DEPARTMENT OF EDUCATION

UPPER SECONDARY SCHOOL CERTIFICATE EXAMINATIONS

ADVANCE MATHEMATICS

Paper 1

Monday

19th October 2015

Time allowed:

2 hours and 30 minutes

(8:00am – 10:30 am)

NO EXTRA TIME

(NO OTHER TIME)

Candidates are advised to fully utilise the allocated time.

M_{A_1}

INSTRUCTIONS TO CANDIDATES

To be read by the external invigilator to all candidates.

1. The subject code for Advance Mathematics is **3.**

2. There are **7** printed pages in the question booklet.

3. An Electronic Answer Sheet for Part A, 2 pages Part B Answer Booklet and a 1 page formula sheet are inserted in the question booklet.

4. There are two parts in this paper. Answer **ALL** questions.

   <u>Part A</u>: **Multiple Choice   (Questions 1-30)   30 Marks**

   This part will be electronically marked.

   All answers to the Multiple Choice Part MUST be answered on the ELECTRONIC ANSWER SHEET provided.

   Carefully following the instructions, fill in your Candidate Information and Subject Information.

   Choose A or B or C or D from the alternatives given and use a HB pencil to shade in the correct letter to each question on the Electronic Answer Sheet.

   If you make a mistake, rub the shading out completely using an eraser and shade in your correct alternative clearly.

   <u>Part B</u>: **Short Answers   (Questions 31- 50)   20 Marks**

   Write your name, your school name and your 10-digit candidate number on the Part B Answer Booklet provided.

5. You are required to write only the correct answer in the space provided on the Answer Sheet.

6. Calculators may be used.

7. Answer all questions on the answer sheet. Answers on any other paper including rough work paper and the question paper <u>will not be marked.</u>

8. Correction fluid is <u>not allowed</u>. Where you have made an error, cross out all the working and start on a new line.

9. Graphical Calculators are <u>not permitted</u>.

Penalty For Cheating Or Assisting To Cheat In National Examinations Is   Non-Certification.

DO NOT TURN OVER THE PAGE AND DO NOT WRITE UNTIL YOU ARE TOLD TO START.

PART A: MULTIPLE CHOICE   **(QUESTIONS 1 to 30)**   **30 MARKS**

Answer each question by shading in with HB pencil the circle directly under the correct alternative A, B, C or D. If you make a mistake, rub it out completely using an eraser rubber and shade the correct answer on the Electronic Answer Sheet.

QUESTION 1

The decimal 0.00205 expressed in standard index form is

A. 0.0205×10^{-2} B. 2.05×10^{-3}

C. 20.5×10^{-1} D. 205.0×10^{-5}

QUESTION 2

The factorised form of $x^2 - 3x + 2$ is

A. $(x-2)(x+1)$ B. $(x+2)(x-1)$

C. $(x+2)(x+1)$ D. $(x-2)(x-1)$

QUESTION 3

Given that $x, y, z \neq 0$, the expression $\left(\dfrac{x^3 y^{-2} z^{\frac{1}{3}}}{yz^{-\frac{2}{3}}} \right)^0$ is

equivalent to

A. $x^3 y^{-3} z$ B. 0

C. $x^3 y^{-3} z^{-\frac{1}{3}}$ D. 1

QUESTION 4

What is the probability of picking a red marble from a bag containing 5 red, 3 blue and 7 black marbles?

A. $\dfrac{5}{7}$ B. $\dfrac{18}{15}$

C. $\dfrac{1}{2}$ D. $\dfrac{1}{3}$

QUESTION 5

The equation of the circle with centre $(1, -3)$ and radius 5 is

A. $(x+1)^2 + (y-3)^2 = 5$

B. $(x-1)^2 + (y+3)^2 = 5$

C. $(x-1)^2 + (y+3)^2 = 25$

D. $(x+1)^2 + (y-3)^2 = 25$

QUESTION 6

The equivalent of $6°$ in radians is

A. $\dfrac{\pi}{20}$ B. $\dfrac{\pi}{30}$

C. $\dfrac{\pi}{60}$ D. $\dfrac{\pi}{6}$

QUESTION 7

The derivative of $x^2 + 2x + 1$ with respect to x is

A. $2x$ B. $2x + 2$

C. x^2 D. $x^2 + 1$

QUESTION 8

A particle is moving in a straight line. Its displacement in metres from a fixed origin O after t seconds is given by $x = t^3 - 3t^2 + 4t - 7$. The velocity of the particle after t seconds is given by

A. $3t^2 - 3t + 4$

B. $3t^2 - 6t^2 + 4t$

C. $3t^2 - 6t + 4t$

D. $3t^2 - 6t + 4$

QUESTION 9

The value of the definite integral $\int_0^1 2x \ dx$ is

A.   0   B.   1

C.   −1   D.   2

QUESTION 10

The recurring decimal $0.1\dot{3}$ expressed as a fraction is

A. $\dfrac{2}{15}$ B. $\dfrac{1}{3}$

C. $\dfrac{13}{100}$ D. $\dfrac{3}{4}$

QUESTION 11

$\log_2 x - \log_2 x^2 + \log_2 x^3$ is equivalent to

A. $\log_2 x$ B. $\log_2 x^3$

C. $2\log_2 x$ D. $-\log_2 x$

QUESTION 12

The graph of the parabola $y = x^2 - 2x + 1$ has a horizontal tangent at the point where

A. $x = -1$ B. $x = 1$

C. $x = 0$ D. $x = 2$

QUESTION 13

The solution of the quadratic equation $2x^2 + 3x - 2 = 0$ is

A. $x = 1 \ or \ x = 2$

B. $x = -\dfrac{1}{2} \ or \ x = -2$

C. $x = \dfrac{1}{2} \ or \ x = 2$

D. $x = \dfrac{1}{2} \ or \ x = -2$

QUESTION 14

The average of the scores 12, 12, 13, 13 and x is 13.

What is the value of the missing score x?

A.   13   B.   15

C.   12   D.   10

QUESTION 15

How many combinations of 3 digit numbers can be obtained from the numbers 2, 3, 5, 7 and 9?

A.   20   B.   5

C.   10   D.   15

QUESTION 16

What is the probability of picking a blue marble, if the first pick was a black marble, which was **not** replaced into the bag originally containing 5 red, 3 blue and 7 black marbles?

A. $\dfrac{3}{14}$ B. $\dfrac{1}{5}$

C. $\dfrac{3}{10}$ D. $\dfrac{1}{4}$

QUESTION 17

Two triangles shown below are similar and all lengths are in centimetres.

The side marked x in centimetres is

A.   9   B.   1

C.   4   D.   5

QUESTION 18

The circle $(x-1)^2 + (y+3)^2 = 9$, expressed in the form $ax^2 + by^2 + cx + dy + e = 0$ is

A. $x^2 + y^2 + 2x + 6y + 1 = 0$

B. $x^2 + y^2 - 2x + 6y + 1 = 0$

C. $x^2 + y^2 - 2x + 6y + 9 = 0$

D. $x^2 + y^2 - 2x - 6y + 9 = 0$

QUESTION 19

The sum of the first 5 terms of the geometric series with $a = 25$ and $r = 0.5$ is

A.   25.50   B.   48.4375

C.   34.375   D.   57.375

QUESTION 20

Interest is paid at 5% p.a. on a deposit of K2, 500. If the amount is compounded yearly, what is the interest in the fifth year?

A.   K3, 190.70   B.   K3, 038.77

C.   K144.70   D.   K151.94

QUESTION 21

The limiting sum of the sequence $25, \dfrac{75}{4}, \dfrac{225}{16}, \ldots$ is

A.   100   B.   50

C.   75   D.   200

QUESTION 22

The value of $\sin\left(\dfrac{\pi}{3}\right)$ is equivalent to

A. $\sin\left(\dfrac{\pi}{6}\right)$ B. $\sin\left(\dfrac{5\pi}{3}\right)$

C. $\sin\left(\dfrac{2\pi}{3}\right)$ D. $\sin\left(\dfrac{4\pi}{3}\right)$

QUESTION 23

A 10 metre long ladder leaning on the wall makes an angle of 30° with the horizontal from the wall base.

How high, in metres will the ladder reach?

A.   10   B.   4

C.   5   D.   6

QUESTION 24

In the diagram below determine the value of x correct to one decimal place.
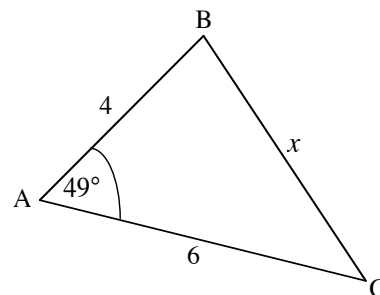
A.   4.5   B.   20.5

C.   10.6   D.   15.8

QUESTION 25

The value 5 in logarithmic form is equivalent to

A. $\log_{10} 50$ B. $\log_{10} 500{,}000$

C. $5\log_{10} 10$ D. $\log_{10} 1000$

QUESTION 26

The sum of two positive numbers x and y is equivalent to the difference of the reciprocal of y from the reciprocal of x. The equivalent statement using algebraic expression is

A. $x + y = y - x$

B. $x + y = \dfrac{1}{y} - \dfrac{1}{x}$

C. $x + y = \dfrac{1}{x} - \dfrac{1}{y}$

D. $x + y = x - y$

QUESTION 27

The graph of the parabola $y = 3 - x^2$ and the straight line $y = -2x$ intersect at the points

A. $(1,-2)$ *and* $(2,-4)$

B. $(2,-4)$ *and* $(-3,6)$

C. $(-1,2)$ *and* $(3,-6)$

D. $(-3,6)$ *and* $(1,-2)$

QUESTION 28

15 males and 20 females were interviewed. It was found that 10 males and 5 females were smokers.

What is the probability of picking a non-smoker from the group interviewed?

A. $\dfrac{3}{7}$ B. $\dfrac{4}{7}$

C. $\dfrac{1}{4}$ D. $\dfrac{3}{4}$

QUESTION 29

The sum of the first seven terms of an arithmetic series is 49.

The sum of the first 6 terms is 48.

The sum of the first 5 terms is 45.

What is the fourth term?

A.   7   B.   5

C.   9   D.   3

QUESTION 30

Which of the sketches below represents the graph of the function $y = (3x + 1)(x - 2)$?
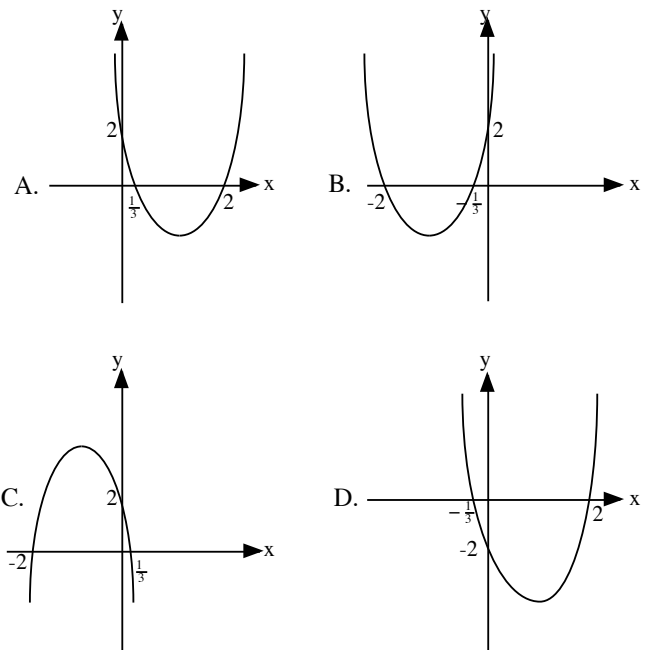
PART B: SHORT ANSWERS   20 MARKS

Write your answers on the Answer Sheet provided.

QUESTION 31

Rationalize the surd $\dfrac{1}{2\sqrt{3}}$.

QUESTION 32

Simplify the expression $\dfrac{w^4 \times w^{-2} \times y^2 \times y^{-3}}{wyz}$.

QUESTION 33

If 5 people can consume one 20 kg rice bag in 8 days, how many days will it take 8 people to consume a similar 20 kg rice bag? Assume the rate of eating is constant.

QUESTION 34

Find the remainder when $f(x) = x^3 + 4x^2 + x - 5$ is divided by $(x + 2)$

QUESTION 35

Determine the slope of the straight line that has the general equation $3x - 6y + 5 = 0$

QUESTION 36

Find the domain of the function $f(x) = \sqrt{3x + 6}$

QUESTION 37

A family of six attended a movie night.

How many ways can the family be seated in a row of 6 seats?

QUESTION 38

A ball is drawn at random from a bag that contained 7 red, 8 blue and 5 green balls.

Find the probability that a red ball or a green ball is picked.

QUESTION 39

Triangle LMN is isosceles with LM = LN, X and Y are points on LM and LN respectively, such that LX = LY.
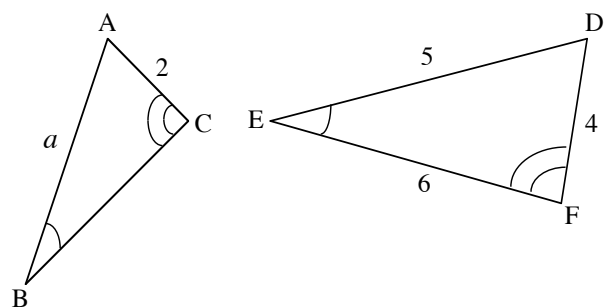
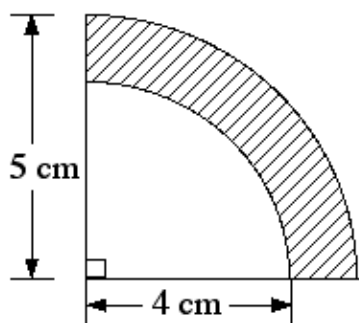Triangle LMY and triangle LNX are congruent under what rule.
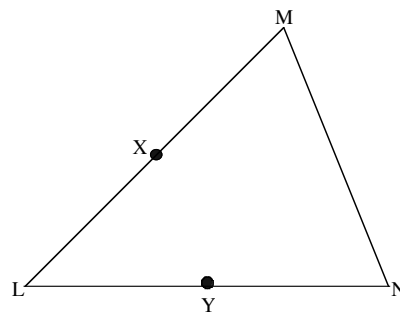
QUESTION 40

Given that the two triangles below are similar.

Calculate the side marked *a*. All lengths are in centimetres.

QUESTION 41

Find the area of the shaded region correct to two decimal places. Assume the shape is a sector of a circle.

QUESTION 42

What is the fourth term of the expansion of the expression $(2a+b)^5$?
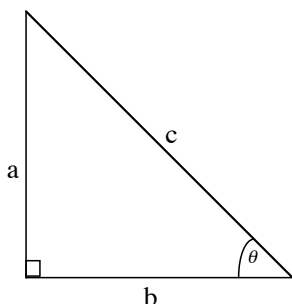
QUESTION 43

A fair die is rolled 3 times.

What is the probability of getting three sixes?

QUESTION 44

Evaluate the determinant $\begin{vmatrix} 3 & -2 \\ 2 & 5 \end{vmatrix}$.

QUESTION 45

Consider the right-triangle below.

Evaluate $\sin^2\theta + \cos^2\theta$. Leave your answer in its most simplified form.

QUESTION 46

Given that $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + \mathrm{k}$ and $\mathbf{w} = \mathbf{i} + \mathbf{k}$, compute $\mathbf{v} + \mathbf{w}$.

QUESTION 47

A plane flies north at a speed of 150 km/hr. Its journey is affected by strong wind **in** the north-easterly direction at the speed of 80 km/hr.

Draw a full vector diagram required to compute the resultant direction and the speed of the plane.

QUESTION 48

Find the gradient of the tangent line to the curve $y = -x^2 + 3x$ at the point $(2,2)$.

QUESTION 49

Given the curve $y = 2x^2 - 2x - 3$, at what value of x is y-minimum?

QUESTION 50

Evaluate the definite integral $\int_0^{\frac{\pi}{6}} \cos x \ dx$.

END OF EXAMINATION

Write your 10-digit candidate number, your name and your school name in the spaces provided below.

Year		Province		School			Candidate		
1	5								

Candidate Name: _____

School Name: _____

This answer booklet is for you to write the answers to Part B only.

All Multiple Choice Answers should be on the Electronic Mark Sheet.

All answers must be written <u>neatly</u> in the appropriate spaces in this booklet. **Answers written elsewhere on the question paper (or any other paper) will not be marked.**

✤✧✤✧✤✧✤✧✤✧✤✧✤✧✤✧✤✧✤✧✤✧✤✧✤

TOTAL SCORE

Recorded by: _____

Checked by: _____

ANSWERS TO PART B ONLY

Question 31	
Question 32	
Question 33	
Question 34	
Question 35	

Question 41	
Question 42	
Question 43	
Question 44	
Question 45	

Question 36	
Question 37	
Question 38	
Question 39	
Question 40	

Question 46	
Question 47	
Question 48	
Question 49	
Question 50	

MENSURATION

Arc Length $\qquad L = \dfrac{\theta}{360}2\pi r = \dfrac{\theta}{360}\pi d$

Area of Sector $\qquad A = \dfrac{\theta}{360}\pi r^2$

Surface Area of Cylinder $\qquad A = 2\pi r^2 + 2\pi rh$

Surface Area of Sphere $\qquad A = 4\pi r^2$

Curved Surface Area of Cone $\qquad A = \pi rL$

Volume of Sphere $\qquad A = \dfrac{4}{3}\pi r^3$

Interior Angle Sum of Polygon $\qquad S_n = (n-2)\times 180$

INTEREST

Compound Interest $\qquad A = P\left(1+\dfrac{r}{100}\right)^n$

TRIGONOMETRY

Sin Rule $\qquad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule $\qquad c^2 = a^2 + b^2 - 2ab\cos C$

Area of Triangle $\qquad A = \dfrac{1}{2}ab\sin C$

Conversion $\qquad \pi^c = 180°$

Arc Length $\qquad L = r\theta^c$

Area of Sector $\qquad A = r^2\theta^c$

Area of Minor Segment $\qquad A = \dfrac{1}{2}r^2\left(\theta^c - \sin\theta^\circ\right)$

PERMUTATION AND COMBINATION

PERMUTATION $\qquad {}^nP_r = \dfrac{n!}{(n-r)!}$

COMBINATION $\qquad {}^nC_r = \dfrac{n!}{r!(n-r)!}$

SERIES

Arithmetic Progression $\qquad T_n = a+(n-1)d$

$\qquad S_n = \dfrac{n}{2}(a+l) \qquad$ or $\qquad S_n = \dfrac{n}{2}(2a+[n-1]d)$

Geometric progression $\qquad T_n = ar^{n-1}$

$\qquad S_n = \dfrac{a(r^n-1)}{r-1} = \dfrac{a(1-r^n)}{1-r}, \; for \; r \neq 1$

$\qquad S_\infty = \dfrac{a}{1-r}, \; for \; -1 < r < 1$

ALGEBRA AND CALCULUS

Quadratic Formula $\qquad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

First Derivative $\qquad f'(x) = \lim_{h\to\infty}\dfrac{f(x+h)-f(x)}{h} = \lim_{\Delta x\to\infty}\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$

ANALYTIC GEOMETRY

Distance between two points $\qquad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Mid-point of Interval $\qquad \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

Gradient of a Line $\qquad m = \dfrac{y_2 - y_1}{x_2 - x_1} = \tan\theta$

ABSOLUTE VALUE $\qquad |x| = \begin{cases} -x, \text{ if } x < 0 \\ x, \text{ if } x \geq 0 \end{cases}$

BINOMIAL EXPANSION

$$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \ldots + y^n \quad where \quad \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$$