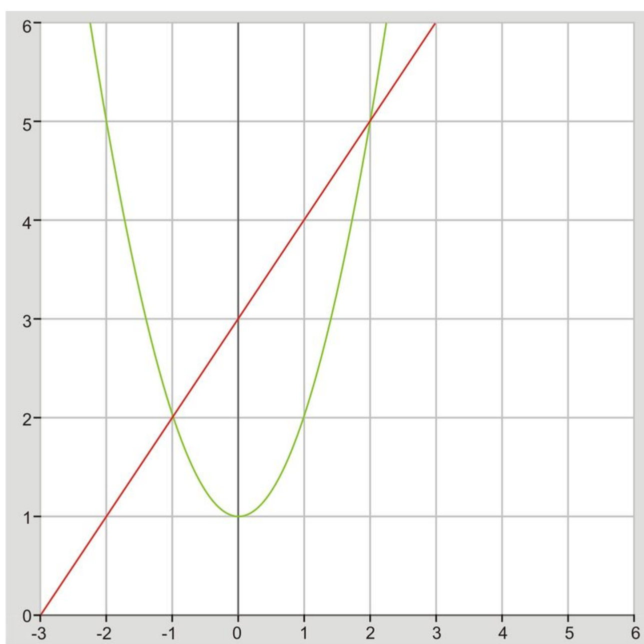
5.2 Volumes

Note: The graphs in this set of answers graph the functions on the coordinate plane so that you can find the limits of integration. They do not show the actual rotation.

1. The curve $y = \sqrt{9-x^2}$ is the top half of a circle with radius 3. The limits of integration are $x = -3$ and $x = 3$. When rotated around the $x-$axis, we get a sphere of radius 3.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(3)^3 \\ &= 36\pi \end{aligned}$$

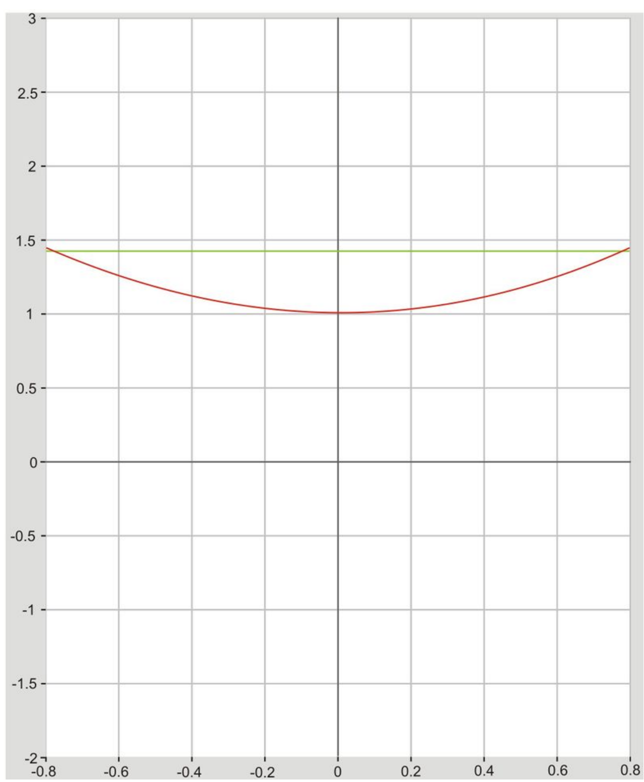
2.



$3+x \geq 1+x^2$ for x on $[-1,2]$. Then

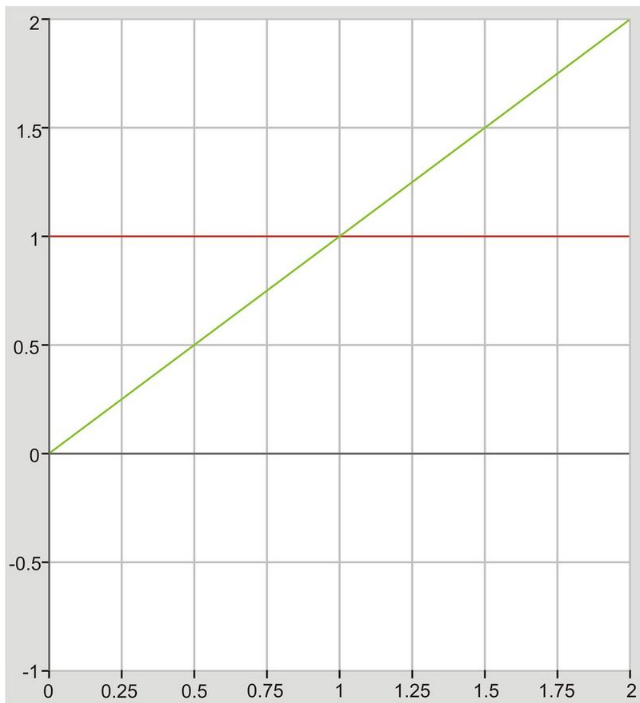
$$\begin{aligned}
V &= \int_{-1}^{2} \pi[(3+x)^2-(1+x^2)^2]dx \\
&= \int_{-1}^{2} \pi[9+6x+x^2-1-2x^2-x^4]dx \\
&= \pi\left[9x+3x^2+\frac{x^3}{3}-x-\frac{2x^3}{3}-\frac{x^5}{5}\right]\Bigg|_{-1}^{2} \\
&= \pi\left[18+12+\frac{8}{3}-2-\frac{16}{3}-\frac{32}{5}+9-3+\frac{1}{3}-1-\frac{2}{3}-\frac{1}{5}\right] \\
&= \frac{117\pi}{5}
\end{aligned}$$

3.



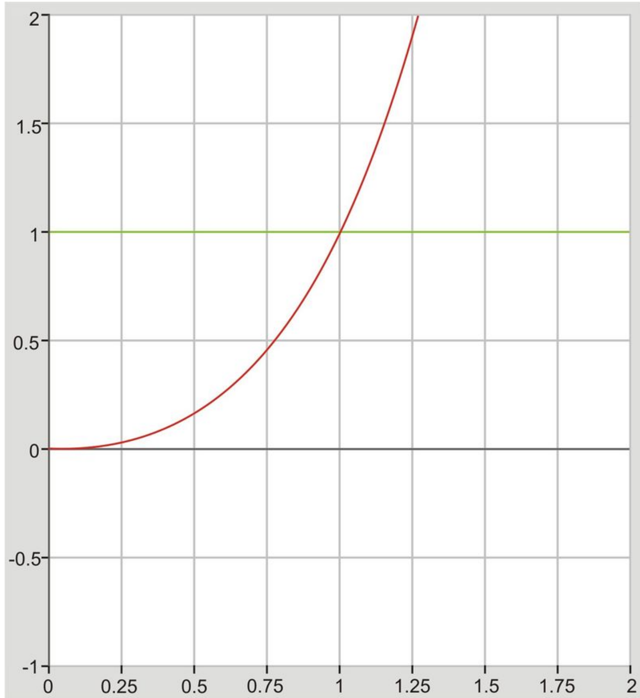
$$\begin{aligned}
V &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \pi\left[\left(\sqrt{2}\right)^2 - \sec^2 x\right] dx \\
&= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \pi[2 - \sec^2 x\, dx] \\
&= [2\pi x - \pi \tan x]\Big|_{\frac{\pi}{4}}^{-\frac{\pi}{4}} \\
&= 2\pi\left(\frac{\pi}{4}\right) - \pi\tan\frac{\pi}{4} - \left(2\pi\left(-\frac{\pi}{4}\right) - \pi\tan\left(-\frac{\pi}{4}\right)\right) \\
&= \pi^2 - \pi - \pi \\
&= \pi^2 - 2\pi
\end{aligned}$$

4.



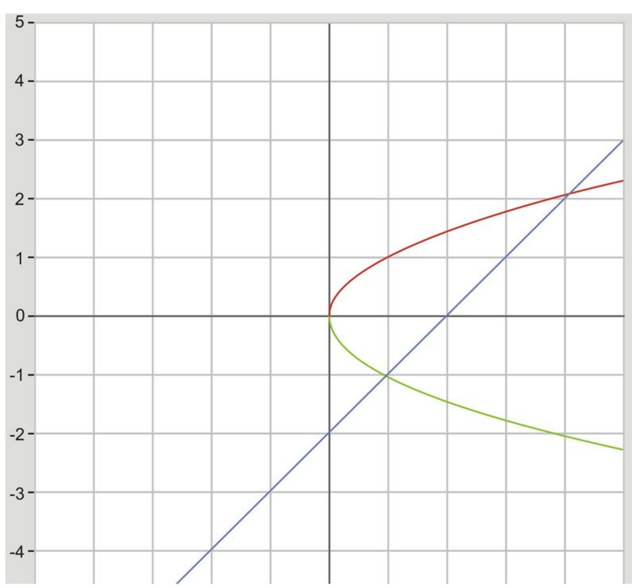
$$\begin{aligned}
V &= \int_0^1 \pi[(1)^2 - x^2]dx \\
&= \left[\pi x - \frac{\pi x^3}{3}\right]_0^1 \\
&= \pi - \frac{\pi}{3} \\
&= \frac{2\pi}{3}
\end{aligned}$$

5.



$$\begin{aligned}V &= \int_0^1 \pi\left[\sqrt[3]{y}\right]^2 dy\\ &= \pi \frac{y^{\frac{5}{3}}}{\frac{5}{3}}\Bigg|_0^1\\ &= \frac{3\pi}{5} y^{\frac{5}{3}}\Bigg|_0^1\\ &= \frac{3\pi}{5}\end{aligned}$$

6.

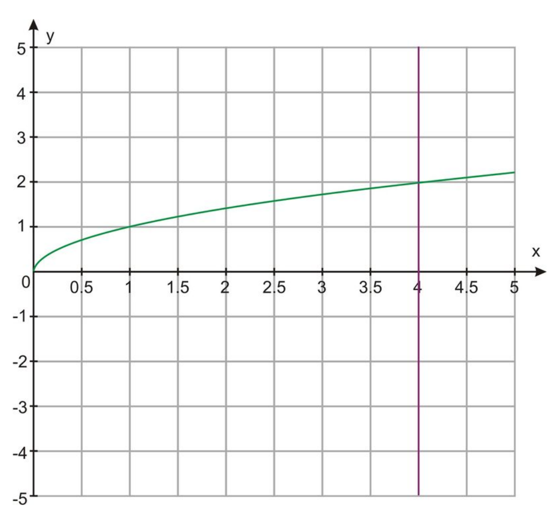


$$
\begin{aligned}
V &= \int_{-1}^{2} \pi\left([y+2]^2 - [y^2]^2\right) dy \\
&= \int_{-1}^{2} \pi(y^2 + 4y + 4 - y^4) dy \\
&= \pi\left(\frac{y^3}{3} + \frac{4y^2}{2} + 4y - \frac{y^5}{5}\right)\Bigg|_{-1}^{2} \\
&= \pi\left[\frac{8}{3} + 8 + -\frac{32}{5} - \left(-\frac{1}{3} + 2 - 4 + \frac{1}{5}\right)\right]\Bigg| \\
&= \pi\left(\frac{72}{5}\right) \\
&= \frac{72\pi}{5}
\end{aligned}
$$

7.

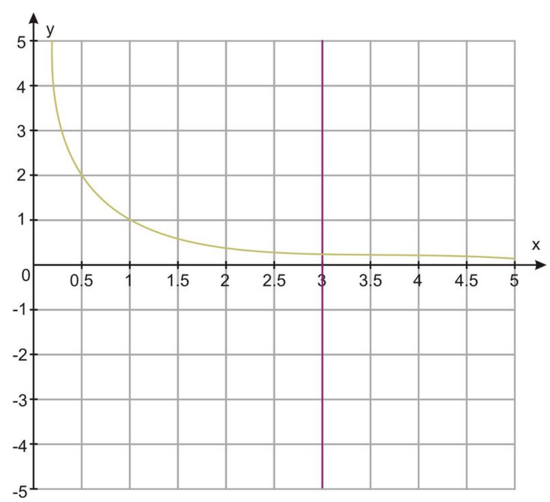
$$
\begin{aligned}
V &= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \pi(\csc^2 y - 0^2) dy \\
&= -\pi \cot y\Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \\
&= -\pi \cot\left(\frac{3\pi}{4}\right) - \left[-\pi \cot\left(\frac{\pi}{4}\right)\right] \\
&= \pi + \pi \\
&= 2\pi
\end{aligned}
$$

8.



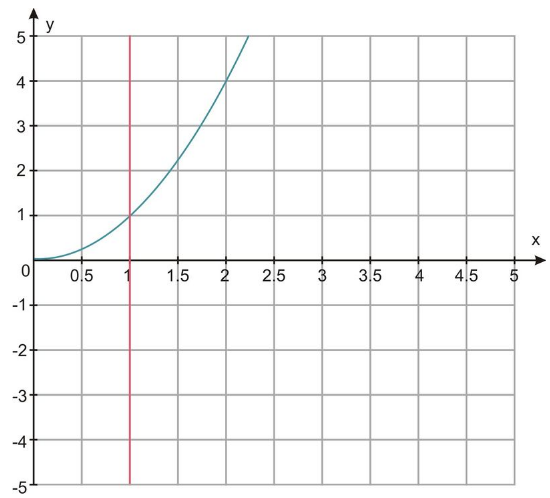
$$V = \int_0^2 \pi \left(4^2 - (y^2)^2\right) dy$$
$$= \int_0^2 \pi(16 - y^4) dy$$
$$= \pi \left(16y - \frac{y^5}{5}\right) \Big|_0^2$$
$$= \pi \left(32 - \frac{32}{5}\right)$$
$$= \frac{128\pi}{5}$$

9.



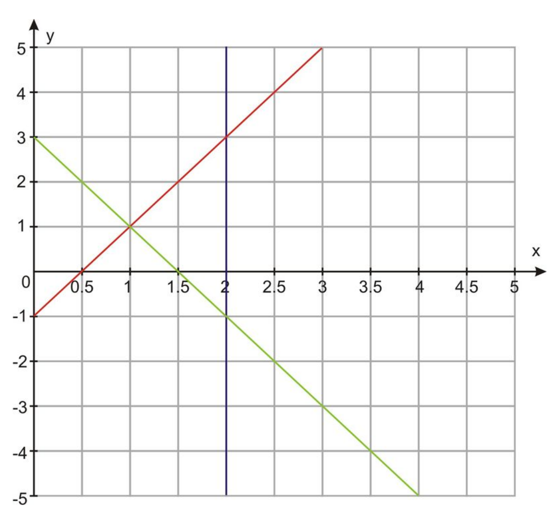
$$V = \int_1^3 2\pi y \left(\frac{1}{y}\right) dy$$
$$= 2\pi y|_1^3$$
$$= 2\pi(3) - 2\pi$$
$$= 4\pi$$

10.



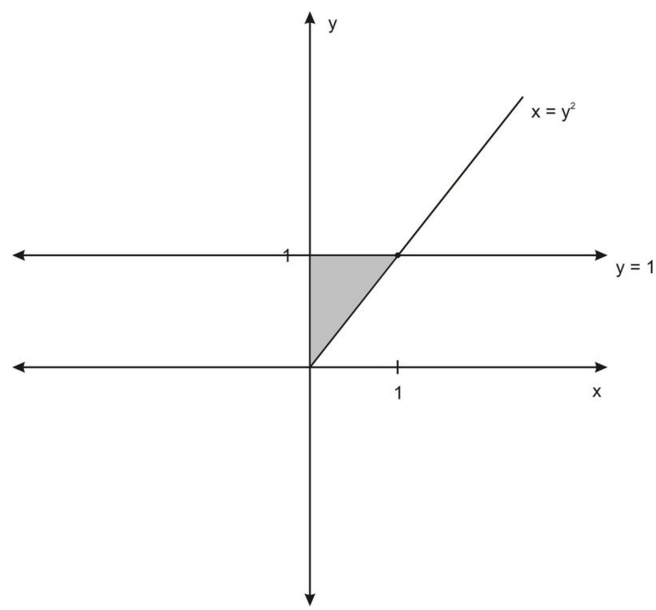
$$\begin{aligned}V &= \int_0^1 2\pi y(\sqrt{y})\,dy\\ &= \int_1^3 2\pi y^{\frac{3}{2}}\,dy\\ &= 2\pi\frac{2}{5}y^{\frac{5}{2}}\Big|_0^1\\ &= \frac{4}{5}\pi(1)\\ &= \frac{4\pi}{5}\end{aligned}$$

11.



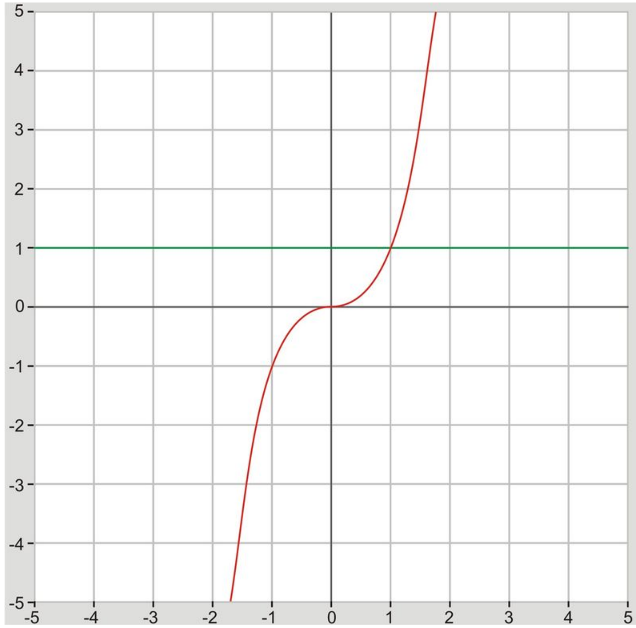
$$
\begin{aligned}
V &= \int_{1}^{2} 2\pi x(4x-4)dx \\
&= \int_{1}^{3} 2\pi(4x^2-4x)dy \\
&= 2\pi \times \left(\frac{4x^3}{3} - \frac{4x^2}{2}\right)\Bigg|_{1}^{2} \\
&= 2\pi\left(\frac{32}{3} - 8 - \frac{4}{3} + 2\right) \\
&= 2\pi\left(\frac{10}{3}\right) \\
&= \frac{20\pi}{3}
\end{aligned}
$$

12.



$$
\begin{aligned}
V &= \int_{0}^{1} 2\pi y(y^2)dy \\
&= \int_{1}^{3} 2\pi y^3 dy \\
&= 2\pi \times \frac{y^4}{4}\Bigg|_{0}^{1} \\
&= 2 \times \frac{\pi}{4} \times (1) \\
&= \frac{\pi}{2}
\end{aligned}
$$

13.



$$V = 2\pi \int_0^1 (1-y)y^{\frac{1}{3}}\,dy$$
$$= 2\pi \int_0^1 \left(y^{\frac{1}{3}} - y^{\frac{4}{3}}\right) dy$$
$$= 2\pi \left[\frac{3}{4}y^{\frac{4}{3}} - \frac{3}{7}y^{\frac{7}{3}}\right]_0^1$$
$$= 2\pi \left(\frac{3}{4} - \frac{3}{7}\right)$$
$$= \frac{18\pi}{28}$$
$$= \frac{9\pi}{14}$$