use Pythagoras' theorem to find the radius: $r = \sqrt{12^2 - 10^2} = \sqrt{144 - 100} = \sqrt{44} \text{ cm}$

   ii. Calculate volume: $V = \frac{1}{3}\left(\frac{22}{7}\right)\left(\sqrt{44}\right)^2(10) = \frac{22}{21}(44)(10) = 460.95 \text{ cm}^3$

c. **Step 1.** Find the angle of the rotation in the circular face: $360° - 120° = 240°$

   Step 2. Multiply the volume formula by the fraction of a full rotation that is in the solid $\left(\frac{240}{360} = \frac{2}{3}\right)$: $V = \frac{2}{3}\pi r^2 \times h = \frac{2}{3}\left(\frac{22}{7}\right) 7^2 \times 16 = 1{,}642.7 \text{ cm}^3$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L134 in the Pupil Handbook.

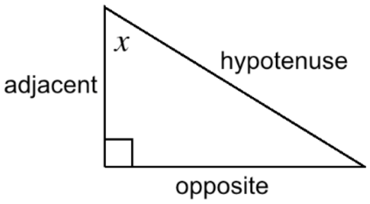
Lesson Title: Trigonometry	**Theme:** Review	
Lesson Number: M3-L135	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply trigonometric ratios to solve triangles.	**Preparation** Bring trigonometry tables to class, and ask pupils to bring trigonometry tables if they have them. Alternatively, a calculator may be used.	

Opening *(2 minutes)*

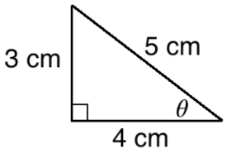
1. Write on the board: SOHCAHTOA
2. Ask volunteers to explain what this means. Ask them to write the associated trigonometric ratios on the board. (Answer: $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$)
3. Explain: This lesson is on solving triangles using the trigonometric ratios.

Teaching and Learning *(22 minutes)*

1. Draw the triangle on the board, and make sure pupils understand how to identify adjacent and opposite angles for the ratios. →



2. Draw the triangle on the board, labelled as shown:



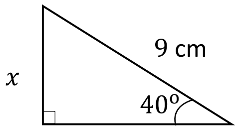
3. Apply the trigonometric ratios to angle θ the triangle. For each one, point out the relevant sides in the triangle on the board and make sure pupils understand.

$$\sin\theta\ \frac{\text{O}}{\text{H}} = \frac{3}{5}$$

$$\cos\theta\ \frac{\text{A}}{\text{H}} = \frac{4}{5}$$

$$\tan\theta\ \frac{\text{O}}{\text{A}} = \frac{3}{4}$$

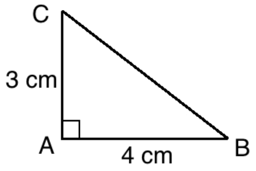
4. Write the following problem on the board: Find the measure of missing side x:



5. Discuss: Which trigonometric ratio can we use to solve this problem? Why? (Answer: Sine, because it is the ratio for opposite side and hypotenuse.)
6. Solve on the board, explaining each step:

$$\begin{aligned}\sin\theta &= \frac{\text{O}}{\text{H}}\\ \sin 40^{\text{o}} &= \frac{x}{9}\\ 9\times\sin 40^{\text{o}} &= x\\ 9\times 0.6428 &= x\\ x &= 5.7852\\ x &= 5.8\text{ cm to 1 d.p.}\end{aligned}$$

7. Show pupils how to find $\sin 40^{\text{o}}$ using a trigonometric table if they are available.
8. Find 40° in the table. It is the first number given (0.6428), because there is no decimal on the degree.
9. Explain:
   - The inverse of a function is its opposite. It's another function that can undo the given function.
   - The **inverse** trigonometric functions allow us to **find missing angles** of a right-angled triangle if we are given the sides.
   - You can use the trigonometric tables or calculators.
   - Using trigonometric tables, you will work backwards. Find the decimal number in the chart, and identify the angle that it corresponds to.
10. Write on the board and make sure pupils understand: Inverse sine "undoes" sine: $\sin^{-1}(\sin\theta)=\theta$
11. Write the following problem on the board: Find the measure of angle B:



12. Solve for angle B, explaining each step:

Step 1. Identify which function to use. The opposite and adjacent sides are known, so we will use $\tan^{-1}$.

Step 2. Find the tangent ratio. This is the ratio that you will "undo" with $\tan^{-1}$ to find the angle:

$\tan B=\frac{3}{4}=0.75$

Step 3. Find $\tan^{-1}$ of both sides to find the angle measure:

$$\begin{aligned}\tan B &= 0.75\\ \tan^{-1}(\tan B) &= \tan^{-1}(0.75)\\ B &= \tan^{-1}(0.75)\end{aligned}$$

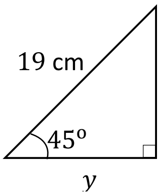
Calculate $\mathbf{\tan^{-1}(0.75)}$ using the tangent table: Look for 0.75 in the table. It is not there, but 0.7481 is there. If we add 0.0018 to 0.7481, it will give us 0.75. Find 18 in the 'add differences' table, and it corresponds to 7. Therefore, the angle is 36.87.

If you do not have trigonometric tables, solve using a calculator.

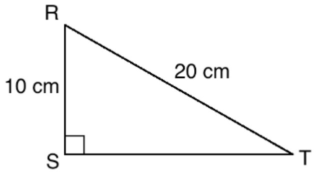
Write the answer on the board:

$$B = 36.87°$$

13. Write the following problems on the board:
    a. Find the measure of y:



    b. Find the measure of angle R:



14. Ask pupils to work with seatmates to solve the problems.
15. Invite volunteers to write the solutions on the board.
    a. Find the measure of y:

$$\begin{aligned}
\cos \theta &= \frac{\text{A}}{\text{H}} \\
\cos 45^{\text{o}} &= \frac{y}{19} \\
19 \times \cos 45^{\text{o}} &= y \\
19 \times 0.7071 &= y \\
y &= 13.4349 \\
y &= 13.4 \text{ cm to 1 d.p.}
\end{aligned}$$

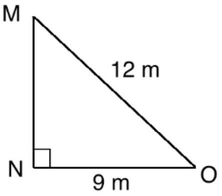
    b. Find the measure of R:

$$\begin{aligned}
\cos R &= \frac{A}{H} \\
\cos R &= \frac{10}{20} = \frac{1}{2} \\
\cos R &= 0.5 \\
\text{s}^{-1}(\cos R) &= \cos^{-1}(0.5) \\
R &= 60°
\end{aligned}$$

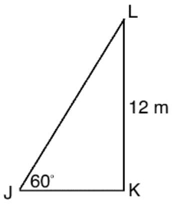
Practice *(15 minutes)*

1. Write on the board: Find the missing sides and angles of the triangles:

a.



b.



2. Ask pupils to work with independently to solve the problems. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board. All other pupils should check their own work.

Solutions:

a. Calculate $\angle M$:

$$\begin{aligned}
\sin M &= \frac{9}{12} = 0.75 \\
\sin^{-1}(\sin M) &= \sin^{-1}(0.75) \\
M &= 48.59°
\end{aligned}$$

Calculate $\angle O$:

$$180° - 90° - 48.59° = 41.41°$$

Calculate $|MN|$:

$$|MN|^2 + 9^2 = 12^2$$

Calculate $|JK|$:

$$\begin{aligned}
\tan 60° &= \frac{12}{|JK|} \\
|JK| &= \frac{12}{\tan 60°} \\
|JK| &= \frac{12}{\sqrt{3}}
\end{aligned}$$

Calculate $|JL|$:

$$\sin 60° = \frac{12}{|JL|}$$

$$
\begin{aligned}
|MN|^2 + 81 &= 144 \\
|MN|^2 &= 63 \\
|MN| &= \sqrt{63} \\
|MN| &= 3\sqrt{7}
\end{aligned}
$$

$$
\begin{aligned}
|JL| &= \frac{12}{\sin 60°} \\
|JL| &= \frac{12}{\frac{\sqrt{3}}{2}} \\
|JL| &= \frac{24}{\sqrt{3}}
\end{aligned}
$$

Pythagoras' theorem may also be used to find $|JL|$.

Calculate $\angle L$:

$$180° - 90° - 60° = 30°$$

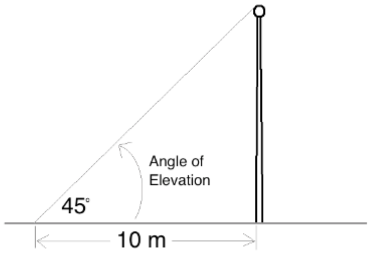
Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L135 in the Pupil Handbook.

Lesson Title: Angles of elevation and depression	**Theme:** Review	
Lesson Number: M3-L136	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems on angles of elevation and depression.	**Preparation** Write the problem in Opening on the board.	

Opening *(2 minutes)*

1. Write the following problem on the board: At a point 10 metres away from a flag pole, the angle of elevation of the top of the pole is 45°. What is the height of the pole?
2. Ask pupils to work with seatmates to draw a diagram for the problem.
3. Ask a group of pupils with a correct diagram to draw it on the board.
4. Explain that today's lesson is angles of elevation and depression.

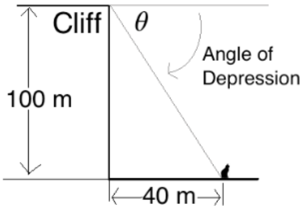


Teaching and Learning *(18 minutes)*

1. Explain:
    - "Elevation" is related to height. Problems on angles of elevation handle the angle that is associated with the height of an object.
    - Angle of elevation problems generally deal with 3 measures: the angle, the distance from the object, and the height of the object.
2. Discuss: Looking at the diagram, how would you find the height of the flag pole? (Answer: Apply trigonometry; we can use the tangent ratio.)
3. Solve the problem on the board, involving pupils in each step:

$$\begin{aligned} \tan 45° &= \frac{h}{10} && \text{Set up the equation} \\ 1 &= \frac{h}{10} && \text{Substitute } \tan 45° = 1 \\ 10 \text{ m} &= h \end{aligned}$$

5. Write the following problem on the board: A cliff is 100 metres tall. At a distance of 40 metres from the base of the cliff, there is a cat sitting on the ground. What is the angle of depression of the cat from the cliff?
6. Ask pupils to work with seatmates to draw a diagram for the problem.
7. Ask a group of pupils with a correct diagram to draw it on the board.



4. Explain:
    - "Depression" is the opposite of elevation. An angle of depression is an angle in the downward direction.

- The angle of depression is the angle made with the **horizontal** line. In this example, the horizontal line is at the height of the cliff.
- Angle of depression problems generally deal with 3 measures: the angle, the horizontal distance, and the depth of the object.
- Depth is the opposite of height. It is the distance downward.

5. Solve the problem on the board, explaining each step:

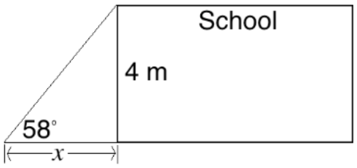
$$\begin{aligned} \tan\theta &= \tfrac{100}{40} = 2.5 && \text{Set up the equation} \\ \tan^{-1}(\tan\theta) &= \tan^{-1}(2.5) && \text{Take the inverse tangent} \\ \theta &= 68.2 && \text{Use the tangent tables} \end{aligned}$$

The angle of depression is 68.2°.

6. Write the following problems on the board:
   a. A school building is 4 metres tall. At a point x metres away from the building, the angle of elevation is 58°. Find x.
   b. A hospital is 5 metres tall. A point is x metres away from the building, and the angle of depression is 17.35°. Find x.
7. Ask pupils to work with seatmates to draw diagrams and solve each problem.
8. Invite volunteers to write the solutions and diagrams on the board.

Solutions:

a. Diagram:

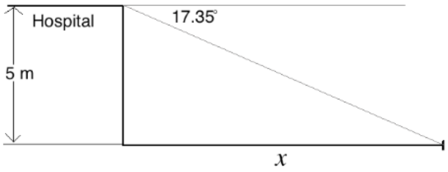


Solution:

$$\begin{aligned} \tan 58° &= \tfrac{4}{x} \\ 1.6 &= \tfrac{4}{x} \\ x &= \tfrac{4}{1.6} \\ x &= 2.5\text{ m} \end{aligned}$$

The point is 2.5 metres away.

b. Diagram:



Solution:

$$\begin{aligned} \tan 17.35° &= \tfrac{5}{x} \\ 0.3125 &= \tfrac{5}{x} \\ x &= \tfrac{5}{0.3125} \\ x &= 16\text{ m} \end{aligned}$$

The point is 16 metres away.

Practice *(19 minutes)*

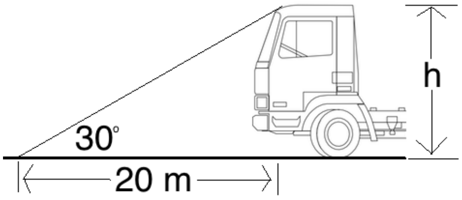
1. Write the following problems on the board:
   a. At a point 20 metres away from a truck, the angle of elevation of the top of the truck is 30°. What is the height of the truck?
   b. A house is 2 metres tall. At a distance d metres away from the house, the angle of elevation is 50.2°. Find d.
   c. A child kicked a football off the top of a tower that is 3 metres tall. The ball landed on the ground. The angle of depression of the ball from the top of the tower is 7.12°. How far is the ball from the tower?

d. A point X is on the same horizontal level as the base of a building. If the distance from X to the building is 10 m and the height of the building is 23 m, calculate the angle of depression of X from the top of the building. Give your answer to the nearest degree.

2. Ask pupils to work with independently or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board. All other pupils should check their own work.

Solutions:

a. Diagram:



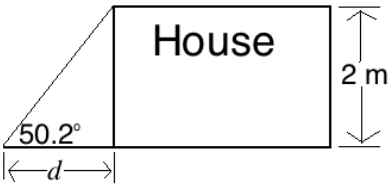
Solution:

Using special angle 30:

$$\begin{aligned} \tan 30° &= \frac{h}{20} \\ \frac{\sqrt{3}}{3} &= \frac{h}{20} \\ h &= \frac{20\sqrt{3}}{3} \text{ m} \end{aligned}$$

Alternatively, pupils can use $\tan 30° = 0.5774$, and find $h = 11.548$ m.

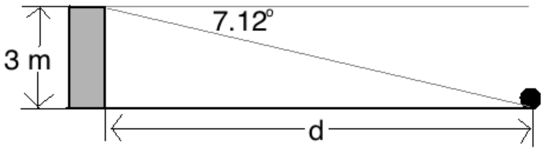
b. Diagram:



Solution:

$$\begin{aligned} \tan 50.2° &= \frac{2}{d} \\ 1.2 &= \frac{2}{x} \\ x &= \frac{2}{1.2} \\ x &= 1.7 \text{ m} \end{aligned}$$

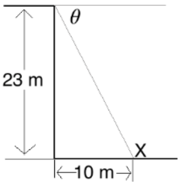
c. Diagram:



Solution:

$$\begin{aligned} \tan 7.12° &= \frac{3}{d} \\ 0.125 &= \frac{3}{d} \\ d &= \frac{3}{0.125} \\ d &= 24 \text{ m} \end{aligned}$$

d. Diagram:



Solution:

$$\begin{aligned} \tan\theta &= \frac{23}{10} = 2.3 \\ \tan^{-1}(\tan\theta) &= \tan^{-1}(2.3) \\ \theta &= 66.5 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L136 in the Pupil Handbook.

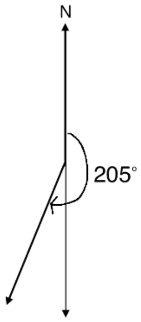
Lesson Title: Bearings and distances	**Theme:** Review	
Lesson Number: M3-L137	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems on bearings and distance.	**Preparation** Bring a protractor and trigonometry tables, and ask pupils to bring them if available.	

Opening *(1 minute)*

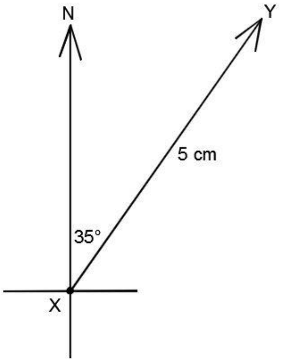
1. Discuss: What are bearings? What are they used for? (Example answer: Bearings are a navigational tool used to determine the direction and distance between 2 points.)
2. Explain that today's lesson is on finding bearings.

Teaching and Learning *(25 minutes)*

1. Explain: Three-figure bearings are bearings given in 3 digits, from 000° to 360°. These 3 digits give the angle of the bearing from geographic north.
2. Use a protractor to draw and label 2 examples on the board, 009° and 205°. Make sure pupils understand.

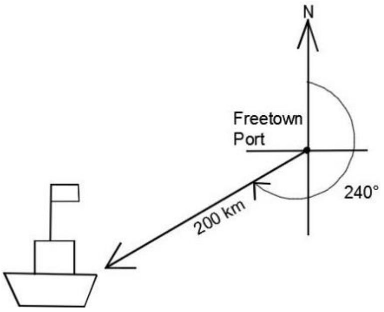


3. Draw another example on the board:

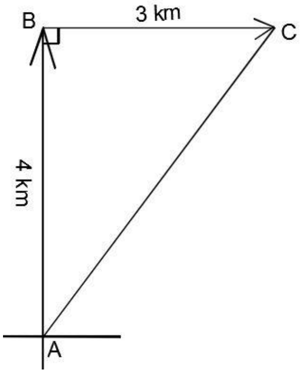


4. Write on the board: $\overrightarrow{XY} = (5\text{ cm}, 035°)$
5. Explain:
    - The position of point Y from point X is described by these 2 numbers.
    - To describe the relationship between two points, give the distance and then the three-point bearing in brackets.

6. Write on the board: The position of a point Q from another point P can be represented by $\overrightarrow{PQ} = (r, \theta)$, where r is the distance between the 2 points, and θ is the three-point bearing from P to Q.
7. Write the following problem on the board: A boat sailed from Freetown port at a bearing of 240°. It is now 200 km from Freetown. Write the ship's bearing and draw a diagram.
8. Invite a volunteer to write the bearing on the board. (Answer: $(200\text{ km}, 240°)$)
9. Ask pupils to work with seatmates to draw the diagram.
10. Walk around to check for understanding and clear misconceptions.
11. Ask volunteers to share their drawings with the class. Accept accurate diagrams.
    Diagram:



12. Write the following problem on the board: Hawa walked 4 km from point A to B in the north direction, then 3 km from point B to C in the east direction. Draw a diagram and find the bearing from point A to point C.



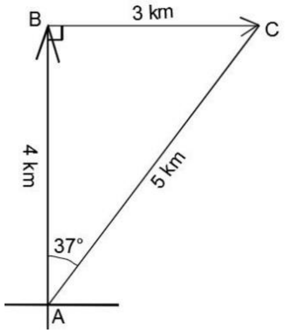
13. Ask pupils to work with seatmates to draw the diagram.
14. Invite a volunteer to draw it on the board. →
15. Solve the problem as a class on the board:
    Step 1. Find the distance from A to C:

$$\begin{aligned} |AB|^2 + |BC|^2 &= |AC|^2 && \text{Apply Pythagoras' theorem} \\ 4^2 + 3^2 &= |AC|^2 && \text{Substitute known lengths} \\ 16 + 9 &= |AC|^2 && \text{Simplify} \\ 25 &= |AC|^2 && \\ \sqrt{25} &= \sqrt{|AC|^2} && \text{Take the square root of both sides} \\ 5\text{ km} &= |AC| && \end{aligned}$$

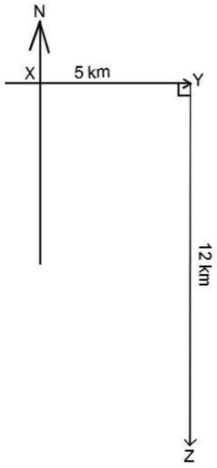
   Step 2. Find the angle of the bearing from A to C:

$$\begin{aligned} \tan A &= \tfrac{3}{4} = 0.75 && \text{Apply tangent ratio} \\ \tan^{-1}(\tan\ A) &= \tan^{-1}(0.75) && \text{Take inverse tangent of both sides} \\ A &= \tan^{-1}(0.75) && \\ A &= 36.87° && \text{From the tangent table} \end{aligned}$$

16. Label the diagram with the length and angle you have calculated:



17. Write in distance-bearing form: $\overrightarrow{AC} = (5\text{ km}, 037°)$)
18. Write the following problem on the board: A ship traveled 5 km due east from point X to point Y, then 12 km due south from point Y to point Z.
    a. Draw a diagram for the problem.
    b. Find the distance from point X to point Z.
    c. Find the bearing from point X to point Z.
19. Ask pupils to work with seatmates to draw the diagram.
20. Walk around to check for understanding and clear misconceptions.
21. Invite a volunteer to draw the diagram on the board. Diagram →



22. Ask pupils to work with seatmates to solve b. and c.
23. Walk around to check for understanding and clear misconceptions.
24. Invite volunteers to write the solutions and label the diagram on the board.

Solutions:

b. Use Pythagoras' theorem:

$$\begin{aligned} |XY|^2 + |YZ|^2 &= |XZ|^2 && \text{Apply Pythagoras' theorem} \\ 5^2 + 12^2 &= |XZ|^2 && \text{Substitute known lengths} \\ 25 + 144 &= |XZ|^2 && \text{Simplify} \\ 169 &= |XZ|^2 && \\ \sqrt{169} &= \sqrt{|XZ|^2} && \text{Take the square root of both sides} \\ 13\text{ km} &= |XZ| && \end{aligned}$$

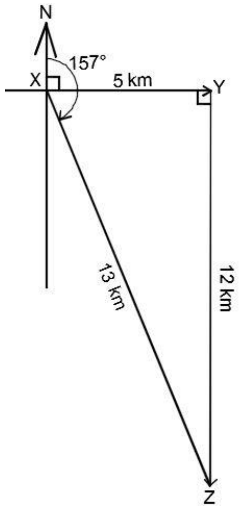
c. The angle of the bearing from X to Z is more than 90°. Find the angle of X in the triangle XYZ, and add this to 90°.

$$\begin{aligned} \tan X &= \tfrac{12}{5} = 2.4 && \text{Apply tangent ratio} \\ \tan^{-1}(\tan\ X) &= \tan^{-1}(2.4) && \text{Take inverse tangent of both sides} \\ X &= \tan^{-1}(2.4) && \\ X &= 67.38° && \text{From the tangent table} \end{aligned}$$

Round to the nearest degree, and add to 90°: $90° + 67° = 157°$

The bearing from X to Z is $\overrightarrow{XZ} = (13\text{ km}, 157°)$)

Labelled diagram:



Practice *(13 minutes)*

1. Write the following problem on the board: A farmer travels 10 km due north to reach his land. He then travels 24 km due east to bring his harvest to a market.
    a. Draw a diagram for the problem.
    b. Find the distance from his starting point to the market.
    c. Find the bearing from his starting point to the market.
2. Ask pupils to work with independently or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board. All other pupils should check their own work.

Solutions:

a. Diagram: See below. Points may be labelled with any letter of the pupil's choice. In the example diagram, O, F and M are used.

b. Find OM:

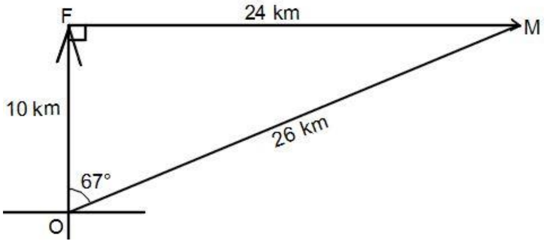
$$\begin{aligned} |OF|^2 + |FM|^2 &= |OM|^2 \\ 10^2 + 24^2 &= |OM|^2 \\ 100 + 576 &= |OM|^2 \\ 676 &= |OM|^2 \\ \sqrt{676} &= \sqrt{|OM|^2} \\ 26 \text{ km} &= |OM| \end{aligned}$$

c. Find angle O inside the triangle:

$$\begin{aligned} \tan O &= \tfrac{10}{26} = 0.3846 \\ \text{:an}^{-1}(\tan\ O) &= \tan^{-1}(0.3846) \\ O &= \tan^{-1}(0.3846) \\ O &= 21.04° \end{aligned}$$

Bearing: $\overrightarrow{OM} = (26 \text{ km}, 021°)$

Diagram:



Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L137 in the Pupil Handbook.

Lesson Title: Probability	**Theme:** Review	
Lesson Number: M3-L138	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve problems on probability.	**Preparation** None	

Opening *(1 minute)*

1. Discuss: What types of probability problems have you learned to solve? (Example answer: The probability of mutually exclusive events occurring; the probability of independent events occurring.)
2. Explain that today's lesson is on solving various probability problems of the type that appear on the WASSCE exam.

Teaching and Learning *(18 minutes)*

1. Write the following problem on the board: A letter is selected at random from the letters of the English alphabet. What is the probability that the letter selected is from the word TEACHER?
2. Ask volunteers to explain in their own words how to solve the problem. (Example answer: Count the letters in teacher, and divide by the number of letters in the alphabet, 26.)
3. Write the solution on the board and explain:
   List the letters in "teacher": t, e, a, c, h, r
   There are 6 letters.
   Pr(choosing a letter in TEACHER)$=\frac{6}{26}=\frac{3}{13}$
4. Write the following problem on the board: There are 50 balls in a bag. 20 are red, 18 are blue, and 12 are green. If a ball is selected at random, what is the probability that:
   a. It is red.
   b. It is either red or blue.
5. Discuss:
   - How would you solve part a.? (Answer: Divide the number of red balls by the total number of balls.)
   - How would you solve part b.? Why? (Answer: The word "**or**" tells us to **add** the probability of selecting a red ball and the probability of selecting a blue ball.)
6. Ask pupils to work with seatmates to solve the problem.
7. Walk around to check for understanding and clear misconceptions.
8. Invite volunteers to write the solutions on the board and explain.
   Solutions:
   a. Pr(choosing a red ball)$=\frac{20}{50}=\frac{2}{5}$

   b. Pr(choosing a red or blue ball)= $\frac{20}{50}+\frac{18}{50}=\frac{38}{50}=\frac{19}{25}$

9. Write the following problem on the board: The table below gives the ages of 50 pupils in a university course.

Age (years)	18	19	20	21
Frequency	5	11	21	14

If 2 pupils are selected at random, what is the probability that:

   a. They are both 21 years old.
   b. At least one of them is 21 years old.
   c. They are both younger than 20.

10. Discuss and make sure pupils understand:
    - How would you solve part a.? (Answer: Apply the multiplication law of probability – **"both"** can be interpreted as **"and"**, which tells us to **multiply**.)
    - How would you solve part b.? (Answer: Find the probability that neither of them are 21, and subtract it from 1.)
    - How would you solve part c.? (Answer: Find the probability of selecting a pupil less than 20 (18- or 19-year old pupil). Multiply for "both" of them being in this age group.)
11. Ask volunteers to give the steps needed to solve each part. Solve the problem on the board as they explain.

    Solution:

    a. Probability that they are both 21 years old:

       Pr(a 21-year-old is selected) $=\frac{14}{50}=\frac{7}{25}$

       Pr(both are 21 years old) $=\frac{7}{25}\times\frac{7}{25}=\frac{49}{625}$

    b. Probability that at least 1 is 21 years old:

       First, find the probability that a pupil selected is **not** 21:

       Pr(Not 21) $=1-\frac{14}{50}=1-\frac{7}{25}=\frac{18}{25}$

       Multiply to find the probability that neither is 21 (in other words, that **both** are **not** 21):

       Pr(Both not 21) $=\frac{18}{25}\times\frac{18}{25}=\frac{324}{625}$

       Subtract to find the probability that at least 1 is 21:

       Pr(at least 1 is 21) $=1-\frac{324}{625}=\frac{625}{625}-\frac{324}{625}=\frac{301}{625}$

    c. **Add** to find the probability of selecting an 18 year old **or** a 19 year old:

       Pr(a pupil under 20 is selected) $=\frac{5}{50}+\frac{11}{50}=\frac{16}{50}=\frac{8}{25}$

       Multiply to find the probability that they are **both** younger than 20:

       Pr(Both are under 20) $=\frac{8}{25}\times\frac{8}{25}=\frac{64}{625}$

Practice *(20 minutes)*

1. Write the following problems on the board:

   a. A letter is chosen at random from the alphabet. Find the probability that it is in either the word PENCIL or BOOK.
   b. A fair six-sided die is thrown. Find the probability of getting:
      i. 5 or 6
      ii. An even number
      iii. A number less than 3
   c. There are 20 pupils in a class. Three are 15 years old, 12 are 16 years old, and 5 are 17 years old. If a pupil is selected at random, find the probability that:
      i. He/she is 16 years old.
      ii. He/she is less than 17 years old.
      iii. He/she is either 15 years old or 17 years old.
2. Ask pupils to work with independently or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board. All other pupils should check their own work.

Solutions:

a. Pr(letter in PENCIL) $= \frac{6}{26}$ and Pr(letter in BOOK) $= \frac{3}{26}$.
   The word "or" tells us to add:
   Pr(letter in PENCIL or BOOK) $= \frac{6}{26} + \frac{3}{26} = \frac{9}{26}$

b. i. Pr(5) $= \frac{1}{6}$ and Pr(6) $= \frac{1}{6}$
   The word "or" tells us to add:
   Pr(5 or 6) $= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$
   ii. List the even numbers: 2, 4, 6. There are 3 even numbers on a die.
   Pr(even) $= \frac{3}{6} = \frac{1}{2}$
   iii. List the numbers less than 3: 1, 2. There are 2 numbers less than 3.
   Pr(less than 3)$= \frac{2}{6} = \frac{1}{3}$

c. i. Pr(16 years old))$= \frac{12}{20} = \frac{3}{5}$
   ii. Find the total number of pupils less than 17: $3 + 12 = 15$
   Pr(less than 17) $= \frac{15}{20} = \frac{3}{4}$
   iii. "or" tells us to add the probabilities:
   Pr(15 or 17) $= \frac{3}{20} + \frac{5}{20} = \frac{8}{20} = \frac{2}{5}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L138 in the Pupil Handbook.

Lesson Title: Statistics – ungrouped data	**Theme:** Review	
Lesson Number: M3-L139	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve statistics problems with ungrouped data.	**Preparation** None	

Opening *(1 minute)*

1. Discuss: What is ungrouped data? What is grouped data? (Example answer: Ungrouped data is concerned with single data points, while grouped data gives frequency of data that fall into class intervals.)
2. Explain that today's lesson is on solving statistics problems with ungrouped data.

Teaching and Learning *(18 minutes)*

1. Write the following problem on the board: The amount of money that Fatu spent each day this week is Le 11,000.00, Le 6,000.00, Le 22,000.00, Le 5,000.00 and Le 6,000.00. Find the mean, median, and mode of her spending.
2. Ask pupils to work with seatmates to solve the problem.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board and explain.

   Solutions:

   Mean: Add the quantities, then divide by the number of days.

   $$\text{Mean} = \frac{11{,}000+6{,}000+22{,}000+5{,}000+6{,}000}{5} = \frac{50{,}000}{5} = \text{Le } 10{,}000.00$$

   Median: Write the numbers in ascending order, and choose the middle value:

   5,000, 6,000, **6,000**, 11,000, 22,000

   $\text{Median} = \text{Le } 6{,}000.00$

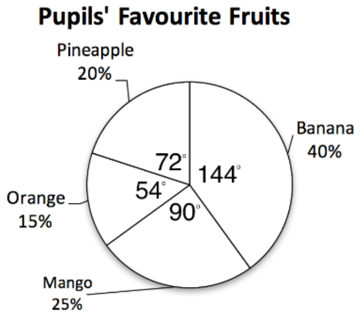
   Mode: The number that occurs with the greatest frequency is mode: $\text{Le}6{,}000.00$.
5. Explain:
   - Ungrouped data can also be presented in bar charts and pie charts.
   - It is important that you are able to draw and interpret both types of charts.
6. Write the following problem on the board: A group of pupils was surveyed to find their favourite fruits. The data is in the table below. Create a pie chart for this data.

Favourite Fruit	**Frequency**	**Percentage**
Banana	16	40%
Mango	10	25%
Orange	6	15%
Pineapple	8	20%
TOTAL	40	100%

7. Ask volunteers to give the steps needed to draw the pie chart. Solve the problem on the board as they explain.

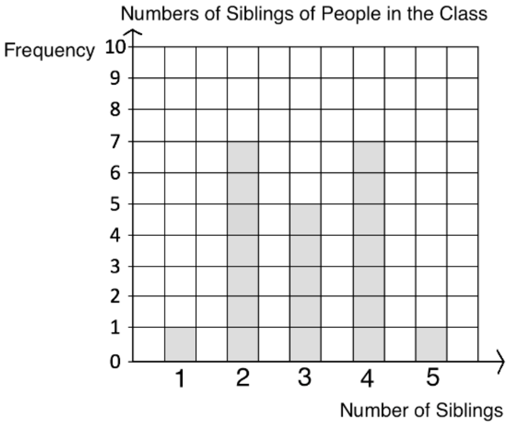
Solution:

First, calculate the degree measure for each fruit. Give each frequency as a fraction of the whole (40), and multiply that fraction by 360°. Draw the pie chart using the degrees you find.



$$\text{Banana} = \frac{16}{40} \times 360° = 144°$$
$$\text{Mango} = \frac{10}{40} \times 360° = 90°$$
$$\text{Orange} = \frac{6}{40} \times 360° = 54°$$
$$\text{Pineapple} = \frac{8}{40} \times 360° = 72°$$

8. Write the following problem on the board: A survey was conducted in a class to determine the number of siblings each pupil has. The information is presented in the bar chart. Find: a. The number of pupils in the class; b. Mean; c. Median; d. Mode.
9. Ask volunteers to give the steps needed to interpret the bar chart. Solve the problem on the board as they explain.



Solutions:

- a. Find the number of pupils by adding the frequencies: $1 + 7 + 5 + 7 + 1 = 21$
- b. **Mean:**

  Find the sum of the siblings: $1(1) + 7(2) + 5(3) + 7(4) + 1(5) = 63$

  Find the number of pupils: $1 + 7 + 5 + 7 + 1 = 21$

  Divide: $63 \div 21 = 3$ siblings

  Show pupils that this is the same as applying the formula below. Make sure they understand how the formula works.

  $\frac{\sum fx}{\sum f} = \frac{1(1)+7(2)+5(3)+7(4)+1(5)}{1+7+5+7+1} = \frac{63}{21} = 3$ siblings
- c. **Median:** The 11th pupil is in the middle of the 21 pupils (this can be found using logic, or the formula $\frac{n+1}{2}$). Count up to 11 in the bar chart. The 11th pupil has 3 siblings, which is the median.
- d. **Mode:** The bars for 2 siblings and 4 siblings both have a height of 7. Thus, there are 2 modes. Mode = 2, 4.

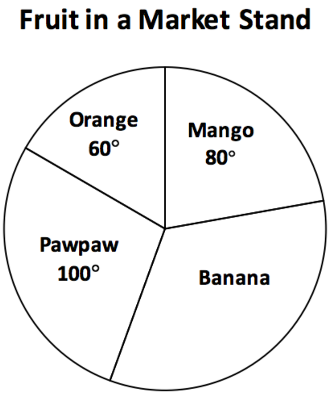
Practice *(20 minutes)*

1. Write the following problems on the board:
   - a. The table below shows the distribution of marks on a test written by a certain class.
     - i. Draw a bar chart for the distribution.

ii. If the pass mark is 6, how many pupils failed the test?
iii. What is the mean?
iv. What is the mode?

Marks	1	2	3	4	5	6	7	8	9	10
Frequency	1	0	1	3	2	6	10	8	7	3

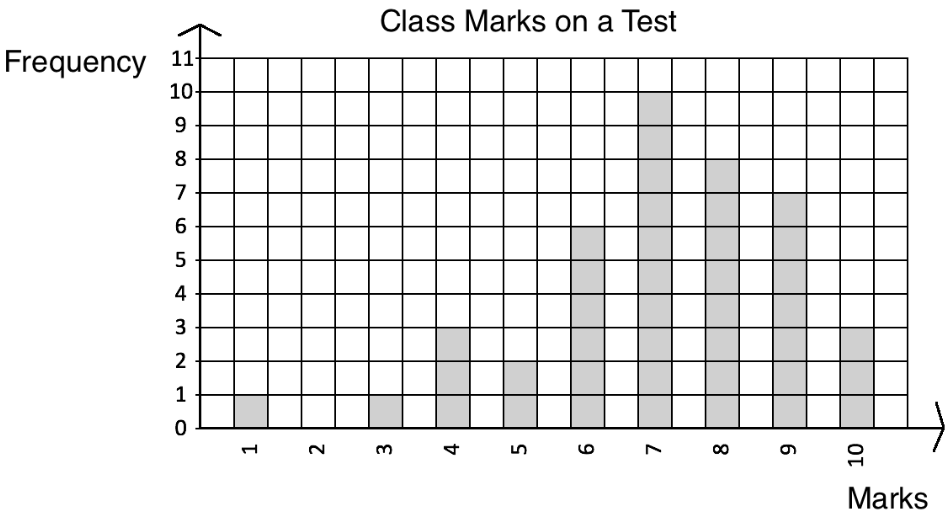
b. The pie chart represents the pieces of fruit for sale in a market stand. If there are 60 mangoes, how many bananas are there?



2. Ask pupils to work with independently or with seatmates to solve the problems.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board. All other pupils should check their own work.

Solutions:

a. i. Bar chart:



ii. If the pass mark is 6, all pupils scoring 1-5 failed. Add the frequencies: $1 + 0 + 1 + 3 + 2 = 7$ pupils failed.

iii. Find the mean using the formula:

$$\frac{\sum fx}{\sum f} = \frac{1(1)+1(3)+3(4)+2(5)+6(6)+10(7)+8(8)+7(9)+3(10)}{1+1+3+2+6+10+8+7+3} = \frac{1+3+12+10+36+70+64+63+30}{41} = \frac{289}{41} = 4.0$$

iv. The mode is the tallest bar, 7 marks.

b. This problem is solved in multiple steps.

Step 1. Find the angle measure of banana:

Banana’s measure: $360° - (100° + 60° + 80°) = 360° - 240°$

$= 120°$

Step 2. Use the fact that there are 60 mangoes to find the total number of fruit. Recall that we would have found 60 mangoes by multiplying the proportion of fruits that are mangoes by the total number of fruits. Set up the equation:

Number of mangoes $=\ 60 = \frac{80}{360} \times F$, where F is the total number of fruit.

Solve for F:

$$\begin{aligned} 60 &= \frac{80}{360} \times F && \\ 60 &= \frac{2}{9} \times F && \text{Simplify} \\ 540 &= 2 \times F && \text{Multiply throughout by 9} \\ 270 &= F && \text{Divide throughout by 2} \end{aligned}$$

There are 270 pieces of fruit in total.

Step 3. Find the number of bananas. Multiply the proportion that are bananas by the total number of fruit:

$$\begin{aligned} \text{Number of bananas} &= \frac{120}{360} \times F && \\ &= \frac{1}{3} \times 270 && \text{Simplify} \\ &= 90 && \text{Multiply} \end{aligned}$$

Answer: There are 90 bananas in the market stand.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L139 in the Pupil Handbook.

Lesson Title: Statistics – grouped data	**Theme:** Review	
Lesson Number: M3-L140	**Class:** SSS 3	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve statistics problems with grouped data.	**Preparation** None	

Opening *(1 minute)*

1. Discuss: What are some ways of presenting grouped data? (Example answers: Grouped frequency table, histogram, frequency polygon.)
2. Explain that today's lesson is on solving statistics problems with grouped data.

Teaching and Learning *(18 minutes)*

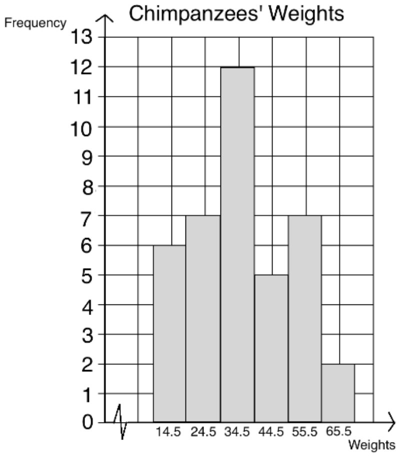
1. Write the following problem on the board: A chimpanzee reserve houses 39 chimpanzees that have been rescued from hunters and people who kept them as pets. The weights of the chimpanzees are displayed in the frequency table:

Weight (kg)	10-19	20-29	30-39	40-49	50-59	60-69
Frequency	6	7	12	5	7	2

   a. Draw a histogram for the data.
   b. What is the median class?
   c. What is the modal class?
   d. Estimate the mode.

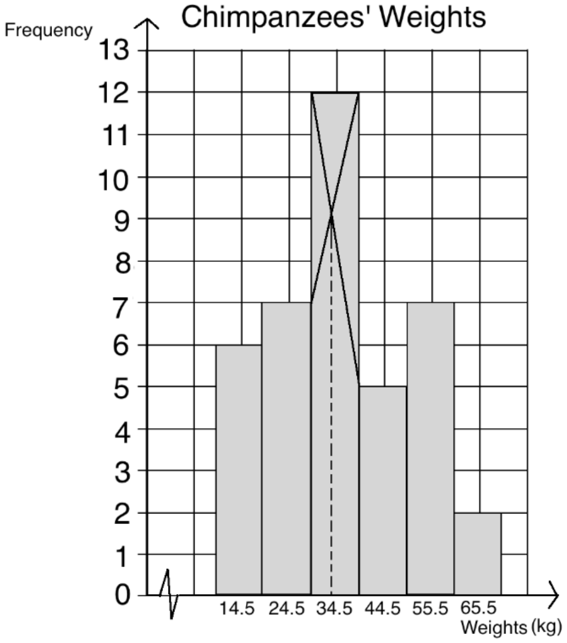
2. Discuss: What is a histogram and what does it look like? (Answer: A histogram looks like a bar chart, but it is used to display frequencies in class intervals.)
3. Find the class mid-point for each class interval. Remind pupils that these are used to plot the histogram:

Weight (kg)	10-19	20-29	30-39	40-49	50-59	60-69
Frequency	6	7	12	5	7	2
Class mid-point	14.5	24.5	34.5	44.5	54.5	64.5



4. Draw the axes for the histogram.
5. Plot 1-2 bars on the histogram, and ask volunteers to plot the others.
6. Discuss:
   - How can we find the median class? (Answer: the median class is the class interval that the median falls into; its position can be found by counting or using the formula $\frac{n+1}{2}$)
   - How can we find the modal class? (Answer: The modal class is the class interval with the tallest bar.)

7. Solve parts b. through d. of the problem on the board, explaining each step.
    a. The median class is the class where the chimpanzee with the median weight falls. The 20th chimpanzee has the median weight $\left(\frac{n+1}{2} = \frac{39+1}{2} = \frac{40}{2} = 20\right)$. This is in class interval 30-39, which is the median class.
    b. The modal class is the tallest bar, which is 30-39.
    c. To estimate the mode, draw lines as shown on the histogram. The mode is approximately 34 kg.



8. Write the following problem on the board: The table below shows 32 pupils' marks on a Maths test.

Marks	51-60	61-70	71-80	81-90	91-100
Frequency	5	7	9	8	3

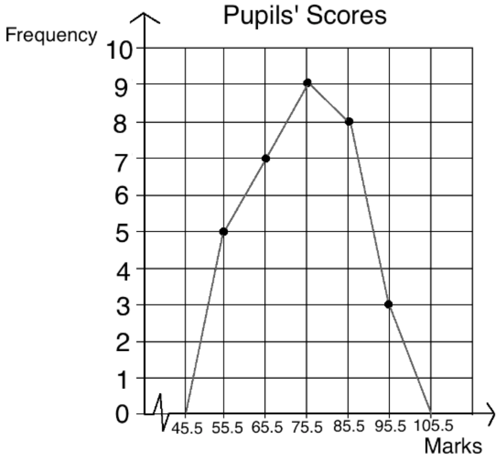
    a. Draw a frequency polygon to display the data.
    b. What is the modal class?
    c. What is the median class?
9. Discuss: What is a frequency polygon and what does it look like? (Answer: A frequency polygon is used for a similar purpose as a histogram, but it has a line instead of bars.)
10. Solve the problem on the board as a class. Make sure pupils understand.
    a. Draw a third row on the frequency table and ask volunteers to fill it with the class mid-points:

Marks	51-60	61-70	71-80	81-90	91-100
Frequency	5	7	9	8	3
Class Mid-point	55.5	65.5	75.5	85.5	95.5

Draw and label the axes on the board (see chart below). Plot each point in the table using the frequency and class mid-point. Connect the points as shown. Extend the line of the frequency polygon to the mid-point of what would be the next interval, if that interval existed in the data.

    b. The modal class is 71-80, because that class interval has the greatest frequency.

c. Find the position of the median: $\frac{n+1}{2} = \frac{32+1}{2} = \frac{33}{2} = 16.5$. The median is the average of the 16th and 17th positions, which both fall in the interval 71-80. Thus, the median class is 71-80.



Practice *(20 minutes)*

1. Write the following problems on the board:
   a. The heights of 20 pupils in centimetres are: 179, 180, 161, 163, 170, 182, 168, 172, 175, 164, 168, 157, 158, 169, 159, 178, 164, 175, 167, 183.
      i. Draw a frequency table using class intervals 156-160, 161-165, 166-170, 171-175, 176-180, 181-185.
      ii. Draw a frequency polygon to display the data.
      iii. What is the modal class?
      iv. What is the median class?
      v. How many pupils are 170 cm or shorter?
      vi. What percentage of the pupils are taller than 175 cm?
   b. In one village, 15 farmers have just harvested their pepper. The table below shows the amount of pepper they harvested in kilogrammes. Estimate, correct to 2 decimal places, how much pepper each farmer harvested on average.

Farmers’ Harvests

Pepper (kg)	**Frequency**
0 – 4	2
5 – 9	5
10 – 14	4
15 – 19	3
20 – 24	1
Total	15

2. Ask pupils to work with independently or with seatmates to solve the problems. Remind them to look at the solved examples in the Pupil Handbook if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board. All other pupils should check their own work.

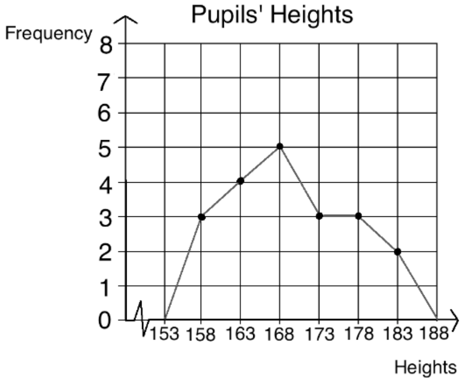
   Solutions:

   a. Write the numbers in ascending order before counting and grouping them: 157, 158, 159, 161, 163, 164, 164, 167, 168, 168, 169, 170, 172, 175, 175, 178, 179, 180, 182, 183.

i. Table:

Pupils' Heights	
Heights	**Frequency**
156-160	3
161-165	4
166-170	5
171-175	3
176-180	3
181-185	2

ii. Frequency polygon:



ii. The modal class is 166-170.

iii. The median class is where the 10th and 11th pupils fall, which is 166-170.

iv. Add the frequencies of the first 3 class intervals: $3 + 4 + 5 = 12$ pupils.

v. Find the number of pupils taller than 175 cm as a percentage of 20.

Pupils taller than 175: $3 + 3 = 5$. As a percentage of 20: $\frac{5}{20} \times 100\% = 25\%$ of pupils.

b. To apply the formula for mean ($\bar{x} = \frac{\sum fx}{\sum f}$), first find the value to use for the data points, x. This can be done by finding the mid-point of each class interval.

Farmers' Harvests		
Pepper (kg)	**Frequency**	**Mid-point**
0 – 4	2	2
5 – 9	5	7
10 – 14	4	12
15 – 19	3	17
20 – 24	1	22
Total	15	

Apply the formula for mean of grouped data:

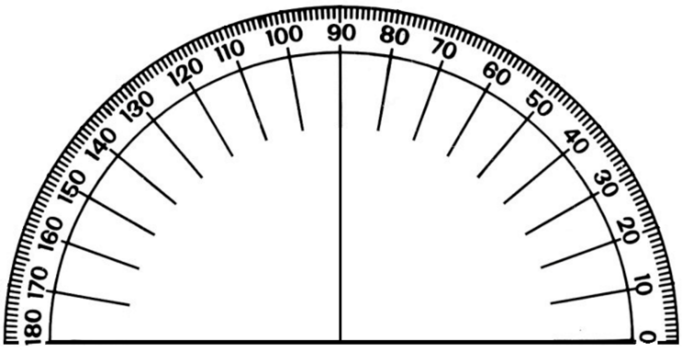
$$\begin{aligned} \bar{x} = \frac{\sum fx}{\sum f} &= \frac{(2\times2)+(5\times7)+(4\times12)+(3\times17)+(1\times22)}{2+5+4+3+1} \\ &= \frac{4+35+48+51+22}{15} \\ &= \frac{160}{15} \\ &= 10.67 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity of PHM3-L140 in the Pupil Handbook.

Appendix I: Protractor

You can use a protractor to measure angles. If you do not have a protractor, you can make one with paper. Trace this protractor with a pen onto another piece of paper. Then, cut out the semi-circle using scissors.



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