4.5 Evaluating Definite Integrals

1.

$$\begin{aligned}\int_4^9 \left(\frac{3}{\sqrt{x}}\right) dx &= \int_4^9 3x^{-\frac{1}{2}} dx \\ &= 3\int_4^9 x^{-\frac{1}{2}} dx \\ &= 3\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\Bigg]_4^9 \\ &= 6\left(\sqrt{9} - \sqrt{4}\right) \\ &= 6(3-2) \\ &= 6\end{aligned}$$

2.

$$\begin{aligned}\int_0^1 \left(t - t^2\right) dt &= \int_0^1 t\ dt - \int_0^1 t^2\ dt \\ &= \frac{t^2}{2}\Bigg]_0^1 - \frac{t^3}{3}\Bigg]_0^1 \\ &= \left(\frac{1^2}{2} - 0\right) - \left(\frac{1^3}{3} - 0\right) \\ &= \frac{1}{2} - \frac{1}{3} \\ &= \frac{1}{6}\end{aligned}$$

3.

$$\begin{aligned}\int_{2}^{5}\left(\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{2}}\right)dx &= \int_{2}^{5} x^{\frac{-1}{2}}\,dx+\int_{2}^{5}\frac{1}{\sqrt{2}}dx\\ &= \frac{x^{\frac{1}{2}}}{\frac{1}{2}}\Bigg]_{2}^{5}+\frac{1}{\sqrt{2}}x\Bigg]_{2}^{5}\\ &= 2\sqrt{x}\Big]_{2}^{5}+\frac{1}{\sqrt{2}}x\Bigg]_{2}^{5}\\ &= \left(2\sqrt{5}-2\sqrt{2}\right)+\left(\frac{5}{\sqrt{2}}-\frac{2}{\sqrt{2}}\right)\\ &= 2\sqrt{5}-2\sqrt{2}+\frac{3\sqrt{2}}{2}\end{aligned}$$

4.

$$\begin{aligned}\int_{0}^{1}\left(x^2-1\right)\left(x^2+1\right)dx &= 4\int_{0}^{1}\left(x^4-1\right)dx\\ &= 4\left[\frac{x^5}{5}\Bigg]_{0}^{1}-x]_{0}^{1}\right]\\ &= 4\left[\frac{1}{5}-1\right]\\ &= \frac{4}{5}-4\\ &= \frac{16}{5}\end{aligned}$$

5.

$$\begin{aligned}\int_{2}^{8}\left(\frac{4}{x}+x^2+x\right)dx &= \int_{2}^{8}\frac{4}{x}dx+\int_{2}^{8}x^2dx+\int_{2}^{8}x\,dx\\ &= 4\ln x]_{2}^{8}+\frac{x^3}{3}\Bigg]_{2}^{8}+\frac{x^2}{2}\Bigg]_{2}^{8}\\ &= 4\ln 8-44\ln 2+\frac{512}{3}-\frac{8}{3}+32-2\\ &= 203.55\end{aligned}$$

6. $\int_{2}^{4} e^{3x}dx = \frac{e^{3x}}{3}\Big]_{2}^{4} = \frac{e^{12}}{3}-\frac{e^{6}}{3}$

7.

$$\int_1^4 \frac{2}{x+3}dx = 2\ln(x+3)\Big]_1^4$$
$$= 2\ln(7) - 2\ln(4)$$

8.

$$F(9) - F(1) = \int_1^9 \sqrt{x}dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Bigg]_1^9 = \frac{2}{3}\left(\sqrt{x}\right)^3\Bigg]_1^9 = \frac{2}{3}\left[\left(\sqrt{9}\right)^3 - \left(\sqrt{1}\right)^3\right] = \frac{2}{3}(27-1) = \frac{52}{3}$$
$$\frac{F(9)-F(1)}{9-1} = \frac{\frac{52}{3}}{8} = \frac{52}{24} = \frac{13}{6}$$

9. By the Mean Value Theorem for derivatives there exits a c in $[a,b]$ such that

$$\int_1^4 f(x)\,dx = f(c)(b-a)$$
$$9 = f(c)(4-1)$$
$$9 = f(c)(3)$$
$$3 = f(c)$$

10. He is partially correct. The definite integral $\int_0^{2\pi} \sin x\, dx$ computes the net area under the curve. However, the area between the curve and the $x-$axis is given by $\int_0^{2\pi} \sin x\, dx - \cos x]_0^{\pi} = 2$.