Free Quality School Education

Ministry of Basic and Senior Secondary Education

Lesson Plans for

Senior Secondary *Mathematics*

SSS I

Term II

STRICTLY NOT FOR SALE

Foreword

These Lesson Plans and the accompanying Pupils' Handbooks are essential educational resources for the promotion of quality education in senior secondary schools in Sierra Leone. As Minister of Basic and Senior Secondary Education, I am pleased with the professional competencies demonstrated by the writers of these educational materials in English Language and Mathematics.

The Lesson Plans give teachers the support they need to cover each element of the national curriculum, as well as prepare pupils for the West African Examinations Council's (WAEC) examinations. The practice activities in the Pupils' Handbooks are designed to support self-study by pupils, and to give them additional opportunities to learn independently. In total, we have produced 516 lesson plans and 516 practice activities – one for each lesson, in each term, in each year, for each class. The production of these materials in a matter of months is a remarkable achievement.

These plans have been written by experienced Sierra Leoneans together with international educators. They have been reviewed by officials of my Ministry to ensure that they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognized techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make the best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new lesson plans. It is really important that the Lesson Plans and Pupils' Handbooks are used, together with any other materials they may have.

This is just the start of educational transformation in Sierra Leone as pronounced by His Excellency, the President of the Republic of Sierra Leone, Brigadier Rtd Julius Maada Bio. I am committed to continue to strive for the changes that will make our country stronger and better.

I do thank our partners for their continued support. Finally, I also thank the teachers of our country for their hard work in securing our future.

Timbo

Mr. Alpha Osman Timbo

Minister of Basic and Senior Secondary Education

The policy of the Ministry of Basic and Senior Secondary Education, Sierra Leone, on textbooks stipulates that every printed book should have a lifespan of three years.

To achieve thus, DO NOT WRITE IN THE BOOKS.

Table of Contents

Introduction
to the Lesson Plans

These lesson plans are based on the National Curriculum and the West Africa Examination Council syllabus guidelines, and meet the requirements established by the Ministry of Basic and Senior Secondary Education.

1. The lesson plans will not take the whole term, so use spare time to review material or prepare for examinations.
2. Teachers can use other textbooks alongside or instead of these lesson plans.
3. Read the lesson plan before you start the lesson. Look ahead to the next lesson, and see if you need to tell pupils to bring materials for next time.
4. Make sure you understand the learning outcomes, and have teaching aids and other preparation ready – each lesson plan shows these using the symbols on the right.
5. If there is time, quickly review what you taught last time before starting each lesson.
6. Follow the suggested time allocations for each part of the lesson. If time permits, extend practice with additional work.
7. Lesson plans have a mix of activities for the whole class and for individuals or in pairs.
8. Use the board and other visual aids as you teach.
9. Interact with all pupils in the class – including the quiet ones.
10. Congratulate pupils when they get questions right! Offer solutions when they don't, and thank them for trying.

Learning outcomes

Preparation

KEY TAKEAWAYS FROM SIERRA LEONE'S PERFORMANCE IN WEST AFRICAN SENIOR SCHOOL CERTIFICATE EXAMINATION – GENERAL MATHEMATICS[1]

This section, seeks to outline key takeaways from assessing Sierra Leonean pupils' responses on the West African Senior School Certificate Examination. The common errors pupils make are highlighted below with the intention of giving teachers an insight into areas to focus on, to improve pupil performance on the examination. Suggestions are provided for addressing these issues.

Common errors

1. Errors in applying principles of BODMAS
2. Mistakes in simplifying fractions
3. Errors in application of Maths learned in class to real-life situations, and vis-a-versa.
4. Errors in solving geometric constructions.
5. Mistakes in solving problems on circle theorems.
6. Proofs are often left out from solutions, derivations are often missing from quadratic equations.

Suggested solutions

1. Practice answering questions to the detail requested
2. Practice re-reading questions to make sure all the components are answered.
3. If possible, procure as many geometry sets to practice geometry construction.
4. Check that depth and level of the lesson taught is appropriate for the grade level.

[1] This information is derived from an evaluation of WAEC Examiners' Reports, as well as input from their examiners and Sierra Leonean teachers.

FACILITATION STRATEGIES

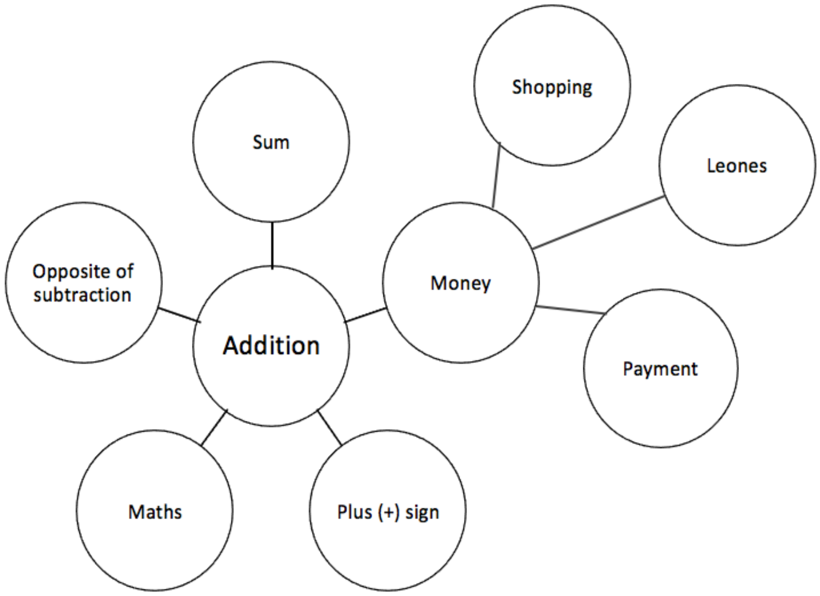
This section includes a list of suggested strategies for facilitating specific classroom and evaluation activities. These strategies were developed with input from national experts and international consultants during the materials development process for the Lesson Plans and Pupils' Handbooks for Senior Secondary Schools in Sierra Leone.

Strategies for introducing a new concept

- **Unpack prior knowledge:** Find out what pupils know about the topic before introducing new concepts, through questions and discussion. This will activate the relevant information in pupils' minds and give the teacher a good starting point for teaching, based on pupils' knowledge of the topic.
- **Relate to real-life experiences:** Ask questions or discuss real-life situations where the topic of the lesson can be applied. This will make the lesson relevant for pupils.
- **K-W-L:** Briefly tell pupils about the topic of the lesson, and ask them to discuss 'What I **k**now' and 'What I **w**ant to know' about the topic. At the end of the lesson have pupils share 'What I **l**earned' about the topic. This strategy activates prior knowledge, gives the teacher a sense of what pupils already know and gets pupils to think about how the lesson is relevant to what they want to learn.
- **Use teaching aids from the environment:** Use everyday objects available in the classroom or home as examples or tools to explain a concept. Being able to relate concepts to tangible examples will aid pupils' understanding and retention.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, can be used to activate prior knowledge and engage pupils in the content which is going to be taught in the lesson.

Strategies for reviewing a concept in 3-5 minutes

- **Mind-mapping:** Write the name of the topic on the board. Ask pupils to identify words or phrases related to the topic. Draw lines from the topic to other related words. This will create a 'mind-map', showing pupils how the topic of the lesson can be mapped out to relate to other themes. Example below:



- **Ask questions:** Ask short questions to review key concepts. Questions that ask pupils to summarise the main idea or recall what was taught is an effective way to review a concept quickly. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Brainstorming:** Freestyle brainstorming, where the teacher writes the topic on the board and pupils call out words or phrases related that topic, is an effective way to review concepts as a whole group.
- **Matching:** Write the main concepts in one column and a word or a phrase related to each concept in the second column, in a jumbled order. Ask pupils to match the concept in the first column with the words or phrases that relate to in the second column.

Strategies for assessing learning without writing

- **Raise your hand:** Ask a question with multiple-choice answers. Give pupils time to think about the answer and then go through the multiple-choice options one by one, asking pupils to raise their hand if they agree with the option being presented. Then give the correct answer and explain why the other answers are incorrect.
- **Ask questions:** Ask short questions about the core concepts. Questions which require pupils to recall concepts and key information from the lesson are an effective way to assess understanding. Remember to pick volunteers from all parts of the classroom to answer the questions.
- **Think-pair-share:** Give pupils a question or topic and ask them to turn to seatmates to discuss it. Then, have pupils volunteer to share their ideas with the rest of the class.
- **Oral evaluation:** Invite volunteers to share their answers with the class to assess their work.

Strategies for assessing learning with writing

- **Exit ticket:** At the end of the lesson, assign a short 2-3 minute task to assess how much pupils have understood from the lesson. Pupils must hand in their answers on a sheet of paper before the end of the lesson.
- **Answer on the board:** Ask pupils to volunteer to come up to the board and answer a question. In order to keep all pupils engaged, the rest of the class can also answer the question in their exercise books. Check the answers together. If needed, correct the answer on the board and ask pupils to correct their own work.
- **Continuous assessment of written work:** Collect a set number of exercise books per day/per week to review pupils' written work in order to get a sense of their level of understanding. This is a useful way to review all the exercise books in a class which may have a large number of pupils.
- **Write and share:** Have pupils answer a question in their exercise books and then invite volunteers to read their answers aloud. Answer the question on the board at the end for the benefit of all pupils.
- **Paired check:** After pupils have completed a given activity, ask them to exchange their exercise books with someone sitting near them. Provide the answers, and ask pupils to check their partner's work.
- **Move around:** If there is enough space, move around the classroom and check pupils' work as they are working on a given task or after they have completed a given task and are working on a different activity.

Strategies for engaging different kinds of learners

- For pupils who progress faster than others:
  - Plan extension activities in the lesson.
  - Plan a small writing project which they can work on independently.
  - Plan more challenging tasks than the ones assigned to the rest of the class.
  - Pair them with pupils who need more support.

- For pupils who need more time or support:
  - Pair them with pupils who are progressing faster, and have the latter support the former.
  - Set aside time to revise previously taught concepts while other pupils are working independently.
  - Organise extra lessons or private meetings to learn more about their progress and provide support.
  - Plan revision activities to be completed in the class or for homework.
  - Pay special attention to them in class, to observe their participation and engagement.

Lesson Title: Powers and roots of logarithms – Numbers greater than 1	**Theme:** Numbers and Numeration	
Lesson Number: M1-L049	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate powers and roots of numbers greater than 1 using logarithms.	**Preparation** Bring logarithm and antilogarithm tables, available in term 1 Lesson Plans.	

Opening *(3 minutes)*

1. Write a revision problem on the board: Use logarithms to evaluate $\frac{36.88}{2.08}$.
2. Ask pupils to solve the problems in their exercise books.
3. Invite a volunteer to write the solution on the board.
   Solution:

Numbers	Logarithms
36.88 2.08	1.5668 $-\ 0.3181$
Subtract	1.2487

   Antilog $1.2487 = 17.73$

4. Remind pupils of the steps to divide logarithms if needed: find the logarithms, subtract, then find the antilog.
5. Explain to pupils that today's lesson is on calculating powers and roots of numbers greater than 1 using logarithms.

Teaching and Learning *(24 minutes)*

1. Explain the **finding powers with logarithms:**
   - For very large numbers or decimal numbers, it is difficult to find the power without a calculator.
   - The product rule allows us to find the powers of numbers using logarithms.
   - We can find any power of numbers in the logarithm using the following steps:
     - Find the logarithm of the numbers.
     - Multiply the result by the power.
     - Find the antilogarithm of the result.
2. Write the following problem on the board: Use logarithms to evaluate 138^2.
3. Explain the solution to pupils on the board.
   Solution:
   Step 1. Find the logarithm: $\log 138 = 2.1399$
   Step 2. Multiply by the power: $2.1399 \times 2 = 4.2798$
   Step 3. Find the antilogarithm: $\text{antilog}\, 4.2798 = 19{,}040$. Since the characteristic is 4, the integer part should be 5 digits.

You can also show the work in a table:

Number	Logarithm
138^2	$2.1399 \times 2 = 4.2798$

$\text{antilog } 4.2798 = 19{,}040$

Therefore, $138^2 = 19{,}040$

4. Write another problem on the board: Evaluate 2.13^5.
5. Ask volunteers to give the steps to solve the problem. As they give them, solve on the board.
   Solution:
   Step 1. Find the logarithm: $\log 2.13 = 0.3284$
   Step 2. Multiply by the power: $0.3284 \times 5 = 1.6420$
   Step 3. Find the antilogarithm: $\text{antilog } 1.6420 = 43.85$. Since the characteristic is 1, the integer part should be 2 digits.
6. Explain **finding roots with logarithms:**
   - It is also difficult to find the root of many numbers without a calculator. We can use logarithms to find the roots.
   - We can find the square, cube or other roots of a logarithm using the following steps:
     - Find the logarithm of the number.
     - Divide the result by the root.
     - Find the antilogarithm of the result.
7. Write the following problem on the board: Use logarithms to evaluate $\sqrt{5}$.
8. Explain the solution to the pupils on the board.
   Solution:
   Step 1. Find the logarithm: $\log 5 = 0.6990$
   Step 2. Divide by the root: $0.6990 \div 2 = 0.3495$
   Step 3. Find the antilogarithm: $\text{antilog } 0.3495 = 2.236$
   Therefore $\sqrt{5} = 2.236$
   You can also show the work in a table:

Number	Logarithm
$\sqrt{5}$	$0.6990 \div 2 = 0.3495$

$\text{antilog } 0.3495 = 2.236$

Therefore, $\sqrt{5} = 2.236$

9. Write another problem on the board: Use the logarithm table to evaluate $\sqrt[3]{46.5}$.
10. Ask volunteers to give the steps to solve the problem. As they give them, solve on the board.
    Solution:
    Step 1. Find the logarithm: $\log 46.5 = 1.6675$
    Step 2. Divide by the root: $1.6675 \div 3 = 0.5558$

Step 3. Find the antilogarithm: $\text{antilog } 0.5558 = 3{,}596$

Since the characteristic is 0, the integer part has 1 digit.

Therefore, $\sqrt[3]{46.5} = 3.596$

11. Write another problem on the board: Use logarithm tables to evaluate:
    a. $245.5^2 \times 1.08^3$
    b. $\sqrt[4]{(42.5)^2}$
12. Discuss: How do you think we will solve these problems? What steps will we take?
13. Allow pupils to share ideas, then explain:
    a. We apply the order of operations (BODMAS), and use the steps we know for solving powers, roots, and multiplication using logarithms.
    b. We can find the logarithms of the numbers, and apply all of the operations necessary before applying the antilogarithm to the result.
    c. For problem a., we will apply the powers to both numbers, then apply multiplication.
    d. For problem b., we will apply the power, then apply the root.
14. Solve problem a. on the board, explaining each step.

Step 1. Draw the table and write the 2 terms of concern under "Number".

Step 2. In the second column, find the logarithms, and write the multiplication for the powers.

Step 3. In the 3rd column, write the results of the multiplication and add them.

Step 4. Find the antilogarithm of the result.

Number	Logarithm	
245.5^2 1.08^3	$2.3901 \times 2 =$ $0.0334 \times 3 =$	4.7802 $+\ 0.1002$
		4.8804

$\text{antilog } 4.8804 = 75{,}930$

Therefore, $245.5^2 \times 1.08^3 = 75{,}930$

15. Solve problem b. on the board, explaining each step.

Step 1. Find the logarithm of 42.5.

Step 2. Find 42.5^2 by multiplying the logarithm by 2.

Step 3. Find $\sqrt[4]{42.5^2}$ by dividing the result by 4.

Step 4. Find the antilogarithm of the result.

Number	Logarithm
42.5	1.6285
42.5^2	$1.6285 \times 2 = 3.257$
$\sqrt[4]{42.5^2}$	$3.257 \div 4 = 0.8143$

$\text{antilog } 0.8143 = 6.521$

Therefore, $\sqrt[4]{(42.5)^2} = 6.521$

Practice *(12 minutes)*

1. Write the following two problems on the board: Use the logarithm tables to evaluate each of the following. Use a calculator to verify your results.
    a. 378.2^2
    b. $\sqrt[3]{85}$
    c. $\sqrt[5]{100^3}$
2. Ask pupils to solve the problems independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to simultaneously solve the problems on the board. Other pupils should check their work.

Solutions:

a.

Number	Logarithm
387.2^2	$2.5879 \times 2 = 5.1758$

$\text{antilog}\ 5.1758 = 149{,}900$

Therefore, $387.2^2 = 149{,}900$

b.

Number	Logarithm
$\sqrt[3]{85}$	$1.9294 \div 3 = 0.6431$

$\text{antilog}\ 0.6431 = 4.396$

Therefore, $\sqrt[3]{85} = 4.396$

c.

Number	Logarithm
100	2.0000
100^3	$2.0000 \times 3 = 6.000$
$\sqrt[5]{100^3}$	$6.000 \div 5 = 1.2000$

$\text{antilog}\ 1.2000 = 15.85$

Therefore, $\sqrt[5]{100^3} = 15.85$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L049 in the Pupil Handbook.

<table>
<tr><td>Lesson Title: Logarithms – Numbers less than 1</td><td colspan="2">Theme: Numbers and Numeration</td></tr>
<tr><td>Lesson Number: M1-L050</td><td>Class: SSS 1</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to find the logarithms of numbers less than 1 using logarithm tables.</td><td colspan="2">Preparation
Write the problem in Opening on the board.</td></tr>
</table>

Opening *(3 minutes)*

1. Write the following revision problems on the board: Find the logarithms of the following numbers: a. 82.47 b. 209.3
2. Ask pupils to solve the problems independently.
3. Invite volunteers to write the answers on the board. (Answers: a. $\log 82.47 = 1.9163$; b. $\log 209.3 = 2.3207$)
4. Explain to pupils that today's lesson is on finding the logarithms of numbers less than 1 using logarithm tables.

Teaching and Learning *(22 minutes)*

1. Explain:
   - We will consider logarithms of numbers which lie between 0 and 1. In other words, decimal numbers less than 1.
   - Here again, we bring in the standard form of the number, where the power of 10 gives us the characteristic.
2. Write on the board: $0.0314 = 3.14 \times 10^{-2}$
3. Explain:
   - 0.0314 is a number between 0 and 1.
   - When we write it in standard form, -2 is the power on 10.
   - Therefore, -2 is the characteristic of the log of 0.0314.
   - An easy rule that works for decimals is that the characteristic is equal to the number of zeros trailing the decimal plus 1. The number has 1 zero after the decimal, so the characteristic is -2.
4. Write on the board: $\log 0.0314 = \bar{2}$ ...
5. Explain:
   - The characteristic is not written with a negative sign.
   - We put the minus sign on top of the characteristic, thus $\bar{2}$ is pronounced **"bar" 2**, but not "minus" 2.
6. Write the general rule on the board: A decimal number should be expressed in its standard form $a \times 10^{-n}$ in which case the characteristic of the logarithm is $\bar{n}$.
7. Explain: The mantissa is found using the logarithm tables, in the same way as with numbers greater than 1.

8. Use the logarithm table to find the mantissa. Remind pupils to find the row marked 31, and the column marked 4. The result is 0.4969.
9. Explain: We must write this mantissa to the negative number that we found for the characteristic.
10. Write on the board: $\log 0.0314 = \bar{2}.4969$
11. Write a problem on the board: Find $\log 0.0512$
12. Invite a volunteer to express the figures in standard form. **(**Answer: $0.0512 = 5.12 \times 10^{-2}$)
13. Invite another volunteer to write down the characteristic. (Answer: Characteristic = $\bar{2}$).
14. Ask pupils to find the mantissa in their logarithm tables. After a moment, invite a volunteer to write the mantissa on the board. (Answer: 0.7093)
15. Write the solution on the board: $\log 0.0512 = \bar{2}.7093$.
16. Write the following problem on the board: Find the logarithm of 0.0009887.
17. Invite volunteers to write down the standard form and characteristic on the board. (Answer: Standard form: $0.0009887 = 9.887 \times 10^{-4}$; Characteristic: $\bar{4}$).
18. Ask pupils to find the mantissa in their logarithm tables. After a moment, invite a volunteer to write the mantissa on the board (Answer: 0.9951).
19. Ask pupils to write the complete solution in their exercise book. Remind them to add the characteristic and mantissa.
20. Invite a volunteer to write the solution on the board. (Answer: $\log 0.0009887 = \bar{4}.9951$)
21. Write another problem on the board: Find the logarithm of 0.1234.
22. Invite volunteers to write the standard form and characteristic on the board. (Answers: $0.1234 = 1.234 \times 10^{-1}$; characteristic $= \bar{1}$)
23. Ask pupils to solve with seatmates.
24. Invite a volunteer to write the solution on the board. (Solution: The mantissa given in the log table is 0.0913. Thus, $\log 0.1234 = \bar{1}.0913$).

Practice *(14 minutes)*

1. Write four problems on the board: Find the logarithms of the following numbers:
   a. 0.000419
   b. 0.0842
   c. 0.062
   d. 0.0075
2. Ask pupils to solve the problems independently in their exercise books. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite four volunteers one at a time to write their answers on the board.
   Answers:
   a. $\log 0.000419 = \bar{4}.6222$
   b. $\log 0.0842 = \bar{2}.9253$
   c. $\log 0.062 = \bar{2}.7924$
   d. $\log 0.0075 = \bar{3}.8751$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L050 in the Pupil Handbook.

[NOTE]

There is another way of finding the logarithm of a decimal number. Explain this to pupils as needed.

You will find that if you use a scientific calculator, you get a different result than the results given in this lesson. Below, another method of calculating logarithms is given, which gives the same answer as the scientific calculator. For the purpose of these lessons and the WASSCE exam, the method described in the above lesson plan is preferable. This makes calculations with logarithms in later lessons more straightforward.

Example problems and solutions using the alternative method:

1. Find the logarithm of 0.0314.
   Solution:
   Step 1. Express the number in standard form: 3.14×10^{-2}

   Step 2. Write -2 as the characteristic.

   Step 3. Use the logarithm table to find the mantissa. This is in the row marked 31, and the column marked 4. The result is 0.4969.

   Step 4. Add the mantissa to the negative number that we found for the characteristic: $-2 + 0.4969 = -1.5031 = \bar{1}.5031$

   Answer: $\log 0.0314 = \bar{1}.5031$

2. Find $\log 0.0512$.
   Solution:

   Step 1. Express the number in standard form: 5.12×10^{-2}

   Step 2. Write -2 as the characteristic.

   Step 3. Use the logarithm table to find the mantissa. This is in the row marked 51, and the column marked 2. The result is 0.7093.

   Step 4. Add the mantissa to the negative number that we found for the characteristic: $-2 + 0.7093 = -1.2907 = \bar{1}.2907$.

   Answer: $\log 0.0512 = \bar{1}.2907$

Lesson Title: Antilogarithms – Numbers less than 0	**Theme:** Numbers and Numeration	
Lesson Number: M1-L051	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to find the antilogarithms of numbers less than 0 using antilogarithm tables.	**Preparation** Write the problems in Opening on the board.	

Opening *(4 minutes)*

1. Write revision problems on the board: Find the logarithm of the following numbers:
   a. 0.592 b. 0.00786
2. Ask pupils to solve the problems independently in their exercise books.
3. Invite volunteers to write their answers on the board and explain. (Answers: a. $\log 0.592 = \bar{1}.7723$; b. $\log 0.00786 = \bar{3}.8954$)
4. Explain to the pupils that today's lesson is in finding antilogarithms of numbers less than 1 using logarithm tables.

Teaching and Learning *(22 minutes)*

1. Explain:
   - Recall that an antilogarithm is the exact opposite of a logarithm of a number.
   - Antilog tables are used for determining the inverse value of the mantissa.
   - The position of the decimal point can be determined from the characteristic.
2. Write a problem on the board: Find the antilog of $\bar{1}.9087$.
3. Explain the steps for finding the inverse value of the mantissa. This is done in the same way as with the antilogarithms of numbers greater than 1:
   - Find 0.9087 in the antilogarithm table. It gives the value 8,104.
   - We know that there is 1 integer digit in the antilog, because the integer in the problem (the characteristics) is $\bar{1}$. We get 0.8104.
4. Write on the board: $\text{antilog}\bar{1}.9087 = 0.8104$
5. Explain how to determine the placement of the decimal point:
   - We move the decimal point base on the characteristic in the problem (the integer digit). Write the number of zeros after the decimal based on the characteristic. Remember that there are 1 fewer zeros than the characteristic.
   - In the example above, the characteristic was $\bar{1}$. Therefore, we change $8{,}104$ to 0.8104. Remember that a characteristic of $\bar{1}$ means that there are no zeroes between the decimal point and 8.
6. Write a problem on the board: Find the antilog of $\bar{7}.5231$.

7. Explain the solution to the pupils. Hold up the antilog tables to show how to find the answer, and write the answer on the board.
   Solution:
   - The fractional part of $\bar{7}.5231$ is 0.5231. 0.5231 in the antilog tables gives 3335 (demonstrate this).
   - The integer part of $\bar{7}.5231$ is $\bar{7}$. This shows that we are going to write down six zeros between the decimal point and 5.
   - Therefore, antilog $\bar{7}.5231 = 0.0000003335$
8. Write on the board: Find the number whose logarithm is $\bar{2}.4875$.
9. Write this on the board as both a logarithm and antilog: $\log x = \bar{2}.4875$, $x = \text{antilog}\ \bar{2}.4875$
10. Explain the solution to the pupils using the logarithm table, and write the answer on the board.
    - The fractional part of $\bar{2}.4875$ is 0.4875. 0.4875 in the antilog tables gives 3073.
    - The integer $\bar{2}$ shows that we should write 1 zero between the decimal and 3.
    - Therefore, the number whose logarithm is $\bar{2}.4875$ is 0.03073.
11. Write 2 problems on the board: Find the antilogarithms of the following numbers:
    - $\bar{3}.4785$ b. $\bar{2}.3045$
12. Ask pupils to work with seatmates to solve the problems.
13. Walk around to check for understanding and clear misconceptions.
14. Invite two volunteers to write the answers on the board and explain using the antilog tables.
    (Answer: a. Antilog $\bar{3}.4785 = 0.003009$ b. Antilog $\bar{2}.3045 = 0.02016$)

Practice *(13 minutes)*

1. Write on the board: Find the antilogarithms of the following numbers:
   a. $\bar{1}.4739$
   b. $\bar{3}.8743$
   c. $\bar{2}.7630$
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. After 10 minutes, ask them to exchange their exercise books.
5. Invite three volunteers to write their answers on the board.
   Answers:
   a. $\bar{1}.4739 = 0.2978$
   b. $\bar{3}.8743 = 0.007487$
   c. $\bar{2}.7630 = 0.05794$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-T2-W13-L051 in the Pupil Handbook.

[NOTE]

There is another way of finding the antilogarithm of a number less than 0. Explain this to pupils as needed.

You will find that if you use a scientific calculator, you get a different result from the results given in this lesson. Below, another method of calculating antilogarithms is given, which gives the same answer as the scientific calculator. For the purpose of these lessons and the WASSCE exam, the method described in the above lesson plan is preferable. This makes calculations with antilogarithms in later lessons more straightforward.

Example problems and solutions using the alternative method:

Example 1. Find the antilog of $\bar{1}.9087$.

Solution:
Make $\bar{1}.9087$ positive by adding 2, but also subtract 2 so it's equivalent.

$$-1.9087 = -2 + (2 - 1.9087)$$

$$= -2 + 0.0913$$

$$= -2.0913$$

Using the table, antilog of $-2.0913 = 0.01234$.

Therefore, $\text{antilog}\bar{1}.9087 = 0.01234$.

Example 2. Find the antilog of $\bar{7}.5231$.

Solution:
Make $\bar{7}.5231$ positive by adding 8, but also subtract 8 so it's equivalent.

$$-7.5231 = -8 + (8 - 7.5231)$$

$$= -8 + 0.4769$$

$$= -8.4769$$

Using the table, antilog of $-8.4769 = 0.00000002998$.

Therefore, $\text{antilog}\bar{7}.5231 = 0.00000002998$.

Lesson Title: Multiplication and division using logarithms – Numbers less than 1	Theme: Numbers and Numeration	
Lesson Number: M1-L052	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to multiply and divide numbers less than 1 using logarithms.	**Preparation** Write the problems in Opening on the board.	

Opening *(3 minutes)*

1. Review logarithms and antilogarithms. Write the following problems on the board:
    a. Find the logarithm of 0.00648
    b. Find the anti-logarithm of $\bar{1}.0877$
2. Ask pupils to solve the problems independently.
3. Ask volunteers to call out their answers. Write them on the board. (Answers: a. $\bar{3}.8116$; b. 0.1224)
4. Revise log and antilog tables briefly if needed.
5. Explain to the pupils that today's lesson is on the multiplication and division of numbers less than 1 using logarithms.

Teaching and Learning *(22 minutes)*

1. Write a problem on the board: Evaluate 0.056×0.95
2. Remind pupils how to multiply numbers using logarithms.
    - We can multiply 2 numbers using logarithms.
    - There are 3 steps:
        - Find the logarithms of the numbers.
        - Add the logarithms.
        - Find the antilogarithm of the result.
3. Solve the problem on the board. Explain each step to pupils.
   Solution:
   Step 1 and **Step 2.** Find the logarithms and add. Use the table as shown.

Numbers	Logarithms
0.056 0.95	$\bar{2}.7482$ $+\ \bar{1}.9777$
Add the logs for multiplication	$\bar{2}.7259$

   Step 3. Find the antilog of $\bar{2}.7259$.
   From the table, we have 5320. There is 1 zero after the decimal place, since the characteristic is $\bar{2}$. Therefore, the solution is 0.0532.
4. Write another problem on the board: Evaluate $0.0978 \div 0.06$

5. Explain the **division rule:**
   i. We can also divide 2 numbers using logarithms.
   ii. There are 3 steps:
      - Find the logarithms of the numbers.
      - Subtract the logarithm of the denominator from the numerator.
      - Find the antilogarithm of the result.
6. Explain: Note that for multiplication, the second step is to add the logarithms. For division, the second step is to subtract the logarithms.
7. Solve the problem on the board. Explain each step to pupils.
   Solution:
   Step 1. and **Step 2.** Find the logarithms and subtract. Use the table as shown.

Numbers	Logarithms
0.0978 0.06	$\bar{2}.9903$ $-\ \bar{2}.7782$
Subtract the logs for division	0.2121

   Step 3. Find the antilog of 0.2121.
   From the table, we have $1{,}629$. The integer part should be 1 digit, since the characteristic is 0. Therefore, the solution is 1.629.
8. Explain:
   - Calculation using logarithms follows the laws of indices.
   - Recall that the logarithms we are handling are based on powers of 10, although there is no need to write out the base 10 every time.
   - Thus, when we add the logarithms for multiplication and subtract for division, this is similar to applying the multiplication and division rules for indices.
9. Write a problem on the board: Evaluate $\frac{0.093\times0.062}{0.0036}$
10. Explain: When you multiply and divide more numbers, follow the same 3 steps.
11. Ask volunteers to give the 3 steps needed to solve the problem. (Answer: Find the logarithms of the numerator, add, find the logarithm of the denominator, subtract the answer of the denominator from the numerator and find the antilogarithm.)
12. Ask pupils to solve the problem with seatmates.
13. Invite a volunteer to write the solution on the board and explain.
    Solution:

Numbers	Logarithms
0.093 0.062 0.0036	$\bar{2}.9685$ $\underline{+\bar{2}.7924}$ $\bar{3}.7609$ $\underline{-\bar{3}.5563}$
	0.2046

    Using the antilog table for 0.2046, we find $1{,}602$. The solution is 1.602.

14. Write another problem on the board: Evaluate $\frac{0.008547}{0.367}$
15. Ask pupils to work with seatmates to solve the problem.
16. Invite a volunteer to write the solution on the board.

Solution:

Numbers	Logarithms
0.008547	$\bar{3}.9319$
0.367	$-\ \bar{1}.5647$
	$\bar{2}.3672$

Using the antilog table for $\bar{2}.3672$, we find 2329. The solution is 0.02329.

Practice *(14 minutes)*

1. Write the following two problems on the board:
    a. Evaluate 0.7685×0.03415
    b. Evaluate $\frac{0.05427\times0.00462}{0.09876\times0.04}$
2. Ask pupils to solve the problems independently in their exercise books.
3. Explain to the pupils about question b. above. Apply the normal order of operations. Find a single logarithm representing the numerator and a single logarithm representing the denominator. Then apply division (subtract).
4. Walk around to check for understanding and clear misconceptions.
5. Invite two volunteers, one at a time, to write their solutions on the board. Other pupils to check their work.

Solutions:

a.

Number	Logarithm
0.7685	$\bar{1}.8857$
0.03415	$\bar{2}.5334$
	$\bar{2}.4191$

Using the antilog table for $\bar{2}.4191$, we find $2{,}625$. The solution is 0.02625.

b.

Multiply the numerator:

Number	Logarithm	
0.05427	$\bar{2}.7346$	
0.00462	$+\ \bar{3}.6646$	
		$\bar{4}.3992$
0.09876	$\bar{2}.9946$	
0.04	$+\ \bar{2}.6021$	
		$-\bar{3}.5967$
		$\bar{2}.8025$

Using the antilog table for $\bar{2}.8025$, we find 6346. The solution is 0.06346.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L052 in the Pupil Handbook.

[NOTE]

There is another way of multiplying and dividing logarithms, which uses the methods described in the notes at the end of lessons 50 and 51. The result of the calculations will be the same whether you use the method described in the lesson above, or that described below.

Example 1. Evaluate: $\frac{0.093 \times 0.062}{0.0036}$

Solution:

Number	Logarithm
0.093	$\bar{1}.0315$
0.062	$+\ \bar{1}.2076$
	$\bar{\bar{2}}.2391$
0.0036	$-\ \bar{2}.4437$
	0.2046

Using the antilog table for 0.2046, we find 1602. The solution is 1.602.

Example 2. Evaluate 0.7685×0.03415

Solution:

Number	Logarithm
0.7685	$\bar{0}.1144$
0.03415	$+\ \bar{1}.4666$
	$\bar{\bar{1}}.5810$

Make $\bar{1}.5810$ positive by adding 2, but also subtract 2 so it's equivalent.

$$\begin{aligned} -1.5810 &= -2 + (2 - 1.5810) \\ &= -2 + 04190 \\ &= \bar{2}.4190 \end{aligned}$$

Using the antilog table for $\bar{2}.4190$, we find 2625. The solution is 0.02625.

Lesson Title: Powers and roots of logarithms – Numbers less than 1	**Theme:** Numbers and Numeration	
Lesson Number: M1-L053	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to calculate powers and roots of numbers less than 1 using logarithms.	**Preparation** Bring logarithm and antilogarithm tables, available in the Term 1 Lesson Plans.	

Opening *(3 minutes)*

1. Write the following revision problem on the board: Use logarithms to evaluate 0.005219×0.0964 .
2. Ask pupils to solve the problems in their exercise books.
3. Invite a volunteer to write the solution on the board.
   Solution:

Numbers	Logarithms
0.005219 0.0964	$\bar{3}.7175$ $+ \quad \bar{2}.9841$
Add	$\bar{4}.7016$

   Using the antilog table for $\bar{4}.7016$, we have 5030. The solution is 0.000503.

4. Remind pupils of the steps to divide logarithms if needed: find the logarithms, subtract, then find the antilog.
5. Explain to pupils that today's lesson is on calculating powers and roots of numbers less than 1 using logarithms.

Teaching and Learning *(24 minutes)*

1. Explain **finding powers with logarithms:**
   - For very small numbers or decimal numbers, it is difficult to find the power without a calculator.
   - The product rule allows us to find the powers of numbers using logarithms.
   - We can find any power of numbers in a logarithm using the following steps:
     - Find the logarithm of the numbers.
     - Multiply the result by the power.
     - Find the antilogarithm of the result.
2. Write a problem on the board: Use logarithms to evaluate 0.085^3.
3. Explain the solution to the pupils on the board.
   Solution:
   Step 1. Find the logarithm: $\log 0.085 = \bar{2}.9294$
   Step 2. Multiply by the power: $\bar{2}.9294 \times 3 = \bar{4}.7882$

Step 3. From the antilog table, we find 6141 for $\bar{4}.7882$. Since the characteristic is $\bar{4}$, we add 3 zeros after the decimal. $\text{antilog}\,\bar{4}.7882 = 0.0006141$.

You can also show the work in a table:

Number	Logarithm
0.085^3	$\bar{2}.9294$
	$\times \quad 3$
	$\bar{4}.7882$

From the antilog table we find $6{,}141$. Therefore, $0.085^3 = 0.0006141$.

4. Write another problem on the board: Evaluate 0.59^2.
5. Ask volunteers to give the steps to solve the problem. As they give them, solve on the board.
   Solution:
   Step 1. Find the logarithm: $\log 0.59 = \bar{1}.7709$
   Step 2. Multiply by the power: $\bar{1}.7709 \times 2 = \bar{1}.5418$
   Step 3. Find the antilogarithm: $\text{Antilog}\,\bar{1}.5418 = 3481$. The solution is 0.3481.
6. Explain **finding roots with logarithms:**
   - It is also difficult to find the roots of many numbers without a calculator. We can use logarithms to find roots.
   - We can find the square, cube or other roots of a number using the following steps:
     - Find the logarithm of the number.
     - Divide the result by the root.
     - Find the antilogarithm of the result.
7. Write a problem on the board: Use logarithms to evaluate $\sqrt[4]{0.0007}$.
8. Write the solution on the board, explaining each steps to pupils:
   Step 1. Find the logarithm: $\log 0.0007 = \bar{4}.8451$
   Step 2. Divide by the root: $\bar{4}.8451 \div 4 = \bar{1}.2113$
   Step 3. Find the antilogarithm: $\text{antilog}\,\bar{1}.2113 = 0.1627$
   Therefore $\sqrt[4]{0.0007} = 0.1627$

   You can also show the work in a table:

Number	Logarithm
$\sqrt[4]{0.0007}$	$\bar{4}.8451$
	$\div \quad 4$
	$\bar{1}.2113$

$\text{antilog}\,\bar{1}.2113 = 0.1627$; Therefore, $\sqrt[4]{0.0007} = 0.1627$

9. Write another problem on the board: Use the logarithm table to evaluate $\sqrt{0.00429}$.

10. Invite volunteers to give the steps to solve the problem. As they give them, solve on the board.

Solution:

Step 1. Find the logarithm: $\log 0.00429 = \bar{3}.6325$

Step 2. Divide by the root: $\bar{3}.6325 \div 2 = \bar{2}.8163$

Step 3. Find the antilogarithm: $\text{Antilog}\ \bar{2}.8163 = 0.06551$

Therefore, $\sqrt{0.00429} = 0.06551$

11. Write another problem on the board: Use logarithm tables to evaluate:
    c. $0.25^2 \times 0.417^3$
    d. $\sqrt[3]{(0.398)^2}$
12. Discuss: How do you think we will solve these problems? What steps will we take?
13. Allow pupils to share ideas, then explain:
    - We apply the order of operations (BODMAS), and use the steps we know for solving powers, roots, and multiplication using logarithms.
    - We can find the logarithms of the numbers, and apply all of the operations necessary before applying the antilogarithm for the result.
    - For problem a., we will apply the powers to both numbers, then apply multiplication.
    - For problem b., we will apply the power, then apply the root.
14. Solve problem a. on the board, explaining each step.

Step 1. Draw the table and write the 2 terms of concern under "Number".

Step 2. In the second column, Find the logarithms, and write the multiplication for the powers.

Step 3. In the 3rd column, write the results of the multiplication and add them.

Step 4. Find the antilogarithm of the result.

Number	Logarithm	
0.25^2 0.417^3	$\bar{1}.3979 \times 2 =$ $\bar{1}.6201 \times 3 =$	$\bar{2}.7958$ $+\ \bar{2}.8603$
		$\bar{3}.6561$

$\text{Antilog}\ \bar{3}.6561 = 0.004530$

Therefore, $0.25^2 \times 0.417^3 = 0.00453$

15. Solve problem b. on the board, explaining each step.

Step 1. Find the logarithm of 0.398.

Step 2. Find 0.398^2 by multiplying the logarithm by 2.

Step 3. Find $\sqrt[3]{0.398^2}$ by dividing the result by 3.

Step 4. Find the antilogarithm of the result.

Number	Logarithm
0.398	$\bar{1}.5999$
0.398^2	$\bar{1}.5999 \times 2 = \bar{1}.1998$
$\sqrt[3]{0.398^2}$	$\bar{1}.1998 \div 3 = \bar{1}.7333$

Antilog $\bar{1}.7333 = 0.5412$

Therefore, $\sqrt[3]{(0.398)^2} = 0.5412$

Practice *(12 minutes)*

1. Write two problems on the board: Use logarithm tables to evaluate each of the following. Use a calculator to verify your results.
   a. $(0.267)^5$
   b. $\sqrt[3]{0.03887}$
2. Ask pupils to solve the problems independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to solve the problems on the board at the same time. Other pupils should check their work.

   Solutions:

   a.

Number	Logarithm
0.267^5	$\bar{1}.4265$
	$\underline{\times \quad 5}$
	$\bar{3}.1325$

Antilog $\bar{3}.1325 = 0.001357$

Therefore, $(0.267)^5 = 0.001357$

b.

Number	Logarithm
$\sqrt[3]{0.03887}$	$\bar{2}.5896$
	$\underline{\div \quad 3}$
	$\bar{1}.5299$

Antilog $\bar{1}.5299 = 0.3388$

Therefore, $\sqrt[3]{0.03887} = 0.3388$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L053 in the Pupil Handbook.

Lesson Title: Laws of Logarithms – Part 1	Theme: Numbers and Numeration	
Lesson Number: M1-L054	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to identify that $\log_{10}(pq) = \log_{10} p + \log_{10} q$.	**Preparation** Bring logarithm tables.	

Opening *(3 minutes)*

1. Write the following revision problem on the board: Simplify $6x^2 \times 4x^3$
2. Ask pupils to work with seatmates and solve the problem.
3. Walk around to check for understanding and clear misconceptions.
4. Invite a volunteer to write the solution on the board (Answer: $6x^2 \times 4x^3 = (6 \times 4)x^{2+3} = 24x^5$).
5. Explain to the pupils that today's lesson is on the first law of logarithms.

Teaching and Learning *(22 minutes)*

1. Discuss: What is the rule when you multiply two indices with the same base together, for example $x^2 \times x^3$? (Answer: The rule is that you keep the base and add the exponents.)
2. Write on the board: $\log_{\mathrm{a}} xy = \log_{\mathrm{a}} x + \log_{\mathrm{a}} y$
3. Explain:
   - This is the first law of logarithms.
   - Remember that logarithms are like exponents.
   - The log of a product is the sum of the logarithms of the 2 factors that give the product when multiplied.
   - The logarithms should all have the same base. Remember that when there is not a base given, we are working in base 10.
4. Write a problem on the board: Simplify $\log 3 + \log 4$
5. Write the solution on the board and explain: $\log 3 + \log 4 = \log(3 \times 4) = \log 12$
6. Explain:
   - The log of a sum is NOT the sum of the logs.
   - The log of a sum **cannot** be simplified ($\log_{\mathrm{a}}(x + y) \neq \log_{\mathrm{a}} x + \log_{\mathrm{a}} y$).
7. Write the following problem on the board: Simplify $\log_{10} 3 + \log_{10} 7$.
8. Ask pupils to work with seatmates to solve the problem.
9. Invite a volunteer to write the solution on the board. (Answer: $\log_{10} 3 + \log_{10} 7 = \log_{10}(3 \times 7) = \log_{10}(21)$)
10. Write on the board: Expand $\log_2 20$
11. Discuss:

- What do you think is meant by expand?
- Can you think of another way to write this logarithm?

12. Allow pupils to share ideas and explain: We can write it as the sum of the logarithms of any of its factors that multiply to give 20.
13. Invite volunteers to write the possible answers on the board. Example answers:
    - $\log_2 4 + \log_2 5$
    - $\log_2 2 + \log_2 2 + \log_2 5$
    - $\log_2 10 + \log_2 2$
    - $\log_2 20 + \log_2 1$
14. Write on the board: Expand $\log_5 9x$.
15. Ask pupils to work with seatmates to solve the problem.
16. Walk around to check for understanding and clear misconceptions.
17. Invite a volunteer to write the answer on the board. (Answer: $\log_5 9x = \log_5 9 + \log_5 x$)
18. Write 4 more problems on the board. Simplify the following if possible. If not possible, write "impossible":
    a. $\log_4 x + \log_4 y$
    b. $\log_3 8 + \log_3 5$
    c. $\log_2(6p + 7)$
    d. $\log_4 3 + \log_7 5$
19. Ask pupils to solve the problems with seatmates.
20. Walk around to check for understanding and clear misconceptions.
21. Invite volunteers to write the solutions on the board. (Answers: a. $\log_4 xy$; b. $\log_3 40$; c. impossible; d. impossible)
22. Explain:
    - For problem c., remember that we cannot simplify the logarithm of a sum.
    - For problem d., remember that we cannot combine logarithms with different bases. The bases must be the same.
23. Write another problem on the board: Given that $\log_{10} 5 = 0.6990$ and $\log_{10} 7 = 0.8451$, calculate $\log_{10} 35$.
24. Ask any volunteer to expand $\log_{10} 35$ on the board using the logarithms given in the problem. (Answer: $\log_{10} 35 = \log_{10} 5 + \log_{10} 7$)
25. Ask pupils to work with seatmates to substitute the values of $\log_{10} 5$ and $\log_{10} 7$, and simplify.
26. Invite a volunteer to write the solution on the board. (Answer: $\log_{10} 35 = \log_{10} 5 + \log_{10} 7 = 0.6990 + 0.8451 = 1.5441$)
27. Ask pupils to find $\log_{10} 35$ using the logarithm table. They should find the same answer, 1.5441.

Practice *(14 minutes)*

1. Write the following problems on the board: Given that $\log_5 2 = 0.431$ and $\log_5 3 = 0.682$, find the values of: a. $\log_5 6$ b. $\log_5 8$ c. $\log_5 12$
2. Ask pupils to solve the problem independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Invite three volunteers, one at a time, to write the solutions on the board.

   Solutions:

   a. $$\begin{aligned} \log_5 6 &= \log_5(2 \times 3) \\ &= \log_5 2 + \log_5 3 \\ &= 0.431 + 0.682 \\ &= 1.113 \end{aligned}$$

   b. $$\begin{aligned} \log_5 8 &= \log_5(2 \times 2 \times 2) \\ &= \log_5 2 + \log_5 2 + \log_5 2 \\ &= 0.431 + 0.431 + 0.431 \\ &= 1.293 \end{aligned}$$

   c. $$\begin{aligned} \log_5 12 &= \log_5(2 \times 2 \times 3) \\ &= \log_5 2 + \log_5 2 + \log_5 3 \\ &= 0.431 + 0.431 + 0.682 \\ &= 1.544 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L054 in the Pupil Handbook.

<table>
<tr><td>Lesson Title: Laws of Logarithms – Part 2</td><td colspan="2">Theme: Numbers and Numeration</td></tr>
<tr><td>Lesson Number: M1-L055</td><td>Class: SSS 1</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to identify that $\log_{10}\left(\frac{p}{q}\right) = \log_{10} p - \log_{10} q$.</td><td colspan="2">Preparation
Write the problem in Opening on the board.</td></tr>
</table>

Opening *(4 minutes)*

1. Write the following revision problem on the board: Find the value of r: $\log_5 0.04 = r$
2. Ask pupils to solve the problem independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Invite a volunteer to write the solution on the board.
   Solution:

$$\begin{aligned} r &= \log_5 0.04 \\ 5^r &= 0.04 && \text{Write in index form} \\ 5^r &= \tfrac{1}{25} \\ 5^r &= 25^{-1} \\ 5^r &= 5^{-2} \\ r &= -2 \end{aligned}$$

5. Explain to the pupils that today's lesson is on the division rule of logarithms.

Teaching and Learning *(22 minutes)*

1. Discuss: What is the rule when you divide two values with the same base (Example: $a^5 \div a^2$)? (Answer: Keep the base and subtract the exponents.)
2. Write on the board: $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. Explain:
   - This is the second law of logarithms.
   - Remember that logarithms are like exponents.
   - When there is a quotient in the logarithm, you can subtract the logarithm of the divisor from the logarithm of the dividend.
   - Logarithms should have the same base to apply this law.
4. Write the following problem on the board: Simplify $\log 8 - \log 2$
5. Write the solution on the board: $\log 8 - \log 2 = \log\left(\frac{8}{2}\right) = \log 4$
6. Explain:
   - The log of a difference is NOT the difference of the logs.

- The log of a difference **cannot** be simplified ($\log_a(x-y) \neq \log_a x - \log_a y$).

7. Write the following problem on the board: Expand $\log_{10} \frac{10}{3}$.
8. Write the solution on the board and explain: $\log_{10}\frac{10}{3} = \log_{10} 10 - \log_{10} 3$
9. Write another problem on the board: Express as a single logarithm: $\log_3 21 - \log_3 7$
10. Ask pupils to work with seatmates to solve the problem.
11. Invite a volunteer to write the solution on the board. (Solution**:** $\log_3 21 - \log_3 7 = \log_3(21 \div 7) = \log_3 3$)
12. Write 4 more problems on the board: Simplify the following if possible. If not possible, write "impossible":
    a. $\log_{10} 2x - \log_{10} x$
    b. $\log_2 7 - \log_3 7$
    c. $\log_7 81 - \log_7 9$
    d. $\log(2x - y)$
13. Ask pupils to solve the problems with seatmates.
14. Walk around to check for understanding and clear misconceptions.
15. Invite volunteers to write the solutions on the board. (Solutions: a. $\log_{10}\frac{2x}{x} = \log_{10} 2$; b. impossible; c. $\log_7\left(\frac{81}{9}\right) = \log_7 9$; d. impossible)
16. Write a problem on the board: Solve for x if $x = \log_5 1 - \log_5 25$
17. Discuss: How can we solve this problem? What steps would you take?
18. Allow pupils to share ideas, then explain:
    - Simplify using the second law of logarithms, then apply what we know about the relationship between logarithms and indices.
    - We will solve using the steps we used in the opening problem.
19. Write the solution on the board, explaining each step:

    Solution:

    Step 1. Simplify: $x = \log_5 1 - \log_5 25 = \log_5 \frac{1}{25}$

    Step 2. Write in index form: $5^x = \frac{1}{25}$

    Step 3. Solve for x:

$$
\begin{aligned}
5^x &= \tfrac{1}{25} \\
5^x &= 25^{-1} \\
5^x &= 5^{-2} \\
x &= -2
\end{aligned}
$$

20. Write another problem on the board: Given that $\log_5 2 = 0.431 \text{ and } \log_5 3 = 0.682$, find the value of $\log_5\left(\frac{2}{3}\right)$.
21. Ask pupils to solve the problems with seatmates.
22. Walk around to check for understanding and clear misconceptions.

23. Invite a volunteer to write the solution on the board. (Answer: $\log_5 2 - \log_5 3 = 0.431 - 0.682 = -0.251$)

Practice *(13 minutes)*

1. Write 4 problems on the board:
    a. Simplify $\log_2 12 - \log_2 4$.
    b. Expand: $\log 0.24$.
    c. Given that $\log_{10} 7 = 0.8451$ and $\log_{10} 3 = 0.4771$, find $\log_{10}\left(\frac{9}{7}\right)$.
    d. Find y if $y = \log_3 6 + \log_3 6 - \log_3 3$
2. Ask pupils to solve the problems independently in their exercise books. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write the solutions on the board.
   Solutions:

a.
$$\log_2 12 - \log_2 4 = \log_2 \frac{12}{4} = \log_2 3$$

b.
$$\begin{aligned} \log 0.24 &= \log_5\left(\frac{24}{100}\right) \\ &= \log 24 - \log 100 \end{aligned}$$

c.
$$\begin{aligned} \log_{10}\left(\frac{9}{7}\right) &= \log_{10} 9 - \log_{10} 7 \\ &= \log_{10}(3 \times 3) - \log_{10} 7 \\ &= \log_{10} 3 + \log_{10} 3 - \log_{10} 7 \\ &= 2\log_{10} 3 - \log_{10} 7 \\ &= 2(0.4771) - 0.8451 \\ &= 0.9542 - 0.8451 \\ &= 0.1091 \end{aligned}$$

d.
$$\begin{aligned} y &= \log_3 6 + \log_3 6 - \log_3 4 \\ &= \log_3 \frac{6 \times 6}{4} \\ &= \log_3 \frac{36}{4} \\ y &= \log_3 9 \\ \text{Index form: } 3^y &= 9 \\ 3^y &= 3^2 \\ \therefore y &= 2 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L055 in the Pupil Handbook.

Lesson Title: Laws of Logarithms – Part 3	**Theme:** Numbers and Numeration	
Lesson Number: M1-L056	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to identify that $\log_{10}(p^n) = n\log_{10}p$.	**Preparation** Write the problem in Opening on the board.	

Opening *(4 minutes)*

1. Write on the board: Given that $\log 10 = 1$ and $\log 3 = 0.4771$, find $\log 0.3$.
2. Discuss: What steps would you take? (Example answer: Rewrite $\log 0.3$ as logarithms of 10 and 3 using the second law of indices, then substitute the given values.)
3. Ask pupils to solve the problem independently.
4. Walk around to check for understanding and clear misconceptions.
5. Invite a volunteer to write the solution on the board.
   Solution:

$$\begin{aligned} \log 0.3 &= \log \frac{3}{10} \\ &= \log 3 - \log 10 \\ &= 0.4771 - 1 \\ &= -0.5229 \end{aligned}$$

6. Explain to the pupils that today's lesson is on powers within a logarithm.

Teaching and Learning *(23 minutes)*

1. Write on the board: $\log_a m^r = r \log_a m$
2. Explain:
   - When a logarithmic term has an exponent, the third law of logarithms says that we can transfer the exponent to the front of the logarithm.
   - In the statement on the board, the log m with exponent r is r times the log of m.
   - This rule can also be used for expanding and simplifying logarithms.
3. Write the following problem on the board: Simplify $\log_{10} 3^5$
4. Write the answer on the board: $\log_{10} 3^5 = 5 \log_{10} 3$
5. Write another problem on the board: Simplify $\log_8 x^6$.
6. Ask pupils to write the answer in their exercise books. Invite a volunteer to write the answer on the board. (Answer: $\log_8 x^6 = 6\log_8 x$)
7. Write another problem on the board: Evaluate $\log_3 9$
8. Write the solution on the board, explaining each step:

Solution:

$$\begin{aligned}\log_3 9 &= \log_3 3^2 && \text{Substitute } 9 = 3^2\\ &= 2\log_3 3 && \text{Apply the power rule}\\ &= 2 \times 1 && \text{Substitute } \log_3 3 = 1\\ &= 2\end{aligned}$$

9. Remind pupils of these basic rules for simplifying logarithms:
    - $\log_{\mathrm{a}} a = 1$; the logarithm of any number to the base of the same number is 1.
    - $\log_{\mathrm{a}} 1 = 0$; the logarithm of one (1) in any base is zero.
10. Write the following problem on the board: Simplify $\log 0.9$ if $\log 3 = 0.4771$.
11. Write the solution on the board, explaining each step.

Solution:

$$\begin{aligned}\log 0.9 &= \log \tfrac{9}{10} && \text{Convert to a fraction}\\ &= \log 9 - \log 10 && \text{Apply the division rule}\\ &= \log 3^2 - \log 10 && \text{Substitute } 9 = 3^2\\ &= 2\log 3 - \log 10 && \text{Apply the power rule}\\ &= 2(0.4771) - 1 && \text{Substitute } \log 3 = 0.4771 \text{ and } \log 10 = 1\\ &= 0.9542 - 1\\ &= -0.0458\end{aligned}$$

12. Write another problem on the board: Simplify $2\log_5 \left(\frac{4}{5}\right)$.
13. Ask pupils to give the steps to solve the problem. As they give the steps, write the solution on the board.

Solution:

$$\begin{aligned}2\log_5 \left(\frac{4}{5}\right) &= \log_5 \left(\frac{4}{5}\right)^2\\ &= \log_5 \frac{4^2}{5^2}\\ &= \log_5 2^4 - \log_5 5^2\\ &= 4\log_5 2 - 2\log_5 5\\ &= 4\log_5 2 - 2\end{aligned}$$

Practice *(12 minutes)*

1. Write the following two problems on the board:
    a. Given that $\log_{10} 2 = 0.3010$ and $\log_{10} 7 = 0.8451$, calculate $\log_{10} 2.8$.
    b. Find the value of $\frac{\log_x 64}{\log_x 4}$.
2. Ask pupils to solve the problems independently or with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite two volunteers to write their answers on the board at the same time.

Answers:

a. $$\begin{aligned}\log_{10} 2.8 &= \log_{10}\left(\frac{28}{10}\right)\\ &= \log_{10}\left(\frac{7\times4}{10}\right)\\ &= \log_{10} 7 + \log_{10} 4 - \log_{10} 10\\ &= \log_{10} 7 + \log_{10} 2^2 - \log_{10} 10\\ &= \log_{10} 7 + 2\log_{10} 2 - \log_{10} 10\\ &= 0.8451 + 2(0.3010) - 1\\ &= 0.8451 + 0.6020 - 1\\ &= 0.4771\end{aligned}$$

b. $\frac{\log_x 64}{\log_x 4} = \frac{\log_x 4^3}{\log_x 4} = \frac{3\cancel{\log_x 4}}{\cancel{\log_x 4}} = 3$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L056 in the Pupil Handbook.

Lesson Title: Define and describe sets and elements of a set	**Theme:** Numbers and Numeration	
Lesson Number: M1-L057	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use various ways of writing and describing sets in terms of their members or elements.	**Preparation** Write the words in Opening on the board.	

Opening *(3 minutes)*

1. Write these words on the board: banana, lemon, cabbage, onion, apple, lettuce, okra, orange, cherry, black beans, legumes and mango.
2. Ask pupils to work with seatmates to sort the objects on the board into groups. They may use any property they have in common.
3. Ask volunteers to raise their hands and call out the groups of objects and state the common property.
   Example answers:
      Fruit: banana, lemon, apple, orange, cherry, and mango
      Vegetables: cabbage, onion, lettuce, okra, black beans, and legumes
   Note that pupils may also sort by colour, shape, size or another characteristic.
4. Explain to the pupils that today's lesson is on ways of writing and describing sets in terms of their members or elements.

Teaching and Learning *(25 minutes)*

1. Explain:
   - A **set** is a collection of objects that have something in common or follow a rule.
   - For example, the common groups of food we just listed are referred to as sets because they have common properties.
   - We can have a set of vegetables and a set of fruit. We could have a set of green foods, or a set of round foods.
2. Ask volunteers to give the districts in Sierra Leone that start with letter "k". Write them on the board as a set:
   $$\mathrm{K} = \{\mathrm{Kambia}, \mathrm{Kono}, \mathrm{Kailahun}, \mathrm{Kenema}, \mathrm{Koinadugu}\}$$
3. Explain:
   - This is the set K, which is the set of districts in Sierra Leone that start with the letter K. These districts share a common property, which is their first letter.
   - When we talk about sets, we use a special language and notation.
   - The objects which make up a set are called **elements** or members of the set.
   - We list the elements of a set inside curly brackets.

- We usually use a capital letter for the name of the set (A, B, C, …) and sometimes use lowercase letters (a, b, c, …) for the elements of the set.

4. Write on the board: $\text{Kono} \in K$
5. Explain: The symbol ($\in$) means "is a member of". The element Kono is a member of set K.
6. Write on the board: $B = \{2, 4, 6, 8\}$
7. Write on the board: $2 \in B$
8. Ask pupils to write statements that the other elements are members of B in their exercise books.
9. Invite volunteers to write the statements on the board. (Answers: $4 \in B$, $6 \in B$, $8 \in B$)
10. Write on the board: $3 \notin A$
11. Explain: This statement says that 3 is not a member of A. The symbol $\notin$ means "is not a member of".
12. Write the following problem on the board: M is the set of months of the year with an even number of letters. Write the set M.
13. Ask pupils to work with seatmates to list the set in their exercise books.
14. Invite any volunteer to write the answer on the board. (Answer: M={February , June, July, August, November, December})
15. Write on the board: $n(M) = 6$.
16. Explain:
    - A "cardinality of a set" or the "cardinal number of a set" is the number of elements in the set. It is denoted like this, with the letter n and brackets.
    - This statement says that the set M has 6 elements.
17. Ask pupils to work with seatmates to find the cardinality of K, the set of districts that start with K.
18. Invite a volunteer to write the answer on the board. (Answer: $n(K) = 5$)
19. Write on the board: $A = \{1, 2, 3, 4, \ldots, 1{,}000\}$
20. Discuss: What is the cardinality of this set? (Answer: $n(A) = 1{,}000$)
21. Explain: Sometimes the members of set may be too many for us to list them all. For example, the set A of integers from 1 to $1{,}000$. It is tedious to list all its members so we usually list few of the first elements continue with three dots $(\ldots)$ and write the last number.
22. Write on the board: $B = \{2, 4, 6,\ 8, \ldots\}$
23. Explain: This is the set B of all even numbers. It continues forever, so does not have a cardinality.
24. Write three problems on the board:
    a. Write the set P of prime numbers between 1 and 15.
    b. List all the elements of the set of odd prime numbers between 1 and 20.
    c. Describe the set $\{10, 20, 30, \ldots\}$ in words.
25. Ask pupils to solve the problem with seatmates.

26. Walk around to check for understanding and clear misconceptions.
27. Invite three volunteers to write the answers on the board one after another.

    Answers:
    a. $P = \{2, 3, 5, 7, 11, 13\}$
    b. $\{3, 5, 7, 11, 13, 17, 19\}$
    c. The set of multiples of 10.

Practice *(11 minutes)*

1. Write the following three problems on the board:
    a. Describe the set in words: $\{1, 2, 3, \ldots\}$
    b. List the elements of set Q and determine its cardinality:
    $$Q = \{\text{even numbers between 7 and 29}\}$$
    c. If $P = \{1, 2, 3, 4, 5, 6, 7, 8\}$, insert the appropriate symbol (∈ or ∉) in the blank spaces:
    i. 1 __ P   ii. 0 __ P   iii. 7 __ P   iv. 9 __ P   v. -5 __ P
2. Ask pupils to solve the problems independently.
3. Allow them to solve the problems in their exercise books.
4. Walk around to check for understanding and clear misconceptions.
5. Invite three volunteers to write their answers on the board at the same time. (Answers: a. Example answers: The set of the counting numbers; positive integers; positive whole numbers. b. $Q = \{8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28\}$; $n(Q) = 11$; c. i. $1 \in P$, ii. $0 \notin P$, iii. $7 \in P$, iv. $9 \notin P$, v. $-5 \notin P$)

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L057 in the Pupil Handbook.

Lesson Title: Define and describe sets and elements of a set	Theme: Numbers and Numeration	
Lesson Number: M1-L058	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to use various ways of writing and describing sets in terms of their members or elements.	**Preparation** Write the exercise from Opening on the board.	

Opening *(3 minutes)*

1. Write two problems on the board:
    a. Write the set A of counting numbers greater than 50 but less than 1001.
    b. List all the members of the set of months of the year with an odd number of letters.
2. Ask pupils to solve the problem independently in their exercise book.
3. Invite any volunteers to solve the problem simultaneously on the board. (Answers: a. $A = \{51, 52, 53, \dots 1000\}$; {January, March, April, May, September, October})
4. Explain to the pupils that today's lesson is on writing and interpreting sets using set notation.

Teaching and Learning *(22 minutes)*

1. Write on the board: $Q = \{x \mid x \text{ is a district in Sierra Leone}\}$
2. Read the statement aloud and have pupils repeat: Q is the set of x such that x is a district in Sierra Leone.
3. Explain:
    - Here, the set Q is given by a property, instead of listing the members.
    - This is another way of writing a set. We call this "set-builder notation".
    - Set-builder notation involves describing a set in terms of its characteristic or property, where only those with that property are considered elements.
    - The members of set Q are all of the districts in Sierra Leone.
    - In this example, x is used as a symbol to represent a member of the set under consideration.
    - The vertical line | is read as "such that".
4. Write on the board: $P = \{x \mid x > 0\}$ $\quad P = \{x : x > 0\}$
5. Explain: These statements say the same thing. We can write set-builder notation using either a vertical bar (|) or a colon (:).
6. Discuss: What are the elements of set P? (Answer: P is numbers greater than 0; it is the set of all positive numbers.)
7. Write another example on the board: $A = \{n \mid -3 < n < 3\}$

8. Ask pupils to work with seatmates to list the elements of A.
9. Invite a volunteer to write the answer on the board. (Answer: $A = \{-2, -1, 0, 1, 2\}$)
10. Write the following problem on the board: Use set-builder notation to describe the set of Sierra Leonean tribes.
11. Invite any volunteer to write the answer on the board. (Answer: $\{x: x$ is a Sierra Leonean tribe$\}$)
12. Write the sets on the board:
    - $A = \{x: x > 0\}$
    - $B = \{x | x \neq 11\}$
    - $C = \{x: x < 5\}$
13. Read each set out loud, and have pupils repeat. They are read as:
    - A: The set of all x such that x is a number greater than zero.
    - B: The set of all x such that x is any number except 11.
    - C: The set of all x such that x is any number less than 5.
14. Write on the board: List the members of the following sets and determine the cardinality of each:
    a. $A = \{x: x \text{ is a multiple of 5 but less than } 60\}$.
    b. $B = \{x: 2 \leq x \leq 15$, where x is an integer$\}$
15. Ask pupils to work with seatmates to solve the problems.
    Invite volunteers to write the answers on the board. (Answers: a. $A = \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55\}$; $n(A) = 11$; b. $B = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$; $n(B) = 14$)
16. Write on the board: Write in set-builder notation: $A = \{-3, -2, -1, \ldots, 3\}$
17. Ask pupils to solve the problem with seatmates.
18. Invite a volunteer to write the answer on the board. (Answer: $A = \{x: x$ is an integer between -4 and $4\}$ or $A = \{x: -4 < x < 4\}$.)

Practice *(14 minutes)*

1. Write the following problems on the board:
    a. List the elements of each set and determine the number of elements:
        i. $E = \{x: 18 < x \leq 23\}$.
        ii. $Q = \{x: x$ is an even number between 5 and $35\}$
        iii. $C = \{x: x$ is a multiple of 5 but less than $50\}$
    b. Use set-builder notation to describe the set of prime integers.
    c. List the elements of the set $A = \{x: x$ is a month of the year that doesn't have 31 days$\}$.
    d. Write in set-builder notation: $B = \{0, 1, 2, 3, \ldots, 100\}$
2. Ask pupils to solve the problems independently.
3. Allow them to solve the problems in their exercise books.

4. Walk around to check for understanding and clear misconceptions.
5. Invite volunteers to write their answers on the board at the same time.

   Answers:

   a. i. $E = \{19, 20, 21, 22, 23\}; n(E) = 5$

      ii. $Q = \{6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34\}$; $n(Q) = 15$

      iii. $C = \{5, 10, 15, 20, 25, 30, 35, 40, 45\}$; $n(C) = 9$

      b. $\{x: x$ is a prime number$\}$

      c. $A = \{\text{February, April, June, September, November}\}$

   d. $B = \{x \mid 0 \leq x < 100\}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L058 in the Pupil Handbook.

Lesson Title: Finite and infinite sets	**Theme:** Numbers and Numeration	
Lesson Number: M1-L059	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to define and identify finite and infinite sets.	**Preparation** Write the exercise in Opening on the board.	

Opening *(4 minutes)*

1. Write a revision problem on the board: List the elements of the set $Q = \{x: x$ is a prime number less than $15\}$ and determine the number of elements of the set.
2. Ask pupils to write the answer in their exercise books.
3. Invite a volunteer to write the answer on the board. (Answer: $A = \{2, 3, 5, 7, 11, 13, \}$; $n(A) = 6$)
4. Explain to the pupils that today's lesson is on finite and infinite sets.

Teaching and Learning *(23 minutes)*

1. Discuss and allow pupils to share their ideas:
   - What is the meaning of finite? (Answer: Something is finite if it has an end, or a definite number.)
   - What is the meaning of infinite? (Answer: Something is infinite if it continues forever without ending.)
2. Write 2 sets on the board: $P = \{1, 2, 3, 4, 5, 6, 7\}$ $\quad Q = \{0, 1, 2, 3, \ldots\}$
3. Discuss:
   - How many elements are there in P? (Answer: 7 elements)
   - How many elements are there in Q? (Answer: An infinite number of elements.)
4. Explain:
   - Set P is finite, because it has an end. Set Q is infinite, because it continues on forever.
   - A set is finite if it comes to an end, and infinite if it does not end.
   - An infinite set is uncountable, and the last element cannot be found.
   - We add "ellipses" (…) at the end of infinite sets to show that the elements go on forever.
5. Write on the board: $n(P) = 7$
6. Explain: We can only use set notation to give the cardinality of a finite set.
7. Write on the board: $Q = \{natural\ numbers\ less\ than\ 25\}$.
8. Ask pupils: Is this a finite or infinite set? How do you know?
9. Give a few moments for pupils to discuss this with seatmates.
10. Have any volunteer to share their answer. (Answer: It is finite because we can list and count all its members.)

11. Ask a volunteer to tell how many elements there are in Q. Write the answer on the board. (Answer: $n(Q) = 24$).
12. Write on the board: $Q = \{1, 2, 3, 4, \dots, 24\}$.
13. Remind pupils that this is how we can list out the elements of Q. We use ellipses (…) in the middle of finite sets to write a long list of elements.
5. Write two examples of infinite sets on the board:
   a. $Z = \{3, 6, 9, 12, \dots\}$
   b. $W = \{0, 1, 2, 3, \dots\}$
6. Ask volunteers to describe the sets in their own words. For example:
   a. The set of all positive multiples of 3
   b. The set of numbers equal to or greater than 0.
7. Write 2 problems on the board:
   a. If A is the set of letters in the word "hello", write the elements of set A and find $n(A)$.
   b. C is the set of numbers which are multiples of 4. Write down the elements of C and state whether C is finite or infinite.
8. Ask pupils to discuss the problem with seatmates and write down the answers.
9. Ask volunteers to share their answers with the class. Allow discussion.
   Answers:
   a. $A = \{h, e, l, o\}; n(A) = 4$. Note hat l is not repeated in set notation, and A is a finite set.
   b. $C = \{4, 8, 12, 16, \dots\}$ and C is an infinite set.
10. Write another problem on the board: State which of the following sets are finite and which are infinite:
   a. The set of integers divisible by 5.
   b. The set of counting numbers greater than 11.
   c. $\{x\colon x$ is a positive integer, $x < 10\}$
   d. $\{2, 4, 6, \dots, 100\}$
11. Ask pupils to solve the problems with seatmates
12. Walk around to check for understanding and clear misconceptions.
13. Ask any four volunteers to write their answers on the board. (Answers: a. infinite; b. infinite; c. finite; d. finite.)

Practice *(12 minutes)*

1. Write on the board: State whether each of the following sets is finite or infinite. For finite sets, list the elements and write the cardinality.
   a. $S = \{x\colon x$ is a multiple of $10\}$
   b. $T = \{1, 2, 3, \dots\}$
   c. $A = \{-2 < x \leq 3\}$

d. $Q = \{x: x$ is a vowel in the English alphabet$\}$

2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite volunteers one at a time to write their answers on the board. (Answers: a. infinite; b. infinite; c. finite, $= \{-2, -1, 0, 1, 2, 3\}$, $n(A) = 6$; d. finite, $Q = \{a, e, i, o, u\}$; $n(Q) = 5$.)

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L059 in the Pupil Handbook.

<table>
<tr><td>Lesson Title: Null/empty, unit and universal sets</td><td colspan="2">Theme: Numbers and Numeration</td></tr>
<tr><td>Lesson Number: M1-L060</td><td>Class: SSS 1</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to define and identify null/empty sets, unit sets, and universal sets.</td><td colspan="2">Preparation
Write the exercise in Opening on the board.</td></tr>
</table>

Opening *(3 minutes)*

1. Write on the board: State whether each set is finite or infinite:
   a. $\{a, b, c, d, \ldots, z\}$
   b. $s = \{s: x^2 - 81 = 0\}$
   c. $T = \{x: x \; is \; a \; triangle\}$
   d. $Q = \{\ldots, 3, -2, -1, 0, 1, 2, \ldots\}$
2. Ask pupils to solve the problems independently in their exercise books.
3. Ask four volunteers to call out their answers. (Answers: a. finite; b. finite; c. infinite; d. infinite)
4. Explain to the pupils that today's lesson is on defining and identifying null/empty set, unit set and universal sets.

Teaching and Learning *(22 minutes)*

1. Write on the board: $M = \{\ \}$ $\quad M = \emptyset$
2. Explain **null/empty set:**
   - The 2 statements on the board say the same thing. The set M doesn't have any elements.
   - M is called a null set, or empty set. A null or empty set is a set that does not contain anything.
   - It can be symbolised with empty, curly brackets or the null symbol ($\emptyset$).
3. Give examples of null sets, and encourage volunteers to give their own examples:
   - A set of dogs with six legs.
   - The sets of integers which are both even and odd numbers.
4. Write the problems on the board: Find the elements of $A =$ {odd numbers divisible by 2}
5. Allow 1 minute for the pupils to discuss with seatmates.
6. Allow pupils to share their ideas with the class, then explain that there is no odd number that can be divided by 2.
7. Write on the board: $A = \{\ \}$
8. Write on the board: $B = \{0\}$

9. Discuss: What kind of set is B? How many elements does it have?
10. Allow pupils to share their ideas, then explain **unit sets:**
    - B is not empty. It has 1 element, which is the number 0.
    - B is known as a unit set, or a singleton. This is a set with exactly one element.
11. Give examples of unit sets, and encourage volunteers to give their own examples:
    - Months with 28 days.
    - Current presidents of Sierra Leone.
12. Write on the board: $U =$ {all students in the school}
13. Explain **universal set:**
    - A universal set is the set of all elements or objects under consideration.
    - We can use sets to discuss different groups of pupils in the school. For example, we have Form one, Form two and Form three students in the school. We have female and male students in the school. The universal set is the set of all pupils under consideration; in other words, all pupils in the school.
    - A universal set is denoted by $\mathcal{E}\ or\ U$.
14. Write on the board. Given that $U = \{5, 6, 7, 8, 9, 10, 11, 12\}$, list the elements of the following sets:
    a. $A = \{x: x\ is\ a\ factor\ of\ 60\}$
    b. $B = \{x: x\ is\ a\ prime\ number\}$.
15. Ask pupils to work with seatmates to write the elements of set A, the factors of 60, using **only** the elements of the universal set on the board.
16. Invite a volunteer to write the answer on the board. (Answer: $A = \{5, 6, 10, 12\}$)
17. Ask pupils to work with seatmates to write the elements of set B, the set of prime numbers, using **only** the elements of the universal sets. (Answer: $B = \{5,7,11\}$).
18. Discuss and make sure pupils understand the difference between the 3 types of sets:
    - Remember the set written as { } is an empty set. It contains nothing. The set written as $\{0\}$ is not an empty set. It is a unit set containing one element. The universal set is the set of all elements under consideration.
19. Write three problems on the board:
    a. Let $U = \{1, 2, 3, 4, \ldots, 10\}$. List the elements of the set $A =$ {numbers less than 7}.
    b. State whether each of the following is a null set or unit set:
       i. $\{\text{Sierra Leoneans who are 4 m tall}\}$
       ii. $\{x: x \text{ is an integer, and } 3 + x = 10\}$
       iii. $\{\text{secondary students of age 3 years}\}$
20. Ask pupil to solve the problems with seatmates.
21. Go round and check to see what they are doing. Clear any misconception.
22. Invite volunteers to write down their answers on the board. (Answer: a. $A = \{1, 2, 3, 4, 5, 6\}$; b. i. Null set; ii. Unit set; iii. Null set)

Practice *(14 minutes)*

1. Write the following problems on the board:
   - Decide whether each of the following is an empty set or a unit set:
     i. {teachers with two heads}
     ii. {months with 32 days}
     iii. {days of the week that start with M}
     iv. {whole numbers which have only one factor}
     v. {months with 3 letters}
   - Given that $U = \{a, b, c, d, e, f, g, h, i, j\}$, list the elements of the following:
     i. $V = \{v\text{owels}\}$
     ii. $C = \{\text{letters of the word "cabbag"}\}$
     iii. $T = \{\text{letters of the word "teacher"}\}$
2. Ask pupil to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite volunteers to write their answers on the board. (Answers: a. i. empty set; ii. empty set; iii. unit set; iv. unit set; v. unit set; b. i. $V = \{a, e, i, \}$; ii. $C = \{c, a, b, g, e\}$; iii. $T = \{e, a, c, h\}$)

Closing *(1minute)*

1. For homework, have pupils do the practice activity PHM1-L060 in the Pupil Handbook.

Lesson Title: Equivalent and equal sets	**Theme:** Numbers and Numeration	
Lesson Number: M1-L061	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to define and identify equivalent and equal sets.	**Preparation** Write the exercise problem in the practice section on the board.	

Opening *(3 minutes)*

1. Write on the board: Define the following terms in one sentence:
   a. Null/empty set.   b. Unit set   c. Universal Set
2. Ask pupils to write the sentences in their exercise books.
3. Ask volunteers to read out their answers. Allow multiple volunteers to share and discuss.
   Example answers:
   a. A null/empty set is a set that does not contain any element in it.
   b. A unit set is a set with exactly one element.
   c. A universal set is a set of all elements under consideration.
4. Explain that today's lesson is on defining and identifying equivalent and equal sets.

Teaching and Learning *(22 minutes)*

1. Write on the board: $A = \{1, 5, 7\}$ and $B = \{a, b, c\}$.
2. Discuss: What do you notice about the two sets on the board? Are there any similarities?
3. Allow volunteers to share their ideas. (Answer: They have the same number of elements; $n(A) = 3$ and $n(B) = 3$.).
4. Explain **Equivalents Sets:**
   - Two sets are equivalent if they have the same number of elements. The examples A and B on the board are equivalent.
   - Another way to say this is that they have the same cardinality. There is one-to-one correspondence, which means that for each element in the set A, there exists an element in the set B till the sets get exhausted.
5. Write on the board: $A \leftrightarrow B$
6. Explain: This is the symbol we use to show that 2 sets are equivalent
7. Explain:
   - All the null / empty sets are equivalent to each other.
   - Not all infinite sets are equivalent to each other. For example, consider the set of all real numbers (which includes decimals, fractions, surds, and so on.) and the set of integers. It seems that there are more real numbers, although both sets are infinite.

8. Write on the board: If $A = \{x: x \text{ is a natural number less than 6}\}$ and $B = \{x: x \text{ is a vowel of English alphabet}\}$, show that the sets are equivalent.
9. Ask pupils to solve the problem with seatmates.
10. Invite a volunteer to write the answer on the board. (Answer: $A = \{1, 2, 3, 4, 5\}$ $B = \{a, e, i, o, u\}$; $n(A) = n(B) = 5$).
11. Write on the board: $P = \{1, 2, 3\}$ and $Q = \{2, 3, 1\}$
12. Discuss: What do you notice about the two sets on the board? Are there any similarities?
13. Allow volunteers to share their ideas. (Answer: They have the same elements; $P = Q$)
14. Explain **Equal Sets:**
    - Equal sets are sets which have the same members. The examples P and Q on the board are equivalent because they contain exactly the same elements.
    - The order in which the members of a set are written does not matter.
15. Write on the board: $P = Q$
16. Explain: We use an equals sign to show that 2 sets are equal.
17. Write on the board: $A = \{\text{dog}, \text{cat}, \text{horse}\}$ and $B = \{\text{cat}, \text{horse}, \text{dog}, \text{dog}\}$.
18. Discuss: Can you say if they are equal sets or equivalent sets?
19. Ask pupils to think about this for 1 minute and write down any ideas they have.
20. Ask any volunteers to share their ideas with the class, then explain:
    - Set A has 3 unique elements and set B also has 3 unique elements and can be written as $\{\text{cat}, \text{horse}, \text{dog}\}$. Therefore, the 2 sets are equal because they have exactly the same elements.
    - To be equal, every element of set A must be in set B. Every element in set B must be in set A. This is true for these sets.
    - Set A and B are also equivalent because they have the same number elements $n(A) = n(B) = 3$.
21. Write on the board: All equal sets are equivalent but not all equivalent sets are equal.
22. Write on the board. $\{P = \{1, 2, 3, 4, 5\}$ $Q = \{2, 1, 4, 5, 3\}$, $R = \{a, b, c, d, e\}$. $S = \{\text{Monday}, \text{Tuesday}, \text{Wednesday}, \text{Thursday}, \text{Friday}\}$.
23. Ask pupils to discuss with seatmates which of these sets are equal and which are equivalent to each other.
24. Have pupils from around the classroom volunteer to give their answers. (Answer: P, Q, R, S are equivalents sets because $n(P) = n(Q) = n(R) = n(S) = 5$;. P and Q are equal sets.).
25. Write on the board: Identify the equal and equivalent sets from the following:
    a. $\{1, 2, 3, 4, 5\}$ and $\{7, 6, 3, 4, 5\}$
    b. $\{3, 5, 7, 9, \ldots\}$ and $\{\text{prime numbers}\}$
    c. $\{x: 3x = 9\}$ and $\{3\}$
    d. $\{\text{odd numbers from 1 to 10}\}$ and $\{1, 2, 3, 4, 5\}$
    e. $\{\text{months with 5 letters}\}$ and $\{\text{March}, \text{April}\}$

f. $\{ \ \}$ and $\{2\}$
g. $\{\text{days of the week}\}$ and $\{2, 3, 5, 7, 11, 13, 15\}$
h. $\{x \mid -1 \leq x < 3\}$ and $\{-1, 0, 1, 2, 3\}$

26. Ask pupils to solve the problems with seatmates.
27. Walk around to check for understanding and clear misconceptions.
28. Ask volunteers to share their answers with the class. Make sure everyone understands. (Answers: Equivalent sets: a, c, d, e, g, h; Equal sets: c, e, h)

Practice *(14 minutes)*

1. Write the following problems on the board:
   a. The 4 sets $A, B, C, \text{and } D$ are described below:
      $A = \{2, 4, 6, 8\}$
      $B = \{x: x$ is a letter of the word $\text{LOVE}\}$
      $C = \{x: x$ is a letter of the word $\text{ALLOY}\}$
      $D = \{x: x$ is a letter of the word $\text{LOYAL}\}$
      Use ↔ (is equivalent to) or = (is equal to) to write as many statements as you can using these 4 sets.
   b. What is the difference between equivalent and equal sets? Write 1 sentence.
2. Ask pupil to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write their answers on the board. (Answers: a. All of the sets are equivalent because they all have 4 elements: $A \leftrightarrow B$; $A \leftrightarrow C$; $A \leftrightarrow D$; $B \leftrightarrow C$; $B \leftrightarrow D$; $C \leftrightarrow D$; Only C and D are equal, $C = D$; b. Two sets are said to be equivalent if they have exactly the same number of elements while two sets are said to be equal if they have exactly the same members.)

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L061 in the Pupil Handbook.

<table>
<tr><td>Lesson Title: Subsets</td><td colspan="2">Theme: Numbers and Numeration</td></tr>
<tr><td>Lesson Number: M1-L062</td><td>Class: SSS 1</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Describe and identify subsets of a given set.
2. Represent subsets with Venn diagrams.
3. Use the correct symbols for intersection.</td><td colspan="2">Preparation
Write the exercise in the practice section on the board.</td></tr>
</table>

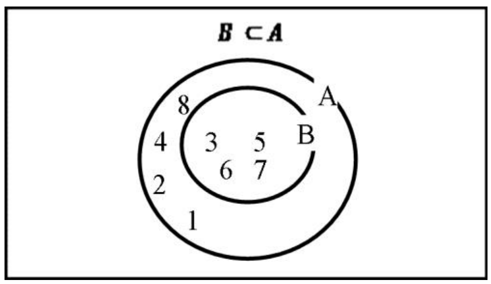
Opening *(3 minutes)*

1. Write a problem on the board. List the elements of the following sets: $P = \{x: x$ is a multiple of 4 less than $30\}$, $Q = \{x: x$ is an even number between 7 and $29\}$.
2. Allow time for pupils to list the elements of the two sets.
3. Ask volunteers to raise their hands to share answers. Write the answers on the board. (Answer: $P = \{4, 8, 12, 16, 20, 24, 28\}$; $Q = \{8, 10,12, 14, 16, 18, 20, 22, 24, 26, 28\}$).
4. Explain to the pupils that today's lesson is to describe, identify and represent subsets with Venn diagrams.

Teaching and Learning *(22 minutes)*

1. Write on the board: $A = \{1,2,3,4, 5\}$ and $B = \{1,2,3\}$.
2. Ask pupils what they notice about the 2 sets, and allow them to share their ideas.
3. Explain:
   - If every element in set B is present in set A, then B is a subset of A.
   - In this example, the 3 elements of B (1, 2, and 3) are in the set A. Therefore, B is a subset of A.
   - We use a special notation for this relationship between the two sets.
4. Write on the board: $\{1, 2, 3\} \subset \{1, 2, 3, 4, 5\}$ ($B \subset A$)
5. Explain:
   - This is the symbol that shows subset.
   - This is read as "set B is a subset of set A".
   - It can also be read as "B contained A".
6. Write on the board: $\subset$ $\not\subset$
7. Explain: The symbol $\subset$ means "subset of" and $\not\subset$ means "not a subset of".
6. Write on the board: $P = \{1, 3, 4\}$ and $Q = \{1, 4, 3, 2\}$
7. Discuss: Are either of these sets a subset of the other?

8. Allow pupils to share ideas, then explain: P is a subset of Q, because every element in P is also in Q.
9. Write on the board: $P \subset Q$ but $Q \not\subset P$
10. Explain:
    - This says that P is a subset of Q but Q is not a subset of P.
    - There is an element in Q (2) that is not in P.
11. Write on the board: $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $B = \{3, 5, 6, 7\}$.
12. Ask pupils to write a subset statement in their exercise books. Invite a volunteer to write the statement on the board. (Answer: $B \subset A$)
13. Explain:
    - We can draw a Venn diagram showing the relationship between sets A and B.
    - B is a subset of A, so it will be inside of A.
14. Draw the Venn diagram as shown:



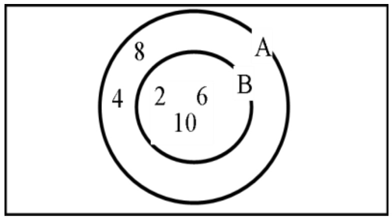
15. Explain:
    - The elements 3, 5, 6, 7 are in A and B.
    - The elements 1, 2, 4, 8 are in A but they are not in B.
16. Write on the board: $C = \{5, 8\}$
17. Discuss: Is this a subset of A or B? (Answer: It is a subset of A, because both 5 and 8 appear in A.)
18. Ask pupils to write down 3 more subsets of set A in their exercise books.
19. Invite volunteers to write their answers on the board. (Example answers: $\{1, 2, 4, 8\}, \{3, 4\}, \{6\}, \{4\}$)
20. Write on the board: $\{\ \} \subset A$.
21. Explain:
    - This says "the empty set is a subset of A".
    - The empty set has nothing in it, and one of the subset of set A also has nothing in it. Therefore, the empty set $\{\ \}$ is a subset of A.
22. Write the following problem on the board. If $A = \{2, 4, 6, 8, 10,\}$ and $B = \{2, 6, 10\}$
    a. Use the set notation to write B is the subset of A.
    b. Draw a Venn diagram to illustrate the relationship between sets A and B.
23. Ask pupils to solve the problem with seatmates.
24. Walk around to check their answers and clear misconceptions.

25. Invite a volunteer to write the answer on the board.

Answers:

a. $B \subset A$

b.

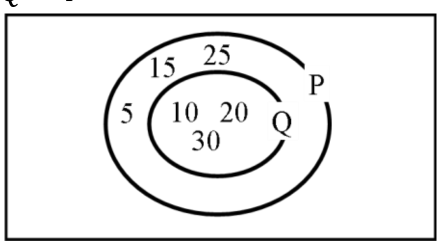


Practice *(14 minutes)*

1. Write two problems on the board.
   a. $P = \{5, 10, 15, 20, 25, 30\}$ and $Q = \{10, 20, 30\}$. Use set notation to write that set Q is a subset of set P and illustrate the two sets in a Venn diagram.
   b. Write all the subsets you can find for the set $\{p, q, r\}$.
2. Ask pupils to answer the question independently in their exercise books.
3. Walk around the classroom to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite two volunteers, one at a time, to write their answers on the board and explain.

Answers:

a. $Q \subset P$



b. $\{\ \}, \{\mathrm{p}\}, \{\mathrm{q}\}, \{\mathrm{r}\}, \{\mathrm{p}, \mathrm{q}\}, \{\mathrm{p}, \mathrm{r}\}, \{\mathrm{q}, \mathrm{r}\}, \{\mathrm{p}, \mathrm{q}, \mathrm{r}\}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L062 in the Pupil Handbook.

<table>
<tr><td>Lesson Title: Intersection of 2 Sets</td><td colspan="2">Theme: Numbers and Numeration</td></tr>
<tr><td>Lesson Number: M1-L063</td><td>Class: SSS 1</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Describe and identify the intersection of 2 sets.
2. Represent the intersection of 2 sets with a Venn diagram.
3. Use the correct symbols for intersection.</td><td colspan="2">Preparation
Write the exercise in Opening on the board.</td></tr>
</table>

Opening *(3 minutes)*

1. Write the following problem on the board:
   a. Write the following in symbols:
      i. C is a subset of M.
      ii. {apple} is an element of the universal set.
   b. Write the following in words:
      i. A ⊂ B ii. D ∉ F
2. Allow pupils to solve the problems in their exercise books.
3. Invite volunteers to write the answers on the board. (Answers: a. i. $C \subset M$; ii. apple $\in U$; b. i. A is a subset of B; ii. D is not an element of F).
4. Explain to the pupils that today's lesson is describing, identifying and representing the intersection of two sets.

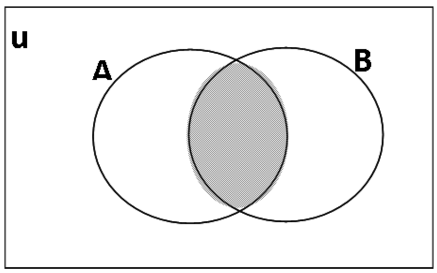
Teaching and Learning *(26 minutes)*

1. Explain: The intersection of the two sets A and B is the set that contains all elements of A that also belongs to B (or all elements of B that also belongs to A) but no other elements.
2. Write on the board:
   A: {1 orange, 1 pineapple, 1 banana, 1 apple}
   B: {1 spoon, 1 orange, 1 knife, 1 fork, 1 apple}
3. Discuss: Do A and B have any common elements? What are they? (Answer: They have common elements "1 orange" and "1 apple")
4. Write on the board: $A \cap B =$ {1 orange, 1 apple}
5. Explain:
   - The intersection of the two sets A and B is the set of elements that are common to both set A and set B.
   - It is denoted with the symbol on the board (∩) and is read "A intersection B".

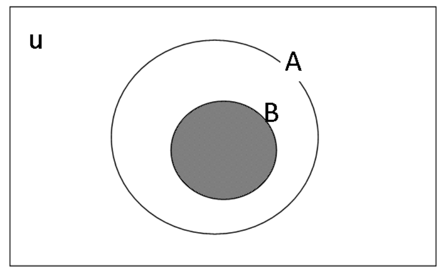
- Basically, we can find the intersection by looking for the elements that 2 sets have in common.

6. Explain:
   - The intersection of the two sets can be drawn using a Venn diagram.
   - A Venn diagram is an important tool allowing relationships between sets to be visualised graphically.
7. Draw the Venn diagrams on the board:

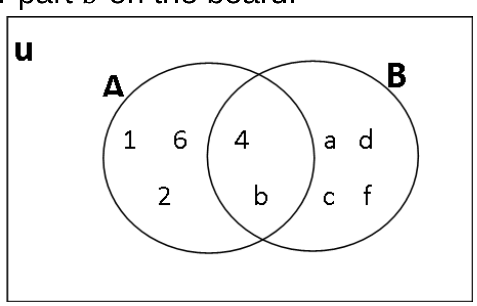
   a. $A \cap B$



   b. $B \subset A$

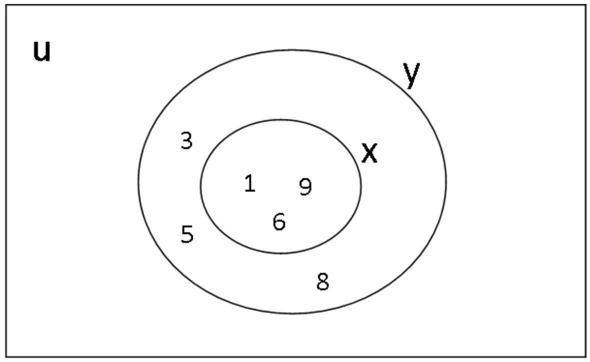


8. Explain:
   - These Venn diagrams show intersections of sets. The shaded portion shows the intersection of the two sets A and B.
   - In Venn diagram b, B is also a subset of A. Their intersection is the entire set B.
9. Write a problem on the board: Consider the following two sets A and B.
   $A = \{b, 1, 2, 4, 6\}$ and $B = \{4, a, b, c, d, f\}$
   a. Find $A \cap B$
   b. Draw one Venn diagram to illustrate both sets.
10. Ask pupils to write in their exercise books the elements that are common in set A and B. Invite a volunteer to write the answer on the board. (Answer: $A \cap B = \{4, b\}$)
11. Draw the Venn diagram for part b on the board:



12. Write on the board: $X \subset Y$, then $X \cap Y = X$.
13. Explain: This is what we saw in Venn diagram b; if one set is the subset of another, then the intersection is the entire subset.
14. Write on the board: $\mathrm{X} = \{1, 6, 9\}$ and $\mathrm{Y} = \{1,3,5,6,8,9\}$. Find $\mathrm{X} \cap \mathrm{Y}$, and draw a Venn diagram to show the relationship.
15. Ask volunteers to give the elements of $\mathrm{X} \cap \mathrm{Y}$, and write it on the board. (Answer: $\mathrm{X} \cap \mathrm{Y} = \{1, 6, 9\}$)

16. Explain to the pupils that $X \cap Y = \{1, 6, 9\}$ is equal to set X. This shows that $X \cap Y = X$.
17. Draw the Venn diagram on the board to illustrate the relationship.



18. Write another problem on the board: Consider the following sets:

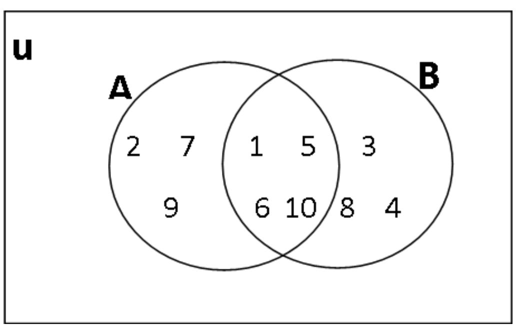
$A = \{1, 2, 5, 6, 7, 9, 10\}$

$B = \{1, 3, 4, 5, 6, 8, 10\}$

   a. Find $A \cap B$
   b. Draw a Venn diagram to illustrate both sets.
19. Ask pupils to solve the problem with seatmates.
20. Walk around to check for understanding and clear misconceptions.
21. Invite a volunteer to write the answer on the board.

Answers:

   c. $A \cap B = \{1, 5, 6, 10\}$
   d.



Practice *(10 minutes)*

1. Write a problem on the board: Use the sets to answer the questions:

$A = \{x: x \text{ is a number bigger than 4 and smaller than 8}\}$

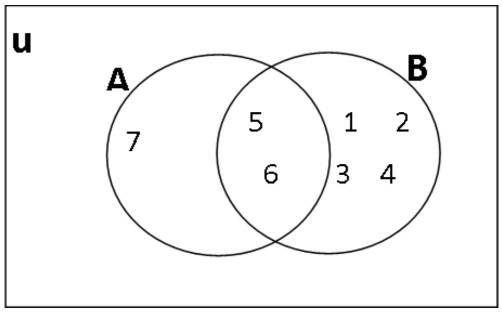
$B = \{x: x \text{ is a positive number smaller than 7}\}$

   a. Find the elements of set A and set B.
   b. Find $A \cap B$.
   c. Draw a Venn diagram to illustrate the relationship of set A and set B.
2. Ask pupils to work independently to solve the problem on the board.

3. Allow pupils to discuss their answers.
4. Ask volunteers to give their answers and explain to the class. All other pupils should check their work.

   Answers:

   a. $A = \{5, 6, 7\}; B = \{1, 2, 3, 4, 5, 6\}$
   b. $A \cap B = \{5, 6\}$
   c.



Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L063 in the Pupil Handbook.

Lesson Title: Intersections of 3 sets	**Theme:** Numbers and Numeration	
Lesson Number: M1-L064	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Describe and identify the intersection of 3 sets. 2. Represent the intersection of 3 sets with a Venn diagram.	**Preparation** Write the problem in Opening on the board.	

Opening *(4 minutes)*

1. Write a problem on the board. Write down the intersection of the sets: $\mathrm{A} = \{10, 20, 30, \ldots, 100\}$ and $\mathrm{B} = \{\text{multiples of 5 between 1 and 50}\}$
2. Allow pupils to share ideas and ask them to solve the problem in their exercise books.
3. Invite a volunteer to write the solution on the board and explain. (Answer: if $\mathrm{A} = \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$ and $\mathrm{B} = \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$; then $\mathrm{A} \cap \mathrm{B} = \{10, 20, 30, 40, 50\}$.)
4. Explain to the pupils that today's lesson is on identifying the intersection of three (3) sets and representing the intersection of 3 sets with a Venn diagram.

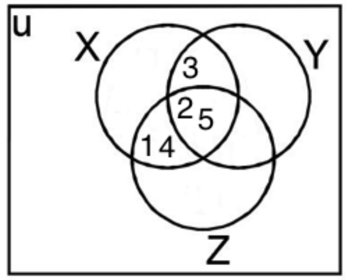
Teaching and Learning *(21 minutes)*

1. Explain: The intersection of three sets X, Y and Z is the set of elements that are common to sets X, Y and Z.
2. Write on the board: $\mathrm{X} \cap \mathrm{Y} \cap \mathrm{Z}$
3. Explain:
   - This is read as "X intersection Y intersection Z".
   - It denotes the set of elements that are common to all 3 sets.
4. Write on the board:

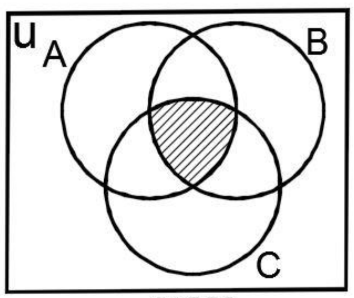
   $\mathrm{X}: \{1,2,3,4, 5\}$

   $\mathrm{Y}: \{2, 3, 5\}$

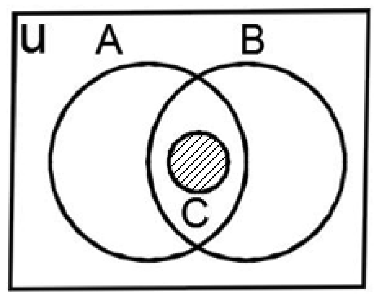
   $\mathrm{Z}: \{1, 2, 4, 5\}$
5. Ask pupils to identify and write down the elements that are common in the sets X, Y and Z.
6. Invite a volunteer to write the answer on the board. (Answer: 2 and 5)
7. Write on the board: $\mathrm{X} \cap \mathrm{Y} \cap \mathrm{Z} = \{2,5\}$.
8. Explain:
   - This conclusion is reached by looking for the elements that are common in the three sets.
   - The intersection of the three sets can be shown using a Venn diagram.
9. Draw on the board:



10. Explain:
    - This Venn diagram shows the intersection of X, Y, and Z.
    - The space in the middle where all 3 circles intersect contains 2 and 5.
    - The other numbers (1, 3, and 4) are each contained in 2 sets. They are in the spaces where only 2 sets overlap.
11. Draw the illustrative Venn diagrams of intersections of 3 sets on the board:

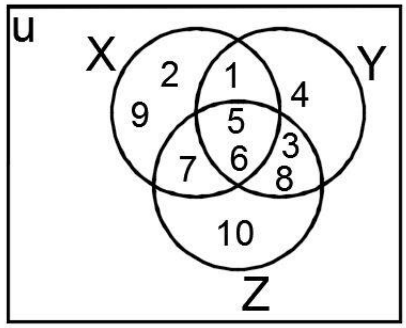


A∩B∩C

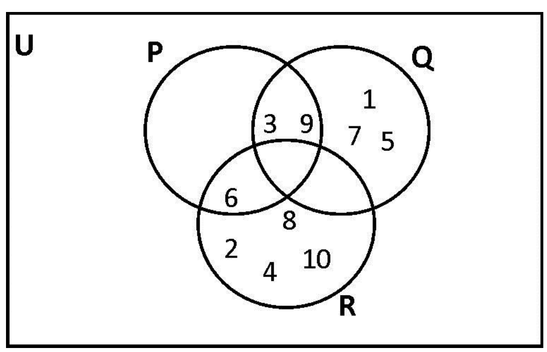


12. Explain:
    - The shaded parts show the intersection of 3 sets on a Venn diagram.
    - In the second Venn diagram, set C is contained in A and B. This means that all of the elements in C also appear in A and B.
13. Write a problem on the board. Consider the following sets: $X = \{1, 2, 5, 6, 7, 9\}$; $Y = \{1, 3, 4, 5, 6, 8\}$ and $Z = \{3, 5, 6, 7, 8, 10\}$.
    a. Find $X \cap Y \cap Z$.
    b. Draw a Venn diagram to illustrate the 3 sets.
14. Invite any volunteer to write down the elements that are common in set X, Y, and Z on the board. (Answer: $X \cap Y \cap Z = \{5, 6\}$).
15. Invite a volunteer to write down the elements in $X \cap Y$. (Answer: $X \cap Y = \{1, 5, 6\}$).
16. Invite a volunteer to write down the elements in $Y \cap Z$. (Answer: $Y \cap Z = \{3, 5, 6, 8\}$).
17. Invite another volunteer to write down the elements in $X \cap Z$. (Answer: $X \cap Z = \{5, 6, 7\}$).
18. Explain: To draw a Venn diagram, start by filling in the elements in the intersection first. Work outwards, filling in the elements that are in only one set last.
19. Draw the Venn diagram for part b. on the board as a class. Invite volunteers to come to the board and identify where each element belongs.

Answer:



20. Write another problem on the board. P, Q and R are subsets of the universal set U, where $\mathrm{U} = \{n: 1 \leq n \leq 10\}$ and n is an integer. P consists of multiples of 3; Q consists of odd numbers and R consists of even numbers. Draw a Venn diagram to illustrate $\mathrm{U}, \mathrm{P}, \mathrm{Q}$ and R.
21. Invite a volunteer to write down the elements of U on the board. (Answer: $\mathrm{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$)
22. Explain: The universal set is $1 \leq n \leq 10$. This means that you will need to locate the integers from 1 and 10 in the Venn diagram.
23. Ask pupils to solve the problem with seatmates.
24. Walk around to check for understanding and clear misconceptions.
25. Invite volunteers to write down the elements of set P, Q and R on the board. (Answer: $\mathrm{P} = \{3, 6, 9\}$, $\mathrm{Q} = \{1, 3, 5, 7, 9\}$, and $\mathrm{R} = \{2, 4, 6, 8, 10\}$)
26. Ask another pupil to write down the elements of the intersections. (Answer: $\mathrm{P} \cap \mathrm{Q} = \{3, 9\}; \mathrm{Q} \cap \mathrm{R} = \{\ \}; \mathrm{P} \cap \mathrm{R} = \{6\}; \mathrm{P} \cap \mathrm{Q} \cap \mathrm{R} = \{\ \}.)$
27. Invite another volunteer to draw the Venn diagram to illustrate the information.



Practice *(14 minutes)*

1. Write a problem on the board:

   U is the universal set consisting of all positive integers x such that $\{1 \leq x \leq 18\}$. P, Q and R are subsets of U such that $\mathrm{P} = \{x: x$ is a factor of $18\}$, $\mathrm{Q} = \{x: x$ is a multiple of 4 less than $18\}$ and $\mathrm{R} = \{10 < x < 18\}$. Draw a Venn diagram to illustrate $\mathrm{U}, \mathrm{P}, \mathrm{Q}$ and R.

2. Ask pupils to work independently to solve the problem in their exercise books.
3. Allow the pupils to discuss their answers with seatmates when they finish.
4. Ask volunteers to give each set and intersection, and the Venn diagram. They should explain to the class. All other pupils should check their work.
   Answers:

$\mathrm{U} = \{1, 2, 3, 4, 5, 6, 7 ,8, 9 ,10, 11, 12 ,13, 14, 15\ 16, 17, 18\}$

$\mathrm{P} = \{1, 2, 3, 6, 9, 18\}$

$\mathrm{Q} = \{4, 8, 12, 16\}$

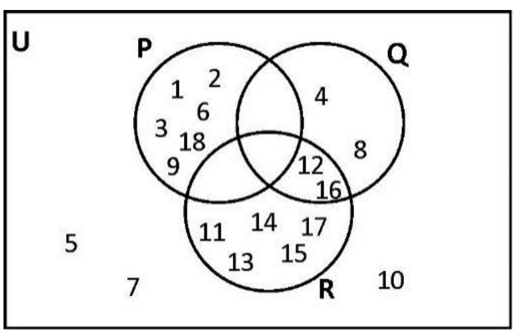
$\mathrm{R} = \{11, 12, 13, 14, 15, 16, 17\}$

$\mathrm{P} \cap \mathrm{Q} = \{\ \}$

$\mathrm{P} \cap \mathrm{Q} = \{\ \}$

$\mathrm{Q} \cap \mathrm{R} = \{12, 16\}$

$\mathrm{P} \cap \mathrm{Q} \cap \mathrm{R} = \{\ \}$



Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L064 in the Pupil Handbook.

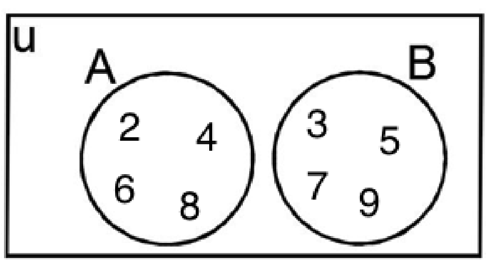
Lesson Title: Disjoint sets	Theme: Numbers and Numeration	
Lesson Number: M1-L065	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Describe and identify disjoint sets. 2. Represent disjoint sets with a Venn diagram.	**Preparation** Write the problem in Opening on the board.	

Opening *(4 minutes)*

1. Write on the board: A, B and C are sets such that $\mathrm{A} = \{y: 0 \leq y \leq 7\}$, $\mathrm{B} = \{4, 6, 8, 10, 12\}$ and $\mathrm{C} = \{y: 1 < y < 7\}$ where y is an integer. Find $\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}$.
2. Ask volunteers to list down the elements of A and C on the board (Example Answer: $\mathrm{A} = \{0, 1, 2, 3, 4, 5, 6, 7\}$, $\mathrm{C} = \{\ 2, 3, 4, 5, 6, \}$)
3. Ask pupils to work with seatmates to find the intersection of the 3 sets.
4. Ask a volunteer to share the answer. (Answer: $\mathrm{A} \cap \mathrm{B} \cap \mathrm{C} = \{4, 6\}$)
5. Explain to the pupils that today's lesson is on describing and identifying disjoint sets.

Teaching and Learning *(17 minutes)*

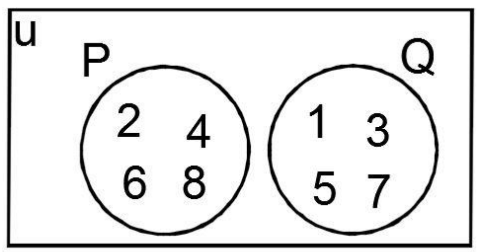
1. Explain: Two sets are said to be disjoint if they have no element in common.
2. Write on the board: $\mathrm{A}: \{2,4, 6, 8\}$ $\qquad$ $\mathrm{B}: \{3, 5, 7, 9\}$
3. Ask pupils to discuss with seatmates what they noticed about the two sets.
4. Explain:
   - The elements in set A are different from the elements in set B. They have no element in common.
   - A and B are disjoint sets.
   - The intersection of 2 disjoint sets is the empty set.
5. Write on the board: $\mathrm{A} \cap \mathrm{B} = \emptyset$
6. Draw the Venn diagram showing A and B:



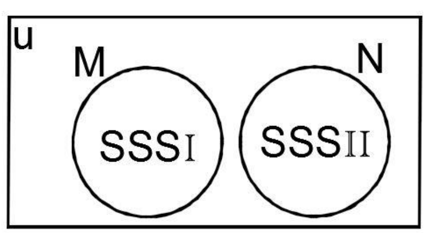
7. Explain: Disjoint sets are shown on a Venn diagram by 2 circles that do not intersect at any point.
8. Explain:
   - "Disjoint events" are used in probability, and are related to "disjoint sets".

- Disjoint events are events that cannot happen at the same time.

9. Discuss some examples of disjoint events. Allow pupils to share their own ideas:
    - A football game can't be held at the same time as a rugby game on the same field.
    - Heading East and West at the same time is impossible.
    - Tossing a coin and getting a head and a tail at the same time.
    - Day and night to occur at the same time.
    - Throwing a die and getting a six and a two at the same time.
10. Write a problem on the board. $\mathrm{P} = \{2, 4, 6, 8\}$ and $\mathrm{Q} = \{1, 3, 5, 7\}$. Explain why P and Q are disjoint sets and illustrate the two sets on a Venn diagram.
11. Ask volunteers to explain why sets P and Q are disjoint. Allow discussion. (Answer: Sets P and Q are disjoint sets because no number of P belongs to Q, i.e. $\mathrm{P} \cap \mathrm{Q} = \emptyset$.)
12. Draw the Venn diagram on the board as a class, illustrating sets P and Q. Ask volunteers to write the numbers in the diagram.
    Answer:



13. Write another problem on the board.
    If $\mathrm{M} = \{\text{set of pupils in SSS I}\}$ and
    $\mathrm{N} = \{\text{set of pupils in SSS II}\}$. Show that set M and N are disjoint using set notation.
    Draw a Venn diagram to illustrate the 2 sets.
14. Ask pupils to solve the problem with seatmates.
15. Walk around to check for understanding and clear misconceptions.
16. Invite a volunteer to write the answer on the board.
    Answer: Since no pupil can be common to both the classes; therefore set M and N are disjoint and $\mathrm{M} \cap \mathrm{N} = \emptyset$.
    Venn diagram:



Practice *(18 minutes)*

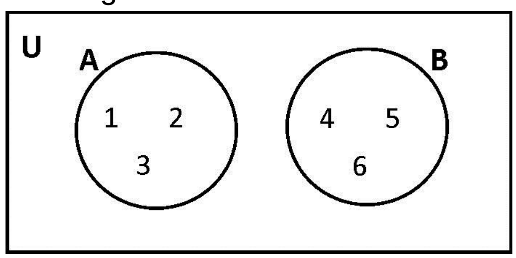
1. Write 3 problems on the board.

a. If $\mathrm{A} = \{1, 2, 3\}$ and $\mathrm{B} = \{4, 5, 6\}$, show that the two sets are disjoint and illustrate the 2 sets on a Venn diagram.
b. If $\mathrm{P} = \{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\}$ and $\mathrm{Q} = \{1, 2, 3, 4, 5\}$, show that P and Q are disjoint sets and illustrate them on a Venn diagram.
c. Write your own example of 2 disjoint sets.

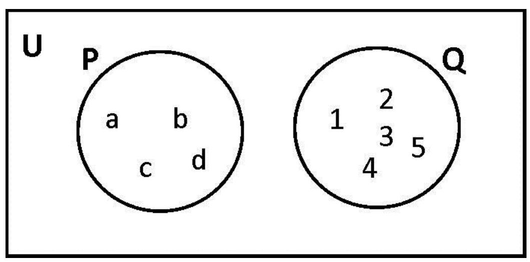
2. Ask pupils to work independently to solve the problems in their exercise books.
3. Allow the pupils to discuss their answers with seatmates when they finish.
4. Ask volunteers to gives their answers and explain to the class. All other pupils should check their work.

Answers:

a. Since no element is common to both set A and B, then the two sets are disjoint sets, $\mathrm{A} \cap \mathrm{B} = \emptyset$. Venn diagram:



b. Since no element is common to both sets P and Q, then the two sets are disjoint sets. $\mathrm{P} \cap \mathrm{Q} = \emptyset$.



c. Ask different volunteers to share their examples and allow discussion.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L065 in the Pupil Handbook.

<table>
<tr><td>Lesson Title: Union of two sets</td><td colspan="2">Theme: Numbers and Numeration</td></tr>
<tr><td>Lesson Number: M1-L066</td><td>Class: SSS 1</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Describe and identify the union of two sets.
2. Represent the union of two sets with a Venn diagram.
3. Use the correct symbols for union.</td><td colspan="2">Preparation
None</td></tr>
</table>

Opening *(4 minutes)*

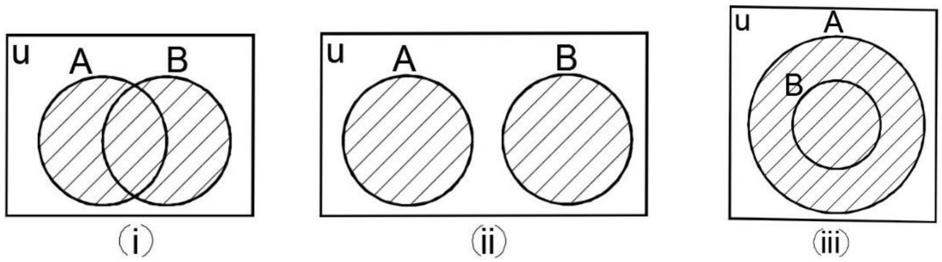
1. Discuss each question:
    a. What is a disjoint set? b. Give 3 examples of disjoint events.
2. Allow pupils to share ideas, then explain:
    a. Two sets are said to be disjoint if they have no element in common.
    b. i. Heading to the North and South at the same time.
    ii. Attending a sports meeting at the National stadium and Lumley Beach at the same time.
    iii. Throwing a die and getting a three and a four at the same time.
3. Explain to the pupils that today's lesson is on describing and identifying the union of two sets.

Teaching and Learning *(20 minutes)*

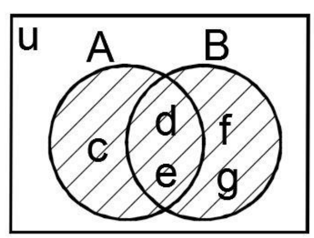
1. Explain: The union of the two sets A and B is the set that contains all of the elements of both sets.
2. Write on the board:
    $\mathrm{A}: \{1 \text{ orange}, 1 \text{ pineapple}, 1 \text{ banana}, 1 \text{ apple}\}$
    $\mathrm{B}: \{1 \text{ spoon}, 1 \text{ knife}, 1 \text{ fork}\}$
3. Discuss:
    - The union of two sets A and B is the set formed by putting the elements of the two sets together.
    - We find the union of the sets by writing all the elements in A and B.
4. Write on the board: $\mathrm{A} \cup \mathrm{B}$
5. Explain: This is how we write the union of the two sets. It is read "A union B".
6. Write $\mathrm{A} \cup \mathrm{B}$ on the board as a class. Ask volunteers to write the elements.
    (Answer: $\mathrm{A} \cup \mathrm{B} = \{1 \text{ orange}, 1 \text{ pineapple}, 1 \text{ banana}, 1 \text{ apple}, 1 \text{ spoon}, 1 \text{ knife}, 1 \text{ fork}\}$)
7. Explain:
    - If a member appears in both sets, it is listed only once.

- Members of a set are usually listed or written in ascending order (smallest members first).
- The union of the two sets can be drawn using the Venn diagram.

8. Draw the illustrative Venn diagrams of unions on the board:

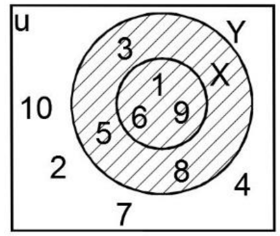


9. Explain: These 3 diagrams show the union of sets. The shaded portion shows the union of the two sets A and B.
10. Write a problem on the board. Consider the following two sets $\mathrm{A} = \{\mathrm{c}, \mathrm{d}, \mathrm{e}\}$ and $\mathrm{B} = \{\mathrm{d}, \mathrm{e}, \mathrm{f}, \mathrm{g}\}$.
    a. Find: $\mathrm{A} \cup \mathrm{B}$
    b. Draw a Venn diagram to illustrate both sets.
11. Invite a volunteer to write down the elements of A and B in a list on the board. (Answer: c, d, e, d, e, f, g).
12. Remind pupils that they should write the elements in ascending order and if a member appears in both sets, it is listed once.
13. Invite volunteers to write down $\mathrm{A} \cup \mathrm{B}$ on the board. (Answer: $\mathrm{A} \cup \mathrm{B} = \{\mathrm{c}, \mathrm{d}, \mathrm{e}, \mathrm{f}, \mathrm{g}\}$).
14. Draw the Venn diagram for part b on the board:

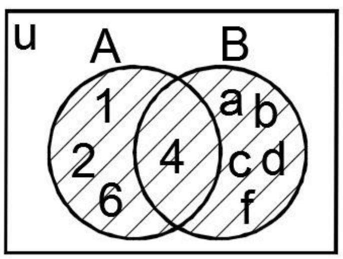


15. Explain: If X is a subset of Y, then the union of the 2 sets is simply the set Y. We can illustrate this relationship on a Venn diagram.
16. Write on the board. The sets $\mathrm{X} = \{1, 6, 9\}$ and $\mathrm{Y} = \{1, 3, 5, 6, 8, 9\}$ are in the universal set $\mathrm{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Find $\mathrm{X} \cup \mathrm{Y}$ and draw a Venn diagram to illustrate $\mathrm{X} \cup \mathrm{Y}$.
17. Invite a volunteer to write $\mathrm{X} \cup \mathrm{Y}$ on the board. (Answer: $\mathrm{X} \cup \mathrm{Y} = \{1, 3, 5, 6, 8, 9\}$).
18. Explain to pupils that $\mathrm{X} \cup \mathrm{Y} = \{1, 3, 5, 6, 8, 9\}$ is equal to set Y. This shows that $\mathrm{X} \cup \mathrm{Y} = \mathrm{Y}$.
19. Write on the board and make sure this is clear: If $\mathrm{X} \subset \mathrm{Y}$ then $\mathrm{X} \cup \mathrm{Y} = \mathrm{Y}$.

20. Draw the Venn diagram on the board to illustrate the relationship:

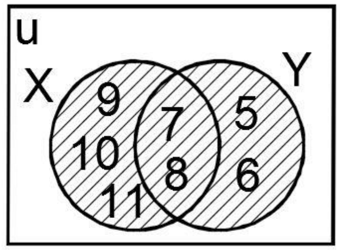


21. Write a problem on the board.
    Consider the following sets: $\mathrm{A} = \{1, 2, 4, 6\}$ and $\mathrm{B} = \{4, \mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{f}\}$.
    a. Find: $\mathrm{A} \cup \mathrm{B}$
    b. Draw a Venn diagram to illustrate both sets.
22. Ask pupils to solve the problem with seatmates.
23. Walk around to check for understanding and clear misconceptions.
24. Invite volunteers to write the answer on the board.
    Answers:
    a. $\mathrm{A} \cup \mathrm{B} = \{1, 2, 4, 6, \mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{f}\}$
    b.



Practice *(15 minutes)*

1. Write the following problem on the board: $\mathrm{X} = \{6 < x < 12\}$ and $\mathrm{Y} = \{4 < x < 9\}$, where x is an integer.
   a. List the elements of sets X and Y.
   b. Find $\mathrm{X} \cup \mathrm{Y}$.
   c. Draw a Venn diagram to illustrate the relationship between sets X and Y.
2. Ask pupils to work independently to solve the problem in their exercise books.
3. Allow the pupils to discuss their answers with seatmates when they finish.
4. Ask volunteers to gives their answers and explain to the class. All other pupils should check their work.
   Answers:
   a. $\mathrm{X} = \{7, 8, 9, 10, 11\}$ and $\mathrm{Y} = \{5, 6, 7, 8, \}$
   b. $\mathrm{X} \cup \mathrm{Y} = \{5, 6, 7, 8, 9, 10, 11\}$

c. Diagram: 

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L066 in the Pupil Handbook.

<table>
<tr><td>Lesson Title: Complement of a set</td><td colspan="2">Theme: Numbers and Numeration</td></tr>
<tr><td>Lesson Number: M1-L067</td><td>Class: SSS 1</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcomes
By the end of the lesson, pupils will be able to:
1. Describe and identify the complement of a set.
2. Represent the complement of a set with a Venn diagram.</td><td colspan="2">Preparation
Write the problem in Opening on the board.</td></tr>
</table>

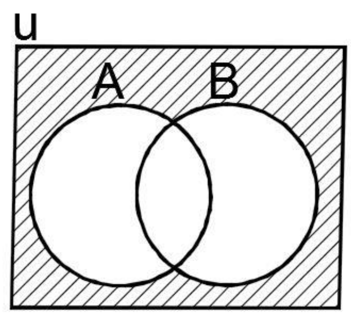
Opening *(4 minutes)*

1. Write a problem on the board: If $\mathrm{P} = \{2, 3,4\}$, $\mathrm{Q} = \{3, 4,5\}$ and $\mathrm{R} = \{3, 4, 7\}$, find $\mathrm{P} \cup \mathrm{Q} \cup \mathrm{R}$.
2. Ask pupils to solve the problem independently.
3. Invite a volunteer to write the answer on the board. (Answer: $\mathrm{P} \cup \mathrm{Q} \cup \mathrm{R} = \{2, 3, 4, 5, 7\}$)
4. Explain that today's lesson is on describing and identifying the complement of a set.

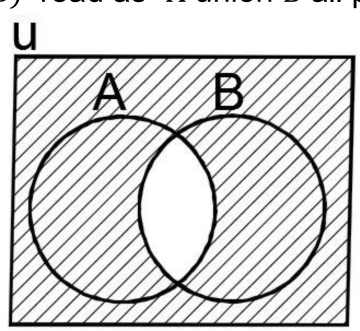
Teaching and Learning *(21 minutes)*

1. Explain:
   - The **complement** of a set A refers to elements not in A.
   - If A is a subset of the universal set U, then the complement of A is the set of members which belong to the universal set U but do not belong to A.
2. Write on the board: $\mathrm{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $\mathrm{A} = \{1, 3, 5, 7, 9\}$
3. Discuss: The complement of a set is equal to elements in the universal set minus elements in the set.
4. Write on the board: A'
5. Explain: The complement of A is written with an apostrophe like this, and is read as "A prime".
6. Ask volunteers to give the elements in the complement of A, given U and A on the board. Write the complement on the board as they give its elements (Answer: $\mathrm{A}' = \{2, 4, 6, 8\}$).
7. Explain: If we did not specify the universal set U, there would be rather a lot of elements not in A. Thus, the elements of U are normally given in the problem.
8. Write a problem on the board: If $\mathrm{U} = \{-5 < x \leq 5\}$ and $\mathrm{A} = \{-2 < x \leq 3\}$. Find A'.
9. Invite a volunteer to write down the elements of the universal set U on the board. (Answer: $\mathrm{U} = \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$).
10. Ask another volunteer to write down the elements of set A on the board. (Answer: $\mathrm{A} = \{-1, 0, 1, 2, 3\}$)
11. Ask pupils to work with seatmates to write the complement of A in their exercise books.

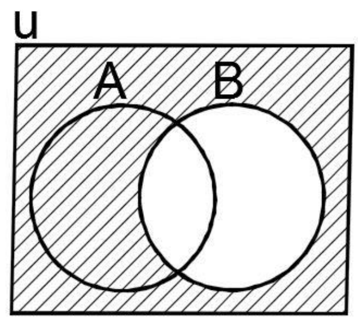
12. Invite a volunteer to write the elements of A' on the board. (Answer: $A' = \{-4, -3, -2, 4, 5\}$).
13. Explain: The complement of the complement of a set A is the set itself.
14. Write on the board and make sure pupils understand: $(A')' = A$. For example, $U = \{2, 4, 6, 8, 10\}$ and $A = \{2, 6, 10\}$ then $A' = \{4, 8\}$. Again taking the complement of A' we get $(A')' = A$.
15. Explain each of the following properties of sets, and write the set notation given in brackets on the board:
    - The complement of the universal set is the empty set. The complement of U is the set which does not contain any element ($U' = \{\ \}$).
    - The complement of an empty set is the universal set under consideration. ($\emptyset' = U$).
    - The union of a set and its complement give the universal set ($A \cup A' = U$).
    - The intersection of a set and its complement is an empty set; there are no elements that exist in both sets ($A \cap A' = \emptyset$).
16. Draw on the board the following Venn diagrams representing complement of a set. Make sure pupils understand each one:



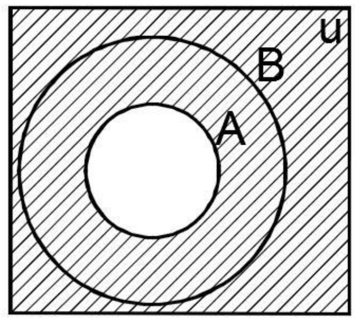
$(A \cup B)'$ read as "A union B all prime"



$(A \cap B)'$ read as "A intersection B all prime"



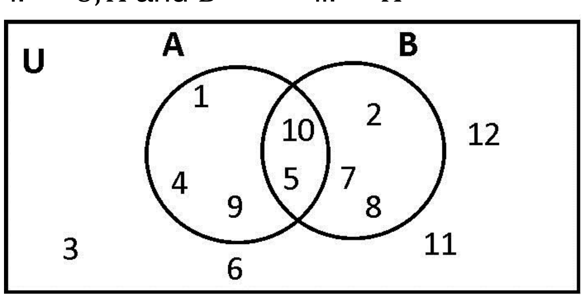
B' read as "B prime"



$(\mathrm{A} \subset \mathrm{B})'$ read as "prime of A subset B".

17. Write on the board: If $\mathrm{A} = \{11, 12, 13, 14\}$ and $\mathrm{B} = \{13, 14, 15, 16, 17\}$ are subsets of $\mathrm{U} = \{10, 11, 12, 13, 14, 15, 16, 17, 18\}$, then list the elements of:
    i. A' ii. B' iii. $\mathrm{A}' \cap \mathrm{B}'$
18. Ask pupils to solve the problem with seatmates.
19. Remind pupils that $\mathrm{A}' \cap \mathrm{B}'$ is the common elements in A' and B'.
20. Walk around the class to check for understanding and clear misconceptions.
21. Invite three volunteers, one at a time, to write the answers on the board.

    Answers:
    i. $\mathrm{A}' = \{10, 15, 16, 17, 18\}$
    ii. $\mathrm{B}' = \{10, 11, 12, 18\}$
    iii. $\mathrm{A}' \cap \mathrm{B}' = \{10, 18\}$
22. Write another problem on the board. Using the Venn diagram below, write the following sets:
    i. U, A and B ii. A' iii. B'

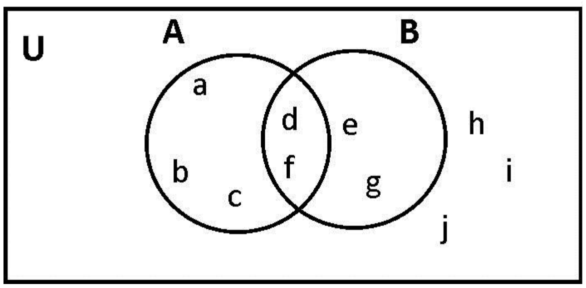


23. Ask pupils to solve the problem with seatmates.
24. Walk around the class to check for understanding and clear misconceptions.

25. Invite three volunteers to write the answer on the board.
Answers:
  i. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
  $A = \{1, 4, 5, 9, 10\}$
  $B = \{2, 5, 7, 8, 10\}$
  ii. $A' = \{2, 3, 6, 7, 8, 11, 12\}$
  iii. $B' = \{1, 3, 4, 6, 9, 11, 12\}$

Practice *(14 minutes)*

1. Write on the board: Use the Venn diagram to write the following sets:
  i. A' ii. B' iii. $A' \cap B'$



2. Ask pupils to work independently to solve the problem in their exercise books.
3. Allow the pupils to discuss their answers with seatmates when they finish.
4. Ask volunteers to give the answers. All other pupils should check their work.
Answers:
Use the fact that $U = \{a, b, c, d, e, f, g, h, i, j\}$, $A = \{a, b, c, d, f\}$ and $B = \{d, e, f, g\}$.
  i. $A' = \{e, g, h, i, j\}$ i.e. elements of U which are not in A.
  ii. $B' = \{a, b, c, h, i, j\}$ i.e. elements of U which are not in B.
  iii. $A' \cap B' = \{h, i, j\}$ i.e. elements common in A' and B'.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L067 in the Pupil Handbook.

Lesson Title: Problem solving with 2 sets	**Theme:** Numbers and Numeration	
Lesson Number: M1-L068	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to diagram and solve problems involving 2 sets, including real-life problems.	**Preparation** Write the problem in Opening on the board.	

Opening *(4 minutes)*

1. Write a problem on the board. If $\mathrm{U} = \{1,2,3,6,9,18\}$, $\mathrm{X} = \{2,6,18\}$ and $\mathrm{Y} = \{1,3,6\}$, list the elements of: a. $\mathrm{X}' \cap \mathrm{Y}$ b. $\mathrm{X} \cup \mathrm{Y}'$ c. $(\mathrm{X} \cup \mathrm{Y})'$
2. Ask pupils to solve the problem independently in their exercise books.
3. Invite three volunteers to write their answers on the board at the same time.
   Answers:
   Note that $\mathrm{X}' = \{1,3,9\}$ and $\mathrm{Y}' = \{2,9,18\}$.
   a. $\mathrm{X}^1 \cap \mathrm{Y} = \{1,3,9\} \cap \{1,3,6\} = \{1,3\}$
   b. $\mathrm{X} \cup \mathrm{Y}' = \{2,6,18\} \cup \{2,9,18\} = \{2,6,9,18\}$
   c. $(\mathrm{X} \cup \mathrm{Y})'$ is the complement of $\mathrm{X} \cup \mathrm{Y}$.
   $\mathrm{X} \cup \mathrm{Y} = \{1,2,3,6,18\}$, therefore $(\mathrm{X} \cup \mathrm{Y})' = \{9\}$.
4. Explain that today's lesson is on solving real life problems involving 2 sets.

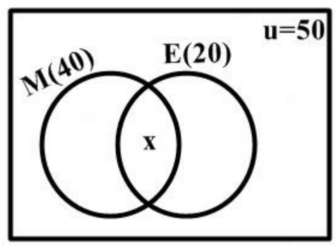
Teaching and Learning *(21 minutes)*

1. Write a question on the board: If A and B are two finite sets such that $n(\mathrm{A}) = 20$, $n(\mathrm{B}) = 28$ and $n(\mathrm{A} \cup \mathrm{B}) = 36$, find $n(\mathrm{A} \cap \mathrm{B})$.
2. Discuss: What does the letter n tell us in set notation? (Answer: It gives the number of elements in a set.)
3. Explain:
   - $n(\mathrm{A} \cup \mathrm{B})$ is the number of elements present in either of the sets A or B.
   - $n(\mathrm{A} \cap \mathrm{B})$ is the number of elements present in both the sets A and B.
4. Discuss: How can you interpret this question? What is it asking? (Example answer: It is asking for the number of elements that are in both A and B, or the intersection of A and B.)
5. Explain:
   - We are given the number of elements in both A and B, and we are given the number of elements in their union.
   - We can use this information to find the number of elements in their intersection.
6. Write on the board: For two sets A and B: $n(\mathrm{A} \cup \mathrm{B}) = n(\mathrm{A}) + n(\mathrm{B}) - n(\mathrm{A} \cap \mathrm{B})$
7. Explain: This formula states that we add the total number of elements in A and B, and subtract the number of elements in their intersection. This is because we do not want to count the elements in their intersection twice.
8. Solve the problem on the board using the formula.

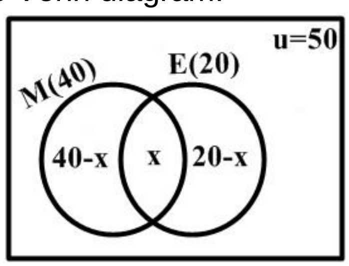
Solution:

$$\begin{aligned} n(A \cap B) &= n(A) + n(B) - n(A \cup B) \\ &= 20 + 28 - 36 \\ &= 48 - 36 \\ &= 12 \end{aligned}$$

9. Write a problem on the board: In a class of 50 pupils, 40 read Mathematics and 20 read English. Each pupil reads at least one subject. How many pupils read both Mathematics and English?
10. Write on the board. Let $\mathrm{U} = \{\text{pupils in the class}\}$, $\mathrm{M} = \{\text{pupils who read Mathematics}\}$, $\mathrm{E} = \{\text{pupils who read English}\}$ and $\mathrm{x} =$ number of pupils who read both subjects.
11. Invite a volunteer to identify the number of pupils in the universal set and write it on the board. (Answer: $n(\mathrm{U}) = 50$).
12. Invite volunteers to identify the number of pupils reading each subject and write them on the board. (Answer: $n(\mathrm{M}) = 40,\ n(\mathrm{E}) = 20$).
13. Draw the Venn diagram for this problem on the board:



14. Discuss: How can we identify the number of pupils who read only Maths or only English? (Answer: Subtract x from the number who read each subject.)
15. Remind pupils that $n(\mathrm{M} \cap \mathrm{E}) = x$.
16. Ask volunteers to identify the number of pupils who read only Maths and only English. (Answer: For Maths only: $(40 - x)$, for English only: $(20 - x)$)
17. Illustrate this information on the Venn diagram:



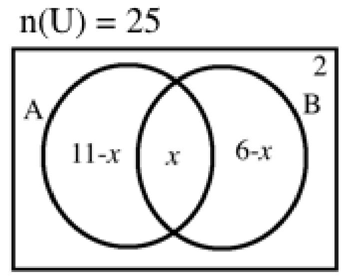
18. Write the solution on the board and explain:
    From the Venn diagram, we have the equation $(40 - x) + x + (20 - x) = 50$.
    Solve the equation for x:

$$\begin{aligned} (40 - x) + x + (20 - x) &= 50 \\ 40 - x + x + 20 - x &= 50 \\ 40 + 20 - x &= 50 \\ x &= 60 - 50 \\ x &= 10 \end{aligned}$$

Therefore, 10 pupils read both Mathematics and English.

19. Explain:
    - Always represent the intersection of the two regions with a variable first.
    - If a variable is not given in the problem, use one such as x or n.
20. Write a problem and diagram on the board:

    In the diagram, two sets A and B are subsets of the universal set U. Given that $n(\mathrm{U}) = 25$, find:
    a. The value of x.
    b. The value of $n(\mathrm{A} \cup \mathrm{B})$.



21. Ask pupils to solve the problem with seatmates.
22. Walk around and check their answers and clear misconceptions.
23. Invite volunteers to write the solutions on the board and explain. Support them as needed.

    Solutions:
    a. To find x, set up an equation that sums up to $n(\mathrm{U}) = 25$. Notice the 2 in the universal set outside of A and B. This must be included too:

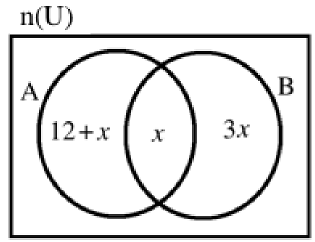
$$\begin{aligned} (11 - x) + x + (6 - x) + 2 &= 25 \\ 11 - x + x + 6 - x + 2 &= 25 \\ 11 + 6 + 2 - x &= 25 \\ 19 + x &= 25 \\ x &= 25 - 19 = 6 \end{aligned}$$

    b. To find $n(\mathrm{A} \cup \mathrm{B})$, add the number of elements in both A and B. Substitute $x = 6$ and evaluate:

$$\begin{aligned} n(\mathrm{A} \cup \mathrm{B}) &= 11 - x + x + 6 - x \\ &= 11 + 6 - x \\ &= 11 + 6 - (6) \\ &= 11 \end{aligned}$$

Practice *(14 minutes)*

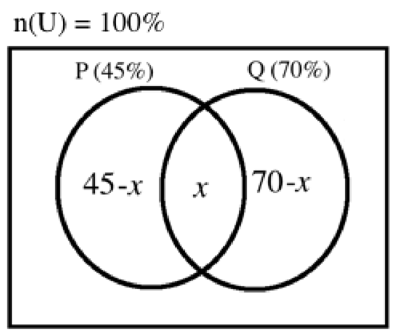
1. Write the following two problems on the board.
    a. In a certain village 45% are Public servants, and 70% are Farmers. Each person is in at least one of the jobs. Find the number of people who are in both jobs.
    b. A and B are two sets and the number of elements are shown in the Venn diagram below. Given that $n(\mathrm{A}) = n(\mathrm{B})$, find:
        i. x ii. $n(\mathrm{A} \cup \mathrm{B})$



6. Ask pupils to work independently or with seatmates to solve the problem in their exercise books. Allow discussion.
7. Ask two volunteers to gives their solutions and explain to the class. All other pupils should check their work.

   Solutions:

   a. Let $\mathrm{U}=$ The people in the village. Then $n(\mathrm{U})=100\%$.
   If $\mathrm{P}=$ Public servants, then $n(\mathrm{P})=45\%$
   If $\mathrm{F}=$ Farmers, then $n(\mathrm{F})=70\%$
   Let $x=$ those in both jobs. Then, $x=n(\mathrm{P}\cap\mathrm{F})$.
   Since each person is in at least one of the jobs, it means there is nobody who does nothing in the village. Draw a Venn diagram:



   The number of people in both jobs is $x=n(\mathrm{P}\cap\mathrm{F})$. We know the 3 parts in the circles of the Venn diagram will sum to 100%. Set up the equation and solve for x:

$$\begin{aligned}100 &= (45-x)+x+(70-x)\\ 100 &= 45-x+x+70-x\\ 100 &= 45+70-x\\ 100 &= 115-x\\ x &= 115-100=15\end{aligned}$$

   Those in both jobs$=15\%$

   b. i. We have $n(\mathrm{A})=n(\mathrm{B})$. Set the number of elements in the 2 sets equal and solve for x.

$$\begin{aligned}12+x+x &= 3x+x\\ 12+2x &= 4x\\ 12 &= 4x-2x\\ 12 &= 2x\\ 6 &= x\end{aligned}$$

$$x = 6$$

ii. To find $n(\text{A} \cup \text{B})$, find the sum of all of the parts inside A and B. Substitute $x = 6$ and evaluate:

$$\begin{aligned} n(\text{A} \cup \text{B}) &= 12 + x + x + 3x \\ &= 12 + 5x \\ &= 12 + 5(6) \\ &= 12 + 30 = 42 \end{aligned}$$

Closing *(1 minute)*

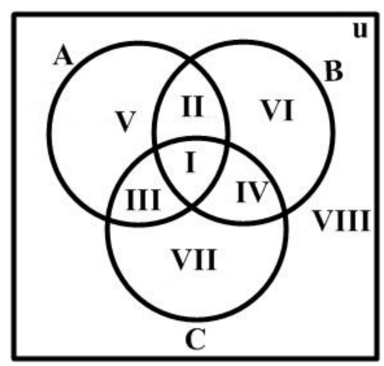
1. For homework, have pupils do the practice activity PHM1-L068 in the Pupil Handbook.

Lesson Title: Problem solving with 3 sets – Part 1	**Theme:** Numbers and Numeration	
Lesson Number: M1-L069	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to diagram and solve problems involving 3 sets, including real-life problems.	**Preparation** Write the problem in Opening on the board.	

Opening *(4 minutes)*

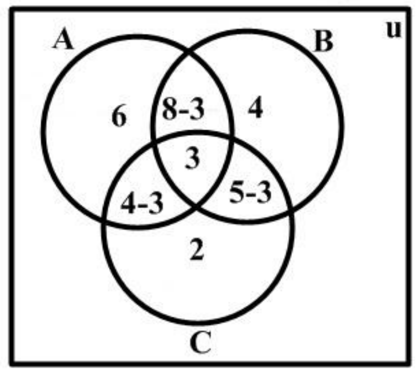
1. Write the following problem on the board. If $\mathrm{U} = \{\mathrm{r, e, v, o, l, t, i, n, g}\}$, $\mathrm{A} = \{\mathrm{l, i, o, n}\}$ and $\mathrm{B} = \{\mathrm{t, i, g, e, r}\}$, list the members of the following sets.
   a. A' b. $\mathrm{A}' \cap \mathrm{B}'$ c. $(\mathrm{A} \cap \mathrm{B})'$
2. Allow pupils to solve the problem independently in their exercise books.
3. Invite volunteers, one at a time, to write their answers on the board.
   (Answer: x' is the complement of x, i.e. those elements in U which are not in x.
   a. $\mathrm{A}' = \{\mathrm{r, e, v, t, g}\}$
   b. $\mathrm{B}' = \{\mathrm{v, o\ , l, n}\}$
   $\mathrm{A}' \cap \mathrm{B}' = \{\ \mathrm{v}\ \}$
   c. $\mathrm{A} \cap \mathrm{B} = \{\ \mathrm{i}\ \}$
   $(\mathrm{A} \cap \mathrm{B})' = \{\mathrm{r, e, v, o, l, t, n, g}\}$
4. Explain to pupils that today's lesson is on solving real life problems involving 3 sets.

Teaching and Learning *(21 minutes)*



1. Draw the Venn diagram on the board:
2. Explain: If A, B and C are subsets of U, the universal set, then the Venn diagram at right shows the various regions of a problem with 3 sets.
3. Discuss the elements in each section of the diagram. You may write each with symbols given on the board if needed.
   - I is the intersection of all the sets. $(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C})$
   - II is the elements of A and B only. $(\mathrm{C}' \cap (\mathrm{B} \cap \mathrm{A}))$
   - III is the elements of A and C only. $(\mathrm{B}' \cap (\mathrm{A} \cap \mathrm{C}))$
   - IV is the elements of B and C only. $(\mathrm{A}' \cap (\mathrm{B} \cap \mathrm{C}))$
   - V is the elements in A only. ($\mathrm{A} \cap (\mathrm{B} \cup \mathrm{C})'$ or $\mathrm{A} \cap \mathrm{B}' \cap \mathrm{C}'$)
   - VI is the elements in B only. ($\mathrm{B} \cap (\mathrm{C} \cup \mathrm{A})'$ or $\mathrm{B} \cap \mathrm{C}' \cap \mathrm{A}'$)
   - VII is the elements in C only. ($\mathrm{C} \cap (\mathrm{B} \cup \mathrm{A})'$ or $\mathrm{C} \cap \mathrm{B}' \cap \mathrm{A}'$)
   - VIII is the elements that are not in any of the 3 sets.

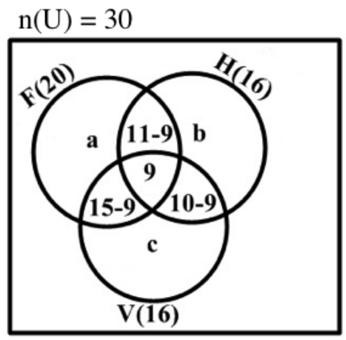
4. Explain: If there are no elements in U that are outside of A, B and C, then the cardinality of U is the cardinality of the union of A, B and C. (If VIII= 0 then $n(\mathrm{U}) = n(\mathrm{A} \cup \mathrm{B} \cup \mathrm{C})$)
5. Write the following problem on the board: Draw a Venn diagram of the universal set U containing three sets A, B and C. Given that $n(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}) = 3$, $n(\mathrm{A} \cap \mathrm{B}) = 8$, $n(\mathrm{A} \cap \mathrm{C}) = 4$, $n(\mathrm{B} \cap \mathrm{C}) = 5$, $n(\mathrm{A} \cap \mathrm{B}' \cap \mathrm{C}') = 6$, $n(\mathrm{A}' \cap \mathrm{B}' \cap \mathrm{C}) = 2$ and $n(\mathrm{A}' \cap \mathrm{B} \cap \mathrm{C}') = 4$. Find $n(\mathrm{A})$, $n(\mathrm{B})$ and $n(\mathrm{C})$.
6. Ask pupils to think about this for 2 minutes and write down any ideas they have in drawing the Venn diagram.
7. Ask volunteers to share their ideas with the class.
8. Draw the Venn diagram on the board and explain to pupils. Example explanations are given:
   - $n(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}) = 3$ tells us that 3 elements are in the middle
   - $n(\mathrm{A} \cap \mathrm{B}) = 8$ tells us that 8 elements are in the intersection of A and B, but we must subtract 3 to give the number in only A and B.
   - $n(\mathrm{A} \cap \mathrm{B}' \cap \mathrm{C}') = 6$ tells us that 6 elements are in A that are not in B or C.



9. Ask any volunteer to add all the values in set A.
   Answer: n(A) = $6 + (8-3) + 3 + (4-3)$
   $= 6 + 5 + 3 + 1$
   $= 15$
10. Ask another volunteer to add the values in sets B.
    Answer: n(B)= $4 + (8-3) + (5-3) + 3$
    $= 4 + 5 + 2 + 3$
    $= 14$
11. Ask another volunteer to add the values of set C.
    Answer: n(C) $= 2 + (4-3) + (5-3) + 3$
    $= 2 + 1 + 2 + 3$
    $= 8$
12. Write another problem on the board: There are 30 pupils in a class. 20 of them play football, 16 play hockey and 16 play volleyball. 9 play all three games, 15 play football and volleyball, 11 play football and hockey, while 10 play hockey and volleyball.
    a. Illustrate the information on a Venn diagram.
    b. Find the number of pupils who play: i. only football; ii. only hockey; iii. only volleyball.
    c. Using your Venn diagram, find the number of pupils who play at least two games.
13. Represent the different games with letters. Write them on the board: U = the pupils in the class, F = football, H = hockey, V = volleyball, a = football only, b = hockey only, c = volleyball only

14. Ask volunteers to give the values for each cardinality of the various games played by pupils. Write them on the board. (Answer: $n(\mathrm{U}) = 30,\ n(\mathrm{F}) = 20,\ n(\mathrm{H}) = 16, n(\mathrm{V}) = 16, n(\mathrm{F} \cap \mathrm{H} \cap \mathrm{V}) = 9,\ n(\mathrm{F} \cap \mathrm{V}) = 15,\ n(\mathrm{F} \cap \mathrm{H}) = 11$ and $n(\mathrm{H} \cap \mathrm{V}) = 10$).
15. Ask pupils to work with seatmates to illustrate the information in a Venn diagram (question a).
16. Walk around to check for understanding and clear misconceptions.
17. Invite a volunteer to draw the Venn diagram on the board.

    Answer:



18. On the board, solve for a, those who play football only:

$$20 = a + 2 + 6 + 9$$
$$20 = a + 17$$
$$a = 20 - 17$$
$$a = 3$$

19. Ask pupils to work with seatmates to find the numbers of pupils who played only hockey and only volleyball.
20. Invite volunteers to write the solutions on the board and explain.

    Solution:

    Those who play hockey only:

$$16 = b + 2 + 9 + 1$$
$$16 = b + 12$$
$$b = 16 - 12$$
$$b = 4$$

    Those play volleyball only:

$$16 = c + 6 + 9 + 1$$
$$16 = c + 16$$
$$c = 16 - 16$$
$$c = 0$$

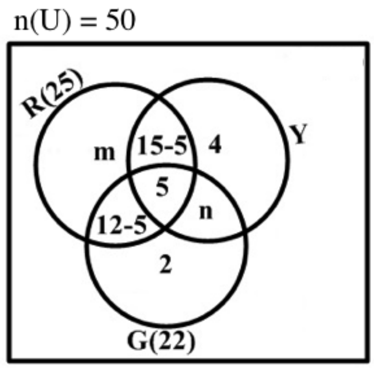
21. Ask pupils to work with seatmates to find the answer to question c., calculate those who play at least two games.
22. Invite a volunteer to write the solution on the board. (Answer: $2 + 6 + 1 + 9 = 18$)

Practice *(14 minutes)*

1. Write the following problem on the board: A survey of 50 girls was conducted. Each girl was asked which of the three colours, red, yellow and green, they liked. 5 of them said they liked all three colours, 25 liked red and 22 liked green. 15 liked red and yellow, 12 liked red and green, 4 liked only yellow and 2 liked only green.
    a. Illustrate the information on a Venn diagram.
    b. Calculate any missing numbers inside the circles.
    c. How many girls did not like any of the three colours?
2. Ask pupils to solve the problem with seatmates.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write their solutions on the board and explain:

   Solutions:

   a. The Venn diagram where U =Total number of girls, R =red, Y =yellow, G =green:

   b. Use the given numbers to find the missing values in the diagram.
   Let those who liked only yellow and green $= n$. Use the total number of girls who like green (22) to find the value of n:

   $$\begin{aligned} 22 &= n+7+2+5 \\ n &= 22-14 \\ n &= 8 \end{aligned}$$

   Let those who liked only red $= m$. Use the total number of girls who like red (25) to find the number who like only red:

   $$\begin{aligned} 25 &= m+7+10+5 \\ m &= 25-22 \\ m &= 3 \end{aligned}$$

   c. Let those who did not like any of the three colours $= x$. We know the sum of all sections is 50. Set up an equation and solve for x:

   $$\begin{aligned} 50 &= 10+5+7+4+2+8+3+x \\ 50 &= 39+x \\ x &= 50-39 \\ x &= 11 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L069 in the Pupil Handbook.

Lesson Title: Problem solving with 3 sets – Part 2	**Theme:** Numbers and Numeration	
Lesson Number: M1-L070	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to diagram and solve problems involving 3 sets, including real-life problems.	**Preparation** Write the problem in Opening on the board.	

Opening *(4 minutes)*

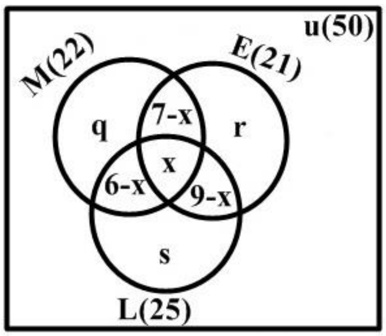
1. Write the following problem on the board:
   If $\mathrm{U} = \{natural\ numbers\ less\ than\ 20\}$, $\mathrm{E} = \{prime\ numbers\ less\ than\ 20\}$ and $\mathrm{F} = \{odd\ numbers\ less\ than\ 20\}$, find: a. $\mathrm{E} \cup \mathrm{F}$ b. $\mathrm{E} \cap \mathrm{F}$ c. $(\mathrm{E} \cup \mathrm{F})'$
2. Ask pupils to solve the problem independently in their exercise books.
3. Ask pupils to exchange their exercise books.
4. Invite a volunteer to write the solutions on the board.
   Solutions:
   List the sets: $\mathrm{E} = \{2, 3, 5, 7, 11, 13, 17, 19\}$, $\mathrm{F} = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$
   d. $\mathrm{E} \cup \mathrm{F} = \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$
   e. $\mathrm{E} \cap \mathrm{F} = \{3, 5, 7, 11, 13, 17, 19\}$
   f. $(\mathrm{E} \cup \mathrm{F})' = \{4, 6, 8, 10, 12, 14, 16, 18\}$.
5. Explain to pupils that today's lesson is on solving real life problems involving 3 sets.

Teaching and Learning *(21 minutes)*

1. Write on the board: For sets A, B and C, $n(\mathrm{A} \cup \mathrm{B} \cup \mathrm{C}) = n(\mathrm{A}) + n(\mathrm{B}) + n(\mathrm{C}) - n(\mathrm{A} \cap \mathrm{B}) - n(\mathrm{A} \cap \mathrm{C}) - n(\mathrm{B} \cap \mathrm{C}) + n(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C})$.
2. Explain:
   - The number of elements in the union of 3 sets can be found with this formula.
   - Add the cardinalities of each of the 3 sets. Subtract the cardinalities of the intersections of 2 sets (for example, $n(\mathrm{A} \cap \mathrm{B})$). Then, add back the number in the intersection of all 3 sets, $n(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C})$.
3. Write the following problem on the board: Mathematics, English and Life skills books were distributed to 50 pupils in a class, such that each pupil got at least 1 book. 22 got Mathematics books, 21 got English books and 25 got Life Skills books. 7 got both Mathematics and English books, 6 got both Mathematics and Life Skills books and 9 got both English and Life Skills books. Find the number of pupils who had:
   a. All three books
   b. Exactly two books
   c. Only Life Skills books
4. Ask pupils to discuss the problems for 2 minutes with seatmates.
5. Assign variables to the sets:

U = Pupil in the class M = Mathematics books
E = English books L = Life Skills books
q = Mathematics books only r = English books only
s = Life skills books only x = Those who have all the three books

6. Ask pupils to work with seatmates to write the cardinality of the sets in the problem using symbols. (For example, $n(\mathrm{U}) = 50$)
7. Invite volunteers to write the cardinalities on the board. (Answer: $n(\mathrm{U}) = 50,\ n(\mathrm{M}) = 22,\ n(\mathrm{E}) = 21, n(\mathrm{L}) = 25, n(\mathrm{M} \cap \mathrm{E}) = 7,\ n(\mathrm{M} \cap \mathrm{L}) = 6, n(\mathrm{E} \cap \mathrm{L}) = 9, n(\mathrm{M} \cap \mathrm{E} \cap \mathrm{L}) = x$.
8. Work as a class to illustrate the above information in a Venn diagram. Allow volunteers to write the labels:



9. Solve the problem on the board, involving pupils. Find the missing values. As you find them, write them in the appropriate space on the diagram.

Solution:

Let q represent only M

$$\begin{aligned} 22 &= q + 6 - x + x + 7 - x \\ 22 &= 13 - x + q \\ q &= x + 9 \end{aligned}$$

Let s represent only L

$$\begin{aligned} 25 &= s + 6 - x + x + x + 9 - x \\ 25 &= 15 - x + s \\ s &= x + 10 \end{aligned}$$

Let r represent only E

$$\begin{aligned} 21 &= r + 7 - x + x + 9 - x \\ 21 &= 16 - x + r \\ r &= x + 5 \end{aligned}$$

Using the formula for cardinality of the union of 3 sets, we have $50 = 7 - x + 6 - x + 9 - x + x + r + q + s$

Substituting for r, q and s, we have

$$\begin{aligned} 50 &= 22 - 2x + x + 5 + x + 9 + x + 10 \\ 50 &= 46 + x \\ x &= 50 - 46 = 4 \end{aligned}$$

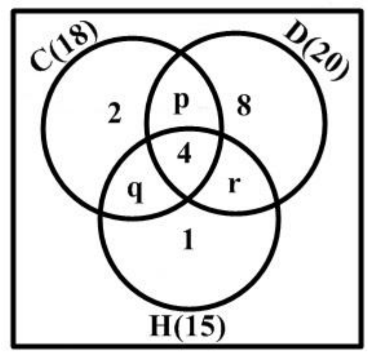
a. Those who had all three books are 4.
b. To find those who had exactly two books, add the values in the intersections of only 2 sets:

$$= 7 - x + 6 - x + 9 - x$$

$$= \quad 22 - 3x$$
$$= \quad 22 - 3(4) \qquad \text{Substitute for } x = 4$$
$$= \quad 22 - 12 = 10$$

c. Those who had only Life Skills books: $s = 10 + x = 10 + 4 = 14$

10. Write the following problem on the board: 40 pupils were chosen to study their choice of vocation and it was revealed that 18 chose Catering (C), 20 chose Dressmaking (D), and 15 chose Hairdressing (H), 2 chose catering only, 8 chose dress making only, 1 chose hairdressing only and 4 chose all three vocations. Find the number of pupils who chose:
    a. Dressmaking and Catering only
    b. None of the three vocations
11. Ask pupils to solve the problem with seatmates.
12. Walk around to check their answers and clear misconceptions.
13. Invite a volunteer to write the solutions on the board.



Solutions: The Venn diagram can be labelled as shown, where $p = \mathrm{C} \cap \mathrm{D}$, $r = \mathrm{H} \cap \mathrm{D}$, and $q = \mathrm{C} \cap \mathrm{H}$.

Use the total number of pupils in each set to set up an equation with variables p, q, and r. You can then solve the equations to find the missing variables:

From C: $18 = 2 + 4 + p + q$

$p + q = 12 \ldots\ldots\ldots (1)$

From D: $20 = 8 + 4 + p + r$

$p + r = 8.\ldots\ldots\ldots (2)$

From H: $15 = 1 + 4 + q + r$

$q + r = 10 \ldots\ldots\ldots (3)$

From equation (1), $p = 12 - q$

Substitute p into equation (2)

$$\begin{aligned} p + r &= 8 \\ 12 - q - 4 &= 8 \\ r &= q - 4 \ldots\ldots\ldots (4) \end{aligned}$$

Substitute r from equation (4) into equation (3)

$$\begin{aligned} q + r &= 10 \\ q + q - 4 &= 10 \\ 2q &= 14 \\ q &= 7 \end{aligned}$$

From equation (1): $p = 12 - 7 = 5$

From equation (5): $r = 7 - 4 = 3$

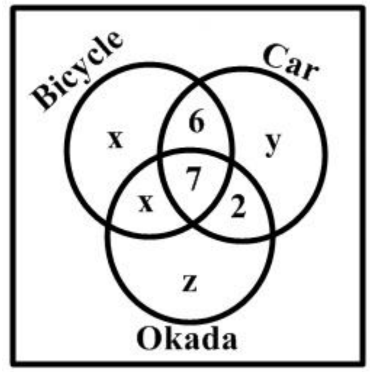
a. Those that chose dressmaking and catering only: $p = 5$

b. To find those that chose none of the three vocations, subtract all of the numbers inside the sets from 40, the total number of pupils:

$$= \quad 40 - (2 + 8 + 1 + 4 + p + q + r)$$

$$
\begin{aligned}
&= 40 - (15 + p + q + r) \\
&= 40 - (15 + 5 + 7 + 3) \\
&= 40 - 30 = 10
\end{aligned}
$$

Practice *(14 minutes)*



1. Write on the board: A number of travellers were questioned about the transport they used on a particular day. Each of them used one or more of the methods shown in the Venn diagram. Given that 35 people used bicycles and 25 people used cars, find:
   a. The value of x.
   b. The number who travelled by car only.
   c. The number who travelled by at least two methods of transport.
   d. Given also that 85 people were questioned altogether; calculate the number who travelled by okada only.
2. Ask pupils to solve the problem independently or with seatmates.
3. Walk around to check their answers and clear misconceptions.
4. Invite volunteers to write the solutions on the board.

   Solutions:

   a. If 35 people used bicycles, then:

$$
\begin{aligned}
x + x + 6 + 7 &= 35 \\
2x + 13 &= 35 \\
2x &= 35 - 13 \\
2x &= 22 \\
x &= 11
\end{aligned}
$$

   11 people travelled by bicycle only.

   b. Let y people travel by cars only. If 25 people used cars, then:

$$
\begin{aligned}
6 + 7 + 2 + y &= 25 \\
15 + y &= 25 \\
y &= 25 - 15 = 10
\end{aligned}
$$

   10 people travelled by bus only.

   c. The number who travelled by at least two methods (i.e. those using two or three methods) $= x + 6 + 2 + 7 = 11 + 15 = 26$

   d. Let z people travelled by okada only, then:

$$
\begin{aligned}
x + x + 6 + 7 + 2 + y + z &= 85 \\
11 + 11 + 15 + 10 + z &= 85 \\
47 + z &= 85 \\
z &= 85 - 47 \\
z &= 38
\end{aligned}
$$

   38 people travelled by okada only.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L070 in the Pupil Handbook.

Lesson Title: Use of Variables	**Theme:** Numbers and Numeration	
Lesson Number: M1-L071	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify that variables represents unknown numbers. 2. Identify the values of variables in simple algebraic expressions (e.g. $2 + x = 5$)	**Preparation** Write the problems in Opening on the board.	

Opening *(3 minutes)*

1. Write on the board: Simplify the following:
   a. $-6 + 8 + 15 - 2$
   b. $-2 - 1 - 4 + 7$
   c. $6 - 4 - 5$
2. Ask pupils to solve the problems independently.
3. Ask volunteers to call out their answers. (Answer: a. 15; b. 0; c. -3)
4. Explain to pupils that today's lesson is how to identify the values of variables in simple algebraic expressions.

Teaching and Learning *(23 minutes)*

1. Write on the board: $2 + x = 5$
2. Explain:
   - A variable is a letter that is used to represent a number.
   - In this expression on the board, the variable x represents an unknown number that will equal 5 when added to 2.
   - While x is a most commonly used variable, any letter can be a variable.
3. Write on the board: $x + x + y = 20$
4. Explain:
   - Each x in this expression is equal to the same amount.
   - The other variable, y, may be equal to different amount.
5. Write another problem on the board: Identify the variables in $5\pi r^2 h$.
6. Ask pupils to identify the variables.
7. Allow them to share, then explain:
   - r and h are variables. Their values are unknown and can change depending on the problem.
   - π is not a variable because it has a constant, known value.
8. Explain: An algebraic expression is a collection of variables, numbers, and operations.

9. Write the following five problems on the board: If x represents an unknown number, write each as an algebraic expression:
    a. 8 more than the number
    b. 7 less than the number
    c. Half the number
    d. The number subtracted from 4
    e. y times the number
10. Write the answers on the board and explain to the pupils. (Answers: a. $x + 8$ b. $x - 7$ c. $\frac{x}{2}$ or $\frac{1}{2}x$ d. $4 - x$ e. xy)
11. Write an expression on the board: $2x$
12. Explain:
    - In this algebraic expression, 2 is the **coefficient**.
    - The number placed in front of a variable or group of variables is called the coefficient, and is multiplied by the variable.
13. Write another example on the board: $3x - 2y + 5$
14. Discuss:
    - What are the variables? (Answer: x, y)
    - What are the coefficients? (Answer: 3, -2)
15. Explain:
    - A number on its own is called a constant term.
    - In the expression on the board, 5 is a constant term.
16. Write on the board: Identify the coefficient and constant term in $6x + 9$
17. Ask pupils to discuss with seatmates for a moment, then ask volunteers to share the answer with the class. (Answer: The coefficient of x is 6 and the constant is 9)
18. Write on the board: Re-write the sentence as an algebraic expression: Twice a certain number minus 8 is equal to twenty-five.
19. Ask pupils to work with seatmates to write the expression.
20. Invite a volunteer to write out the answer on the board. (Answer: $2m - 8 = 25$; accept any variable in place of m.)
21. Write on the board: $2 + x = 5$
22. Discuss: What do you think is the value of x? How do you know? (Answer: The value of x must be 3, because when we add 3 to 2, the result is 5)
23. Write on the board: $2y = 8$
24. Discuss: What do you think is the value of y? How do you know? (Answer: The value of y must be 4, because when we multiply 4 by 2, the result is 8.)
25. Write 2 more examples on the board:
    a. $7 = z + 3$
    b. $3m = 9$
26. Ask pupils to solve the problems with seatmates.
27. Walk around to check for understanding and clear misconceptions.

28. Ask three volunteers to give their answers and explain using logic (do not ask them to use algebraic techniques such as transposing or dividing). (Answers: $z = 4$, $m = 3$)

Practice *(13 minutes)*

1. Write 5 problems on the board:
    a. Circle the coefficients in the expression $5m^3 + 3n^2 - 7p + 2$
    b. Write as an algebraic expression: If 4 times a certain number is subtracted from 3, the result is 7.
    c. John is x years old now. How old was he 6 years ago?
    d. Find the value of a if $3 + a = 10$.
    e. Find the value of b if $2b = 6$.
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Ask pupils to exchange their exercise books to check their neighbor's answers.
5. Invite volunteers one at a time to write their answer on the board. (Answers: a. coefficients 5, 3, and -7 should be circled; b. $3 - 4m = 7$; c. $x - 6$ years; d. $a = 7$; $b = 3$).

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM-L071 in the Pupil Handbook.

Lesson Title: Simplification - grouping terms	**Theme:** Numbers and Numeration	
Lesson Number: M1-L072	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to simplify algebraic expressions by grouping like terms.	**Preparation** Write the problems in Opening on the board.	

Opening *(4 minutes)*

1. Write two problems on the board:
    a. Circle the coefficients in these expressions:
        i. $9g + 6r + 4$
        ii. $2a - 5p + 8$
    b. Find the value of x if $2x = 20$.
2. Ask pupils to solve the problems independently in their exercise books.
3. Invite volunteers to write the answers on the board. (Answers: a. i. 9 and 6 should be circled; ii. 2 and -5 should be circled; b. $x = 10$)
4. Explain to pupils that today's lesson is on simplifying algebraic expressions by grouping like terms.

Teaching and Learning *(22 minutes)*

1. Write on the board: $2x + 3x + 5y - y$
2. Explain:
    - Each part of the expression that is separated by an operation is a term. There are 4 terms in this expression.
    - In algebra, **like terms** are terms that have the same variables and powers.
3. Ask volunteers to identify the like terms in the expression on the board. (Answer: $2x$ and $3x$, $5y$ and $-y$)
4. Write another expression on the board: $2x^2 + 5 + 3x + 4x^2 - 1$
5. Ask volunteers to identify the like terms in the expression on the board. (Answer: $2x^2$ and $4x^2$, 5 and -1)
6. Explain:
    a. The variables must also have the same power. So $2x^2$ and $4x^2$ are like terms, but $3x$ is not a like term. $3x$ does not have a like term in this expression.
    b. Terms that are simply numbers are also considered like terms, although they do not have any variable.
7. Write on the board:

$$a = 1a$$
$$a + a = 2a$$

$a + a + a = 3a$
$2a - a = a$

8. Explain:
   - A variable without a number has a coefficient of 1. a is the same as $1a$.
   - When a coefficient and variable are written together, they are multiplied by each other. However, remember that multiplication is repetitive addition.
   - So we could extend the statements on the board for even greater coefficients.
9. Explain:
   - We can add or subtract like terms to give a single term. This only works for like terms.
   - When you add or subtract like terms it is often called **collecting like terms**.
   - In collecting like terms, it is possible to have negative values.
10. Write the following problem on the board: Simplify $5p + 7p + p$.
11. Explain:
    a. Each term has a variable p with power 1. Therefore, we can combine all of them.
    b. We add the coefficients of the terms.
12. Write the solution on the board and explain: $5p + 7p + p = (5 + 7 + 1)p = 13p$
13. Write another problem on the board: Simplify $7a - 3a$
14. Discuss: How do you think we will solve this problem? (Answer: Combine the terms; subtract the coefficients)
15. Write the solution on the board: $7a - 3a = (7 - 3)a = 4a$
16. Write another problem on the board: Simplify $2a + 7b + 5a - 2b$
17. Ask volunteers to identify the like terms. (Answer: $2a$ and $5a$, $7b$ and $-2b$)
18. Write the solution on the board and explain:
    Solution:
    Step 1. Collect like terms together:
    $2a + 5a + 7b - 2b$
    Step 2. Combine like terms:
    $(2 + 5)a + (7 - 2)b = 7a + 5b$
19. Explain: Do not try to combine the a's and the b's together when you are adding and taking away. They are different variables.
20. Write the following problem on the board: Simplify $5x + 4 - 9y + 3x + 2y - 7$
21. Ask volunteers to identify the like terms. (Answer: $5x$ and $3x$, $-9y$ and $2y$, 4 and -7)
22. Invite a volunteer to come to the board and collect the terms together (Step 1). (Answer: $5x + 3x - 9y + 2y + 4 - 7$)
23. Invite another volunteer to combine the terms together. (Answer: $(5 + 3)x + (-9 + 2)y + (4 - 7) = 8x - 7y - 3$)
24. Write the following problem on the board: Simplify $4x + 2b$

25. Discuss: Can this expression be simplified? Why or why not? (Answer: No, It has unlike terms because the variables are x and b. Unlike terms cannot be added or subtracted.)
26. Write the following two problems on the board:
    a. Simplify $7m + 8n - m$
    b. Simplify $5y + 4b - 5y - 4b$
27. Ask pupils to solve the problems with seatmates.
28. Walk around to check for understanding and clear misconceptions.
29. Invite volunteers to write the solutions on the board.

    Solutions:

    a. $7m + 8n - m = 7m - m + 8n = (7 - 1)m + 8n = 6m + 8n.$
    b. $5y + 4b - 5y - 4b = 5y - 5y + 4b - 4b = (5 - 5)y + (4 - 4)b = 0y + 0b = 0$

Practice *(13 minutes)*

1. Write four problems on the board: Simplify by combining the like terms:
    a. $7e + 2f - e - f$
    b. $k + 5g - k - g$
    c. $4m - 6m + 2m - m$
    d. $6j + 5k + 3j - 2k$
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Invite four volunteers to write the solutions on the board.

    Answers:

    a. $7e + 2f - e - f = 7e - e + 2f - f = (7 - 1)e + (2 - 1)f = 6e + f$
    b. $k + 5g - k - g = k - k + 5g - g = (1 - 1)k + (5 - 1)g = 0k + 4g = 4g$
    c. $4m - 6m + 2m - m = (4 - 6 + 2 - 1)m = -m$
    d. $6j + 5k + 3j - 2k = 6j + 3j + 5k - 2k = (6 + 3)j + (5 - 2)k = 9j + 3k$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L072 in the Pupil Handbook.

Lesson Title: Simplification - removing brackets	**Theme:** Algebraic Processes	
Lesson Number: M1-L073	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to simplify algebraic expression by removing brackets.	**Preparation** Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write a problem on the board: Simplify: $6k + m + 2k + 12m + 3k$.
2. Ask pupils to solve the problem in their exercise books.
3. Invite a volunteer to write the solution on the board. (Answer: $6k + m + 2k + 12m + 3k = 6k + 2k + 3k + m + 12m = (6 + 2 + 3)k + (1 + 12)m = 11k + 13m$)
4. Explain to pupils that today's lesson is on simplifying algebraic expressions by removing brackets.

Teaching and Learning *(23 minutes)*

1. Write the following problem on the board: Remove the brackets: $2(5x - 4)$
2. Explain:
   - Brackets are used to group terms together.
   - In removing brackets, multiply the term outside the bracket by each of the terms inside the bracket.
3. Write the solution on the board, and explain:

$$\begin{aligned} 2(5x-4) &= 2(5x) + 2(-4) && \text{Multiply each term in brackets by 2} \\ &= 10x - 8 \end{aligned}$$

4. Explain:
   - Take care with the signs when removing brackets.
   - When there is a (+) sign before the brackets, the sign inside the bracket does not change when the brackets are removed.
   - When there is (–) in front of the brackets, the signs inside the brackets change when the brackets are removed.
   - This is because of the rules of multiplication. Remember that multiplying a negative by a positive gives a negative answer. Multiplying a negative and a negative gives a positive answer.
5. Write a problem on the board. Simplify the expression $-2(3y - 5)$
6. Explain the solution on the board.

   Solution:

$$-2(3y-5) = (-2)(3y) + (-2)(-5) \quad \text{Multiply each term by } -2$$

$$= \quad -6y + 10$$

7. Write another problem on the board: remove the brackets from $-(3x+1)$.
8. Discuss:
    a. What is the number outside of the brackets? (Answer: -1)
    b. How can we solve this problem?
9. Allow pupils to share their ideas, then explain: When you see a negative sign outside the brackets, treat it as -1. Multiply each term inside the brackets by -1.
10. Solve the problem on the board, explaining each step:
    Solution:

$$\begin{aligned} -(3x+1) &= (-1)(3x) + (-1)(1) && \text{Multiply each term by } -1 \\ &= -3x + (-1) \\ &= -3x - 1 \end{aligned}$$

11. Explain:
    - When you have to simplify an expression with the bracket in it, remove the brackets first and then collect or group the like terms together.
    - When there are brackets inside brackets, always remove the inside bracket first.
12. Write another problem on the board: Simplify $7(x+3) + 5(x+4)$
13. Solve on the board, explaining each step:

$$\begin{aligned} 7(x+3) + 5(x+4) &= 7x + 21 + 5x + 20 && \text{Remove the brackets} \\ &= 7x + 5x + 21 + 20 && \text{Collect like terms} \\ &= 12x + 41 && \text{Combine like terms} \end{aligned}$$

14. Write the following problem on the board: Simplify $3(4m+1) - 2(m-6)$
15. Ask pupils to work with seatmates to simplify the expression.
16. Walk around to check for understanding and clear misconceptions.
17. Invite a volunteer to solve the problem on the board.
    Solution:

$$\begin{aligned} 3(4m+1) - 2(m-6) &= 12m + 3 - 2m + 12 && \text{Remove the brackets} \\ &= 12m - 2m + 3 + 12 && \text{Collect like terms} \\ &= 10m + 15 && \text{Combine like terms} \end{aligned}$$

18. Write another problem on the board: Simplify $5a[(2a+b) + 3(4a-2b)]$
19. Solve on the board, explaining each step:

$$\begin{aligned} 5[(2a+b) + 3(4a-2b)] &= 5[2a + b + 12a - 6b] && \text{Remove inside brackets} \\ &= 10a + 5b + 60a - 30b && \text{Remove outside brackets} \\ &= 10a + 60a + 5b - 30b && \text{Collect like terms} \\ &= 70a - 25b && \text{Combine like terms} \end{aligned}$$

Practice *(13 minutes)*

1. Write the following three problems on the board: Simplify the following:
    a. $-(4m + 7n)$
    b. $3(p - q) + 2r(p - q)$
    c. $5[(a - 4b) - 5(2a - 3b)]$
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. After several minutes, ask pupils to exchange their exercise books.
5. Invite three volunteers one at a time to write their answer on the board.

Solutions:

a.

$$\begin{aligned} -(4m + 7n) &= (-1)(4m) + (-1)(7n) && \text{Multiply each term by } -1 \\ &= -4m - 7n \end{aligned}$$

b. $3(p - q) + 2r(p - q) = 3p - 3q + 2pr - 2qr$; there are no like terms.

c.

$$\begin{aligned} 5[(a - 4b) - 5(2a - 3b)] &= 5[a - 4b - 10a + 15b] && \text{Remove inside brackets} \\ &= 5a - 20b - 50a + 75b && \text{Remove outside brackets} \\ &= 5a - 50a - 20b + 75b && \text{Collect like terms} \\ &= -45a + 55b && \text{Combine like terms} \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L073 in the Pupil Handbook.

Lesson Title: Simplification - expanding brackets	**Theme:** Algebraic Processes	
Lesson Number: M1-L074	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to simplify algebraic expressions by expanding brackets.	**Preparation** Write the problems in Opening on the board.	

Opening *(4 minutes)*

1. Write the following problems on the board: Simplify:
   a. $y(x-1)-x(x-2)$
   b. $12b-(5+2b)-7$
2. Remind pupils to remove brackets before combining like terms.
3. Ask pupils to solve the problems independently.
4. Invite two volunteers to write the solutions on the board at the same time.
   Solutions:
   a. $y(x-1)-x(x-2)=xy-y-x^2+2x$; no like terms
   b.
   $$\begin{array}{rcll} 12b-(5+2b)-7 & = & 12b-5-2b-7 & \text{Remove the brackets} \\ & = & 12b-2b-5-7 & \text{Collect like terms} \\ & = & 10b-12 & \text{Combine like terms} \end{array}$$
5. Explain to the pupils that today's lesson is on simplifying algebraic expressions by expanding brackets.

Teaching and Learning *(22 minutes)*

1. Explain:
   - In expanding an expression containing brackets, we rewrite the expression in an equivalent form without any brackets in it.
   - In other words, expanding brackets is the same as removing brackets.
2. Write the following problem on the board: Expand $3(x+2)$
3. Explain to the pupils that the 3 outside must multiply both terms inside the brackets. $3(x+2)=3x+6$
4. Write another problem on the board: Expand $-3a^2(3-b)$
5. Explain to pupils that both terms inside the brackets must be multiplied by $-3a^2$.
6. Invite a volunteer to write the solution on the board. (Answer: $-3a^2(3-b)=-9a^2+3a^2b$)
7. Write on the board: $(x+5)(x+10)$
8. Explain:
   - In this expression, 2 binomials are multiplied. Binomials are algebraic expressions with 2 terms.
   - Multiply each term in one bracket by each term in the other bracket.

- After removing the brackets, always collect like terms together and simplify.

9. Write out the solution on the board:

$(x+5)(x+10)$	$= x(x+10)+5(x+10)$	Multiply each term of the first binomial by the second binomial
	$= x^2+10x+5x+50$	Remove the brackets
	$= x^2+15x+50$	Combine like terms

10. Write another problem on the board: Expand $(x-7)(x-10)$
11. Write the solution on the board.

Solution:

$(x-7)(x-10)$	$= x(x-10)-7(x-10)$	Multiply
	$= x^2-10x-7x+70$	Remove the brackets
	$= x^2-17+70$	Combine like terms

12. Write another problem on the board. Expand $(3x-1)(3x+1)$
13. Ask pupils to solve problem with seatmates.
14. Walk around to check for understanding and clear misconceptions.
15. Invite a volunteer to write the solution on the board.

Solution:

$(3x-1)(3x+1)$	$= 3x(3x+1)-1(3x+1)$	Multiply
	$= 9x^2+3x-3x-1$	Remove the brackets
	$= 9x^2-1$	Combine like terms

16. Write another problem on the board: Expand $(x-2)^2$
17. Explain: We will rewrite this as the multiplication of 2 binomials, then we will treat it as we did the previous problems.
18. Rewrite the expression on the board: $(x-2)^2=(x-2)(x-2)$
19. Ask pupils to solve the problem with seatmates.
20. Invite a volunteer to write the solution on the board.

Solution:

$(x-2)^2$	$= (x-2)(x-2)$	
	$= x(x-2)-2(x-2)$	Multiply
	$= x^2-2x-2x+4$	Remove the brackets
	$= x^2-4x+4$	Combine like terms

21. Write another problem on the board: Expand and simplify $(2x+y)(x-y)+(2x-y)(x+y)$
22. Explain:
    - Remember BODMAS. Multiplication comes before addition.
    - First, multiply (expand) each set of binomials. Then, combine the like terms.
23. Ask pupils to work with seatmates to solve the problem. Support them and remind them of the steps as needed.
24. Invite volunteers to write the solution on the board. Support them as needed.

Step 1. Multiply the first set of brackets:

$$\begin{aligned}(2x+y)(x-y) &= 2x(x-y)+y(x-y)\\ &= 2x^2-2xy+xy-y^2\\ &= 2x^2-xy-y^2\end{aligned}$$

Step 2. Multiply the second set of brackets:

$$\begin{aligned}(2x-y)(x+y) &= 2x(x+y)-y(x+y)\\ &= 2x^2+2xy-xy-y^2\\ &= 2x^2+xy-y^2\end{aligned}$$

Step 3. Add the first and second expressions, combining any like terms:

$$\begin{aligned}(2x+y)(x-y)+(2x-y)(x+y) &= (2x^2-xy-y^2)+(2x^2+xy-y^2)\\ &= 2x^2+2x^2-xy+xy-y^2-y^2\\ &= 4x^2-2y^2\end{aligned}$$

Practice *(13 minutes)*

1. Write three problems on the board: Expand and simplify the following expressions:
    a. $(5x-1)(x-5)$
    b. $(k+9)^2$
    c. $(9+8x)(x-4)+(2+x)(x+1)$
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Allow them to exchange their exercise books after 10 minutes.
5. Invite three volunteers one at a time to write their answers on the board.

Solutions:

a. $(5x-1)(x-5)$

$$\begin{aligned}&= 5x(x-5)-(x-5)\\ &= 5x^2-25x-x+5\\ &= 5x^2-26x+5\end{aligned}$$

b. $(k+9)^2$

$$\begin{aligned}&= (k+9)(k+9)\\ &= k(k+9)+9(k+9)\\ &= k^2+9k+9k+81\\ &= k^2+18k+81\end{aligned}$$

c. $(9+8x)(x-4)+(2+x)(x+1)$

$$\begin{aligned}&= 9(x-4)+8x(x-4)+2(x+1)+x(x+1)\\ &= 9x-36+8x^2-32x+2x+2+x^2+x\\ &= 8x^2+x^2+9x-32x+2x+x-36+2\\ &= (8+1)x^2+(9-32+2+1)x-34\\ &= 9x^2-20x-34\end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L074 in the Pupil Handbook.

Lesson Title: Factoring - common factors	**Theme:** Algebraic Processes	
Lesson Number: M1-L075	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to factorise algebraic expressions by determining common factors.	**Preparation** Write the problem in Opening on the board.	

Opening *(4 minutes)*

1. Write a problem on the board: Expand and simplify $(m+3)^2$.
2. Ask pupils to solve the problem in their exercise books.
3. Invite a volunteer to write the solution on the board.
   Solution:

$$\begin{aligned}(m+3)^2 &= (m+3)(m+3)\\ &= m(m+3)+3(m+3)\\ &= m^2+3m+3m+9\\ &= m^2+6m+9\end{aligned}$$

4. Explain to the pupils that today's lesson is on factorising algebraic expressions by determining common factors.

Teaching and Learning *(22 minutes)*

1. Write on the board: $2a+2b$
2. Discuss: Can you think of any other way to write this expression? (Example answers: No, because they are not like terms. Yes, we can divide by or factor 2.)
3. Write on the board: $2(a+b)=2a+2b$
4. Explain:
   - In previous lessons, we simplified expressions like the one on the board using multiplication.
   - Today we will be factoring algebraic expressions. Factoring uses division, so it is the reverse of multiplying.
   - In the expression $2a+2b$, 2 is a common factor because it can divide each term.
   - If we switch sides of the equation on the board, we will have factored $2a+2b$
5. Write on the board: $2a+2b=2(a+b)$
6. Explain: The general method used to factor an algebraic expression is to find the factor common to all the terms in the expression and bring it outside the bracket.
7. Write a problem on the board: Factorise $10a-15b+5$

8. Discuss: Do the terms in this expression have any common factors? What are they? (Answer: The only common factor is 5.)
9. Write the solution on the board: $10a - 15b + 5 = 5(2a - 3b + 1)$
10. Explain: Remember to divide each factor by 5. Write the result inside the brackets.
11. Tell pupils that they can always check factoring by multiplying the right-hand side. It should produce the left-hand side.
12. Write another problem on the board. Factorise the expression $6x^3 + x^2$.
13. Discuss: Do the terms of this expression have any common factors? What are they? (Answers: Accept x, x^2.)
14. Explain:
    a. Variables can also be common factors.
    b. If every term has a power of x, as in the example given, then the lowest power is the highest common factor.
    c. The greatest common factor is x^2. It is important to take the greatest common factor. If you take another common factor, you will have to factorise the expression a second time.
    d. Remember that factoring is the same as division. If you factor a variable from an expression, subtract the exponents.
15. Write the solution on the board: $6x^3 + x^2 = x^2(6x + 1)$
16. Show pupils what happens if they do not choose the greatest factor. Factor x twice: $6x^3 + x^2 = x(6x^2 + x) = x^2(6x + 1)$
17. Write the following problem on the board: Factor $x^7 + 3x^6 + 2x^5 + x^4$
18. Ask pupils to discuss and find the greatest common factor with seatmates.
19. Invite a volunteer to write the greatest common factor on the board. (Answer: x^4)
20. Ask pupils to solve the problem with seatmates.
21. Invite a volunteer to write the solution on the board. (Answer: $x^7 + 3x^6 + 2x^5 + x^4 = x^4(x^3 + 3x^2 + 2x + 1)$)
22. Write another problem on the board: Factor $6y^2 + 2y$.
23. Discuss:
    a. What are the factors of this expression? (Answer: $2, y$)
    b. What is the greatest common factor? (Answer: $2y$)
24. Explain:
    a. We can factor numbers and variables from expressions at the same time.
    b. At times, you might factor a number or variable, and then realise there is more that you can factor. That is fine, just continue with the next factor.
25. Factor the problem on the board: $6y^2 + 2y = 2y(3y + 1)$
26. Write it as two steps so pupils understand that they can continue if they choose either 2 or y as a factor: $6y^2 + 2y = y(6y + 2) = 2y(3y + 1)$
27. Write another problem on the board. Factor the following:
    a. $15x^2 + 12x$

b. $5x^5 - 4x^4 + 3x^3$

28. Ask pupils to solve the problems with seatmates.
29. Walk around to check for understanding and clear misconceptions.
30. Invite two volunteers, one at a time, to solve the problems on the board.
    Solutions:
    a. $15x^2 + 12x = 3x(5x + 4)$
    b. $5x^5 - 4x^4 + 3x^3 = x^3(5x^2 - 4x + 3)$

Practice *(13 minutes)*

1. Write four problems on the board: Factor the following expressions:
    a. $2x + 8$
    b. $16y + 10x$
    c. $5x^5 + 3x^4 - 2x^3$
    d. $8x^2 + 6x$
    e. $x^3y^2 - xy^2$
2. Ask pupils to solve the problems independently in their exercise books. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite four volunteers, one at a time, to write their solutions on the board and explain.
    Solutions:
    a. $2x + 8 = 2(x + 4)$
    b. $16y + 10x = 2(8y + 5x)$
    c. $5x^5 + 3x^4 - 2x^3 = x^3(5x^2 + 3x - 2)$
    d. $8x^2 + 6x = 2x(4x + 3)$
    e. $x^3y^2 - xy^2 = xy^2(x^2 - 1)$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L075 in the Pupil Handbook.

Lesson Title: Factoring - grouping	Theme: Algebraic Processes	
Lesson Number: M1-L076	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to factorise algebraic expressions by grouping common terms.	**Preparation** Write the problems in Opening on the board.	

Opening *(4 minutes)*

1. Write on the board: Simplify the following algebraic expressions:
    d. $16m - 15m + 3m$
    e. $8yz + z + 2yz - 2z - 4z$
2. Ask pupils to solve the problems independently in their exercise books.
3. Invite volunteers to write the answers on the board. (Answer: a. $16m - 15m + 3m = 4m$ b. $8yz + 2yz + z - 2z - 4z = 10yz - 5z$).
4. Explain to pupils that today's lesson is on factoring algebraic expressions by grouping common terms.

Teaching and Learning *(22 minutes)*

1. Write on the board: $x^2 - ax + bx - ab$
2. Discuss:
    - Are there any common factors in this expression? (Answer: No.)
    - Can you think of any way to factor this?
3. Allow pupils to share ideas, then explain:
    - This expression does not have a common factor.
    - Parts of the expression can be factored.
    - We can factor the parts by grouping.
4. Explain the steps required for factoring by grouping:
    - Expressions that we can factor by grouping usually have four terms.
    - Decide if the four terms have anything in common, called the greatest common factor or GCF. If so, factor out the GCF.
    - Create smaller groups within the problem. This is usually done by grouping the first two terms together and the last two terms together. You may need to change the order of the terms.
    - Factor out the GCF from each of the two groups.
    - If the factors inside the bracket are exactly the same, you can factor out what is inside the bracket once. This step will become clear with an example.
5. Write a problem on the board: Factor completely: $x^2 - ax + bx - ab$
6. Explain the solution on the board.

Solution:

$x^2 - ax + bx - ab$	Decide if the four terms have anything in common
$= (x^2 - ax) + (bx - ba)$	Create smaller groups
$= x(x - a) + b(x - a)$	Factorise the GCF from each of the two groups
$= (x - a)(x + b)$	See explanation below

7. Explain the last step:
   - After factoring the GCFs, we have 2 terms: $x(x-a)$ and $b(x-a)$.
   - These two terms have a common factor of $(x-a)$.
   - Factoring out $(x-a)$, we have $x+b$.
   - Thus, the product of these 2 binomials is our answer.
8. Write another problem on the board: Factor completely: $x^3 + 2x^2 - 3x - 6$
9. Ask volunteers to give the steps. As they give them, factor on the board:

Solution:

$x^3 + 2x^2 - 3x - 6$	Decide if the four terms have anything in common
$= (x^3 + 2x^2) + (-3x - 6)$	Create smaller groups
$= x^2(x + 2) - 3(x + 2)$	Factorise the GCF from each of the two groups
$= (x^2 - 3)(x + 2)$	

10. Write another problem on the board: Factor completely: $3p^2 + 2qt + 6qp + pt$
11. Explain:
    - In this problem, we need to change the order of the terms, so that the terms that can be factored are next to each other.
    - Change the positions of the terms. Position the terms with t near each other. We can then factor p from 2 terms, and t from 2 terms.
12. Invite a volunteer to write the solution on the board.

$3p^2 + 2qt + 6qp + pt$	Original numbers given
$= 3p^2 + 6qp + 2qt + pt$	Reposition the terms
$= (3p^2 + 6qp) + (pt + 2qt)$	Create smaller groups
$= 3p(p + 2q) + t(p + 2q)$	Factor out the GCF from each of the two groups
$= (3p + t)(p + 2q)$	

13. Explain:
    - For some problems, there is more than 1 way to factor.
    - For example, in the problem on the board, you could have factored p and q instead of p and t. You would arrive at the same answer.
14. Solve the problem again, factoring p and q this time:

$3p^2 + 2qt + 6qp + pt$	Original numbers given
$= 3p^2 + pt + 6qp + 2qt$	Reposition the terms
$= (3p^2 + pt) + (6qp + 2qt)$	Create smaller groups
$= p(3p + t) + 2q(3p + t)$	Factor out the GCF from each of the two groups
$= (p + 2q)(3p + t)$	

15. Write another problem on the board: Factor completely: $ab - 2 + a - 2b$
16. Ask pupils to work with seatmates to solve the problem.
17. Invite a volunteer to write the solution on the board.

Solution:

$$\begin{aligned} ab - 2 + a - 2b &= ab + a - 2b - 2 \\ &= a(b+1) - 2(b+1) \\ &= (a-2)(b+1) \end{aligned}$$

Practice *(13 minutes)*

1. Write four problems on the board: Completely factor the following:
   a. $x^3 - 3x^2 + 5x - 15$
   b. $9x - 27y + 7x^2 - 21xy$
   c. $ad + bc + ac + bd$
   d. $4b^2 - 27ac - 6ab + 18bc$
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Ask the pupils to exchange their exercise books.
5. Invite four volunteers one at a time to write their solutions on the board.

Solutions:

a. $x^3 - 3x^2 + 5x - 15$
$= x^2(x-3) + 5(x-3)$
$= (x^2+5)(x-3)$

b. $9x - 27y + 7x^2 - 21xy$
$= 9(x-3y) + 7x(x-3y)$
$= (9+7x)(x-3y)$

c. $ad + bc + ac + bd$
$= ac + ad + bc + bd$
$= a(c+d) + b(c+d)$
$= (a+b)(c+d)$

d. $4b^2 - 27ac - 6ab + 18bc$
$= 4b^2 - 6ab + 18bc - 27ac$
$= 2b(2b-3a) + 9c(2b-3a)$
$= (2b+9c)(2b-3a)$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L076 in the Pupil Handbook.

Lesson Title: Substitution of values	Theme: Algebraic Processes	
Lesson Number: M1-L077	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to substitute values into given algebraic expressions.	**Preparation** Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write the following problem on the board: Factor completely: $px - 2qx - 4qy + 2py$.
2. Ask pupils to solve the problem independently.
3. Invite a volunteer to solve the problem on the board.
   Solution:
$$\begin{aligned} px - 2qx - 4qy + 2py &= (px + 2py) - (2qx - 4qy) \\ &= p(x + 2y) - 2q(x + 2y) \\ &= (x + 2y)(p - 2q) \end{aligned}$$
4. Explain to pupils that today's lesson is on substituting values into given algebraic expressions.

Teaching and Learning *(23 minutes)*

1. Explain:
   - In algebra, "substitution" means putting numbers where the letters are.
   - Substitution is a useful tool in algebra to find a value or to rewrite equations in terms of a single variable.
2. Write an example on the board: Find the value of $x - 2$ if $x = 6$.
3. Explain:
   - We can substitute 6 for x.
   - Instead of $x - 2$, we will have $6 - 2$.
4. Write the solution on the board: $6 - 2 = 4$.
5. Write the following problem on the board: If $x = 3$ and $y = 4$, what is $x + xy$?
6. Explain:
   - It doesn't matter how many variables a problem has. We will solve it in the same way, by substituting each value we are given.
   - We put 3 where x is, and 4 where y is.
   - Remember that two variables written together in a term (as in xy) means they are multiplied together.
7. Solve the problem on the board:
$$\begin{aligned} x + xy &= 3 + (3)(4) \\ &= 3 + 12 \\ &= 15 \end{aligned}$$

8. Write another problem on the board: Find $x^2 + x$ if $x = 2$.
9. Ask volunteers to give the steps needed to solve. As they give them, solve on the board:

$$\begin{aligned} x^2 + x &= 2^2 + 2 \\ &= 4 + 2 \\ &= 6 \end{aligned}$$

10. Write another problem on the board: If $x = 5,$ what is $x + \frac{x}{2}$?
11. Ask pupils to work with seatmates to solve the problem.
12. Invite a volunteer to write the solution on the board.

Solution:

$$\begin{aligned} x + \frac{x}{2} &= 5 + \frac{5}{2} \\ &= 5 + 2.5 \\ &= 7.5 \end{aligned}$$

13. Explain:
    - When substituting negative numbers, put brackets around them so you get the calculations right.
    - Two like signs become a positive sign and two unlike signs become a negative sign.
14. Write the following problem on the board: If $x = -2$, what is the value of $1 - x + x^2$?
15. Solve on the board, explaining each step:

$$\begin{aligned} 1 - x + x^2 &= 1 - (-2) + (-2)^2 && \text{Substitute } x = -2 \\ &= 1 + 2 + (-2)^2 && \text{Change the two negative signs to +} \\ &= 1 + 2 + 4 && \text{Simplify} \\ &= 7 \end{aligned}$$

16. Write another problem on the board: Evaluate $\frac{2a^2bc}{2b-c}$ when $a = 3, b = -4$ and $c = -5$
17. Explain:
    - As long as you know how to work out the arithmetic, you can solve the problem.
    - Remember to use the order of operations (BODMAS) when evaluating.
18. Ask pupils to work with seatmates to solve the problem.
19. Ask a volunteer to substitute the value of a, b and c in the expression.
20. Ask another volunteer to complete the solution.

Solution:

$$\frac{2a^2bc}{2b-c} = \frac{2(3)^2(-4)(-5)}{2(-4)-(-5)} = \frac{2\times9\times20}{-8+5} = \frac{360}{-3} = -120$$

21. Write a problem on the board: Evaluate $ab\sqrt{c^2 + b^2}$, given that $a = 2$, $b = -3$ and $c = 4$.
22. Ask pupils to work with seatmates to solve the problem.
23. Invite any volunteer to solve the problem on the board.

Solution:

$$
\begin{aligned}
ab\sqrt{c^2+b^2} &= 2(-3)\sqrt{(4)^2+(-3)^2} \\
&= -6\sqrt{16+9} \\
&= -6\sqrt{25} \\
&= (-6)\times(\pm 5) \\
&= \pm 30
\end{aligned}
$$

24. Remind pupils that the result of a square root can be positive or negative. This is because $(-5)(-5)=25$ and $5\times 5=25$.

Practice *(13 minutes)*

1. Write two problems on the board:
    a. If $x=-3$, find the value of x^2+5.
    b. If $y=3$ and $z=-1$, find the value of $yz+12$.
    c. if $a=-3$, $b=2$, $c=1$ and $d=-4$, find the value of $\left(\frac{c-a}{b-d}\right)^2$.
    d. Evaluate $\frac{a^2}{2b+c}$ when $a=2$, $b=-3$ and $c=-2$.
2. Ask pupils to work independently and solve the problems in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Invite 4 volunteers to write the solutions on the board.
5. Ask the class to check their answers with the answers on the board. Explain the solutions if needed.

    Solutions:

    a. $x^2+5=(-3)^2+5=9+5=14$
    b. $yz+12=(3)(-1)+12=-3+12=9$
    c. $\left(\frac{c-a}{b-d}\right)^2=\left[\frac{1-(-3)}{2-(-4)}\right]^2=\left(\frac{1+3}{2+4}\right)^2=\left(\frac{4}{6}\right)^2=\frac{4^2}{6^2}=\frac{16}{36}=\frac{4}{9}$
    d. $\frac{a^2}{2b+c}=\frac{(2)^2}{2(-3)+(-2)}=\frac{4}{-6-2}=\frac{4}{-8}=-\frac{1}{2}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L077 in the Pupil Handbook.

Lesson Title: Addition of algebraic fractions	Theme: Algebraic Processes	
Lesson Number: M1-L078	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to add algebraic fractions.	**Preparation** Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write a revision problem on the board: Find the value of $3(x+y)$ if $x=-2$ and $y=7$.
2. Ask pupils to solve the problem independently in their exercise books.
3. Invite a volunteer to write the solution on the board. (Solution: $3(x+y)=3(-2+7)=3(5)=15$).
4. Explain to pupils that today's lesson is on addition of algebraic fractions.

Teaching and Learning *(23 minutes)*

1. Write on the board: $\frac{x}{2}$ $\quad \frac{x}{y}$ $\quad \frac{wxy}{xyz}$ $\quad \frac{2+x}{2-x}$
2. Discuss: Ask pupils to describe the expressions on the board. (Example answers: Fractions with variables; algebraic expressions; algebraic fractions.)
3. Explain:
   - Fractions that involve variables are called "algebraic fractions".
   - This lesson is on adding simple algebraic fractions, similar to the ones on the board.
4. Write a problem on the board: Simplify $\frac{x}{3}+\frac{x}{2}$
5. Explain:
   - The process for adding algebraic fractions is similar to the process for adding numerical fractions.
   - To add algebraic fractions:
     - Find the lowest common multiple (LCM) of the denominators.
     - Express all fractions with the LCM in the denominator.
     - Once the fractions have the same denominator, they are **like fractions** and can be added.
     - Add the fractions, and combine like terms if possible.
     - If the answer can be simplified, simplify it.
6. Ask a volunteer to give the LCM of the denominators of the fractions on the board. (Answer: The LCM of 3 and 2 is 6)
7. Explain to pupils how to solve the problem.

   Solution:

   $\frac{x}{3}+\frac{x}{2} \quad = \quad \frac{2\times x}{6}+\frac{3\times x}{6}$ Express the denominators as the LCM

$$= \frac{2x}{6} + \frac{3x}{6} \quad \text{Simplify the numerators}$$

$$= \frac{2x+3x}{6} \quad \text{Add the numerators}$$

$$= \frac{5x}{6}$$

8. Explain: This cannot be simplified further. The answer is also an algebraic fraction, $\frac{5x}{6}$.
9. Write the following problem on the board: Simplify $\frac{x}{12} + \frac{y}{4}$
10. Invite a volunteer to write the LCM of 12 and 4 on the board. (Answer: 12)
11. Solve the problem on the board, explaining each step:

$$\frac{x}{12} + \frac{y}{4} = \frac{x}{12} + \frac{3y}{12} \quad \text{Express the denominators as the LCM}$$

$$= \frac{x+3y}{12} \quad \text{Add the numerators}$$

12. Explain: This cannot be simplified further. Recall that only like terms can be combined.
13. Write the following problem on the board: Simplify $\frac{4x+3}{12} + \frac{2x+5}{8}$
14. Ask any volunteer to write the LCM of 12 and 8 on the board. (Answer: 24)
15. Ask volunteers to give the steps to solve the problem. As they give them, solve the problem on the board:

Solution:

$$\frac{4x+3}{12} + \frac{2x+5}{8} = \frac{2(4x+3)}{24} + \frac{3(2x+5)}{24} \quad \text{Express the denominators as the LCM}$$

$$= \frac{8x+6+6x+15}{24} \quad \text{Simplify and add the numerators}$$

$$= \frac{8x+6x+6+15}{24} \quad \text{Collect like terms}$$

$$= \frac{14x+21}{24} \quad \text{Combine like terms}$$

16. Write 2 problems on the board: Simplify: a. $\frac{y}{5} + \frac{y}{3}$ b. $\frac{3x-2}{5} + \frac{x+1}{3}$
17. Invite a volunteer to write the LCM of 5 and 3 on the board. (Answer: 15)
18. Ask pupils to work with seatmates to solve the problems.
19. Walk around to check for understanding and clear misconceptions.
20. Invite a volunteer to write out the solutions on the board.

Solutions:

a.

$$\frac{y}{5} + \frac{y}{3} = \frac{3y}{15} + \frac{5y}{15} \quad \text{Express the denominators as the LCM}$$

$$= \frac{3y+5y}{15} \quad \text{Add the numerators}$$

$$= \frac{8y}{15} \quad \text{Combine like terms}$$

b.

$$\frac{3x-2}{5} + \frac{x+1}{3} = \frac{3(3x-2)}{15} + \frac{5(x+1)}{15} \quad \text{Express the denominators as the LCM}$$

$$= \frac{3(3x-2)+5(x+1)}{15} \quad \text{Simplify the numerators}$$

$= \frac{9x-6+5x+5}{15}$ Remove the brackets

$= \frac{9x+5x-6+5}{15}$ Collect like terms

$= \frac{14x-1}{15}$ Combine like terms

Practice *(13 minutes)*

1. Write two problems on the board:
   a. Simplify $\frac{3d+6}{4} + \frac{d+3}{2}$
   b. Simplify $\frac{x-1}{3} + \frac{x+3}{2}$
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear any misconception.
4. Ask pupils to exchange their exercise books to check answers.
5. Ask two volunteers one at a time to write their solutions on the board.

   Solutions:

a.	$\frac{3d+6}{4} + \frac{d+3}{2}$	$= \frac{3d+6}{4} + \frac{2(d+3)}{4}$	Express the denominators as the LCM
		$= \frac{3d+6+2(d+3)}{4}$	Add the numerators
		$= \frac{3d+6+2d+6}{4}$	Remove the brackets
		$= \frac{3d+2d+6+6}{4}$	Collect like terms
		$= \frac{5d+12}{4}$	Combine like terms
b.	$\frac{x-1}{3} + \frac{x+3}{2}$	$= \frac{2(x-1)}{6} + \frac{3(x+3)}{6}$	Express the denominators as the LCM
		$= \frac{2(x-1)+3(x+3)}{6}$	Add the numerators
		$= \frac{2x-2+3x+9}{6}$	Remove the brackets
		$= \frac{2x+3x-2+9}{6}$	Collect like terms
		$= \frac{5x+7}{6}$	Combine like terms

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L078 in the Pupil Handbook.

<table>
<tr><td>Lesson Title: Subtraction of algebraic fractions</td><td colspan="2">Theme: Algebraic Processes</td></tr>
<tr><td>Lesson Number: M1-L079</td><td>Class: SSS 1</td><td>Time: 40 minutes</td></tr>
<tr><td>Learning Outcome
By the end of the lesson, pupils will be able to subtract algebraic fractions.</td><td colspan="2">Preparation
Write the problem in Opening on the board.</td></tr>
</table>

Opening *(4 minutes)*

1. Write a revision problem on the board: Simplify $\frac{m}{4}+\frac{m+1}{3}$
2. Ask pupils to solve the problem independently in their exercise books.
3. Invite a volunteer to write the solution on the board.
   Solution:
$$\begin{aligned}\frac{m}{4}+\frac{m+1}{3} &= \frac{3m}{12}+\frac{4(m+1)}{12}\\ &= \frac{3m+4(m+1)}{12}\\ &= \frac{3m+4m+4}{12}\\ &= \frac{7m+4}{12}\end{aligned}$$
4. Explain to the pupils that today's lesson is on subtracting algebraic fractions.

Teaching and Learning *(22 minutes)*

1. Explain:
   - The process for subtracting algebraic fractions is similar to the process for adding algebraic fractions.
   - To subtract algebraic fractions:
     - Find the lowest common multiple (LCM) of the denominators.
     - Express all fractions with the LCM in the denominator.
     - Once the fractions have the same denominator, they are **like fractions** and can be added or subtracted.
     - Subtract the fractions. Subtract each of the like terms in the numerator.
     - If the answer can be simplified, simplify it.
   - Be careful of a minus sign before a bracket.
   - When there is a minus sign before a bracket, the sign(s) inside the bracket change(s).
2. Write the following problem on the board: Simplify $\frac{3x}{4}-\frac{x}{3}$
3. Ask a volunteer to give the LCM of the denominators. (Answer: 12)
4. Solve on the board, explaining each step.
   Solution:

$$\frac{3x}{4} - \frac{x}{3} = \frac{3(3x)}{12} - \frac{4x}{12}$$ Express the denominators as the LCM

$$= \frac{9x}{12} - \frac{4x}{12}$$ Simplify the numerators

$$= \frac{9x-4x}{12}$$ Subtract the numerators

$$= \frac{5x}{12}$$ Combine like terms

5. Write another problem on the board: Simplify $\frac{a-4}{2} - \frac{a-2}{6}$
6. Invite a volunteer to write the LCM of 2 and 6 on the board. (Answer: 6)
7. Ask volunteers to describe the steps to solve the problem. As they describe them, solve on the board.

   Solution:

$$\frac{a-4}{2} - \frac{a-2}{6} = \frac{3(a-4)}{6} - \frac{a-2}{6}$$ Express the denominators as the LCM

$$= \frac{3a-12}{6} - \frac{a-2}{6}$$ Simplify the numerators

$$= \frac{3a-12-(a-2)}{6}$$ Subtract the numerators

$$= \frac{3a-12-a+2}{6}$$ Distribute the negative sign

$$= \frac{3a-a-12+2}{6}$$ Collect like terms

$$= \frac{2a-10}{6}$$ Combine like terms

$$= \frac{2(a-5)}{2(3)}$$ Factor out 2

$$= \frac{a-5}{3}$$ Simplify

8. Write 2 more problems on the board: Simplify:

   a. $\frac{5a}{3} - \frac{a}{2}$ b. $\frac{b+5}{7} - \frac{2-b}{5}$
9. Invite a volunteer to write the LCM of the denominators of each problem on the board. (Answer: a. 6, b. 35)
10. Ask pupils to work with seatmates to solve the problems.
11. Walk around to check for understanding and clear misconceptions.
12. Invite volunteers to write out the solutions on the board.

    Solutions:

    a.

$$\frac{5a}{3} - \frac{a}{2} = \frac{10a}{6} - \frac{3a}{6}$$ Express the denominators as the LCM

$$= \frac{10a-3a}{6}$$ Subtract the numerators

$$= \frac{7a}{6}$$ Combine like terms

b.

$$\frac{b+5}{7} - \frac{2-b}{5} = \frac{5(b+5)}{35} - \frac{7(2-b)}{35}$$ Express the denominators as the LCM

$= \frac{5(b+5)-7(2-b)}{35}$ Subtract the numerators

$= \frac{5b+25-14+7b}{35}$ Simplify the numerators

$= \frac{5b+7b+25-14}{35}$ Collect like terms

$= \frac{12b+11}{35}$ Combine like terms

Practice *(13 minutes)*

1. Write two problems on the board:
   a. Simplify $\frac{3x+2}{3} - \frac{x-1}{4}$ b. Simplify $\frac{2x-5}{7} - \frac{8x-8}{6}$
2. Ask pupils to solve the problems individually.
3. Walk around to check for understanding and clear misconceptions.
4. Ask pupils to exchange their exercise books.
5. Invite two volunteers, one at a time, to write their answers on the board.

   Answers:

   a.

   $\frac{3x+2}{3} - \frac{x-1}{4}$ $= \frac{4(3x+2)}{12} - \frac{3(x-1)}{12}$ Express the denominators as the LCM

   $= \frac{4(3x+2)-3(x-1)}{12}$ Subtract the numerators

   $= \frac{12x+8-3x+3}{12}$ Simplify the numerators

   $= \frac{12x-3x+8+3}{12}$ Collect like terms

   $= \frac{9x+11}{12}$ Combine like terms

   b.

   $\frac{2x-5}{7} - \frac{8x-8}{6}$ $= \frac{6(2x-5)}{7} - \frac{7(8x-8)}{6}$ Express the denominators as the LCM

   $= \frac{6(2x-5)-7(8x-8)}{42}$ Subtract the numerators

   $= \frac{12x-30-56x+56}{42}$ Simplify the numerators

   $= \frac{12x-56x-30+56}{42}$ Collect like terms

   $= \frac{-44x+26}{42}$ Combine like terms

   $= \frac{26-44x}{42}$

   $= \frac{2(13-22x)}{2(21)}$ Factor out 2

   $= \frac{13-22x}{21}$ Simplify

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L079 in the Pupil Handbook.

Lesson Title: Linear equations	**Theme:** Algebraic Processes	
Lesson Number: M1-L080	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve linear equations using the balance method.	**Preparation** None	

Opening *(2 minutes)*

1. Write on the board: Solve for x in the equation $x + 5 = 7$
2. Discuss: What is the value of x in the equation? Can you tell by observing? (Answer: Pupils can use logic to identify that $x = 2$, because $2 + 5 = 7$.)
3. Explain to pupils that today's lesson is on solving linear equations using the balancing method.

Teaching and Learning *(25 minutes)*

1. Explain:
    - The equation on the board is an example of a linear equation in 1 variable.
    - A linear equation in one variable is an equation that can be written in the form $ax + b = c$ where a, b and c are real numbers, x is the variable and $a \neq 0$
    - The highest power of the variable in a linear equation is 1, so linear equations are also called first-degree equations.

    a. Today we will solve for variables using the balancing method.
2. Explain:
    - To use the balancing method, we solve by applying the same operation to both sides of an equals sign. Our goal is to get the variable by itself.
    - These are possible steps in using the balancing method:
        - Add the same quantity to each side.
        - Subtract the same quantity from each side.
        - Multiply each side by the same quantity.
        - Divide each side by the same quantity.
3. Write the solution to the problem on the board, explaining each step:
   Solution:
$$\begin{aligned} x + 5 &= 7 \\ x + 5 - 5 &= 7 - 5 \qquad \text{Subtract 5 from both sides} \\ x + 0 &= 2 \\ x &= 2 \end{aligned}$$
4. Write a problem on the board: Solve for x if $8 = 3x - 7$
5. Explain:

a. When balancing, perform addition or subtraction before multiplication and division.
b. To get x by itself in this problem, we will need to divide by 3 and add 7. Add 7 first, before dividing by 3.

6. Solve the problem on the board, explaining each step:
   Solution:

$$
\begin{aligned}
8 &= 3x - 7 \\
8 + 7 &= 3x - 7 + 7 && \text{Add 7 to both sides} \\
15 &= 3x \\
\frac{15}{3} &= \frac{3x}{3} && \text{Divide both sides by } 3 \\
5 &= x \\
x &= 5
\end{aligned}
$$

7. Explain: After solving for a variable, we can check our answer by using substitution.
8. Show how to check on the board:

$$
\begin{aligned}
8 &= 3(5) - 7 && \text{Substitute } x = 5 \\
8 &= 15 - 7 && \text{Simplify} \\
8 &= 8
\end{aligned}
$$

9. Explain: After checking, the 2 sides of the equation should be the same.
10. Write another problem on the board: Solve for x if $5x - 3 = 3x + 7$
11. Discuss: What will we do first? Then what will we do next?
12. Allow pupils to share their ideas, then explain:
    a. We have x on both sides of the equation. We need to subtract $3x$ from both sides to get x on only one side.
    b. We will perform addition and/or subtraction.
    c. Finally, we will perform division.
13. Write the solution on the board and explain.
    Solution:

$$
\begin{aligned}
5x - 3 &= 3x + 7 \\
5x - 3 - 3x &= 3x + 7 - 3x && \text{Subtract } 3x \text{ from both sides} \\
2x - 3 &= 7 \\
2x - 3 + 3 &= 7 + 3 && \text{Add 3 to both sides} \\
2x &= 10 \\
\frac{2x}{2} &= \frac{10}{2} && \text{Divide both sides by 2} \\
x &= 5
\end{aligned}
$$

14. Ask pupils to work with seatmates to check the answer.
15. Invite a volunteer to write their work on the board.
    Check:

$$
\begin{aligned}
5(5) - 3 &= 3(5) + 7 \\
25 - 3 &= 15 + 7
\end{aligned}
$$

$$22 = 22$$

16. Write another problem on the board: Solve for y in the equation $4y + 1 = 4 - y$
17. Ask volunteers to give each step to solve the problem. As they give the steps, solve on the board.
    Solution:

$$\begin{aligned} 4y + 1 &= 4 - y & \\ 4y + y + 1 &= 4 - y + y & \text{Add } y \text{ to both sides} \\ 5y + 1 &= 4 & \\ 5y + 1 - 1 &= 4 - 1 & \text{Subtract 1 from both sides} \\ 5y &= 3 & \\ \frac{5y}{5} &= \frac{3}{5} & \text{Divide both sides by 5} \\ y &= \frac{3}{5} & \end{aligned}$$

18. Write another problem on the board: Solve $5a - 8 = a + 5$
19. Ask pupils to solve the problem with seatmates.
20. Walk around to check for understanding and clear misconceptions.
21. Invite a volunteer to write the solution on the board.
    Solution:

$$\begin{aligned} 5a - 8 &= a + 5 & \\ 5a - a - 8 &= a + 5 - a & \text{Subtract } a \text{ from both sides} \\ 4a - 8 &= 5 & \\ 4a - 8 + 8 &= 5 + 8 & \text{Add 8 to both sides} \\ 4a &= 13 & \\ \frac{4a}{4} &= \frac{13}{4} & \text{Divide both sides by 4} \\ a &= \frac{13}{4} = 3\frac{1}{4} & \end{aligned}$$

Practice *(12 minutes)*

1. Write the following three problems on the board: Solve the following:
    a. $a + 9 = 16$
    b. $20 = 2x + 4$
    c. $9 + 4x = 3 - 2x$
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Ask pupils to exchange their exercise books.
5. Invite three volunteers to write their solutions on the board.

Solutions:

a.

$$\begin{aligned} a+9 &= 16 \\ a+9-9 &= 16-9 && \text{Subtract } 9 \text{ from both sides} \\ a &= 7 \end{aligned}$$

b.

$$\begin{aligned} 20 &= 2x+4 \\ 20-4 &= 2\mathrm{x}+4-4 && \text{Subtract } 4 \text{ from both sides} \\ 16 &= 2x \\ \tfrac{16}{2} &= \tfrac{2x}{2} && \text{Divide both sides by } 2 \\ 8 &= x \\ x &= 8 \end{aligned}$$

c.

$$\begin{aligned} 9+4x &= 3-2x \\ 9+4x+2x &= 3-2x+2x && \text{Add } 2x \text{ to both sides} \\ 9+6x &= 3 \\ 9-9+6x &= 3-9 && \text{Subtract } 9 \text{ from both sides} \\ 6x &= -6 \\ \tfrac{6x}{6} &= \tfrac{-6}{6} && \text{Divide both sides by } 6 \\ x &= -1 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L080 in the Pupil Handbook.

Lesson Title: Linear equations with brackets	**Theme:** Algebraic Processes	
Lesson Number: M1-L081	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve linear equations that contain brackets.	**Preparation** None	

Opening *(4 minutes)*

1. Write a revision problem on the board: Solve $2y - 5 = y - 2$
2. Ask pupils to solve the problem.
3. Invite a volunteer to write the solution on the board.
   Solution:

$$\begin{aligned} 2y - 5 &= y - 2 & \\ 2y - y - 5 &= y - y - 2 & \text{Subtract } y \text{ from both sides} \\ y - 5 &= -2 & \\ y - 5 + 5 &= -2 + 5 & \text{Add } 5 \text{ to both sides} \\ y &= 3 & \end{aligned}$$

4. Explain to the pupils that today's lesson is on solving linear equations that contain brackets.

Teaching and Learning *(23 minutes)*

1. Write the following problem on the board: Solve $2(p + 1) = 18$
2. Explain:
   - This is also a linear equation, but it contains brackets.
   - To solve the equation containing brackets, we may proceed as follows:
     - Remove the brackets.
     - Balance the equation, as in the previous lesson.
   - Remember that when there is a negative sign outside the bracket, the signs inside the brackets change.
3. Solve the problem on the board, explaining each step.
   Solution:

$$\begin{aligned} 2(p + 1) &= 18 & \\ 2p + 2 &= 18 & \text{Removing the brackets} \\ 2p + 2 - 2 &= 18 - 2 & \text{Subtract 2 from both sides} \\ 2p &= 16 & \\ \frac{2p}{2} &= \frac{16}{2} & \text{Divide both sides by 2} \\ p &= 8 & \end{aligned}$$

4. Write the following problem on the board: Solve $3(m+6)=2(m-3)$
5. Ask volunteers to describe the steps to solve the problem. As they give them, solve on the board.

   Solution:

$$
\begin{aligned}
3(m+6) &= 2(m-3) && \\
3m+18 &= 2m-6 && \text{Remove the brackets} \\
3m-2m+18 &= 2m-2m-6 && \text{Subtract } 2m \text{ from both sides} \\
m+18 &= -6 && \\
m+18-18 &= -6-18 && \text{Subtract 18 from both sides} \\
m &= -24 &&
\end{aligned}
$$

6. Write another problem on the board: Solve $-5(2y-1)=-7y+6$
7. Ask pupils to work with seatmates to remove the brackets.
8. Invite a volunteer to rewrite the equation on the board with the brackets removed. (Answer: $-10y+5=-7y+6$)
9. Ask pupils to work with seatmates to solve the problem.
10. Walk around to check for understanding and clear misconceptions.
11. Invite a volunteer to solve for y on the board.

$$
\begin{aligned}
-5(2y-1) &= -7y+6 && \\
-10y+5 & \quad -7y+6 && \text{Remove the brackets} \\
-10y+5+7y &= -7y+7y+6 && \text{Add } 7y \text{ to both sides} \\
-3y+5 &= 6 && \\
-3y+5-5 &= 6-5 && \text{Subtract 5 from both sides} \\
-3y &= 1 && \\
\tfrac{-3y}{-3} &= \tfrac{1}{-3} && \text{Divide both sides by } -3 \\
y &= -\tfrac{1}{3} &&
\end{aligned}
$$

12. Write a problem on the board: Solve $3(4c-7)-4(4c-1)=0$
13. Ask pupils to solve the problem with seatmates.
14. Walk around to check for understanding and clear misconceptions.
15. Invite a volunteer to solve for c on the board.

    Solution:

$$
\begin{aligned}
3(4c-7)-4(4c-1) &= 0 && \\
12c-21-16c+4 &= 0 && \text{Remove brackets} \\
12c-16c-21+4 &= 0 && \text{Collect like terms} \\
-4c-17 &= 0 && \text{Combine like terms} \\
-4c-17+17 &= 0+17 && \text{Add 17 to both sides} \\
-4c &= 17 && \\
\tfrac{-4c}{-4} &= \tfrac{17}{-4} && \text{Divide both sides by -4} \\
c &= \tfrac{-17}{4} = -4\tfrac{1}{4} &&
\end{aligned}
$$

Practice *(12 minutes)*

1. Write the following two problems on the board: Solve the following equations:
   a. $x + 6 = 3(2x - 2)$
   b. $5(a + 2) - 3(3a - 5) = 1$
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Ask pupils to exchange their exercise books.
5. Invite volunteers to write the solutions on the board.

   Solutions:

   a.

$$\begin{aligned} x + 6 &= 3(2x - 2) && \\ x + 6 &= 6x - 6 && \text{Remove the brackets} \\ x - 6x + 6 &= 6x - 6x - 6 && \text{Subtract } 6x \text{ from both sides} \\ -5x + 6 &= -6 && \\ -5x + 6 - 6 &= -6 - 6 && \text{Add } -6 \text{ from both sides} \\ -5x &= -12 && \\ \frac{-5x}{-5} &= \frac{-12}{-5} && \text{Divide both sides by } -5 \\ x &= \frac{12}{5} = 2\frac{2}{5} && \end{aligned}$$

   b.

$$\begin{aligned} 5(a + 2) - 3(3a - 5) &= 1 && \\ 5a + 10 - 9a + 15 &= 1 && \text{Remove the brackets} \\ 5a - 9a + 10 + 15 &= 1 && \text{Collect like terms} \\ -4a + 25 &= 1 && \\ -4a + 25 - 25 &= 1 - 25 && \text{Subtract } 25 \text{ from both sides} \\ -4a &= -24 && \\ \frac{-4a}{-4} &= \frac{-24}{-4} && \text{Divide both sides by -4} \\ a &= 6 && \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L081 in the Pupil Handbook.

Lesson Title: Linear equations with fractions	**Theme:** Algebraic Processes	
Lesson Number: M1-L082	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve linear equations that contain fractions.	**Preparation** None	

Opening *(3 minutes)*

1. Write a revision problem on the board: Solve $2(3b+4)=26$
2. Ask pupils to solve the problem in their exercise books.
3. Invite a volunteer to solve the problem on the board.
   Solution:

$$\begin{aligned} 2(3b+4) &= 26 & \\ 6b+8 &= 26 & \text{Remove the brackets} \\ 6b+8-8 &= 26-8 & \text{Subtract 8} \\ 6b &= 18 & \\ \tfrac{6b}{6} &= \tfrac{18}{6} & \text{Divide by 6} \\ b &= 3 & \end{aligned}$$

4. Explain to pupils that today's lesson is on solving linear equations that contain fractions.

Teaching and Learning *(24 minutes)*

1. Write on the board: Solve $\frac{x}{6}=8$
2. Discuss: How would you solve this? (Answer: We can multiply both sides by 6 to get x by itself on the left-hand side.)
3. Explain:
   - When an equation contains a fraction, we can multiply to cancel the denominator.
   - Multiply both sides of the equation by the denominator of the fraction.
4. Solve the problem on the board:

$$\begin{aligned} \tfrac{x}{6} &= 8 & \\ 6\left(\tfrac{x}{6}\right) &= 6(8) & \text{Multiply by 6} \\ x &= 48 & \text{6 cancels on the left-hand side} \end{aligned}$$

5. Write another problem on the board: Solve $\frac{x}{6}+\frac{x}{4}=10$
6. Explain:

- When there is more than 1 fraction, they can be canceled by multiplying throughout by the LCM of denominators.
- After multiplying by the LCM, follow the steps from the previous 2 lessons. Remove any brackets, and balance the equation to solve for the variable.

7. Invite a volunteer to give the LCM of the denominators in the problem on the board. (Answer: The LCM of 6 and 4 is 12.)
8. Solve the problem on the board and explain to pupils.
   Solution:

$$\begin{array}{rcll} \frac{x}{6}+\frac{x}{4} & = & 10 & \\ 12\left(\frac{x}{6}\right)+12\left(\frac{x}{4}\right) & = & 12(10) & \text{Multiply each term by the LCM, 12} \\ 2x+3x & = & 120 & \text{Simplify} \\ 5x & = & 120 & \text{Combine like terms} \\ \frac{5x}{5} & = & \frac{120}{5} & \text{Divide both sides by 5} \\ x & = & 24 & \end{array}$$

9. Ask pupils to work with seatmates to check whether $x = 24$ is correct.
10. Remind them to substitute the value of $x = 24$ into $\frac{x}{6}+\frac{x}{4}$, which should result in 10.
11. Invite a volunteer to write their work on the board. (Answer: $\frac{x}{6}+\frac{x}{4}=\frac{24}{6}+\frac{24}{4}=4+6=10$)
12. Write the following problem on the board: Solve the equation $\frac{7}{24}=\frac{x}{8}+\frac{1}{6}$
13. Ask a volunteer to give the LCM of the denominators in the problem. (Answer: The LCM of 24, 8, and 6 is 24.)
14. Explain:
    - It is not necessary to clear every fraction by multiplying by the LCM. We must clear the fractions with a variable.
    - In this problem, we could multiply throughout by 8, the denominator of the algebraic fraction.
    - However, the problem is easier to work if we clear every fraction, so we will use the LCM.
15. Explain the solution to the pupils.
    Solution:

$$\begin{array}{rcll} \frac{7}{24} & = & \frac{x}{8}+\frac{1}{6} & \\ 24\left(\frac{7}{24}\right) & = & 24\left(\frac{x}{8}\right)+24\left(\frac{1}{6}\right) & \text{Multiply by 24, the LCM} \\ 7 & = & 3x+4 & \\ 7-4 & = & 3x+4-4 & \text{Subtract 4} \\ 3 & = & 3x & \end{array}$$

$$\frac{3}{3} = \frac{3x}{3} \qquad \text{Divide by 3}$$
$$1 = x$$
$$x = 1$$

.

16. Write the following problem on the board: Solve the equation $\frac{3}{4}x - 1\frac{2}{3} = \frac{2}{3}x$
17. Ask a volunteer to give the LCM of the denominators in the problem. (Answer: The LCM of 3 and 4 is 12.)
18. Ask volunteers to give the steps needed to solve the problem. As they give them, solve on the board.

Solution:

$\frac{3}{4}x - 1\frac{2}{3}$	$=$	$\frac{2}{3}x$	
$\frac{3}{4}x - \frac{5}{3}$	$=$	$\frac{2}{3}x$	Convert mixed to improper fraction
$12\left(\frac{3}{4}x\right) - 12\left(\frac{5}{3}\right)$	$=$	$12\left(\frac{2}{3}x\right)$	Multiply by the LCM, 12
$9x - 20$	$=$	$8x$	Remove the brackets
$9x - 20 + 20$	$=$	$8x + 20$	Add 20
$9x$	$=$	$8x + 20$	
$9x - 8x$	$=$	$8x + 20 - 8x$	Subtract $8x$
x	$=$	20	

19. Write another problem on the board: Solve $\frac{x+1}{3} + \frac{x-1}{4} = 3$
20. Ask a volunteer to give the LCM of the denominators. (Answer: 12)
21. Ask pupils to solve the problem with their seatmates.
22. Walk around to check for understanding and clear misconceptions.
23. Invite a volunteer to solve the problem on the board.

Solution:

$\frac{x+1}{3} + \frac{x-1}{4}$	$=$	3	
$12\left(\frac{x+1}{3}\right) + 12\left(\frac{x-1}{4}\right)$	$=$	$12(3)$	Multiply each term by the LCM=30
$4(x+1) + 3(x-1)$	$=$	36	Remove the brackets
$4x + 4 + 3x - 3$	$=$	36	Collect like terms
$7x + 1$	$=$	36	
$7x + 1 - 1$	$=$	$36 - 1$	Subtract 1
$7x$	$=$	35	
$\frac{7x}{7}$	$=$	$\frac{35}{7}$	Divide by 7
x	$=$	5	

Practice *(12 minutes)*

1. Write the following problems on the board:
    a. Solve $\frac{x}{2}+\frac{2x}{3}=7$
    b. Solve $\frac{x+1}{3}-\frac{x-1}{8}=4$
2. Ask pupils to solve the problems independently in their exercise books. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers to write their solutions on the board.

Solutions:

a.
$$\begin{aligned} \frac{x}{2}+\frac{2x}{3} &= 7 \\ 6\left(\frac{x}{2}\right)+6\left(\frac{2x}{3}\right) &= 6(7) \\ 3x+4x &= 42 \\ 7x &= 42 \\ \frac{7x}{7} &= \frac{42}{7} \\ x &= 6 \end{aligned}$$

b.
$$\begin{aligned} \frac{x+1}{3}-\frac{x-1}{8} &= 4 \\ 24\left(\frac{x+1}{3}\right)-24\left(\frac{x-1}{8}\right) &= 24(4) \\ 8(x+1)-3(x-1) &= 24(4) \\ 8x+8-3x+3 &= 96 \\ 8x-3x+8+3 &= 96 \\ 5x+11 &= 96 \\ 5x+11-11 &= 96-11 \\ 5x &= 85 \\ \frac{5x}{5} &= \frac{85}{5} \\ x &= 17 \end{aligned}$$

Closing *(1minute)*

1. For homework, have pupils do the practice activity PHM1-L082 in the Pupil Handbook.

Lesson Title: Word problems	**Theme:** Algebraic Processes	
Lesson Number: M1-L083	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to create and solve equations from word problems.	**Preparation** Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write on the board: Write an expression for the following: A certain number plus nine is equal to five times the number.
2. Explain that pupils should use a variable to represent the unknown number.
3. Ask pupils to write the expression with seatmates.
4. Invite a volunteer to write their expression on the board. If other pupils wrote something else, have them write it on the board too. Encourage all pupils, even if their expressions are incorrect.
5. Discuss the expressions as a class before determining the correct expression. (Answer: $x + 9 = 5x$)
6. Explain to the pupils that today's lesson is on solving equations from word problems.

Teaching and Learning *(22 minutes)*

1. Explain: There are many types of word problems which involve relations among known and unknown numbers. These can be written in the form of equations.
2. Explain the steps for solving linear equations from word problems. Ask pupils to look at this section in their Pupil Handbook, or write the steps on the board:
    - Read the problem carefully and note what is given and what is required.
    - Assign a variable to represent each unknown.
    - Identify any value that is multiplied by the variable (the coefficient).
    - Identify any value that is constant.
    - Write the algebraic expression representing the situation.
    - Solve the equation for the unknown variable.
    - Check whether the answer satisfies the conditions of the problem.
3. Write on the board: You make 90.00 Leones for every hour that you work. If you make $1{,}800.00$ Leones in total, how many hours did you work?
4. Ask a volunteer to give the unknown in this problem. (Answer: Hours you worked)
5. Ask a volunteer to choose a variable to represent this unknown. (Example answer: h for hours)
6. Discuss: Which number can be a coefficient of the variable h? Why? (Answer: 90 because if you make 90 Leones for each hour you work, you should multiply 90 by the number of hours worked).

7. Invite a volunteer to write down the algebraic expression on the board. (Answer: $90h$)
8. Discuss? What is the full algebraic equation representing this situation? Why? (Answer: $90h = 1{,}800$, because the total amount of money earned was 1,800.00 Leones.)
9. Solve the following problem on the board:

$$\begin{aligned} 90h &= 1800 \\ \frac{90h}{90} &= \frac{1800}{90} && \text{Divide by 90} \\ h &= 20 \text{ hours} \end{aligned}$$

10. Write on the board: The sum of two numbers is 25. One of the numbers exceeds the other by 9. Find the numbers.
11. Solve on the board, explaining each step:

    Solution:

    Let the number be y

    Then the other number is $y + 9$

    The sum of the two numbers is 25

    In this situation, $y + y + 9 = 25$

    Solve for y:

$$\begin{aligned} y + y + 9 &= 25 \\ 2y + 9 &= 25 \\ 2y + 9 - 9 &= 25 - 9 && \text{Subtract 9 from both sides} \\ 2y &= 16 \\ \frac{2y}{2} &= \frac{16}{2} && \text{Divide both sides by 2} \\ y &= 8 \end{aligned}$$

    Therefore, $y + 9 = 8 + 9 = 17$

    Therefore, the two numbers are 8 and 17.

12. Write another problem on the board: Michael is 3 years older than Fatu. The sum of their ages is 27. How old is Fatu?
13. Discuss:
    - How would you start by solving this problem?
    - What steps would you take?
14. Allow pupils to share their ideas, then explain: We start by assigning a variable. Let's use f to represent Fatu's age.
15. Write on the board:

    Fatu's age: f

    Michael's age: $f + 3$

    Sum of their ages: $f + f + 3 = 27$
16. Ask pupils to work with seatmates to finish the problem by solving for f.
17. Walk around to check for understanding and clear misconceptions.

18. Invite a volunteer to write the solution on the board.

Solution:

$$\begin{aligned} f+f+3 &= 27 \\ 2f+3 &= 27 \\ 2f+3-3 &= 27-3 && \text{Subtract 3 from both sides} \\ 2f &= 24 \\ \frac{2f}{2} &= \frac{24}{2} && \text{Divide both sides by 2} \\ f &= 12 \text{ years old} \end{aligned}$$

19. Write another problem on the board: Abass is 5 years younger than Daniel. Four years later, Daniel will be twice as old as Abass. Find their present ages.
20. Ask pupils to write the equation with seatmates (they should not solve yet).
21. Walk around to check for understanding and clear misconceptions.
22. Invite a volunteer to write the equation on the board and explain how they found it.

Solution:

Let Daniel's present age be m

Then Abass' present age is $m-5$

After 4 years, Daniel's age will be $m+4$

Abass' age will be $m-5+4$

Daniel will be twice as old as Abass, therefore:

$$m+4=2(m-5+4)$$

$$m+4=2(m-1)$$

23. Ask pupils to work with seatmates to solve the problem. Remind them to find both ages.
24. Walk around to check for understanding and clear misconceptions.
25. Invite a volunteer to solve the problem on the board.

$$\begin{aligned} m+4 &= 2(m-1) \\ m+4 &= 2m-2 \\ m+4-4 &= 2m-2-4 && \text{Subtract 4} \\ m &= 2m-6 \\ m-2m &= 2m-2m-6 && \text{Subtract } 2m \\ -m &= -6 \\ -1(-\mathrm{m}) &= -1(-6) && \text{Multiply by -1} \\ m &= 6 \end{aligned}$$

Therefore, Daniel's present age is 6.

Abass present age $= m-5=6-5=1$ year.

Practice *(14 minutes)*

1. Write the following two problems on the board:
   a. The total number of pupils in a school is 126. The number of boys in the school is 14 greater than the number of girls. Find the number of girls.
   b. The total cost of 4 pens and 2 notebooks was Le$11{,}000.00$. The price of one notebook was Le1,000.00 more than the price of one pen. Find the unit price of one pen and one notebook.
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Invite volunteers, one at a time, to write their answer on the board.
   Solutions:
   a. Let the numbers of girls x and the number of boys $= x + 14$

$$\begin{aligned} x + (x + 14) &= 126 \\ x + x + 14 &= 126 \\ 2x + 14 &= 126 \\ 2x &= 126 - 14 \\ 2x &= 112 \\ x &= \frac{112}{2} \\ x &= 56 \end{aligned}$$

The numbers of girls is 56.

   b. Let the price of one pen $= y$ and the price of one note book $y + 1{,}000$

$$\begin{aligned} 4y + 2(y + 1000) &= 11{,}000 \\ 4y + 2y + 2000 &= 11{,}000 \\ 6y + 2000 &= 11{,}000 \\ 6y &= 11{,}000 - 2{,}000 \\ 6y &= 9{,}000 \\ y &= \frac{9{,}000}{6} \\ y &= 1{,}500 \end{aligned}$$

The price of one pen is $Le\ 1{,}500.00$ and the price of one notebook is $1{,}500 + 1{,}000 = Le\ 2{,}500.00$.

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L083 in the Pupil Handbook.

Lesson Title: Substitution in formulae	Theme: Algebraic Processes	
Lesson Number: M1-L084	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to substitute given values into a formula.	**Preparation** Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write on the board: If $x = -7$ and $y = 3$, calculate the value of $\left(\frac{x+y}{x-y}\right)^2$
2. Ask pupils to solve the problem independently in their exercise books.
3. Invite a volunteer to solve the problem on the board.
   Solution:
   $\left(\frac{x+y}{x-y}\right)^2 = \left(\frac{-7+3}{-7-3}\right)^2 = \left(\frac{-4}{-10}\right)^2 = \frac{16}{100} = \frac{4}{25}$
4. Explain to pupils that today's lesson is on substituting given values into a formula.

Teaching and Learning *(23 minutes)*

1. Explain: A formula is an equation with variables that represent specific measures.
2. Write on the board: $P = 2(l + w)$
3. Explain:
   - This is the formula for the perimeter, P, of a rectangle of length l and width w.
   - In this example, P is the subject of the formula. By substituting the values of l and w into the formula, the corresponding value of P can be found.
4. Write on the board: Calculate the perimeter of a rectangle whose length is 5 cm and width is 3 cm.
5. Solve on the board, explaining each step:
   Solution:

P	$=$	$2(l + w)$	Formula
P	$=$	$2(5 + 3)$	Substitute $l = 5$ and $w = 3$
P	$=$	2×8	
P	$=$	16	

   The perimeter is 16 cm.
6. Write on the board: Given that $R = 3$, $d = 5$ and $L = 15$, find the value of k if $k = \frac{Rd^2}{L}$.
7. Ask volunteers to give the steps to solve the problem. As they give the steps, work the problem on the board.
   Solution:

k	$=$	$\frac{Rd^2}{L}$	Formula

$$k = \frac{3(5)^2}{15} \quad \text{Substitute } R = 3, d = 5 \text{ and } L = 15$$
$$k = \frac{3 \times 25}{15} \quad \text{Simplify}$$
$$k = \frac{75}{15}$$
$$k = 5$$

8. Write another problem on the board. The formula $A = \pi r(r + s)$ gives the surface area, A, of a cone of base radius r cm and slant height s cm. Find the surface area of a cone of base radius 10 cm and slant height 11 cm using the value of $\pi = \frac{22}{7}$.
9. Explain:
   - We have not covered surface area in this course. However, we have enough information to solve this problem.
   - You may encounter problems on topics that you do not fully understand, but you will be given enough information to solve the problem. Use your problem-solving skills.
10. Ask volunteers to give the steps to solve the problem. As they give the steps, work the problem on the board.

    Solution:

$$A = \pi r(r + s)$$
$$A = \frac{22}{7}(10)(10 + 11) \quad \text{Substitute } r = 10, s = 11 \text{ and } \pi = \frac{22}{7}$$
$$A = \frac{22}{7}(10)(21)$$
$$A = \frac{22}{7}(210)$$
$$A = 22 \times 30$$
$$A = 660 \text{ cm}^2$$

11. Write two problems on the board:
    a. Given that $C = 2\pi r$, find C when $\pi = 3.142$ and $r = 25$.
    b. If $q = ut + \frac{1}{2}ft$, find q when $u = 4$, $t = 6$ and $f = 15$.
12. Ask pupils to solve the problem with seatmates.
13. Walk around to check for understanding and clear misconceptions.
14. Invite two volunteers to write their answers on the board at the same time.

    Solutions:

    a.

$$C = 2\pi r$$
$$C = 2 \times 3.142 \times 25 \quad \text{Substitute } \pi = 3.142 \text{ and } r = 25$$
$$C = 157.1$$

    b.

$$q = ut + \frac{1}{2}ft$$
$$q = 4(6) + \frac{1}{2}(15 \times 6) \quad \text{Substitute } u = 4, t = 6, f = 15$$

$$
\begin{aligned}
q &= 24 + \tfrac{1}{2} \times 90 \\
q &= 24 + 45 \\
q &= 69
\end{aligned}
$$

Practice *(13 minutes)*

1. Write the following two problems on the board:
   a. Evaluate $s = \frac{4n^3 - 3n^2 + 6}{n}$ for $n = 3$
   b. The formula $A = P\left(1 + \frac{RT}{100}\right)$ gives the total money, A, that a principal, P, amounts to in T years at $R\%$ simple interest per annum. Find the amount that a principal of $Le80{,}000.00$ becomes if invested for 3 years at 4% simple interest per annum.
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Ask pupils to exchange their exercise books.
5. Invite two volunteers, one at a time, to write their answers on the board.
   Answers:
   a.
$$
\begin{aligned}
s &= \frac{4n^3 - 3n^2 + 6}{n} \\
&= \frac{4(3)^3 - 3(3)^2 + 6}{3} && \text{Substitute } n = 3 \\
&= \frac{4(27) - 3(9) + 6}{3} && \text{Simplify} \\
&= \frac{108 - 27 + 6}{3} \\
&= \frac{87}{3} \\
&= 29
\end{aligned}
$$
   b.
$$
\begin{aligned}
A &= P\left(1 + \frac{RT}{100}\right) \\
&= 80{,}000\left(1 + \frac{4 \times 3}{100}\right) && \text{Substitute } P = 80{,}000.00, R = 4\%, T = 3 \\
&= 80{,}000\left(1 + \frac{12}{100}\right) \\
&= 80{,}000\left(\frac{112}{100}\right) \\
&= 80{,}0\cancel{00} \times \frac{112}{1\cancel{00}} \\
&= 800 \times 112 \\
&= \text{Le } 89{,}600.00
\end{aligned}
$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L084 in the Pupil Handbook.

Lesson Title: Change of subject – Part 1	**Theme:** Algebraic Processes	
Lesson Number: M1-L085	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to change the subject of a formula.	**Preparation** Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write a problem on the board: Solve for x if $9x + 2 = 6x$
2. Ask pupils to solve the problem independently.
3. Invite any volunteer to solve the problem on the board.
   Solution:

$$\begin{aligned} 9x + 2 &= 6x && \\ 9x - 6x &= -2 && \text{Transpose } 6x \text{ and } 2 \\ 3x &= -2 && \text{Combine like terms} \\ \frac{3x}{3} &= \frac{-2}{3} && \text{Divide by 3} \\ x &= -\tfrac{2}{3} && \end{aligned}$$

4. Explain to the pupils that today's lesson is on change of subject of a formula.

Teaching and Learning *(25 minutes)*

1. Explain: A formula is a type of equation which shows the relationship between different variables.
2. Write on the board: $x + y = 7$ $\qquad$ $2x - 8 = 0$
3. Explain:
   - For an equation to be a formula it must have more than one variable. For example $x + y = 7$ is a formula because it shows the relationship between x and y.
   - $2x - 8 = 0$ is not a formula because it only has one variable, x.
   - The subject of a formula is the single variable to which everything else in the formula is equal. The subject of a formula will usually be positioned to the left of the equal to sign.
   - In $x + y = 7$, neither variable is the subject.
4. Write on the board: a. $x = 7 - y$ $\qquad$ b. $y = 7 - x$
5. Explain: x is the subject of formula a. y is the subject of formula b.
6. Write a problem on the board: Make x the subject of the formula: $z = 4x + 4y$
7. Explain: We will use the balancing method to get x by itself on one side of the equal to sign.

8. Write the solution on the board, explaining each step:

$$\begin{aligned} z &= 4x + 4y \\ z - 4y &= 4x + 4y - 4y && \text{Subtract } 4y \text{ from both sides} \\ z - 4y &= 4x \\ \frac{z-4y}{4} &= \frac{4x}{4} && \text{Divide throughout by 4} \\ x &= \frac{z-4y}{4} && \text{Write the formula in terms of } x \end{aligned}$$

9. Explain:
   - When changing the subject of a formula, remember the following rules:
     - Adding and subtracting are the opposite of one another.
     - Multiplying and dividing are the opposite of one another.
     - You must perform the same operation to both sides of the equation.
10. Write on the board: Make m the subject of $mt + n = mp + q$.
11. Explain:
    c. The variable m appears twice in this formula.
    d. We want to collect each term with m on one side of the equation.
12. Write the solution on the board, explaining each step:

$$\begin{aligned} mt + n &= mp + q \\ mt - mp &= q - n && \text{Group terms containing } m \\ m(t-p) &= q - n && \text{Factorise } m \\ \frac{m(t-p)}{t-p} &= \frac{q-n}{t-p} && \text{Divide both sides by } t - p \\ m &= \frac{q-n}{t-p} \end{aligned}$$

13. Write another problem on the board. Make q the subject of the formula $p = \frac{4}{7}(q - 28)$.
14. Explain: When you have to change the subject of a formula with a fraction like this, you must first multiply both sides of the formula by the denominator.
15. Write the solution on the board, explaining each step:

$$\begin{aligned} p &= \frac{4}{7}(q - 28) \\ 7(p) &= 7 \times \frac{4}{7}(\mathrm{q} - 28) && \text{Multiply both sides by 7} \\ 7p &= 4(q - 28) \\ 7p &= 4q - 112 && \text{Expand the brackets} \\ 7\mathrm{p} + 112 &= 4\mathrm{q} - 112 + 112 && \text{Add 112 to both sides} \\ 7p + 112 &= 4q \\ \frac{7p+112}{4} &= \frac{4q}{4} && \text{Divide both sides by 4} \\ q &= \frac{7\mathrm{p}+112}{4} \end{aligned}$$

16. Write a problem on the board: Make b the subject of the formula $c = \frac{2b}{8\pi}$

17. Ask volunteers to give the steps to solve the problem. As they explain, solve it on the board:

$$\begin{aligned} c &= \frac{2b}{8\pi} & \\ 8\pi(c) &= 8\pi\left(\frac{2b}{8\pi}\right) & \text{Multiply both sides by } 8\pi \\ 8\pi c &= 2b & \\ \frac{8\pi c}{2} &= \frac{2b}{2} & \text{Divide both sides by 2} \\ b &= \frac{8\pi c}{2} & \end{aligned}$$

18. Write the following two problems on the board:
    a. Make p the subject of the relation $5t - pq = 3p + 2$
    b. Make a the subject of the relation $s = \frac{1}{2}(a + 4)$.
19. Ask pupils to solve the problems with seatmates.
20. Walk around to check for understanding and clear misconceptions.
21. Invite two volunteers to give their answers on the board.

Answer:

a. $5t - pq = 3p + 2$

$$\begin{aligned} 5t - 2 &= 3p + pq & \text{Collect like terms with } p \\ 5t - 2 &= p(3 + q) & \text{Factorise } p \\ \frac{5t-2}{3+q} &= \frac{p(3+q)}{3+q} & \text{Divide both sides by } 3 + q \\ p &= \frac{5t-3l}{3+q} & \end{aligned}$$

b. $s = \frac{1}{2}(a + 4)$

$$\begin{aligned} 2(s) &= 2(\tfrac{1}{2}(a + 4)) & \text{Multiply both sides by 2} \\ 2s &= a + 4 & \\ 2s - 4 &= a & \text{Subtract 4 from both sides} \\ a &= 2s - 4 & \end{aligned}$$

Practice *(11 minutes)*

1. Write the following two problems on the board:
    a. Make v the subject of the relation $f = \frac{1}{3}(v + u)$.
    b. Make x the subject of the relation $ry + 2x = tx + 4$.
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Ask pupils to exchange their exercise books.
5. Invite two volunteers to write their solutions on the board.

Solutions:

a. $f = \frac{1}{3}(v + u)$.

$$\begin{aligned} 3f &= 3 \times \frac{1}{3}(v+u) && \text{Multiply both sides by } 3 \\ 3f &= v + u \\ 3f - u &= v && \text{Subtract } u \text{ from both sides} \\ v &= 3f - u && \text{Factorising } v \end{aligned}$$

b. $ry + 2x = tx + 4$

$$\begin{aligned} 2x - tx &= 4 - ry && \text{Group terms containing } x \\ x(2-t) &= 4 - ry && \text{Factorise } x \\ \frac{x(2-t)}{2-t} &= \frac{4-ry}{2-t} && \text{Divide both sides by } 2 - t \\ x &= \frac{4-ry}{2-t} \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L085 in the Pupil Handbook.

Lesson Title: Change of subject – Part 2	**Theme:** Algebraic Processes	
Lesson Number: M1-L086	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to change the subject of a formula.	**Preparation** Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write on the board: Make d the subject of the formula: $a = \frac{c+dx}{b}$
2. Ask pupils to solve the problem independently in their exercise books.
3. Invite any volunteer to solve the problem on the board.

   Solution:

$$
\begin{aligned}
a &= \frac{c+dx}{b} && \\
b \times a &= b\left(\frac{c+dx}{b}\right) && \text{Multiply both sides by } b \\
ab &= c + dx && \\
ab - c &= dx && \text{Subtract } c \text{ from both sides} \\
\frac{ab-c}{x} &= \frac{\mathrm{d}x}{x} && \text{Divide both sides by } x \\
d &= \frac{ab-c}{x} &&
\end{aligned}
$$

4. Explain to the pupils that today's lesson is also on changing the subject of a formula. We will solve more difficult problems, such as those involving squares and square roots.

Teaching and Learning *(23 minutes)*

1. Write on the board: Make q the subject of the formula $p = \sqrt{q}$
2. Explain: A formula remains balanced if we perform the same operations to both sides of it. Therefore, we can find either the square or square-root of both sides.
3. Explain:
   - We need to get q on its own. To do this, we must find a way of removing the square root sign. This can be achieved by squaring both sides.
   - Remind pupils if needed: $\left(\sqrt{q}\right)^2 = \sqrt{q} \times \sqrt{q} = q^{\frac{1}{2}+\frac{1}{2}} = q$
4. Solve the problem on the board, explaining each step:

$$
\begin{aligned}
p &= \sqrt{q} && \\
p^2 &= \left(\sqrt{q}\right)^2 && \text{Square both sides} \\
p^2 &= q &&
\end{aligned}
$$

$$q = p^2$$

5. Write on the board: Make b the subject of the formula $p = \sqrt{a+b}$
6. Ask volunteers to explain the steps to solve the problem. Solve on the board:

$$p = \sqrt{a+b}$$
$$p^2 = \left(\sqrt{a+b}\right)^2 \quad \text{Square both sides}$$
$$p^2 = a+b$$
$$p^2 - a = b$$

7. Explain: Whenever a given relation involves the square, we can find the square root of both sides of the relation.
8. Write on the board: Make i the subject of the formula $R = (1+i)^2$.
9. Write the solution on the board and explain:

$$R = (1+i)^2$$
$$\sqrt{R} = \sqrt{(1+\mathrm{i})^2} \quad \text{Take the square root of both sides}$$
$$\sqrt{R} = 1+i$$
$$i = \sqrt{R} - 1 \quad \text{Subtract 1 from each side}$$

10. Write another problem on the board: Make x the subject of the formula $v = \frac{1}{3}\pi x^2$.
11. Ask volunteers to explain the steps to solve the problem. Solve on the board:

$$v = \frac{1}{3}\pi x^2$$
$$3v = \pi x^2 \quad \text{Multiply both sides by 3}$$
$$\frac{3v}{\pi} = \frac{\pi x^2}{\pi} \quad \text{Divide both sides by } \pi$$
$$x^2 = \frac{3v}{\pi}$$
$$x = \sqrt{\frac{3\mathrm{v}}{\pi}} \quad \text{Square root both sides}$$

12. Write the following two problems on the board:
    a. Make r, the subject of the relation $v = \frac{1}{3}\pi r^2 h$.
    b. Make x, the subject of the relation $y = \frac{1}{2}\sqrt{5+x}$. Then, find the value of x when $y = 2$.
13. Ask pupils to solve the problem with seatmates.
14. Walk around to check for understanding and clear misconceptions.
15. Invite two volunteers to write the solutions on the board.

Solutions:

a.

$$v = \frac{1}{3}\pi r^2 h$$
$$3v = \pi r^2 h \quad \text{Multiply by 3}$$

$$\frac{3v}{\pi h} = \frac{\pi r^2 h}{\pi h} \quad \text{Divide by } \pi h$$

$$r^2 = \frac{3v}{\pi h}$$

$$r = \sqrt{\frac{3v}{\pi h}} \quad \text{Take the square root}$$

b.

$$\begin{aligned} y &= \tfrac{1}{2}\sqrt{5+x} & \\ 2y &= \sqrt{5+x} & \text{Multiply by 2} \\ (2y)^2 &= 5+x & \text{Square both sides} \\ 4y^2 &= 5+x & \\ 4y^2-5 &= x & \text{Subtract 5} \\ x &= 4y^2-5 & \end{aligned}$$

Substitute $y = 2$:

$$\begin{aligned} x &= 4(2)^2-5 \\ &= 4(4)-5 \\ &= 16-5 = 11 \end{aligned}$$

Practice *(13 minutes)*

1. Write the following two problems on the board:
    a. Make x the subject of the relation $x^2 + y^2 = w^2$.
    b. Make a the subject of the relation $t = \sqrt{\frac{5a}{h}}$. Then, find the value of a when $t = 5$, $h = 4$.
2. Ask pupils to solve the problems independently in their exercise books. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Ask pupils to exchange their exercise books.
5. Invite two volunteers to write the solutions on the board.
   Solutions:

c.

$$\begin{aligned} w^2 &= x^2+y^2 & \\ x^2 &= w^2-y^2 & \text{Subtract } y^2 \text{ from both sides} \\ x &= \pm\sqrt{w^2-y^2} & \text{Square root both sides} \end{aligned}$$

Remember that when you find the square root of a number, the result can be either positive or a negative.

d.

$$t = \sqrt{\frac{5a}{h}}$$

$$t^2 = \frac{5a}{h}$$ Square both sides

$$ht^2 = 5a$$ Multiply by h

$$\frac{ht^2}{5} = a$$ Divide by 5

$$a = \frac{ht^2}{5}$$

Substitute $t = 5$, $h = 4$

$$a = \frac{4(5)^2}{5}$$

$$= \frac{4(25)}{5}$$

$$= \frac{100}{5} = 20$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L086 in the Pupil Handbook.

Lesson Title: Reduction to basic form of surds	**Theme:** Numbers and Numerations	
Lesson Number: M1-L087	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to reduce surds to basic forms.	**Preparation** Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write the following two problems on the board:
   a. Find the value of the following:
      i. 5^2 ii. 4^2
   e. Find the square roots of the following:
      i. 36 ii. 81
2. Ask pupils to solve the problem independently in their exercise books.
3. Ask volunteers to call out their answers and write them on the board. (Answers: a. i. $5^2 = 25$; ii. $4^2 = 16$; b. i. $\sqrt{36} = 6$; ii. $\sqrt{81} = 9$)
4. Explain to pupils that today's lesson is on how to reduce surds to basic form.

Teaching and Learning *(23 minutes)*

1. Write on the board: $\sqrt{2}, \sqrt{3}, \sqrt{5}$
2. Discuss: Can you find the square root of these numbers? How would you do it?
3. Allow pupils to share their ideas, then explain:
   - Surds are numbers that we cannot find a whole number square root of. The examples on the board are surds.
   - Surds are left in root form (with √) to express their exact value.
   - When calculated, surds have an infinite number of non-recurring decimals. Therefore, surds are irrational numbers.
   - The square roots of all prime numbers are surds. They give decimals that never repeat and never end.
4. Write on the board: $\sqrt{8}$
5. Discuss:
   - Can you find the square root of this?
   - Can you simplify it in any way?
6. Allow pupils to share their ideas, then explain:
   - We can simplify $\sqrt{8}$. We need to find the largest perfect square factor in order to simplify surds.

- The largest perfect square factor is found by looking at any possible factors of the number that is being square rooted.

7. Ask volunteers to give perfect square numbers. As they give them, write them on the board. (Example answers: 4, 9, 16, 25, 36)
8. Discuss: Does our surd contain any perfect square factor? (Answer: Yes, 4 is a factor of 8 and a perfect square.)
9. Rewrite the surd on the board: $\sqrt{8} = \sqrt{4 \times 2}$
10. Write on the board: $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$.
11. Explain: This rule helps in simplifying surds to basic form.
12. Apply the rule to the problem on the board: $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$
13. Write a problem on the board: Simplify $\sqrt{18}$ to basic form.
14. Ask a volunteer to find two numbers whose product is 18 and one of the numbers is a perfect square. (Answer: $9 \times 2 = 18$)
15. Solve the problem on the board, explaining each step:
$$\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$$
16. Write another problem on the board: simplify $\sqrt{32}$
17. Ask volunteers to give the steps to simplify the surd. As they give them, solve on the board:
$$\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$$
18. Write a problem on the board: Simplify $\sqrt{800}$
19. Ask a volunteer to identify a perfect square factor of 800. (Example answers: 4, 100, 400)
20. Explain:
    - There is more than one perfect square factor.
    - We may find the square root of the biggest perfect square factor, or we may find the square root of smaller perfect square factors in separate steps.
21. Solve on the board as follows:
$$\sqrt{800} = \sqrt{100 \times 8} = \sqrt{100} \times \sqrt{8} = 10\sqrt{8} = 10\sqrt{4 \times 2} = 10\sqrt{4} \times \sqrt{2} = 10 \times 2 \times \sqrt{2}$$
$$= 20\sqrt{2}$$
22. Explain:
    - After simplifying a surd, always check your result to see if it can be simplified further.
    - In this example we factored out 100, but found that we could still factor out 4.
23. Write the solution again, using fewer steps:
$$\sqrt{800} = \sqrt{400 \times 2} = \sqrt{400} \times \sqrt{2} = 20\sqrt{2}$$
24. Write three problems on the board:
    a. $\sqrt{128}$
    b. $\sqrt{500}$

c. $\sqrt{250}$

25. Ask pupils to solve the problems with seatmates.
26. Walk around to check for understanding and clear misconceptions.
27. Invite 3 volunteers to write the solutions on the board. Solutions may be different if they chose different perfect square factors.

Solutions:

a. $\sqrt{128} = \sqrt{64 \times 2} = \sqrt{64} \times \sqrt{2} = 8\sqrt{2}$
b. $\sqrt{500} = \sqrt{100 \times 5} = \sqrt{100} \times \sqrt{5} = 10\sqrt{5}$
c. $\sqrt{250} = \sqrt{25 \times 10} = \sqrt{25} \times \sqrt{10} = 5\sqrt{10}$

Practice *(13 minutes)*

1. Write 5 problems on the board: Simplify the following surds
   a. $\sqrt{48}$
   b. $\sqrt{28}$
   c. $\sqrt{72}$
   d. $\sqrt{448}$
   e. $\sqrt{15{,}000}$
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Ask five volunteers to write their solutions on the board. Solutions may vary based on which perfect square factors pupils used.

Solutions:

a. $\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$
b. $\sqrt{28} = \sqrt{4 \times 7} = \sqrt{4} \times \sqrt{7} = 2\sqrt{7}$
c. $\sqrt{72} = \sqrt{36 \times 2} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$
d. $\sqrt{448} = \sqrt{64 \times 7} = \sqrt{64} \times \sqrt{7} = 8\sqrt{7}$
e. $\sqrt{15000} = \sqrt{2500 \times 6} = \sqrt{2500} \times \sqrt{6} = 50\sqrt{6}$

Alternatively, pupils may factor in multiple steps:

$$\sqrt{15000} = \sqrt{100 \times 150} = \sqrt{100} \times \sqrt{150} = 10\sqrt{150} = 10\sqrt{25 \times 6} = 10\sqrt{25}\sqrt{6}$$
$$= 10 \times 5\sqrt{6} = 50\sqrt{6}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L087 in the Pupil Handbook.

Lesson Title: Addition and subtraction of Surds – Part 1	**Theme:** Numbers and Numerations	
Lesson Number: M1-L088	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to solve simple problems involving addition and subtraction of surds.	**Preparation** Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write the following two problems on the board:
   a. Simplify $\sqrt{99}$ b. Simplify $\sqrt{50}$
2. Ask pupils to solve the problems.
3. Invite volunteers to write the solutions on the board.
   Solutions:
   a. $\sqrt{99} = \sqrt{9 \times 11} = \sqrt{9} \times \sqrt{11} = 3\sqrt{11}$
   b. $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$
4. Explain to the pupils that today's lesson is on solving simple problems involving addition and subtraction of surds.

Teaching and Learning *(23 minutes)*

1. Explain:
   - Surds that are in the basic form can be added or subtracted.
   - We can only add or subtract like surds. Like surds have the same surd part.
2. Write on the board: $3\sqrt{2} + 7\sqrt{2}$
3. Explain:
   a. To add 2 numbers with the same surd, simply add the rational coefficients (the numbers in front of the surds).
   b. The same surd will appear in the answer.
4. Solve on the board: $3\sqrt{2} + 7\sqrt{2} = (3 + 7)\sqrt{2} = 10\sqrt{2}$)
5. Write another problem on the board: $2\sqrt{3} + 5\sqrt{2}$
6. Explain:
   a. This expression cannot be simplified because the surd part is different.
   b. Mixed surds cannot be added or subtracted. They cannot be simplified further.
   c. The expression on the board is in its simplest form.
7. Write the general rules on the board:
   - Addition: $m\sqrt{k} + n\sqrt{k} = (m + n)\sqrt{k}$
   - Subtraction*:* $m\sqrt{k} - n\sqrt{k} = (m - n)\sqrt{k}$

8. Write another problem on the board: Simplify $10\sqrt{5} - \sqrt{5}$
9. Ask pupils to solve the problem with seatmates.
10. Invite a volunteer to write the solution on the board. (Answer: $10\sqrt{5} - \sqrt{5} = (10 - 1)\sqrt{5} = 9\sqrt{5}$)
11. Write on the board: Add $\sqrt{12} + \sqrt{27}$
12. Discuss: Can we add these surds as they are now? Why or why not? (Answer: No, they are not like surds.)
13. Explain the following steps to add or subtract two or more surds:
    Step 1. Convert each surd to its simplest form.
    Step 2. Add or subtract any like surds.
14. Write a problem on the board: Find the sum of $\sqrt{12}$ and $\sqrt{27}$
15. Write the solution on the board and explain:
    Solution:
    Step 1. Express each surd in its simplest form:
    $$\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$$
    $$\sqrt{27} = \sqrt{9 \times 3} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$$
    Step 2. They are like surds, so we can add:
    $$2\sqrt{3} + 3\sqrt{3} = (2 + 3)\sqrt{3} = 5\sqrt{3}$$
16. Write another problem on the board: Subtract $\sqrt{20}$ from $\sqrt{45}$.
17. Ask volunteers to give the steps to solve the problem.
    Solution:
    Step 1. Express each surd in its simplest form.
    $$\sqrt{45} = \sqrt{9 \times 5} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$$
    $$\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$$
    Step 2. They are like surds, so we can subtract:
    $$\sqrt{45} - \sqrt{20} = 3\sqrt{5} - 2\sqrt{5} = (3 - 2)\sqrt{5} = \sqrt{5}$$
18. Write two problems on the board:
    a. Find the sum of $\sqrt{12}$ and $\sqrt{75}$
    b. Simplify $\sqrt{162} - \sqrt{18}$
19. Ask pupils to solve the problems with seatmates.
20. Walk around to check for understanding and clear misconceptions.
21. Invite two volunteers to write the solutions on the board.
    Solutions:
    a. $\sqrt{12} + \sqrt{75} = \sqrt{4 \times 3} + \sqrt{25 \times 3} = 2\sqrt{3} + 5\sqrt{3} = (2 + 5)\sqrt{3} = 7\sqrt{3}$
    b. $\sqrt{162} - \sqrt{18} = \sqrt{81 \times 2} - \sqrt{9 \times 2} = 9\sqrt{2} - 3\sqrt{2} = (9 - 3)\sqrt{2} = 6\sqrt{2}$

Practice *(13 minutes)*

1. Write two problems on the board:

a. Simplify $\sqrt{24}+\sqrt{54}$
b. Simplify $\sqrt{500}-\sqrt{125}$

2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite 2 volunteers to write the solutions on the board.

Solutions:

a. $\sqrt{24}+\sqrt{54}=\sqrt{4\times 6}+\sqrt{9\times 6}=\sqrt{4}\times\sqrt{6}+\sqrt{9}\times\sqrt{6}=2\sqrt{6}+3\sqrt{6}$
$=(2+3)\sqrt{6}=5\sqrt{6}$

b. $\sqrt{500}-\sqrt{125}=\sqrt{100\times 5}-\sqrt{25\times 5}=\sqrt{100}\times\sqrt{5}-\sqrt{25}\times\sqrt{5}$
$=10\sqrt{5}-5\sqrt{5}=(10-5)\sqrt{5}=5\sqrt{5}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L088 in the Pupil Handbook.

Lesson Title: Addition and subtraction of Surds – Part 2	**Theme:** Numbers and Numerations	
Lesson Number: M1-L089	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson pupils will be able to solve more complicated problems involving addition and subtraction of surds.	**Preparation** Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write the following problem on the board: Simplify $\sqrt{18} + \sqrt{72}$
2. Ask pupils to solve the problem in their exercise books.
3. Invite a volunteer to write the solution on the board. (Solution: $\sqrt{18} + \sqrt{72} = \sqrt{9 \times 2} + \sqrt{36 \times 2} = 3\sqrt{2} + 6\sqrt{2} = (3 + 6)\sqrt{2} = 9\sqrt{2}$)
4. Explain to pupils that today's lesson is on more complicated problems involving addition and subtraction of surds.

Teaching and Learning *(22 minutes)*

1. Remind pupils:
    - We can only add or subtract surds which are like, or have the same surd part.
    - First simplify surds to their basic form, then add or subtract if possible.
2. Write the following problem on the board: Simplify $\sqrt{162} - \sqrt{18} + \sqrt{50}$
3. Ask pupils to work with seatmates to write the 3 surds in basic form.
4. Invite volunteers to rewrite the 3 surds in basic form on the board.

   Answers: $\sqrt{162} = \sqrt{81 \times 2} = \sqrt{81} \times \sqrt{2} = 9\sqrt{2}$

   $\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

   $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$
5. Explain: All 3 are like surds. This means that we can add or subtract them.
6. Solve the problem on the board, explaining each step:

   $\sqrt{162} - \sqrt{18} + \sqrt{50} = 9\sqrt{2} - 3\sqrt{2} + 5\sqrt{2}$ Simplify surds

   $= (9 - 3 + 5)\sqrt{2}$ Add/subtract coefficients

   $= 11\sqrt{2}$ Simplify
7. Write a problem on the board: Simplify $\sqrt{24} - 3\sqrt{6} - \sqrt{216}$
8. Ask pupils to work with seatmates to write the 3 surds in basic form.
9. Invite volunteers to rewrite the 3 surds in basic form on the board.

   Answers: $\sqrt{24} = \sqrt{4 \times 6} = \sqrt{4} \times \sqrt{6} = 2\sqrt{6}$

   $3\sqrt{6}$ is already in its basic form

   $\sqrt{216} = \sqrt{36 \times 6} = \sqrt{36} \times \sqrt{6} = 6\sqrt{6}$

10. Ask pupils to work with seatmates to subtract and complete the problem.
11. Invite a volunteer to write the solution on the board.

$\sqrt{24} - 3\sqrt{6} - \sqrt{216}$	$= 2\sqrt{6} - 3\sqrt{6} - 6\sqrt{6}$	Simplify surds
	$= (2 - 3 - 6)\sqrt{6}$	Subtract coefficients
	$= -7\sqrt{6}$	

12. Write another problem on the board: Find the exact value of x if $\sqrt{300} + \sqrt{27} - \sqrt{75} = x\sqrt{3}$
13. Discuss: How can we solve this problem?
14. Allow pupils to share ideas, then explain:
    a. We can simplify the left-hand side, then add and subtract the surds.
    b. Based on the right-hand side, it seems like we will have $\sqrt{3}$ in our surds on the left-hand side.
15. Solve on the board, explaining each step:

$$\begin{aligned}\sqrt{300} + \sqrt{27} - \sqrt{75} &= \sqrt{100 \times 3} + \sqrt{9 \times 3} - \sqrt{25 \times 3}\\ &= \sqrt{100} \times \sqrt{3} + \sqrt{9} \times \sqrt{3} - \sqrt{25} \times \sqrt{3}\\ &= 10\sqrt{3} + 3\sqrt{3} - 5\sqrt{3}\\ &= (10 + 3 - 5)\sqrt{3}\\ &= 8\sqrt{3}\end{aligned}$$

    Setting the result equal to the right-hand side of the original equation, we have $8\sqrt{3} = x\sqrt{3}$
    Therefore, $x = 8$
16. Write on the board: Find the value of y if $\sqrt{24} + 2\sqrt{54} - \sqrt{600} = y\sqrt{6}$
17. Ask pupils to solve the problem with seatmates.
18. Walk around to check for understanding and clear misconceptions.
19. Invite a volunteer to write the solution on the board.
    Solution:

$\sqrt{24} + 2\sqrt{54} - \sqrt{600}$	$= \sqrt{4 \times 6} + 2\sqrt{9 \times 6} - \sqrt{100 \times 6}$	Simplify surds
	$= 2\sqrt{6} + (2 \times 3)\sqrt{6} - 10\sqrt{6}$	
	$= 2\sqrt{6} + 6\sqrt{6} - 10\sqrt{6}$	
	$= (2 + 6 - 10)\sqrt{6}$	Add and subtract
	$= -2\sqrt{6}$	

    Therefore, the answer is $y = -2$

Practice *(14 minutes)*

1. Write the following three problems on the board:
    a. Simplify $\sqrt{50} + \sqrt{32} - \sqrt{162}$
    b. Simplify $3\sqrt{500} - 2\sqrt{125}$

   c. Simplify $2\sqrt{80}+\sqrt{20}-\sqrt{245}$

2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite three volunteers, one at a time, to write the solutions on the board.

   Solutions:

   a.

$$\begin{aligned}\sqrt{50}+\sqrt{32}-\sqrt{162} &= \sqrt{25\times 2}+\sqrt{16\times 2}-\sqrt{81\times 2}\\ &= 5\sqrt{2}+4\sqrt{2}-9\sqrt{2}\\ &= (5+4-9)\sqrt{2}\\ &= (9-9)\sqrt{2}\\ &= 0\end{aligned}$$

   b.

$$\begin{aligned}3\sqrt{500}-2\sqrt{125} &= 3\sqrt{100\times 5}-2\sqrt{25\times 5}\\ &= 3\times 10\sqrt{5}-2\times 5\sqrt{5}\\ &= 30\sqrt{5}-10\sqrt{5}\\ &= (30-10)\sqrt{5}\\ &= 20\sqrt{5}\end{aligned}$$

   c.

$$\begin{aligned}2\sqrt{80}+\sqrt{20}-\sqrt{245} &= 2\sqrt{16\times 5}+\sqrt{4\times 5}-\sqrt{49\times 5}\\ &= 2\times 4\sqrt{5}+2\sqrt{5}-7\sqrt{5}\\ &= 8\sqrt{5}+2\sqrt{5}-7\sqrt{5}\\ &= (8+2-7)\sqrt{5}\\ &= 3\sqrt{5}\end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L089 in the Pupil Handbook.

Lesson Title: Properties of surds	**Theme:** Numbers and Numeration	
Lesson Number: M1-L090	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to identify properties of surds.	**Preparation** 1. Write the questions in Opening on the board. 2. Write the properties of surds at the start of Teaching and Learning on the board.	

Opening *(3 minutes)*

1. Write the following three questions on the board:
    a. What are real numbers? Give three examples.
    b. What is a rational number?
    c. What is an irrational number?
2. Allow pupils to discuss these problems with seatmates.
3. Ask volunteers to call out their answers.
   Example answers:
    c. Real numbers are those that can be represented by points on a number line. Examples: $-5, \sqrt{4}, 7, \sqrt{2}, 10$
    d. A rational number is a real number that can be expressed as the ratio $\frac{a}{b}$ of two integers, where $b \neq 0$.
    e. An irrational number is a real number that is not rational.
4. Remind pupils that surds are irrational, but they are also real numbers.
5. Explain that today's lesson is on identifying the properties of surds.

Teaching and Learning *(23 minutes)*

1. Write the following properties on the board: Properties of Surds:
    1) $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$
    2) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
    3) $\frac{b}{\sqrt{a}} = \frac{b}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}} = \frac{b\sqrt{a}}{a}$
    4) $a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}$
    5) $\sqrt{a^2} = \sqrt{a} \times \sqrt{a} = a$
    6) $a\sqrt{b} \times \sqrt{c} = a\sqrt{bc}$
    7) $a \times \sqrt{b} = a\sqrt{b}$
    8) $a \times b\sqrt{c} = ab\sqrt{c}$
2. Explain:
    - These are the properties of surds.
    - You have seen some of these in previous lessons. For example, property 4 is on addition and subtraction.
    - You need to remember these properties, because they are used to solve different types of problems with surds.
    - Property 3 is known as rationalising the denominator, and it will be covered in a later lesson. We will use the other properties today.

3. Write a problem on the board. Simplify $\sqrt{18}$.
4. Explain: You have simplified problems like this before. Let's solve this and see which properties we use.
5. Ask volunteers to give the steps to solve. As they give the steps, solve on the board. Tell pupils which property is used at each step.
   Solution:

$$\begin{aligned} \sqrt{18} &= \sqrt{9\times 2} && \\ &= \sqrt{9}\times\sqrt{2} && \text{Using 1) } \sqrt{a\times b}=\sqrt{a}\times\sqrt{b} \\ &= 3\times\sqrt{2} && \\ &= 3\sqrt{2} && \text{Using 7) } a\times\sqrt{b}=a\sqrt{b} \end{aligned}$$

6. Write a problem on the board: Simplify $5\sqrt{6}+4\sqrt{6}$.
   Ask volunteers to give the steps to solve. As they give the steps, solve on the board. Tell pupils which property is used at each step.
   Solution:

$$\begin{aligned} 5\sqrt{6}+4\sqrt{6} &= (5+4)\sqrt{6} && \text{Using 4) } a\sqrt{\mathrm{c}}\pm \mathrm{b}\sqrt{\mathrm{c}}=(\mathrm{a}\pm\mathrm{b})\sqrt{\mathrm{c}} \\ &= 9\sqrt{6} && \end{aligned}$$

7. Write another problem on the board: Simplify $\sqrt{\frac{12}{121}}$
8. Explain the solution on the board. Tell pupils which property is used at each step.

$$\begin{aligned} \sqrt{\frac{12}{121}} &= \frac{\sqrt{12}}{\sqrt{121}} && \text{Using 2) } \sqrt{\frac{\mathrm{a}}{\mathrm{b}}}=\frac{\sqrt{\mathrm{a}}}{\sqrt{\mathrm{b}}} \\ &= \frac{\sqrt{4\times 3}}{11} && \\ &= \frac{\sqrt{4}\times\sqrt{3}}{11} && \text{Using 1) } \sqrt{\mathrm{a}\times\mathrm{b}}=\sqrt{\mathrm{a}}\times\sqrt{\mathrm{b}} \\ &= \frac{2\times\sqrt{3}}{11} && \\ &= \frac{2\sqrt{3}}{11} && \text{Using 7) } a\times\sqrt{b}=a\sqrt{b} \end{aligned}$$

9. Write the following three problems on the board. Simplify:
   a. $4\times 2\sqrt{3}$
   b. $2\sqrt{7}\times\sqrt{2}$
   c. $\sqrt{5}\times\sqrt{3}$
10. Ask pupils to solve the problems with seatmates.
11. Walk around to check for understanding and clear misconceptions.
12. Ask 3 volunteers, one at a time, to write their solutions on the board. They should explain which properties they used.
    Solutions:
    a. $4\times 2\sqrt{3}=8\sqrt{3}$ (Property 7)
    b. $2\sqrt{7}\times\sqrt{2}=2\sqrt{7\times 2}=2\sqrt{14}$ (Property 6)
    c. $\sqrt{5}\times\sqrt{3}=\sqrt{5\times 3}=\sqrt{15}$ (Property 1)

13. Write another problem on the board: Simplify:
    a. $\sqrt{7^2}$
    b. $\sqrt{3^4}$
14. Ask pupils to solve the problem with their seatmates.
15. Go round and see what the pupils are doing. Clear any misconception.
16. Ask two volunteers one at a time to write their answers on the board. They should explain which properties they used.

    Solutions:
    a. $\sqrt{7^2} = \sqrt{7} \times \sqrt{7} = 7$ (Property 5)
    b. $\sqrt{3^4} = \sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} = 3 \times 3 = 9$ (Property 5)

Practice *(13 minutes)*

1. Write the following five problems on the board: Simplify the following surds using the properties of surds:
    a. $\sqrt{50}$
    b. $\sqrt{y^3}$
    c. $\sqrt{\frac{9}{25}}$
    d. $5\sqrt{11} \times \sqrt{2}$
    e. $7 \times 6\sqrt{5}$
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite five volunteers, one at a time, to write their solutions on the board.

    Solutions:
    a. $\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5 \times \sqrt{2} = 5\sqrt{2}$
    b. $\sqrt{\mathrm{y}^3} = \sqrt{\mathrm{y}} \times \sqrt{\mathrm{y}} \times \sqrt{\mathrm{y}} = \mathrm{y} \times \sqrt{\mathrm{y}} = y\sqrt{\mathrm{y}}$
    c. $\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$
    d. $5\sqrt{11} \times \sqrt{2} = 5\sqrt{11 \times 2} = 5\sqrt{22}$
    e. $7 \times 6\sqrt{5} = 42\sqrt{5}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L090 in the Pupil Handbook.

Lesson Title: Multiplication of surds – Part 1	**Theme:** Numbers and Numeration	
Lesson Number: M1-L091	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to multiply surds.	**Preparation** Write the problems in Opening on the board.	

Opening *(3 minutes)*

1. Write two problems on the board. Simplify the following surds:
   a. $\left(\sqrt{m}\right)^5$ b. $\sqrt{8}$
2. Ask pupils to solve the problems.
3. Invite volunteers to write the solutions on the board.
   Solutions:
   a. $\left(\sqrt{m}\right)^5 = \sqrt{m} \times \sqrt{m} \times \sqrt{m} \times \sqrt{m} \times \sqrt{m}$
   $= m \times m \times \sqrt{m}$
   $= m^2\sqrt{m}$
   b. $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$
4. Explain to the pupils that today's lesson is on multiplication of surds.

Teaching and Learning *(24 minutes)*

1. Write on the board:
$$\sqrt{a} \times \sqrt{\mathrm{b}} = \sqrt{ab}$$
$$a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$$
2. Explain:
   - These are the rules for multiplying surds.
   - Remember that the first equation is a property of surds that was covered in the previous class, and used for simplification.
3. Explain how to **multiply surds**:
   - To multiply surds, it is best to express each surd in its simplest mixed form first.
   - Multiply the coefficients, and multiply the surd parts (as shown in the second equation on the board).
   - Make sure the result is in its simplest form, and simplify where possible.
4. Write a problem on the board: Find the product of $7\sqrt{2}$ and $5\sqrt{3}$.
5. Discuss: Is each surd in its simplest form? (Answer: Yes.)
6. Write the solution on the board, explaining each step.
   Solution:
   $7\sqrt{2} \times 5\sqrt{3} = (7 \times 5)\sqrt{2 \times 3}$ Multiply coefficients and numbers in surds

$$= 35\sqrt{6}$$

7. Write a problem on the board: Simplify $\sqrt{5} \times \sqrt{15}$
8. Ask volunteers to give the steps to solve. As they give them, solve on the board.
   Solution:

$\sqrt{5} \times \sqrt{15}$	$=$	$\sqrt{5 \times 15}$	Multiply numbers in surds
	$=$	$\sqrt{75}$	
	$=$	$\sqrt{25 \times 3}$	Simplify $\sqrt{75}$
	$=$	$\sqrt{25} \times \sqrt{3}$	
	$=$	$5\sqrt{3}$	

   Note that the following method is also correct. If necessary, solve using both methods and explain that there is often more than 1 way to simplify surds:

$\sqrt{5} \times \sqrt{15}$	$=$	$\sqrt{5 \times 5 \times 3}$	Multiply numbers in surds
	$=$	$\sqrt{5} \times \sqrt{5}$ $\times \sqrt{3}$	
	$=$	$5 \times \sqrt{3}$	Simplify $\sqrt{75}$
	$=$	$5\sqrt{3}$	

9. Write another problem on the board: Simplify $2\sqrt{12} \times 3\sqrt{8}$.
10. Discuss: Is each surd in its simplest form? (Answer: No)
11. Ask pupils to simplify each surd in their exercise books. Invite 2 volunteers to write the steps on the board. (Answer: $2\sqrt{12} = 2\sqrt{4 \times 3} = 2\sqrt{4}\sqrt{3} = 2 \times 2\sqrt{3} = 4\sqrt{3}$; $3\sqrt{8} = 3\sqrt{4 \times 2} = 3\sqrt{4}\sqrt{2} = 3 \times 2\sqrt{2} = 6\sqrt{2}$)
12. Write the problem on the board with simplified surds: $2\sqrt{12} \times 3\sqrt{8} = 4\sqrt{3} \times 6\sqrt{2}$
13. Ask volunteers to give the steps to solve. As they give them, solve on the board.
    Solution:

$2\sqrt{12} \times 3\sqrt{8}$	$=$	$4\sqrt{3} \times 6\sqrt{2}$	Simplify
	$=$	$(4 \times 6)\sqrt{3 \times 2}$	Multiply coefficients and numbers
	$=$	$24\sqrt{6}$	

14. Explain:
    a. If you do not simplify the surds before solving the problem, you will still be able to find the solution.
    b. If you don't simplify as the first step, you will need to simplify the result, as the last step.
15. Solve the problem again on the board, multiplying first:

$2\sqrt{12} \times 3\sqrt{8}$	$=$	$(2 \times 3)\sqrt{12 \times 8}$	Multiply coefficients and numbers
	$=$	$6\sqrt{96}$	
	$=$	$6\sqrt{16 \times 6}$	Simplify
	$=$	$6\sqrt{16}\sqrt{6}$	

$= \quad 6 \times 4\sqrt{6}$
$= \quad 24\sqrt{6}$

16. Explain:
    a. We reached the same answer ($24\sqrt{6}$) by solving in 2 different ways.
    b. It is often possible to solve surd problems in different ways. However, it is often easiest to simplify the surds first.
17. Write two problems on the board:
    a. Simplify $2\sqrt{3} \times 3\sqrt{2}$
    b. Simplify $\sqrt{27} \times \sqrt{50}$
16. Ask pupils to solve the problems with seatmates.
17. Walk around to check for understanding and clear misconceptions.
18. Invite two volunteers, one at a time, to write the solutions on the board and explain.
    Answers:
    a.
    $2\sqrt{3} \times 3\sqrt{2} = (2 \times 3)\sqrt{3 \times 2}$ Multiply coefficients and numbers in surds
    $= 6\sqrt{6}$

    b.
    $\sqrt{27} \times \sqrt{50} = \sqrt{9 \times 3} \times \sqrt{25 \times 2}$ Simplify surds
    $= \sqrt{9} \times \sqrt{3} \times \sqrt{25} \times \sqrt{2}$
    $= 3\sqrt{3} \times 5\sqrt{2}$
    $= (3 \times 5)\sqrt{3 \times 2}$ Multiply
    $= 15\sqrt{6}$

Practice *(12 minutes)*

1. Write the following four problems on the board:
    a. $\sqrt{8} \times \sqrt{10}$
    b. $\left(2\sqrt{5}\right)^2$
    c. $\sqrt{30} \times \sqrt{5}$
    d. $2\sqrt{18} \times 3\sqrt{20}$
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite four volunteers, one at a time, to write their solutions on the board.
    Solutions:
    a.
    $\sqrt{8} \times \sqrt{10} = \sqrt{80}$ Multiply

$= \sqrt{16 \times 5}$ Simplify

$= \sqrt{16} \times \sqrt{5}$

$= 4\sqrt{5}$

b.

$(2\sqrt{5})^2 = 2\sqrt{5} \times 2\sqrt{5}$ Multiply

$= 2 \times 2(\sqrt{5} \times \sqrt{5})$

$= 4 \times 5 = 20$

c.

$\sqrt{30} \times \sqrt{5} = \sqrt{30 \times 5}$ Multiply numbers in surds

$= \sqrt{150}$

$= \sqrt{25 \times 6}$

$= \sqrt{25} \times \sqrt{6}$

$= 5\sqrt{6}$

d.

$2\sqrt{18} \times 3\sqrt{20} = 2\sqrt{9 \times 2} \times 3\sqrt{4 \times 5}$ Simplify surds

$= 2\sqrt{9} \times \sqrt{2} \times 3 \times \sqrt{4} \times \sqrt{5}$

$= 2 \times 3\sqrt{2} \times 3 \times 2 \times \sqrt{5}$

$= 6\sqrt{2} \times 6\sqrt{5}$ Multiply

$= (6 \times 6)\sqrt{2 \times 5}$

$= 36\sqrt{10}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L091 in the Pupil Handbook.

Lesson Title: Multiplication of surds – Part 2	**Theme:** Numbers and Numeration	
Lesson Number: M1-L092	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to multiply surds.	**Preparation** Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write a problem on the board: Simplify $2\sqrt{12} \times \sqrt{3}$
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around the class to check for understanding and clear misconceptions.
4. Invite a volunteer to write the solution on the board.
   Solution:
   $$\begin{aligned} 2\sqrt{12} \times \sqrt{3} &= 2\sqrt{4}\sqrt{3} \times \sqrt{3} && \text{Simplify } 2\sqrt{12} \text{ first} \\ &= 2 \times 2\sqrt{3} \times \sqrt{3} \\ &= 4\sqrt{3} \times \sqrt{3} && \text{Note that } \sqrt{3} \times \sqrt{3} = 3 \\ &= 4 \times 3 \\ &= 12 \end{aligned}$$
5. Explain to the pupils that today's lesson is on more complicated problems involving multiplication of surds.

Teaching and Learning *(21 minutes)*

1. Write the following problem on the board: Simplify $\sqrt{20} \times \left(\sqrt{5}\right)^3$.
2. Invite any volunteer to simplify $\sqrt{20}$ on the board. (Answer: $\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$)
3. Invite another volunteer to simplify $\left(\sqrt{5}\right)^3$ on the board. (Answer: $\left(\sqrt{5}\right)^3 = \sqrt{5} \times \sqrt{5} \times \sqrt{5} = 5\sqrt{5}$)
4. Solve the problem on the board:
   $$\begin{aligned} \sqrt{20} \times \left(\sqrt{5}\right)^3 &= 2\sqrt{5} \times 5\sqrt{5} && \text{Simplify each surd} \\ &= 2 \times 5\sqrt{5 \times 5} && \text{Multiply} \\ &= 10\sqrt{25} && \text{Simplify} \\ &= 10 \times 5 \\ &= 50 \end{aligned}$$
5. Write a problem on the board: Simplify $\sqrt{10}\left(\sqrt{2} + 3\right)$
6. Explain: We must distribute $\sqrt{10}$. We will multiply it by each term in the brackets.
7. Solve the problem on the board:

$$\sqrt{10}(\sqrt{2}+3) = \sqrt{10\times 2}+3\sqrt{10} \quad \text{Remove bracket}$$
$$= \sqrt{20}+3\sqrt{10}$$
$$= \sqrt{4\times 5}+3\sqrt{10}$$
$$= 2\sqrt{5}+3\sqrt{10}$$

8. Explain:
    a. This is the answer because it cannot be simplified further.
    b. Remember that surds can only be added if they are like surds.
9. Write another problem on the board: Simplify $\sqrt{50}-3\sqrt{2}(2\sqrt{2}-5)-5\sqrt{32}$
10. Explain: This problem looks complicated, but we can just work through it using the order of operations.
11. Solve on the board. Involve pupils by asking them to describe each step.
    Solution:

$$\sqrt{50}-3\sqrt{2}(2\sqrt{2}-5)-5\sqrt{32} = \sqrt{50}-(3\times 2\sqrt{2\times 2})+(5\times 3\sqrt{2})-5\sqrt{32}$$
$$= \sqrt{25\times 2}-(6\times 2)+15\sqrt{2}-5\sqrt{16\times 2}$$
$$= 5\sqrt{2}-12+15\sqrt{2}-5\times 4\sqrt{2}$$
$$= 5\sqrt{2}+15\sqrt{2}-20\sqrt{2}-12$$
$$= (5+15-20)\sqrt{2}-12$$
$$= 0-12$$
$$= -12$$

12. Write another problem on the board: Simplify $\frac{\sqrt{75}\times\sqrt{45}}{\sqrt{200}\times\sqrt{50}}$
13. Explain: Remember the order of operations. To solve this problem, we can work the numerator and denominator first, separately.
14. Ask pupils to work with seatmates to simplify the numerator of the fraction.
15. Invite a volunteer to write the solution on the board.
    Solution:

$$\sqrt{75}\times\sqrt{45} = \sqrt{25\times 5}\times\sqrt{9\times 5}$$
$$= \sqrt{25}\times\sqrt{5}\times\sqrt{9}\times\sqrt{5}$$
$$= 5\sqrt{5}\times 3\sqrt{5}$$
$$= (5\times 3)(\sqrt{5}\times\sqrt{5})$$
$$= 15\times 5 = 75$$

16. Ask pupils to work with seatmates to simplify the denominator of the fraction.
17. Invite a volunteer to write the solution on the board.
    Solution:

$$\sqrt{200}\times\sqrt{50} = \sqrt{100\times 2}\times\sqrt{25\times 2}$$
$$= \sqrt{100}\times\sqrt{2}\times\sqrt{25}\times\sqrt{2}$$
$$= 10\sqrt{2}\times 5\sqrt{2}$$
$$= (10\times 5)(\sqrt{2}\times\sqrt{2})$$

$$= \quad 50 \times 2 = 100$$

18. Solve the problem on the board: $\frac{\sqrt{75}\times\sqrt{45}}{\sqrt{200}\times\sqrt{50}} = \frac{75}{100} = \frac{3}{4}$

Practice *(15 minutes)*

1. Write 3 problems on the board:
   a. Simplify: $-3\sqrt{6}(2\sqrt{5}+\sqrt{6})$
   b. Simplify: $\sqrt{18} \times (\sqrt{2})^3$
   c. Simplify: $2\sqrt{5}(5-2\sqrt{5})$
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around the class to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite volunteers, one at a time, to write their solutions on the board and explain.
   Solutions:
   a.
$$\begin{aligned} -3\sqrt{6}(2\sqrt{5}+\sqrt{6}) &= -3\sqrt{6}\times 2\sqrt{5} - 3\sqrt{6}\times\sqrt{6} \\ &= -3\times 2\sqrt{6\times 5} - 3\times 6 \\ &= -6\sqrt{30} - 18 \end{aligned}$$
   b.
$$\begin{aligned} \sqrt{18}\times(\sqrt{2})^3 &= 3\sqrt{2}\times 2\sqrt{2} \\ &= 3\times 2\sqrt{2}\sqrt{2} \\ &= 6\times 2 \\ &= 12 \end{aligned}$$
   c.
$$\begin{aligned} 2\sqrt{5}(5-2\sqrt{5}) &= 2\times 5\sqrt{5} - 2\times 2\sqrt{5\times 5} \\ &= 10\sqrt{5} - 4\times 5 \\ &= 10\sqrt{5} - 20 \end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L092 in the Pupil Handbook.

Lesson Title: Rationalisation of the denominator of surds – Part 1	**Theme:** Numbers and Numeration	
Lesson Number: M1-L093	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to rationalise the denominator of surds.	**Preparation** None	

Opening *(4 minutes)*

1. Write the following problem on the board: Simplify $\left(3-\sqrt{5}\right)^2$
2. Ask pupils to solve the problem in their exercise books.
3. Invite any volunteer to write the solution on the board.
   Solution:
   $$\begin{aligned}\left(3-\sqrt{5}\right)^2 &= \left(3-\sqrt{5}\right)\left(3-\sqrt{5}\right)\\ &= 3\left(3-\sqrt{5}\right)-\sqrt{5}\left(3-\sqrt{5}\right)\\ &= 9-3\sqrt{5}-3\sqrt{5}+\sqrt{25}\\ &= 9-6\sqrt{5}+5\\ &= 14-6\sqrt{5}\end{aligned}$$
4. Explain to the pupils that today's lesson is on a process called "rationalising the denominator" of surds.

Teaching and Learning *(22 minutes)*

1. Write on the board: $\frac{1}{\sqrt{2}}$
2. Explain:
   - This is a fraction with an irrational number in the denominator.
   - "Rationalising the denominator" is when we move a root from the bottom of the fraction to the top.
   - To be in "Simplest form," the denominator should not be irrational. We can change the denominator to a rational number by multiplying both the numerator and denominator by the denominator.
3. Write on the board: $\frac{\mathrm{a}}{\sqrt{\mathrm{b}}}=\frac{\mathrm{a}}{\sqrt{\mathrm{b}}}\times\frac{\sqrt{\mathrm{b}}}{\sqrt{\mathrm{b}}}=\frac{\mathrm{a}\sqrt{\mathrm{b}}}{\mathrm{b}}$
4. Explain:
   - This is the general rule.
   - Remember that when you multiply a fraction that has the same numerator and denominator, you are actually multiplying by 1.
5. Write a problem on the board: Simplify $\frac{1}{\sqrt{2}}$.
6. Solve on the board, explaining each step:

$$\begin{aligned}\frac{1}{\sqrt{2}} &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} && \text{Multiply top and bottom by } \sqrt{2} \\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

7. Explain:
   - This is the simplified form, because the denominator has a rational number, which is 2.
   - It is okay to have an irrational number in the numerator of a fraction.
8. Write the following problem on the board: Simplify $\frac{8}{\sqrt{2}}$
9. Ask volunteers to give the steps to solve the problem. As they describe them, work the problem on the board:

$$\begin{aligned}\frac{8}{\sqrt{2}} &= \frac{8}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} && \text{Multiply top and bottom by } \sqrt{2} \\ &= \frac{8\sqrt{2}}{2} \\ &= \frac{4\sqrt{2}}{1} && \text{Simplify} \\ &= 4\sqrt{2}\end{aligned}$$

10. Write another problem on the board: Simplify$\frac{10}{\sqrt{12}}$.
11. Explain: When the denominator can be simplified, we should simplify it first.
12. Ask any volunteer to simplify the denominator on the board. (Answer: $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$)
13. Explain: We have $\sqrt{3}$ as the surd in the denominator. Therefore, we multiply the numerator and denominator by this $\sqrt{3}$.
14. Solve the problem on the board:

$$\begin{aligned}\frac{10}{\sqrt{12}} &= \frac{10}{2\sqrt{3}} && \text{Simplify the denominator} \\ &= \frac{10}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} && \text{Multiply top and bottom by } \sqrt{3} \\ &= \frac{10\sqrt{3}}{2\times\sqrt{3}\times\sqrt{3}} \\ &= \frac{10\sqrt{3}}{2\times3} \\ &= \frac{10\sqrt{3}}{6} \\ &= \frac{5\sqrt{3}}{3}\end{aligned}$$

15. Write on the board: $\frac{5\sqrt{3}}{3} = \frac{5}{3}\sqrt{3}$
16. Explain: The answer can be written either way. Both are correct.
17. Write two more problems on the board:
    a. Simplify $\frac{\sqrt{2}}{\sqrt{7}}$
    b. Simplify $\frac{5}{\sqrt{10}}$

18. Ask pupils to solve the problem with seatmates.
19. Walk around the class to check for understanding and clear misconceptions.
20. Invite two volunteers to write their solutions on the board.

Solutions:

a.

$$\begin{aligned}\frac{\sqrt{2}}{\sqrt{7}} &= \frac{\sqrt{2}}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}} && \text{Multiply top and bottom by } \sqrt{7}\\ &= \frac{\sqrt{14}}{7}\end{aligned}$$

b.

$$\begin{aligned}\frac{5}{\sqrt{10}} &= \frac{5}{\sqrt{10}}\times\frac{\sqrt{10}}{\sqrt{10}} && \text{Multiply top and bottom by } \sqrt{10}\\ &= \frac{5\sqrt{10}}{10}\\ &= \frac{\sqrt{10}}{2}\end{aligned}$$

Practice *(13 minutes)*

1. Write 5 problems on the board:
   a. Simplify: $\frac{1}{\sqrt{8}}$
   b. Simplify: $\frac{1}{2\sqrt{3}}$
   c. Simplify: $\frac{4\sqrt{3}}{3\sqrt{2}}$
   d. Simplify: $\frac{18}{\sqrt{6}}$
   e. Simplify: $\frac{2}{\sqrt{32}}$
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around the class to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite five volunteers, one at a time, to write their solutions on the board.

Solutions:

a. $\frac{1}{\sqrt{8}} = \frac{1}{\sqrt{4\times 2}} = \frac{1}{2\sqrt{2}} = \frac{1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2\times 2} = \frac{\sqrt{2}}{4}$

b. $\frac{1}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2\times 3} = \frac{\sqrt{3}}{6}$

c. $\frac{4\sqrt{3}}{3\sqrt{2}} = \frac{4\sqrt{3}}{3\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{6}}{3\times 2} = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3}$

d. $\frac{18}{\sqrt{6}} = \frac{18}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}} = \frac{18\sqrt{6}}{6} = 3\sqrt{6}$

e. $\frac{2}{\sqrt{32}} = \frac{2}{\sqrt{16\times 2}} = \frac{2}{4\sqrt{2}} = \frac{2}{4\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{4\times 2} = \frac{2\sqrt{2}}{8} = \frac{\sqrt{2}}{4}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L093 in the Pupil Handbook.

Lesson Title: Rationalisation of the denominator of surds – Part 2	**Theme:** Numbers and Numeration	
Lesson Number: M1-L094	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to rationalise the denominator of surds.	**Preparation** Write the problem in Opening on the board.	

Opening *(3 minutes)*

1. Write the following problem on the board: Simplify the following: $\frac{\sqrt{5}}{\sqrt{10}}$
2. Ask pupils to solve the problem independently in their exercise books.
3. Invite a volunteer to solve the problem on the board. (Solution: $\frac{\sqrt{5}}{\sqrt{10}} = \frac{\sqrt{5}}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{5\times10}}{10} = \frac{\sqrt{50}}{10} = \frac{\sqrt{25\times2}}{10} = \frac{5\sqrt{2}}{10} = \frac{\sqrt{2}}{2}$)
4. Explain to the pupils that today's lesson is on more complicated problems involving rationalizing the denominator of surds.

Teaching and Learning *(23 minutes)*

1. Write on the board: Simplify: $\frac{1}{3-\sqrt{2}}$
2. Explain:
   - This problem cannot be solved by multiplying the numerator and denominator by $\sqrt{2}$, as we did in the previous lesson.
   - When there are 2 terms added or subtracted in the numerator, we can rationalise by multiplying the numerator and denominator by the **conjugate** of the denominator.
   - The conjugate is found by changing the sign in the middle of two terms.
3. Write on the board: The conjugate of $(3-\sqrt{2})$ is $(3+\sqrt{2})$.
4. Explain:
   - The conjugate can also be called the "rationalising factor". This is because multiplying a surd by its conjugate rationalises it, or makes it a rational number.
   - Conjugates have special properties that make them useful for rationalising surds.
5. Write on the board: $(3-\sqrt{2})(3+\sqrt{2}) = 3^2 - \sqrt{2}^2$
6. Explain: When a binomial is multiplied by its conjugate, we simply square each term. The other terms cancel.
7. Demonstrate why this is true on the board:

$$\begin{aligned}(3-\sqrt{2})(3+\sqrt{2}) &= 3(3+\sqrt{2}) - \sqrt{2}(3+\sqrt{2}) && \text{Multiply}\\ &= 3^2 + 3\sqrt{2} - 3\sqrt{2} - \sqrt{2}^2 && \text{Remove the brackets}\end{aligned}$$

$$= \quad 3^2 - \sqrt{2}^2 \qquad\qquad \text{Middle terms cancel}$$

8. Explain:
   - When you multiply a binomial by its conjugate, the middle terms will always cancel.
   - This fact will save you time when rationalising the denominator.
9. Solve on the board, explaining each step:

$$\frac{1}{3-\sqrt{2}} = \frac{1}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} \qquad \text{Multiply top and bottom by } 3+\sqrt{2}$$

$$= \frac{3+\sqrt{2}}{3^2-\sqrt{2}^2} \qquad \text{Remove the brackets}$$

$$= \frac{3+\sqrt{2}}{9-2} \qquad \text{Simplify}$$

$$= \frac{3+\sqrt{2}}{7}$$

10. Write another problem on the board: Simplify $\frac{2}{3+\sqrt{7}}$
11. Invite a volunteer to write the conjugate of $3+\sqrt{7}$ on the board. (Answer: $3-\sqrt{7}$)
12. Ask volunteers to give the steps to solve the problem. As they give them, solve on the board.

Solution:

$$\frac{2}{3+\sqrt{7}} = \frac{2}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}} \qquad \text{Multiply top and bottom by } 3-\sqrt{7}$$

$$= \frac{2(3-\sqrt{7})}{3^2-\sqrt{7}^2}$$

$$= \frac{6-2\sqrt{7}}{9-7}$$

$$= \frac{6-2\sqrt{7}}{2}$$

$$= \frac{6}{2} - \frac{2}{2}\sqrt{7}$$

$$= 3-\sqrt{7}$$

13. Write another problem on the board. Simplify $\frac{4}{1+2\sqrt{3}}$
14. Ask pupils to work with seatmates to solve the problem.
15. Walk around to check for understanding and clear misconceptions.
16. Invite a volunteer to simplify the problem on the board.

Solution:

$$\frac{4}{1+2\sqrt{3}} = \frac{4}{1+2\sqrt{3}} \times \frac{1-2\sqrt{3}}{1-2\sqrt{3}} \qquad \text{Multiply top and bottom by } 1-2\sqrt{3}$$

$$= \frac{4(1-2\sqrt{3})}{1^2-(2\sqrt{3})^2}$$

$$= \frac{4-8\sqrt{3}}{1-4(3)}$$

$$= \frac{4-8\sqrt{3}}{-11}$$

$$= -\frac{4-8\sqrt{3}}{11}$$

17. Write another problem on the board: Express $\frac{\sqrt{3}+\sqrt{7}}{\sqrt{21}}$ in the form $a\sqrt{3}+b\sqrt{7}$, where a and b are rational numbers.
18. Explain:
    - This problem has 2 terms in the numerator. However, the process is similar to the problems solved in the previous lesson.
    - Rationalise the denominator first by multiplying by $\sqrt{21}$, then simplify.
    - In many cases a problem in this form will naturally simplify to the form given.
19. Solve the problem on the board. Involve pupils by asking them to give the steps.

Solution:

$$\frac{\sqrt{3}+\sqrt{7}}{\sqrt{21}} = \frac{\sqrt{3}+\sqrt{7}}{\sqrt{21}} \times \frac{\sqrt{21}}{\sqrt{21}} \quad \text{Rationalise the denominator}$$
$$= \frac{\sqrt{21}(\sqrt{3}+\sqrt{7})}{\sqrt{21}\times\sqrt{21}}$$
$$= \frac{\sqrt{63}+\sqrt{147}}{21} \quad \text{Multiply the numerator}$$
$$= \frac{\sqrt{9\times7}+\sqrt{49\times3}}{21} \quad \text{Simplify surds}$$
$$= \frac{3\sqrt{7}+7\sqrt{3}}{21}$$
$$= \frac{7\sqrt{3}+3\sqrt{7}}{21} \quad \text{Change the order of the numerator}$$
$$= \frac{7\sqrt{3}}{21} + \frac{3\sqrt{7}}{21} \quad \text{Simplify}$$
$$= \frac{1}{3}\sqrt{3} + \frac{1}{7}\sqrt{7}$$

Where $a = \frac{1}{3}$ and $b = \frac{1}{7}$

Practice *(13 minutes)*

1. Write two problems on the board:
    a. Simplify: $\frac{3}{2+\sqrt{5}}$
    b. Simplify: $\frac{\sqrt{27}+1}{\sqrt{48}}$, leave your answer in the form $p + q\sqrt{n}$ where p and q are rational numbers and n is an integer.
1. Ask pupils to solve the problems independently in their exercise books.
2. Walk around to check for understanding and clear misconceptions.
3. Allow pupils to exchange their exercise books.
4. Invite two volunteers, one at a time, to write their solutions on the board.

Solutions:

a.
$$\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$$
$$= \frac{3(2-\sqrt{5})}{2^2+5}$$

$$= \frac{3(2-\sqrt{5})}{4+5}$$

$$= \frac{3(2-\sqrt{5})}{9}$$

$$= \frac{2-\sqrt{5}}{3}$$

b. $\frac{\sqrt{27}+1}{\sqrt{48}} = \frac{\sqrt{27}+1}{\sqrt{16\times 3}}$

$$= \frac{\sqrt{27}+1}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{81}+\sqrt{3}}{4\times 3}$$

$$= \frac{9+\sqrt{3}}{12}$$

$$= \frac{9}{12} + \frac{1}{12}\sqrt{3}$$

$$= \frac{3}{4} + \frac{1}{12}\sqrt{3}$$

Where $p = \frac{3}{4}$, $q = \frac{1}{12}$ and $n = \sqrt{3}$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L094 in the Pupil Handbook.

Lesson Title: Expansion and simplification of surds	**Theme:** Numbers and Numeration	
Lesson Number: M1-L095	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to expand and simplify expressions involving surds.	**Preparation** None	

Opening *(3 minutes)*

1. Write the following problem on the board: Simplify: $\frac{\sqrt{2}+\sqrt{7}}{\sqrt{14}}$
2. Ask pupils to solve the problem independently.
3. Invite a volunteer to solve the problem on the board.
   Solution:

$$\begin{aligned}
\frac{\sqrt{2}+\sqrt{7}}{\sqrt{14}} &= \frac{\sqrt{2}+\sqrt{7}}{\sqrt{14}} \times \frac{\sqrt{14}}{\sqrt{14}} \\
&= \frac{\sqrt{28}+\sqrt{98}}{14} \\
&= \frac{\sqrt{4\times7}+\sqrt{49\times2}}{14} \\
&= \frac{2\sqrt{7}+7\sqrt{2}}{14} \\
&= \frac{2}{14}\sqrt{7}+\frac{7}{14}\sqrt{2} \\
&= \frac{1}{7}\sqrt{7}+\frac{1}{2}\sqrt{2}
\end{aligned}$$

4. Explain that today's lesson is on expanding and simplifying expressions involving surds.

Teaching and Learning *(20 minutes)*

1. Write another problem on the board: Expand and simplify $(1+\sqrt{5})(1-\sqrt{5})$.
2. Explain:
   - In expanding a bracket that involves surds you use the distributive law.
   - Multiply each term in the first bracket by the expression in the second bracket.
3. Solve the problem on the board, explaining each step:
   Solution:

$(1+\sqrt{5})(1-\sqrt{5})$	$= 1(1-\sqrt{5})+\sqrt{5}(1-\sqrt{5})$	Expand (distribute)
	$= 1-\sqrt{5}+\sqrt{5}-(\sqrt{5})^2$	Simplify
	$= 1-\sqrt{5}+\sqrt{5}-5$	Note that $(\sqrt{5})^2=\sqrt{5}\times\sqrt{5}=5$
	$= 1-5-\sqrt{5}+\sqrt{5}$	Collect like terms
	$= -4$	

4. Write another problem on the board: Expand and simplify $(1+\sqrt{2})(3+\sqrt{2})$.

5. Ask a volunteer to apply the distributive law. (Answer: $(1+\sqrt{2})(3+\sqrt{2}) = 1(3+\sqrt{2}) + \sqrt{2}(3+\sqrt{2})$)
6. Invite other volunteers to give the other steps on the board. They may write them or tell you the steps to wrote.

   Solution:

$$\begin{aligned}(1+\sqrt{2})(3+\sqrt{2}) &= 1(3+\sqrt{2}) + \sqrt{2}(3+\sqrt{2})\\ &= 3+\sqrt{2}+3\sqrt{2}+(\sqrt{2})^2\\ &= 3+\sqrt{2}+3\sqrt{2}+2\\ &= 3+2+\sqrt{2}+3\sqrt{2}\\ &= 5+4\sqrt{2}\end{aligned}$$

7. Write another problem on the board: Expand and simplify: $(\sqrt{7}+\sqrt{3})(\sqrt{7}+\sqrt{3})$
8. Ask pupils to work with seatmates to solve the problem.
9. Walk around to check for understanding and clear misconceptions.
10. Invite any volunteer to solve the problem on the board.

   Solution:

$$\begin{aligned}(\sqrt{7}+\sqrt{3})(\sqrt{7}+\sqrt{3}) &= \sqrt{7}(\sqrt{7}+\sqrt{3}) + \sqrt{3}(\sqrt{7}+\sqrt{3})\\ &= (\sqrt{7})^2 + \sqrt{7\times 3} + \sqrt{3\times 7} + (\sqrt{3})^2\\ &= 7+\sqrt{21}+\sqrt{21}+3\\ &= 7+3+\sqrt{21}+\sqrt{21}\\ &= 10+2\sqrt{21}\end{aligned}$$

11. Write another problem on the board: Expand and simplify $\sqrt{5}\left(\sqrt{45}-\frac{8}{\sqrt{80}}\right)$
12. Explain: For this type of problem, distribute as usual. Multiply the term in front of the brackets by each term inside, then simplify.
13. Write the solution on the board and explain to the pupils.

   Solution:

$$\begin{aligned}\sqrt{5}\left(\sqrt{45}-\frac{8}{\sqrt{80}}\right) &= \sqrt{5\times 45} - \sqrt{5}\times\frac{8}{\sqrt{80}} && \text{Multiply } \sqrt{5} \text{ by each term in the bracket}\\ &= \sqrt{225} - \frac{8\sqrt{5}}{\sqrt{80}}\\ &= 15 - \frac{8\sqrt{5}}{\sqrt{16\times 5}}\\ &= 15 - \frac{8\sqrt{5}}{4\sqrt{5}}\\ &= 15 - \frac{8}{4} && \text{Cancel } \sqrt{5}\\ &= 15-2\\ &= 13\end{aligned}$$

Practice *(16 minutes)*

1. Write the following three problems on the board:
   a. Expand and simplify: $(\sqrt{6}+\sqrt{3})(\sqrt{3}-\sqrt{6})$
   b. Simplify: $(4+\sqrt{2})(5-3\sqrt{2})$
   c. Simplify: $\sqrt{2}\left(\sqrt{50}+\frac{20}{\sqrt{72}}\right)$
2. Ask pupils to solve the problems independently in their exercise books.
3. Walk around to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite any three volunteers, one at a time, to write their solutions on the board.
   Solutions:

a.
$$\begin{aligned}(\sqrt{6}+\sqrt{3})(\sqrt{3}-\sqrt{6}) &= \sqrt{6}(\sqrt{3}-\sqrt{6})+\sqrt{3}(\sqrt{3}-\sqrt{6}) \\ &= \sqrt{6\times 3}-(\sqrt{6})^2+(\sqrt{3})^2-\sqrt{3\times 6} \\ &= \sqrt{18}-6+3-\sqrt{18} \\ &= \sqrt{9\times 2}-3-\sqrt{9\times 2} \\ &= 3\sqrt{2}-3-3\sqrt{2} \\ &= -3+3\sqrt{2}-3\sqrt{2} \\ &= -3\end{aligned}$$

b.
$$\begin{aligned}(4+\sqrt{2})(5-3\sqrt{2}) &= 4(5-3\sqrt{2})+\sqrt{2}(5-3\sqrt{2}) \\ &= 20-12\sqrt{2}+5\sqrt{2}-3(\sqrt{2})^2 \\ &= 20-12\sqrt{2}+5\sqrt{2}-6 \\ &= 20-6-12\sqrt{2}+5\sqrt{2} \\ &= 14-7\sqrt{2}\end{aligned}$$

c.
$$\begin{aligned}\sqrt{2}\left(\sqrt{50}+\frac{20}{\sqrt{72}}\right) &= \sqrt{2\times 50}+\frac{20\sqrt{2}}{\sqrt{72}} \\ &= \sqrt{100}+\frac{20\sqrt{2}}{\sqrt{36}\sqrt{2}} \\ &= 10+\frac{20}{\sqrt{36}} \\ &= 10+\frac{20}{6} \\ &= 10+\frac{10}{3} \\ &= \frac{30+10}{3} \\ &= \frac{40}{3} \\ &= 13\frac{1}{3}\end{aligned}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L095 in the Pupil Handbook.

Lesson Title: Practice of surds	**Theme:** Numbers and Numeration	
Lesson Number: M1-L096	**Class:** SSS 1	**Time:** 40 minutes
Learning Outcome By the end of the lesson, pupils will be able to apply various operations to simplify expressions involving surds.	**Preparation** Write the problems in Opening on the board.	

Opening *(5 minutes)*

1. Write the following two problems on the board:
    a. Simplify: $\sqrt{32} + 3\sqrt{8}$
    b. Simplify: $\frac{13}{9\sqrt{8}}$
2. Ask pupils to solve the problem independently in their exercise books.
3. Invite any two volunteers to write their answer on the board.
   Answers:
    a.
$$\begin{aligned}\sqrt{32} + 3\sqrt{8} &= \sqrt{16 \times 2} + 3\sqrt{4 \times 2} \\ &= \sqrt{16 \times 2} + 3 \times \sqrt{4} \times \sqrt{2} \\ &= 4\sqrt{2} + 3 \times 2\sqrt{2} \\ &= 4\sqrt{2} + 6\sqrt{2} \\ &= 10\sqrt{2}\end{aligned}$$
    b.
$$\begin{aligned}\frac{13}{9\sqrt{8}} &= \frac{13}{9\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} \\ &= \frac{13\sqrt{8}}{9\times 8} \\ &= \frac{13\sqrt{8}}{72} \\ &= \frac{13\times\sqrt{4\times 2}}{72} \\ &= \frac{13\times 2\sqrt{2}}{72} \\ &= \frac{13\sqrt{2}}{36}\end{aligned}$$
4. Explain to pupils that today's lesson is on applying various operations to simplify expressions involving surds.

Teaching and Learning *(23 minutes)*

1. Write a problem on the board: Evaluate: $\frac{\sqrt{60}\times\sqrt{180}}{\sqrt{75}}$
2. Discuss: How would you simplify this problem? (Answer: Simplify the surds, then rationalise the denominator.)
3. Ask pupils to work with seatmates to solve the problem.

4. Walk around to check for understanding and clear misconceptions.
5. Invite a volunteer to write the solution on the board.
   Solution:

$$\begin{aligned}
\frac{\sqrt{60}\times\sqrt{180}}{\sqrt{75}} &= \frac{\sqrt{4\times15}\times\sqrt{36\times5}}{\sqrt{25\times3}} \\
&= \frac{2\sqrt{15}\times6\sqrt{5}}{5\sqrt{3}} \\
&= \frac{(2\times6)\sqrt{15\times5}}{5\sqrt{3}} \\
&= \frac{12\sqrt{75}}{5\sqrt{3}} \\
&= \frac{12\sqrt{25\times3}}{5\sqrt{3}} \\
&= \frac{12\times\cancel{5}\sqrt{\cancel{3}}}{\cancel{5}\sqrt{\cancel{3}}} \\
&= 12
\end{aligned}$$

6. Write a problem on the board: If $a\sqrt{28}+\sqrt{63}-\sqrt{7}=0$, find a.
7. Discuss: How would you solve this problem? (Answer: Simplify the surds and see if they can be combined by adding or subtracting.)
8. Ask pupils to work with seatmates to solve the problem.
9. Walk around to check for understanding and clear misconceptions.
10. Invite any volunteer to write the solution on the board.
    Solution: (note that there are multiple ways to solve, accept any correct solution):

$$\begin{aligned}
a\sqrt{28}+\sqrt{63}-\sqrt{7} &= 0 && \\
a\sqrt{4\times7}+\sqrt{9\times7}-\sqrt{7} &= 0 && \text{Simplify surds} \\
a\times2\sqrt{7}+3\sqrt{7}-\sqrt{7} &= 0 && \\
2a\sqrt{7}+2\sqrt{7} &= 0 && \text{Combine like terms} \\
2a\sqrt{7} &= -2\sqrt{7} && \text{Subtract } 2\sqrt{7} \text{ from both sides} \\
\frac{2a\sqrt{7}}{2\sqrt{7}} &= \frac{-2\sqrt{7}}{2\sqrt{7}} && \text{Divide throughout by } 2\sqrt{7} \\
a &= -1 &&
\end{aligned}$$

11. Write another problem on the board: Given that $\sqrt{2}=1.4142$, evaluate $\sqrt{128}+\frac{1}{3\sqrt{2}}$, correct your answer to four significant figures.
12. Discuss: How would you solve this problem? (Answer: Simplify the expression. The only surd should be $\sqrt{2}$, since the value is given in the problem. Then, substitute the given value and evaluate.)
13. Explain:
    a. Simplify the expression so that the only surd is $\sqrt{2}$.
    b. It is not necessary to simplify the expression completely, but it will make the problem easier. For example, if you rationalise the denominator, the operations on the decimal number will be possible without a calculator.
14. Ask pupils to work with seatmates to solve the problem.

15. Walk around to check for understanding and clear misconceptions.
16. Invite any volunteer to write the solution on the board.

Solution:

$$
\begin{aligned}
\sqrt{128}+\frac{1}{3\sqrt{2}} &= \sqrt{64\times 2}+\frac{1}{3\sqrt{2}} \\
&= 8\sqrt{2}+\frac{1}{3\sqrt{2}} && \text{Note that } \sqrt{2}=1.4142 \text{ can be substituted here, or the expression can be further simplified.} \\
&= \frac{(8\times 3)\sqrt{2\times 2}+1}{3\sqrt{2}} && \text{Add the fractions} \\
&= \frac{24\times 2+1}{3\sqrt{2}} && \text{Simplify} \\
&= \frac{49\sqrt{2}}{3\times 2} && \text{Rationalise the denominator} \\
&= \frac{49(1.4142)}{6} && \text{Substitute } \sqrt{2}=1.4142 \\
&= \frac{69.2958}{6} \\
&= 11.5493 \\
&\simeq 11.55 && \text{Rounded to 4 significant figures}
\end{aligned}
$$

Practice *(13 minutes)*

1. Write two problems on the board:
   a. If $\sqrt{5}=2.2361$, evaluate $\sqrt{245}-\frac{10}{\sqrt{5}}$, correct to 3 decimal places.
   b. Simplify: $\frac{2\sqrt{3}+3}{2-\sqrt{3}}$
2. Ask pupils to solve the problems independently in their exercise books. Allow discussion with seatmates if needed.
3. Walk around to check for understanding and clear misconceptions.
4. Allow pupils to exchange their exercise books.
5. Invite any two volunteers one at a time to write their solutions on the board.

Solutions:

a.

$$
\begin{aligned}
\sqrt{245}-\frac{10}{\sqrt{5}} &= \sqrt{49\times 5}-\frac{10}{\sqrt{5}} \\
&= 7\sqrt{5}-\frac{10}{\sqrt{5}} \\
&= \frac{7\sqrt{5}(\sqrt{5})-10}{\sqrt{5}} \\
&= \frac{35-10}{\sqrt{5}} \\
&= \frac{25}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} \\
&= \frac{25\sqrt{5}}{5} \\
&= 5\sqrt{5}
\end{aligned}
$$

$$= 5(2.2361)$$
$$= 11.1805$$
$= 11.181$ to 3 d.p.

b.

$$\frac{2\sqrt{3}+3}{2-\sqrt{3}} = \frac{2\sqrt{3}+3}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$
$$= \frac{(2\sqrt{3}+3)(2+\sqrt{3})}{2^2-(\sqrt{3})^2}$$
$$= \frac{2\sqrt{3}(2+\sqrt{3})+3(2+\sqrt{3})}{4-3}$$
$$= \frac{4\sqrt{3}+6+6+3\sqrt{3}}{1}$$
$$= 6+6+4\sqrt{3}+3\sqrt{3}$$
$$= 12+7\sqrt{3}$$

Closing *(1 minute)*

1. For homework, have pupils do the practice activity PHM1-L096 in the Pupil Handbook.

Appendix I: Logarithm Table

COMMON LOGARITHM TABLE

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Appendix II: Anti-Logarithm Table

FRITHLOGARTAIM ANTI-LOGARITHMS

FRITHLOGARTAIM ANTI-LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	1	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1696	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2970	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

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