Calculus/Limits/Exercises

Basic Limit Exercises

1. $\lim_{x \to 2} \left[4x^2 - 3x + 1 \right]$

2. $\lim_{x \to 5} \left[x^2 \right]$

One-Sided Limits

Evaluate the following limits or state that the limit does not exist.

3. $\lim_{x \to 0^-} \frac{x^3 + x^2}{x^3 + 2x^2}$

4. $\lim_{x \to 7^-} \left[|x^2 + x| - x \right]$

5. $\lim_{x \to -1^-} \sqrt{1 - x^2}$

6. $\lim_{x \to -1^+} \sqrt{1 - x^2}$

Two-Sided Limits

Evaluate the following limits or state that the limit does not exist.

7. $\lim_{x \to -1} \frac{1}{x - 1}$

8. $\lim_{x \to 4} \frac{1}{x - 4}$

9. $\lim_{x \to 2} \frac{1}{x - 2}$

10. $\lim_{x \to -3} \frac{x^2 - 9}{x + 3}$

11. $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

12. $\lim_{x \to -1} \frac{x^2 + 2x + 1}{x + 1}$

13. $\lim_{x \to -1} \frac{x^3 + 1}{x + 1}$

14. $\lim\limits_{x\to 4} \dfrac{x^2+5x-36}{x^2-16}$

15. $\lim\limits_{x\to 25} \dfrac{x-25}{\sqrt{x}-5}$

16. $\lim\limits_{x\to 0} \dfrac{|x|}{x}$

17. $\lim\limits_{x\to 2} \dfrac{1}{(x-2)^2}$

18. $\lim\limits_{x\to 3} \dfrac{\sqrt{x^2+16}}{x-3}$

19. $\lim\limits_{x\to -2} \dfrac{3x^2-8x-3}{2x^2-18}$

20. $\lim\limits_{x\to 2} \dfrac{x^2+2x+1}{x^2-2x+1}$

21. $\lim\limits_{x\to 3} \dfrac{x+3}{x^2-9}$

22. $\lim\limits_{x\to -1} \dfrac{x+1}{x^2+x}$

23. $\lim\limits_{x\to 1} \dfrac{1}{x^2+1}$

24. $\lim\limits_{x\to 1} \left[x^2+5x-\dfrac{1}{2-x}\right]$

25. $\lim\limits_{x\to 1} \dfrac{x^2-1}{x^2+2x-3}$

26. $\lim\limits_{x\to 1} \dfrac{5x}{x^2+2x-3}$

Limits to Infinity

Evaluate the following limits or state that the limit does not exist.

27. $\lim\limits_{x\to \infty} \dfrac{-x+\pi}{x^2+3x+2}$

28. $\lim\limits_{x\to -\infty} \dfrac{x^2+2x+1}{3x^2+1}$

29. $\lim\limits_{x\to-\infty} \dfrac{3x^2+x}{2x^2-15}$

30. $\lim\limits_{x\to-\infty} \left[3x^2-2x+1\right]$

31. $\lim\limits_{x\to\infty} \dfrac{2x^2-32}{x^3-64}$

32. $\lim\limits_{x\to\infty} 6$

33. $\lim\limits_{x\to\infty} \dfrac{3x^2+4x}{x^4+2}$

34. $\lim\limits_{x\to-\infty} \dfrac{2x+3x^2+1}{2x^2+3}$

35. $\lim\limits_{x\to-\infty} \dfrac{x^3-3x^2+1}{3x^2+x+5}$

36. $\lim\limits_{x\to\infty} \dfrac{x^2+2}{x^3-2}$

Limits of Piecewise Functions

Evaluate the following limits or state that the limit does not exist.

37. Consider the function

$$f(x)=\begin{cases}(x-2)^2 & \text{if } x<2\\ x-3 & \text{if } x\geq 2\end{cases}$$

a. $\lim\limits_{x\to2^-} f(x)$

b. $\lim\limits_{x\to2^+} f(x)$

c. $\lim\limits_{x\to2} f(x)$

38. Consider the function

$$g(x)=\begin{cases}-2x+1 & \text{if } x\leq 0\\ x+1 & \text{if } 0<x<4\\ x^2+2 & \text{if } x\geq 4\end{cases}$$

a. $\lim\limits_{x\to4^+} g(x)$

b. $\lim\limits_{x\to4^-} g(x)$

c. $\lim\limits_{x\to0^+} g(x)$

d. $\lim_{x \to 0^-} g(x)$

e. $\lim_{x \to 0} g(x)$

f. $\lim_{x \to 1} g(x)$

39. Consider the function

$$h(x) = \begin{cases} 2x - 3 & \text{if } x < 2 \\ 8 & \text{if } x = 2 \\ -x + 3 & \text{if } x > 2 \end{cases}$$

a. $\lim_{x \to 0} h(x)$

b. $\lim_{x \to 2^-} h(x)$

c. $\lim_{x \to 2^+} h(x)$

d. $\lim_{x \to 2} h(x)$

External Links

- Limits 1 exercise at Khan Academy (http://www.khanacademy.org/exercise/limits_1)
- Limits 2 exercise at Khan Academy (http://www.khanacademy.org/exercise/limits_2)
