

5.5 Applications from Physics, Engineering, and Statistics

1.

$$\begin{aligned}
 W &= \int_0^{10} \frac{1}{x^2 + 1} dx \\
 &= \tan^{-1}(x) \Big|_0^{10} \\
 &= \tan^{-1}(10) - \tan^{-1}(0) \\
 &= 1.471 \text{ J}
 \end{aligned}$$

2.

$$\begin{aligned}
 W &= \int_1^5 \cos\left(\frac{\pi x}{2}\right) dx \\
 u &= \frac{\pi x}{2} \\
 du &= \frac{\pi}{2} dx \\
 \frac{2}{\pi} du &= dx
 \end{aligned}$$

$$x = 1, \text{ then } u = \frac{\pi}{2}$$

$$x = 5, \text{ then } u = \frac{5\pi}{2}$$

$$\begin{aligned}
 W &= \int_1^5 \cos\left(\frac{\pi x}{2}\right) dx \\
 &= \frac{2}{\pi} \int_{\frac{\pi}{2}}^{\frac{5\pi}{2}} \cos u \, du \\
 &= \frac{2}{\pi} [\sin u]_{\frac{\pi}{2}}^{\frac{5\pi}{2}} \\
 &= \frac{2}{\pi} \left(\sin \frac{5\pi}{2} - \sin \frac{\pi}{2} \right) \\
 &= \frac{2}{\pi} (1 - 1) \\
 &= 0 \text{ J}
 \end{aligned}$$

3. Be sure to convert kilometers to meters.

$$\begin{aligned}
 W &= \int_{6,370}^{35,780} G \frac{mM}{r^2} dr \\
 &= GmM \left(\frac{r^{-1}}{-1} \right) \Big|_{6,370}^{35,780} \\
 &= GmM \left(\frac{1}{35,780,000} + \frac{1}{6,370,000} \right) \\
 &= (6.67 \times 10^{-11})(1000)(6 \times 10^{24}) \left(\frac{1}{35,780,000} + \frac{1}{6,370,000} \right) \\
 &= (5.16 \times 10^{-6})10^{-11} \times 10^3 \times 10^{24} \\
 &= 5.16 \times 10^{10} \text{ J}
 \end{aligned}$$

4. a.

$$\begin{aligned}
 F(1) &= 5 \\
 k(1) &= 5 \\
 k &= 5 \\
 k &= 5 \text{ N/m}
 \end{aligned}$$

b.

$$\begin{aligned}
 W &= \int_0^{1.8} 5x \, dx \\
 &= \left[\frac{5x^2}{2} \right]_0^{1.8} \\
 &= \frac{5(1.8)^2}{2} \\
 &= 8.1 \text{ J}
 \end{aligned}$$

5.

$$\begin{aligned}
 F(3) &= \int 3 \\
 k(3) &= 30 \\
 k &= 10
 \end{aligned}$$

$$\begin{aligned}
 W &= \int_{12}^{20} 10x \, dx \\
 &= \left[\frac{10x^2}{2} \right]_{12}^{20} \\
 &= [5x^2]_{12}^{20} \\
 &= 2000 - 720 \\
 &= 8.1 \text{ J}
 \end{aligned}$$

6. Yes.

$$F = \int_a^b wh(x)L(x)dx$$

Double w to $2w$.

Then the force F becomes $F = \int_a^b 2wh(x)L(x)dx = 2 \int_a^b wh(x)L(x)dx$.

7.

$$\begin{aligned}
 d &= \frac{1}{4}x + 4 \\
 w &= 10 \\
 F &= 9800 \int_0^{16} 10 \left(\frac{1}{4}x + 4 \right) dx \\
 &= 98,000 \int_0^{16} \left(\frac{1}{4}x + 4 \right) dx \\
 &= 98,000 \left(\frac{x^2}{8} + 4x \right)_0^{16} \\
 &= 98,000(32 + 64) \\
 &= 940,800 \text{ N}
 \end{aligned}$$

8. a. The integral represents the probability that a randomly chosen light bulb will have a lifetime between 1000 and 5000 hours.

b. The integral represents the probability that a randomly chosen light bulb will have a lifetime of at least 3000 hours.

9.a.

$$\begin{aligned}
 P(x \leq 3) &= \int_a^b \frac{1}{\mu} e^{-\frac{1}{\mu}x} dx \\
 &= \int_0^3 \frac{1}{8} e^{-\frac{1}{8}x} dx \\
 &= [-e^{-\frac{1}{8}x}]_0^3 \\
 &= -e^{-\frac{3}{8}} - (-1) \\
 &= 0.31 \\
 &= 31\%
 \end{aligned}$$

b. We can safely assume that a customer will not have to wait more than 100 minutes.

$$\begin{aligned}
 P(x > 10) &= \int_a^b \frac{1}{\mu} e^{-\frac{1}{\mu}x} dx \\
 &= \int_{10}^{100} \frac{1}{8} e^{-\frac{1}{8}x} dx \\
 &= [-e^{-\frac{1}{8}x}]_{10}^{100} \\
 &= -e^{-\frac{100}{8}} - (-e^{-\frac{10}{8}}) \\
 &= 0.29 \\
 &= 29\%
 \end{aligned}$$

10. a.

$$\begin{aligned}
 \mu &= 63.4 \text{ inches} \\
 \sigma &= 3.2 \text{ inches}
 \end{aligned}$$

Since 63.4 is the average and the heights form a normal distribution, then $P(x < 63.4)$ covers half of the probability. Thus,

$$\begin{aligned}
 P(x < 63.4) &= \int_0^{63.4} \frac{1}{3.2\sqrt{2\pi}} e^{-\frac{(x-63.4)^2}{(2(3.2)^2)}} dx \\
 &= 50\%
 \end{aligned}$$

b.

$$\begin{aligned}
 P(63 \leq x \leq 65) &= \int_{63}^{65} \frac{1}{3.2\sqrt{2\pi}} e^{-\frac{(x-63.4)^2}{(2(3.2)^2)}} dx \\
 &= 24\%
 \end{aligned}$$

c.

$$\begin{aligned} P(x > 72) &= \int_{72}^{\infty} \frac{1}{3.2\sqrt{2\pi}} e^{-\frac{(x-63.4)^2}{(2(3.2)^2)}} dx \\ &= 0.36\% \end{aligned}$$

d.

$$\begin{aligned} P(x = 60) &= \int_{60}^{60} \frac{1}{3.2\sqrt{2\pi}} e^{-\frac{(x-63.4)^2}{(2(3.2)^2)}} dx \\ &= 0\% \end{aligned}$$