7.1 Integration by Substitution

1. Let $u = x - 8$. Then $du = dx$.

$$\begin{aligned}\int \frac{3}{(x-8)^2}dx &= \int \frac{3}{u^2}du \\ &= \frac{3u^{-1}}{-1} + C \\ &= -\frac{3}{x-8} + C\end{aligned}$$

2. Let $u = 2 + x$. Then $du = dx$.

$$\begin{aligned}\int \sqrt{2+x}dx &= \int (2+x)^{\frac{-1}{2}}dx \\ &= \int u^{\frac{1}{2}}du \\ &= \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C \\ &= \frac{2}{3}u^{\frac{3}{2}} + C \\ &= \frac{2}{3}(2+x)^{\frac{3}{2}} + C\end{aligned}$$

3. Let $u = 2 + x$. Then $du = dx$.

$$\begin{aligned}\int \frac{1}{\sqrt{2+x}}dx &= \int (2+x)^{\frac{-1}{2}}dx \\ &= \int u^{\frac{-1}{2}}du \\ &= \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= 2u^{\frac{1}{2}} + C \\ &= 2\sqrt{2+x} + C\end{aligned}$$

4.

$$\begin{aligned}\frac{x^2}{x+1} &= x - 1 - \frac{1}{x+1} \\ \int \frac{x^2}{x+1}dx &= \int \left(x - 1 - \frac{1}{x+1}\right)dx \\ &= \frac{x^2}{2} - x - \ln|x+1| + C\end{aligned}$$

5. Let $u = e^{-x} + 2$. Then $du = -e^{-x}dx$.

$$\begin{aligned}\int \frac{e^{-x}}{e^{-x}+2}dx &= -\int \frac{1}{u}du \\ &= -\ln|u|+C \\ &= \ln|e^{-x}+2|+C\end{aligned}$$

6.

$$\begin{aligned}\frac{3\sqrt{t}+5}{t} &= \frac{3}{t^{\frac{1}{2}}} + \frac{5}{t} \\ \int \frac{3\sqrt{t}+5}{t}dt &= \int \left(\frac{3}{t^{\frac{1}{2}}} + \frac{5}{t}\right) dt \\ &= \frac{3t^{\frac{1}{2}}}{\frac{1}{2}} + 5\ln|t|+C \\ &= 6\sqrt{t} + 5\ln|t|+C\end{aligned}$$

7. Let $u = 3x - 1$. Then $du = 3\ dx$.

$$\begin{aligned}\int \frac{2}{\sqrt{3x-1}}dx &= 2\int \frac{1}{3}u^{\frac{-1}{2}}du \\ &= \frac{2}{3}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= \frac{4}{3}\sqrt{3x-1} + C\end{aligned}$$

8. Let $u = \sin x$. Then $du = \cos x\ dx$.

$$\begin{aligned}\int \sin x \cos x\ dx &= \int u\ du \\ &= \frac{u^2}{2} + C \\ &= \frac{1}{2}\sin^2 x + C\end{aligned}$$

9. Let $u = \cos x$. Then $du = -\sin x\ dx$.

$$\begin{aligned}\int \cos x \sqrt{1-\cos^2 x}dx &= -\int u\ du \\ &= -\frac{u^2}{2} + C \\ &= -\frac{1}{2}\cos^2 x + C\end{aligned}$$

10. Let $u = \sin x$. Then $du = \cos x dx$.

$$\begin{aligned}\int \sin^5 x \cos x \, dx &= \int u^5 \, du \\ &= \frac{u^6}{6} + C \\ &= \frac{1}{6}\sin^6 x + C\end{aligned}$$

11. Let $u = 4x^4$. Then $du = 16x^3 \, dx$.

$$\begin{aligned}\int x^3 \cos(4x^4) dx &= \frac{1}{16}\int \cos u \, du \\ &= \frac{1}{6}\sin u + C \\ &= \frac{1}{6}\sin\left(4x^4\right) + c\end{aligned}$$

12. Let $u = 2x + 4$. Then $du = 2 \, dx$.

$$\begin{aligned}\int \sec^2(2x+4) dx &= \frac{1}{2}\int \sec^2 u \, du \\ &= \frac{1}{2}\tan u + C \\ &= \frac{1}{2}\tan(2x+4) + C\end{aligned}$$

13. Let $u = x^2$. Then $du = 2x \, dx$.

Then if $x = 0$, then $u = 0$ and if $x = 2$, then $u = 4$.

$$\begin{aligned}\int_0^2 xe^{x^2} dx &= \frac{1}{2}\int_0^4 e^u du \\ &= \frac{1}{2}[e^u]_0^4 \\ &= \frac{1}{2}[e^4 - 1]\end{aligned}$$

14. Let $u = x^2$. Then $du = 2x \, dx$.

If $x = 0$ then $u = 0$. If $x = \sqrt{\pi}$, then $u = \pi$.

$$\begin{aligned}\int_0^{\sqrt{\pi}} x \sin\left(x^2\right) dx &= \frac{1}{2}\int_0^{\pi} \sin u \, du \\ &= -\frac{1}{2}[\cos u]_0^{\pi} \\ &= -\frac{1}{2}(\cos\pi - \cos 0) \\ &= -\frac{1}{2}(-1-1) \\ &= 2\end{aligned}$$

15. Let $u = x+5$. Then $du = dx$ and $x = u-5$.

If $x = 0$, then $u = 5$. If $x = 1$ then $u = 6$.

$$\begin{aligned}
\int_0^1 x(x+5)^4 dx &= \int_5^6 (u-5)u^4\ du \\
&= \int_5^6 (u^5 - 5u^4)\ du \\
&= \left[\frac{u^6}{6} - 5\frac{u^5}{5}\right]_5^6 \\
&= 7776 - 7776 - \left(\frac{5^6}{6} - 5^5\right) \\
&= 520\frac{5}{6}
\end{aligned}$$