5.5 Applications from Physics, Engineering, and Statistics

1.

$$\begin{aligned}W &= \int_0^{10} \frac{1}{x^2+1}dx \\ &= \tan^{-1}(x)|_0^{10} \\ &= \tan^{-1}(10) - \tan^{-1}(0) \\ &= 1.471 \text{ J}\end{aligned}$$

2.

$$\begin{aligned}W &= \int_1^5 \cos\left(\frac{\pi x}{2}\right) dx \\ u &= \frac{\pi x}{2} \\ du &= \frac{\pi}{2}dx \\ \frac{2}{\pi}du &= dx\end{aligned}$$

$x = 1$, then $u = \frac{\pi}{2}$

$x = 5$, then $u = \frac{5\pi}{2}$

$$\begin{aligned}W &= \int_1^5 \cos\left(\frac{\pi x}{2}\right) dx \\ &= \frac{2}{\pi}\int_{\frac{\pi}{2}}^{\frac{5\pi}{2}} \cos u\, du \\ &= \frac{2}{\pi}[\sin\ u]_{\frac{\pi}{2}}^{\frac{5\pi}{2}} \\ &= \frac{2}{\pi}\left(\sin\frac{5\pi}{2} - \sin\frac{\pi}{2}\right) \\ &= \frac{2}{\pi}(1-1) \\ &= 0 \text{ J}\end{aligned}$$

3. Be sure to convert kilometers to meters.

$$\begin{aligned}
W &= \int_{6,370}^{35,780} G\frac{mM}{r^2}dr \\
&= GmM\left(\frac{r^{-1}}{-1}\right)\Bigg|_{6,370}^{35,780} \\
&= GmM\left(\frac{1}{35,780,000}+\frac{1}{6,370,000}\right) \\
&= (6.67\times10^{-11})(1000)(6\times10^{24})\left(\frac{1}{35,780,000}+\frac{1}{6,370,000}\right) \\
&= (5.16\times10^{-6})10^{-11}\times10^{3}\times10^{24} \\
&= 5.16\times10^{10}\text{ J}
\end{aligned}$$

4. a.

$$\begin{aligned}
F(1) &= 5 \\
k(1) &= 5 \\
k &= 5 \\
k &= 5\text{ N/m}
\end{aligned}$$

b.

$$\begin{aligned}
W &= \int_{0}^{1.8} 5x\,dx \\
&= \left[\frac{5x^2}{2}\right]_0^{1.8} \\
&= \frac{5(1.8)^2}{2} \\
&= 8.1\text{ J}
\end{aligned}$$

5.

$$\begin{aligned}
F(3) &= \int 3 \\
k(3) &= 30 \\
k &= 10
\end{aligned}$$

$$\begin{aligned} W &= \int_{12}^{20} 10x\,dx \\ &= \left[\frac{10x^2}{2}\right]_{12}^{20} \\ &= [5x^2]_{12}^{20} \\ &= 2000 - 720 \\ &= 8.1 \text{ J} \end{aligned}$$

6. Yes.

$$F = \int_a^b wh(x)L(x)dx$$

Double w to $2w$.

Then the force F becomes $F = \int_a^b 2wh(x)L(x)dx = 2\int_a^b wh(x)L(x)dx$.

7.

$$\begin{aligned} d &= \frac{1}{4}x + 4 \\ w &= 10 \\ F &= 9800\int_0^{16} 10\left(\frac{1}{4}x+4\right)dx \\ &= 98,000\int_0^{16}\left(\frac{1}{4}x+4\right)dx \\ &= 98,000\left(\frac{x^2}{8}+4x\right)_0^{16} \\ &= 98,000(32+64) \\ &= 940,800 \text{ N} \end{aligned}$$

8. a. The integral represents the probability that a randomly chosen light bulb will have a lifetime between 1000 and 5000 hours.

b. The integral represents the probability that a randomly chosen light bulb will have a lifetime of at least 3000 hours.

9.a.

$$\begin{aligned}
P(x \le 3) &= \int_a^b \frac{1}{\mu} e^{-\frac{1}{\mu}x} dx \\
&= \int_0^3 \frac{1}{8} e^{-\frac{1}{8}x} dx \\
&= [-e^{-\frac{1}{8}x}]_0^3 \\
&= -e^{-\frac{3}{8}} - (-1) \\
&= 0.31 \\
&= 31\%
\end{aligned}$$

b. We can safely assume that a customer will not have to wait more than 100 minutes.

$$\begin{aligned}
P(x > 10) &= \int_a^b \frac{1}{\mu} e^{-\frac{1}{\mu}x} dx \\
&= \int_{10}^{100} \frac{1}{8} e^{-\frac{1}{8}x} dx \\
&= [-e^{-\frac{1}{8}x}]_{10}^{100} \\
&= -e^{-\frac{100}{8}} - (-e^{-\frac{10}{8}}) \\
&= 0.29 \\
&= 29\%
\end{aligned}$$

10. a.

$$\begin{aligned}
\mu &= 63.4 \text{ inches} \\
\sigma &= 3.2 \text{ inches}
\end{aligned}$$

Since 63.4 is the average and the heights form a normal distribution, then $P(x < 63.4)$ covers half of the probability. Thus,

$$\begin{aligned}
P(x < 63.4) &= \int_0^{63.4} \frac{1}{3.2\sqrt{2\pi}} e^{\frac{-(x-63.4)^2}{(2(3.2)^2)}} dx \\
&= 50\%
\end{aligned}$$

b.

$$\begin{aligned}
P(63 \le x \le 65) &= \int_{63}^{65} \frac{1}{3.2\sqrt{2\pi}} e^{\frac{-(x-63.4)^2}{(2(3.2)^2)}} dx \\
&= 24\%
\end{aligned}$$

c.

$$\begin{aligned} P(x > 72) &= \int_{72}^{\infty} \frac{1}{3.2\sqrt{2\pi}} e^{\frac{-(x-63.4)^2}{(2(3.2)^2)}}\, dx \\ &= 0.36\% \end{aligned}$$

d.

$$\begin{aligned} P(x = 60) &= \int_{60}^{60} \frac{1}{3.2\sqrt{2\pi}} e^{\frac{-(x-63.4)^2}{(2(3.2)^2)}}\, dx \\ &= 0\% \end{aligned}$$