

7.2 Integration by Parts

1. Let $u = 3x$ and $dv = e^x dx$.

Then $du = 3 dx$ and $v = e^x$.

$$\begin{aligned}\int 3xe^x dx &= uv - \int v du \\ &= 3xe^x - 3 \int e^x dx \\ &= 3xe^x - 3e^x + C\end{aligned}$$

2. Let $u = x^2$ and $dv = e^{-x} dx$.

Then $du = 2x dx$ and $v = -e^{-x}$.

$$\begin{aligned}\int x^2 e^{-x} dx &= uv - \int v du \\ &= -x^2 e^{-x} + \int 2xe^{-x} dx\end{aligned}$$

Let $u = 2x$ and $dv = e^{-x} dx$.

Then $du = 2 dx$ and $v = -e^{-x}$.

$$\begin{aligned}\int x^2 e^{-x} dx &= -x^2 e^{-x} + \int 2xe^{-x} dx \\ &= -x^2 e^{-x} + (uv - \int v du) \\ &= -x^2 e^{-x} + \left(-2xe^{-x} - \int -2xe^{-x} dx\right) \\ &= -x^2 e^{-x} - 2xe^{-x} + (-[-2(-e^{-x})]) + C \\ &= -x^2 e^{-x} - 2xe^{-x} - 2e^{-x} + C\end{aligned}$$

3. Let $u = \ln(3x+2)$ and $dv = 1 dx$. Then $du = \frac{3}{3x+2} dx$ and $v = x$.

$$\begin{aligned}\int \ln(3x+2) dx &= uv - \int v du \\ &= x \ln|3x+2| - \int x \frac{3}{3x+2} dx \\ &= x \ln|3x+2| - \int \frac{3x}{3x+2} dx\end{aligned}$$

Let $u = 3x+2$ and $x = \frac{u-2}{3}$. Then $du = 3 dx$.

$$\begin{aligned}
 \int \ln(3x+2) dx &= x \ln|3x+2| - \int \frac{3x}{3x+2} dx \\
 &= x \ln|3x+2| - \left(\int \frac{u-2}{3u} du \right) \\
 &= x \ln|3x+2| - \left(\int \left(\frac{1}{3} - \frac{2}{3u} \right) du \right) \\
 &= x \ln|3x+2| - \frac{1}{3}u + \frac{2}{3} \ln|u| \\
 &= x \ln|3x+2| - \frac{1}{3}(3x+2) + \frac{2}{3} \ln|3x+2|
 \end{aligned}$$

4. Let $u = \sin^{-1} x$ and $dv = 1 dx$. Then $du = \frac{1}{\sqrt{1-x^2}} dx$ and $v = x$.

$$\begin{aligned}
 \int \sin^{-1} x dx &= uv - \int v du \\
 &= x \sin^{-1} x - \int \frac{1}{\sqrt{1-x^2}} dx
 \end{aligned}$$

Let $u = 1 - x^2$. Then $du = -2x dx$.

$$\begin{aligned}
 \int \sin^{-1} x dx &= x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx \\
 &= x \sin^{-1} x + \frac{1}{2} \int u^{-\frac{1}{2}} du \\
 &= x \sin^{-1} x + \frac{1}{2} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C \\
 &= x \sin^{-1} x + \sqrt{1-x^2} + C
 \end{aligned}$$

5. $\int \sec^3 x dx = \int \sec^2 x \sec x dx$

Let $u = \sec x$ and $dv = \sec^2 x dx$. Then $du = \tan x dx$ and $v = \tan x$.

$$\begin{aligned}
 \int \sec^3 x dx &= \int \sec^2 x \sec x dx \\
 &= uv - \int v du \\
 &= (\sec x)(\tan x) - \int (\tan^2 x) \sec x dx \\
 &= (\sec x)(\tan x) - \int (\sec^2 x - 1) \sec x dx \\
 &= (\sec x)(\tan x) - \int \sec^3 x dx + \int \sec x dx \\
 2 \int \sec^3 x dx &= (\sec x)(\tan x) + \int \sec x dx \\
 &= (\sec x)(\tan x) + \ln|\sec x + \tan x| \\
 \int \sec^3 x dx &= \frac{1}{2} (\sec x)(\tan x) + \frac{1}{2} \ln|\sec x + \tan x| + C
 \end{aligned}$$

6. Let $u = \ln(3x)$ and $dv = 2x dx$. Then $du = \frac{3}{3x}dx = \frac{1}{x}dx$ and $v = x^2$.

$$\begin{aligned}\int 2x\ln(3x)dx &= uv - \int v du \\ &= x^2\ln(3x) - \int \frac{x^2}{x}dx \\ &= x^2\ln(3x) - \int x dx \\ &= x^2\ln(3x) - \frac{x^2}{2} \\ &= x^2\ln(3x) - \frac{x^2}{2} + C\end{aligned}$$

7. Let $u = \ln x$. Then $du = \frac{1}{x}dx$.

$$\begin{aligned}\int \frac{(\ln x)^2}{x}dx &= \int u^2 du \\ &= \frac{u^3}{3} + C \\ &= \frac{(\ln x)^2}{3} + C\end{aligned}$$

8. Let $u = 5x - 2$. Then $du = 5 dx$ and $x = \frac{u+2}{5}$.

$$\begin{aligned}\int x\sqrt{5x-2}dx &= \frac{1}{5} \int \left(\frac{u+2}{5}\right) \sqrt{u} du \\ &= \frac{1}{5} \int \frac{u^{\frac{3}{2}}}{5} + \frac{2u^{\frac{1}{2}}}{5} du \\ &= \frac{1}{5} \left(\frac{u^{\frac{5}{2}}}{5(\frac{5}{2})} + \frac{2u^{\frac{3}{2}}}{5(\frac{3}{2})} \right) \\ &= \frac{2(5x-2)^{\frac{5}{2}}}{125} + \frac{2(5x-2)^{\frac{5}{2}}}{75} + C\end{aligned}$$

Let $u = x$ and $dv = \sqrt{5x-2}dx$. Then $du = dx$ and $v = \frac{1}{5} \frac{(5x-2)^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{15}(5x-2)^{\frac{3}{2}}$.

$$\begin{aligned}\int x\sqrt{5x-2}dx &= \frac{2}{15}x(5x-2)^{\frac{3}{2}} - \int \frac{2}{15}(5x-2)^{\frac{3}{2}}dx \\ &= \frac{2}{15}x(5x-2)^{\frac{3}{2}} - \frac{2}{15} \times \frac{1}{5} \frac{(5x-2)^{\frac{5}{2}}}{\frac{5}{2}} \\ &= \frac{2}{15}x(5x-2)^{\frac{3}{2}} - \frac{4}{375}(5x-2)^{\frac{5}{2}} + C\end{aligned}$$

9. Let $u = x^2$ and $dv = e^{5x}dx$.

TABLE 7.1:

Alternate signs	u and its derivatives	dv and its antiderivatives
+	x^2	e^{5x}
-	$2x$	$\frac{1}{5}e^{5x}$
+	2	$\frac{1}{25}e^{5x}$
-	0	$\frac{1}{125}e^{5x}$

$$\int x^2 e^{5x} dx = \frac{1}{5}x^2 e^{5x} - \frac{2}{25}x e^{5x} + \frac{2}{125}e^{5x} + C$$

10. Let $u = x^2$ and $dv = e^x dx$.

TABLE 7.2:

Alternate signs	u and its derivatives	dv and its antiderivatives
+	x^2	e^x
-	$2x$	e^x
+	2	e^x
-	0	e^x

$$\begin{aligned} \int_0^1 x^2 e^x dx &= (x^2 e^x - 2x e^x + 2e^x) \Big|_0^1 \\ &= e - 2e + 2e - (2) \\ &= e - 2 \end{aligned}$$

11. Let $u = \ln(x+1)$ and $dv = 1 dx$. Then $du = \frac{1}{x+1} dx$ and $v = x$.

$$\begin{aligned} \int_1^3 \ln(x+1) dx &= x \ln(x+1) \Big|_1^3 - \int_1^3 \frac{x}{x+1} dx \\ &= x \ln(x+1) \Big|_1^3 - \int_1^3 \left(1 - \frac{1}{x+1}\right) dx \\ &= [x \ln(x+1) - x + \ln(x+1)] \Big|_1^3 \\ &= 3 \ln 4 - 3 + \ln 4 - (\ln 2 - 1 + \ln 2) \\ &= 4 \ln 4 - 2 - 2 \ln 2 \\ &= 8 \ln 2 - 2 \ln 2 - 2 \\ &= 6 \ln 2 - 2 \end{aligned}$$