INSTRUCTIONS TO CANDIDATES

To be read by the external invigilator to all candidates

1. The code for Advance Mathematics is **3.**

2. There are **6** printed pages in the question booklet and a **1** page Section B Answer Booklet. A **1 printed page** formula sheet is also inserted in the centre.

3. There are two sections in this paper. Answer all questions.

   <u>Section A</u>: **Multiple Choice Questions - 30 marks**
   This section will be electronically marked.
   Electronic Answer Sheets will be distributed by your external invigilator. All answers to the Multiple Choice Section MUST be answered on this Answer Sheet.
   Carefully following the instructions, fill in your Candidate Information and Subject Information.

   <u>Section B</u>: **Short Answer Questions - 20 marks**
   Write down your name, your school name and your 10 digit candidate number on the Section B Answer Sheet Provided.

4. You are required to only write the correct answer in the space provided.

5. Calculators may be used.

6. Answers written on the question paper will not be marked. Write answers neatly in spaces as allocated on the answer sheet. Answer **ALL** questions.

7. Answer all questions on the answer sheet. Answers on any other paper including rough work paper and the question paper **<u>will not be marked</u>**

8. ALL working must be shown step by step to get full marks. Students may lose marks for writing down final answers only.

9. Enough spaces have been allocated for answers to every question. Questions must be answered in spaces as allocated. Answers all over the answer booklet may not be marked.

10. Correctional Fluid is <u>not allowed</u> on the answer sheet. Where you have made an error, cross out all the working and start on a new line.

11. Graphical Calculators are <u>not permitted</u>.

    Penalty For Cheating Or Assisting To Cheat In National Examinations Is Non-Certification.

    DO NOT TURN OVER THE PAGE AND DO NOT WRITE UNTIL YOU ARE TOLD TO START.

SECTION A: (Questions 1 to 25) : 1 mark each

Answer each question by shading in with HB pencil, the circle directly under the correct alternative A, B, C, D or E. If you make a mistake, rub it out completely using an eraser rubber and shade the correct answer on the Electronic Answer Sheet.

QUESTION 1

Douglas, Renae and Lynette were given K235.00 and told to divide it in the ratio 2:1:3 amongst themselves in order of names listed.

To he nearest toea each will receive

A.   K39.17,   K120.00,   K78.33
B.   K78.33,   K39.17,   K117.50
C.   K117.50, K39.17,   K140.00
D.   None of the above

QUESTION 2

The solution of the quadratic equation $x^2 - 2x - 3 = 0$ is:

A. $x = 3, x = 1$
B. $x = -1, x = 3$
C. $x = -3, x = 1$
D. $x = 2, x = 2$

QUESTION 3

The solution to the equation $8x^{\frac{1}{3}} - 3 = 1$ is

A. $x = \dfrac{1}{2}$ B. $x = \left(\dfrac{1}{2}\right)^{\frac{1}{3}}$

C. $x = \dfrac{1}{8}$ D. $x = \frac{1}{4}$

QUESTION 4

A map has a scale of 1:25000.

What distance is represented by a length of

6.8 cm on the map?

A.   170 km   B.   17.0 km
C.   1.70 km   D.   0.17 km

QUESTION 5

$\dfrac{(x-1)}{(x^2-1)}$ can be simplified to

A. $\dfrac{(x-1)}{(x+1)}$ B. $\dfrac{1}{(x+1)}$

C.   1   D. $x+1$

QUESTION 6

Which expression cannot be simplified any further?

A. $\dfrac{m-n}{n-m}$ B. $\dfrac{2x-2y}{y-x}$

C. $\dfrac{ab^2-ab}{2-2ab}$ D. $\dfrac{r+2s}{4s-2r}$

QUESTION 7

The domain of the function $y = -x^2 + 2$

A. $(-2, 2)$ B. $(-2, 0)$
C. $(0, 2)$ D. $(2, -2)$

QUESTION 8

$t^3 - 8$ can be expressed as

A. $(t-8)^3$
B. $(t-2)(t^2-2t+4)$
C. $(t-2)(t^2+2t+4)$
D. $(t-2)(t^2-t+4)$

QUESTION 9

If $\log_4 x = \dfrac{1}{2}$ then what is the value of x?

A.   4   B. -2
C.   2   D. -4

QUESTION 10

The graph of the rational function $y = \dfrac{x+3}{x-2}$

has a

A.   vertical asymptote at $x = -3$
B.   horizontal asymptote at $y = 2$
C.   vertical asymptote at $x = 2$
D.   horizontal asymptote at $x = -3$

QUESTION 11

Which of these terms does not mean "data item"?

A.   score   B.   value
C.   average   D.   measurement

QUESTION 12

The spread of a frequency distribution is measured by:

A.   mean   B.   median
C.   standard deviation   D.   average

QUESTION 13

The value of 6 factorial (6!) is :

A.   36   B.   720

C.   6   D. 6^6

QUESTION 14

The number of permutations of 5 objects is:

A.   25   B.   5

C. 5^5 D.   120

QUESTION 15

Two events are mutually exclusive if on one trial of an experiment:

A. both must occur
B. exactly one must occur
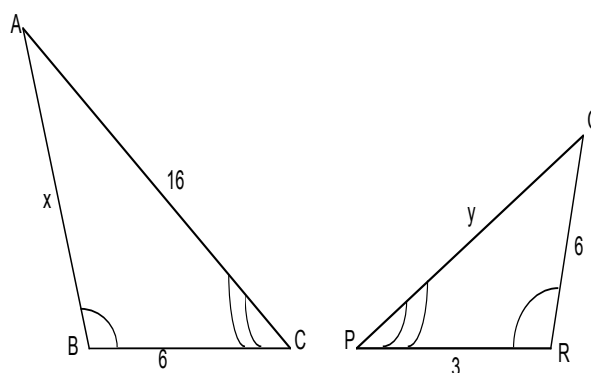C. exactly one may occur
D. both may occur

QUESTION 16

Which of these is the best option?

A congruent shape has all corresponding

A. angles equal
B. sides equal
C. sides and angles equal
D. none of the above

QUESTION 17

Find the side marked y if $A\hat{B}C = P\hat{R}Q$. All measurements are in the same unit.

A.   8   B.   12   C.   10   D.   4

QUESTION 18

A circle $x^2 + y^2 + 2x - 2y - 1 = 0$ expressed in the following form $(x-h)^2 + (y-k)^2 = r^2$ where h, k are the coordinates of the centre and r the radius is given as:

A. $x^2 + y^2 = 4$
B. $(x+2)^2 + (y-2)^2 = 3^2$
C. $(x-2)^2 + (y+2)^2 = 3^2$
D. $(x+1)^2 + (y-1)^2 = 3$

QUESTION 19

Which is a geometric progression?

    A. 1, 3, 4, 7, 9,…….
    B. 1, 3, 5, 7, 9,……
    C. 1, 2, 4, 9, 16,……
    D. 1, 3, 9, 27, 81,……

QUESTION 20

Which is an arithmetic progression?

    A. 1, 3, 4, 7, 9,…….
    B. 1, 2, 4, 8, 16,…….
    C. 1, 3, 9, 27, 81,……
    D. 4, 2, 0, -2, -4,……

QUESTION 21

Which of the following sequence is not geometric?

    A. 1, -1, 1, -1, 1,…….
    B. $\dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16}$,................
    C. -2, 4, -8, 16,…….
    D. -2, 3, -4, 5, -6,……

QUESTION 22

If $D = \{1,3,5\}, E = \{3,4,5\}, F = \{1,5,10\}$, then $(D \cup E) \cup F$ is:

    A. $\{1,3,4,5\}$
    B. $\{1,5\}$
    C. $\{1,3,4,5,10\}$
    D. $\{1,3,5,10\}$

QUESTION 23

If $\cos\theta = \dfrac{2}{3}$, find θ where $180° < \theta < 360°$.

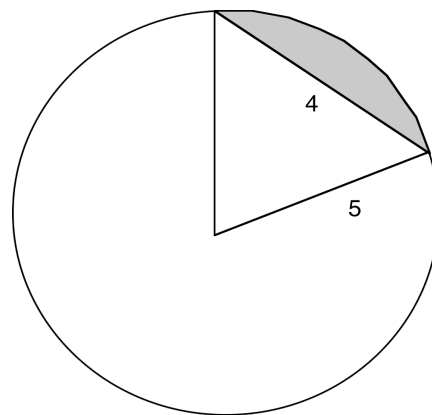   A.   48.18°   B.   221.81°
    C.   213.69°   D.   311.82°

QUESTION 24

If $\vec{OA}$ = 3i + 5j and $\vec{OB}$ = 5i -2j, find $\vec{AB}$ (where O is the origin)

    A. -2i – 7j
    B. 2i – 7j
    C. -2i + 7j
    D. 2i + 7j

QUESTION 25

Find the approximate area of the shaded region, given that the area of the sector is approximately 13.08 square units.

    A.   10   B. $2\sqrt{21}$
    C.   4   D. $4\sqrt{5}$

QUESTION 26

Evaluate the following expression
$(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2$

  A.   2   B.   1   C. $\dfrac{1}{2}$ D.   0

QUESTION 27

The equation of the tangent to the curve $y = (x^2 - 1)x$ at the point (1,0) is

    A. $y = \dfrac{1}{4}x - \dfrac{9}{8}$
    B. $y = \dfrac{1}{4}x + \dfrac{9}{8}$
    C. $y = -\dfrac{1}{4}x + \dfrac{9}{8}$
    D. None of the above

QUESTION 28

The two stationary points on the curve

$y = \dfrac{2x}{x^2 + 1}$ are

    A. (1, -1) and (-1, 1)
    B. (-1, 1) and (1, 1)
    C. (1, 1) and (-1, -1)
    D. none of the above

QUESTION 29

The motion of a particle is described by the law $s(t) = t^3 - 2t^2 + t + 1$ where t is in seconds and s is in metres. Its velocity after two (2) seconds is:

    A. 5 m/s
    B. 3 m/s
    C. 1 m/s
    D. None of the above

QUESTION 30

The first derivative of $y = x^3 + 2x^2 + 3x - 4$ is

    A. $3x^3 + 2x + 3$
    B. $3x^2 + 2x + 3$
    C. $3x^2 + 4x + 3$
    D. $3x^2 + 4^2 x + 3$

SECTION B :   20 SHORT ANSWER QUESTIONS.

Each question is worth 1 mark.

QUESTION 31

Rationalize the denominator $\dfrac{2}{\sqrt{2} + 1}$

QUESTION 32

Write the recurring decimal $0.3\dot{3}$ as a fraction.

QUESTION 33

Factorize $x^2 - 4$

QUESTION 34

Sketch the region satisfied by the inequality $2y + x \leq 1$.

QUESTION 35

Find the positive solution of the equation $x^2 - 2 = 0$.

QUESTION 36

Sketch the graph of $y = \dfrac{1}{x + 2}$.

QUESTION 37

Is it possible to calculate the exact average of a grouped frequency distribution?

QUESTION 38

How many combinations of 5 objects from 7 are there?

QUESTION 39

How many sides does a regular polygon whose angles total 900° have?

QUESTION 40

Find the line that is perpendicular to
$y = \dfrac{-3}{4}x + 2$ and passes through (3,4).

QUESTION 41

Calculate the arc length that subtends an angle of 55° at the centre of a circle with radius r = 5cm.

QUESTION 42

Use the binomial expansion
$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ to expand and simplify $(x-3)^4$.

QUESTION 43

Given that $A = \{1,3,5,7,9\}$ and
$B = \{2,4,6,7,8,9,10\}$. Find $A \cap B$.

QUESTION 44

Find the 10th term of the geometric sequence
16,8,4,2,1,................
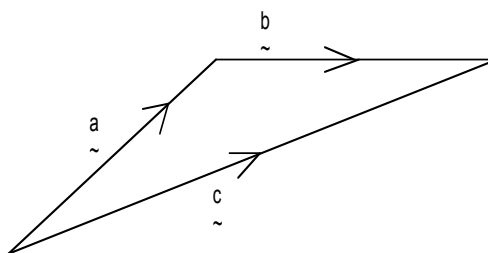
QUESTION 45

If $\cos^{-1}\left(\dfrac{4}{5}\right)$ **is** θ**,** what is $\sin\theta$? $0 < \theta < \dfrac{\pi}{2}$

QUESTION 46

Sine rule and cosine rule are both used in solving trigonometry problems. Give an example of when the cosine rule is more applicable than the sine rule?

QUESTION 47

If $a = 2i - j$ and $b = -i + 3j$, what is the vector labelled c? Write in terms of i, j components.

QUESTION 48

Find $\dfrac{dy}{dx}$ if $y = \dfrac{1}{8}$.

QUESTION 49

Find the gradient of the tangent line to the graph of $y = 3x^2 - x + 1$ at the point where

x = -1.

QUESTION 50

Does the parabola $y = -3x^2 + 2x - 1$ have a maximum or minimum value?

END OF EXAMINATION

INSTRUCTIONS TO CANDIDATES

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2. There are **3** printed pages in the question booklet and **6 printed** pages in the answer booklet. The **1 page formula sheet** is inserted in the question booklet.

3. The answer booklet is enclosed in the centre of this booklet. Take out the answer booklet now.

4. Check that you have the correct number of pages.

5. Write your 10 -digit candidate number, your name and your school name in the spaces provided in the answer booklet.

6. This paper contains 10 Short Answer Questions worth 5 marks each.

Total : 50 marks

Answer **ALL** questions.

7. Calculators, rulers and protractors are allowed.

8. Answer all questions on the answer sheet. Answers on any other paper including rough work paper and the question paper **will not be marked**

9. ALL working must be shown step by step to get full marks. Students may lose marks for writing down final answers only.

10. Enough space has been allocated for the answer to every question. Questions must be answered in spaces as allocated. Answers all over the answer booklet may not be marked.

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QUESTION 1

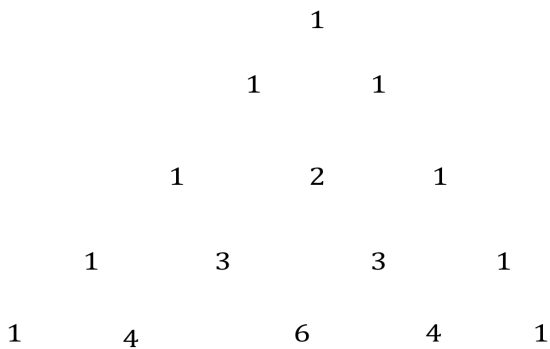If U = {Positive whole numbers ≤ 12} be the universal set.

A = {Positive primes ≤12}

B = {Positive factors of 12}

a)   List the elements of sets A and B *(2 marks)*

b)   Show the sets A, B and U on a Venn diagram.
                                              (1 mark)

c)   List the elements not in A or A'.
                                            (2 marks)

QUESTION 2

This triangle of numbers is called Pascal's triangle

$$1$$
$$1 \qquad 1$$
$$1 \qquad 2 \qquad 1$$
$$1 \qquad 3 \qquad 3 \qquad 1$$
$$1 \qquad 4 \qquad 6 \qquad 4 \qquad 1$$

**a)**   Write the next two rows of Pascal's triangle.
                                          (2 marks)

b)   Hence write down the binomial expression for:
                                          (3 marks)

     i. $(x + y)^3$
     ii. $(p + q)^4$
     iii. $(p - q)^4$

QUESTION 3

a)   Fill in the missing words.

     The angle between a tangent and a chord through the point of contact is equal to the angle _____ by the chord in the alternate segment.   *(1 mark)*

b)   Two circles intersect at points P and Q. Line APB is drawn through P. The tangents at A and B meet at C.
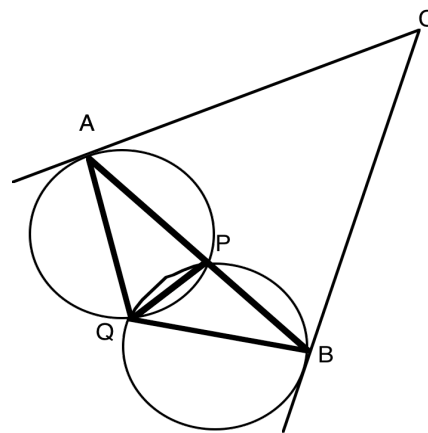
     If $\angle ABC = \beta^0$ and $\angle BAC = \alpha^0$, find expression in terms of α and β for

     i. $\angle PQB$ *(2 marks)*
     ii. $\angle PQA$ *(1 mark)*
     iii. $\angle AQB$ *(1 mark)*

QUESTION 4

An aircraft flies 500 km on a bearing of 100° to point A and then flies 600 km on a bearing of 160° to point B.

a)   Sketch the aircraft's flight from point A to point B.   *(1 mark)*

b)   Find the distance from starting point to finishing point.   *(2 marks)*

c)   Find the bearing from starting point to finishing point.   *(2 marks)*

QUESTION 5   *(5 marks)*

For the following frequency distribution, construct the cumulative distribution (cumulating from the lower end), draw the corresponding cumulative frequency graph and use the graph to estimate the median.

Distribution of Aid Posts per Province, 2010	
No. of Aid Posts	No. of Provinces
Less than 20	1
20 to less than 40	2
40 to less than 60	2
60 to less than 80	3
80 to less than 100	5
100 to less than 120	3
120 to less than 140	2
140 or more	1
TOTAL	**19**

QUESTION 6

Given the function $y = \dfrac{1}{3}x\left(x^2 - 9\right)$.

a)   Locate stationary points.

     (Leave answer in surd form)   *(2 marks)*

b)   Of the points in **a)** above, show which is maximum and which is minimum.

                                     (1 mark)

c)   Sketch the graph, clearly indicating the y intercept and the turning points.

                                       (2 marks)

QUESTION 7   *(5 marks)*

Solve for x in the equation

$$\frac{1}{2}x = \left(1 - \frac{3}{4}x\right)^{0.5}$$

QUESTION 8   *(5 marks)*

Solve the simultaneous equations for x and y.

$$\log_2 x + \log_2 y = 2$$

$$\log_2 x - \log_2 y = 0$$

QUESTION 9   *(5 marks)*

Find constants a and b, given that the polynomial

$P(x) = x^3 + ax^2 - bx - 10$ is divisible by $(x + 1)$

and $(x - 2)$.

QUESTION 10

Consider the function $f(x) = |x - 3| + 1$.

a)   Sate the domain and the range of the function.

                                       (2 marks)

b)   Sketch the graph of $y = f(x)$.

                                       (3 marks)

END OF EXAMINATION

FORMULAE SHEET FOR MATHEMATICS A

MENSURATION

Arc Length
$$L = \frac{\theta}{360} 2\pi x$$

Area of Sector
$$A = \frac{\theta}{360} 2\pi x^2$$

Surface Area of Cylinder
$$A = 2\pi r^2 + 2\pi rh$$

Surface Area of Sphere
$$A = 4\pi r^2$$

Curved Surface Area of Cone
$$A = \pi r L$$

Volume of Sphere
$$V = \frac{4}{3}\pi r^3$$

Interior Angles of Polygon
$$S_n = (n-2) \times 180°$$

INTEREST

Compound Interest
$$A = P(1 + \frac{r}{100})^n$$

TRIGONOMETRY

Sin Rule
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of Triangle
$$A = \frac{1}{2}ab\sin C$$

Conversion
$$\pi^c = 180°$$

Arc Length
$$L = r\theta^c$$

Area of Sector
$$A = \frac{1}{2}r^2\theta$$

Area of Minor Segment
$$A = \frac{1}{2}r^2(\theta^c - \sin\theta^c)$$

SERIES

Arithmetic Progression
$$T_a = a + (n-1)d$$

$$S_n = \frac{n}{2}(a + T_a)$$

$$S_a = \frac{n}{2}\left(2a + [n-1]d\right)$$

Geometric progression
$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{1 - r}, \quad \text{for} \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}, \quad \text{for} \quad -1 < 0 < 1$$

ALGEBRA

Quadratic Formula
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

First Derivative
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

ANALYTIC GEOMETRY

Distance between two points
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Mid-point of Interval
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Gradient of a Line
$$\frac{y_2 - y_1}{x_2 - x_1} = m = \tan\theta \quad \frac{y_2 - y_1}{x_2 - x_1} = m = \tan\theta$$

FORMULAE SHEET FOR MATHEMATICS A

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Arc Length
$$L = \frac{\theta}{360} 2\pi x$$

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Curved Surface Area of Cone
$$A = \pi rL$$

Volume of Sphere
$$V = \frac{4}{3}\pi r^3$$

Interior Angles of Polygon
$$S_n = (n-2) \times 180°$$

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$$A = P(1 + \frac{r}{100})^n$$

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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$$S_a = \frac{n}{2}\left(2a + [n-1]d\right)$$

Geometric progression
$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{1 - r}, \quad \text{for} \quad r \neq 1$$

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Quadratic Formula
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

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$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Mid-point of Interval
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Gradient of a Line
$$\frac{y_2 - y_1}{x_2 - x_1} = m = \tan\theta \quad \frac{y_2 - y_1}{x_2 - x_1} = m = \tan\theta$$