

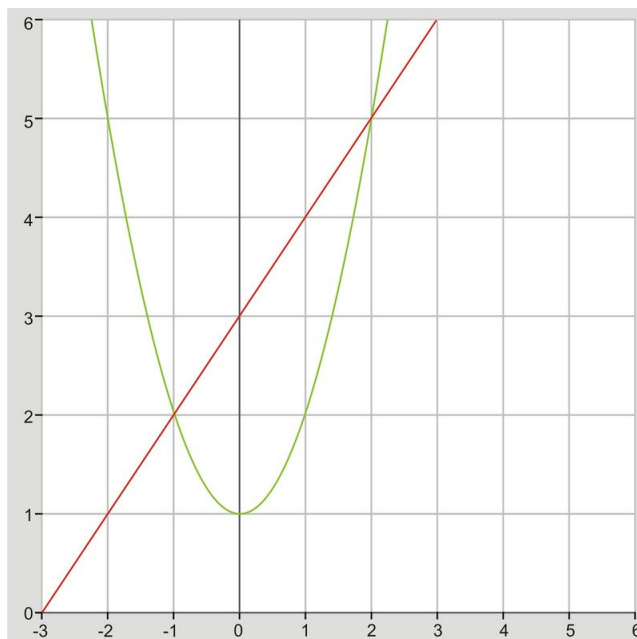
5.2 Volumes

Note: The graphs in this set of answers graph the functions on the coordinate plane so that you can find the limits of integration. They do not show the actual rotation.

1. The curve $y = \sqrt{9 - x^2}$ is the top half of a circle with radius 3. The limits of integration are $x = -3$ and $x = 3$. When rotated around the x -axis, we get a sphere of radius 3.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(3)^3 \\ &= 36\pi \end{aligned}$$

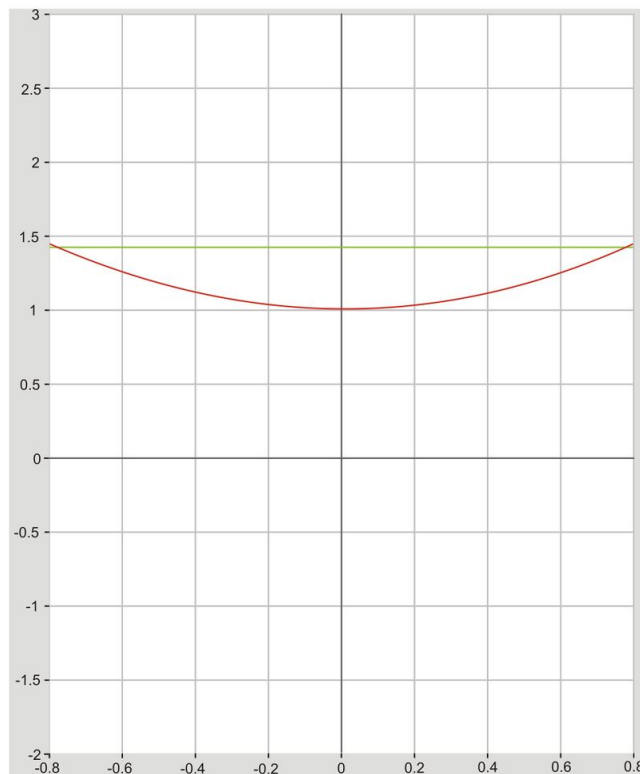
2.



$3 + x \geq 1 + x^2$ for x on $[-1, 2]$. Then

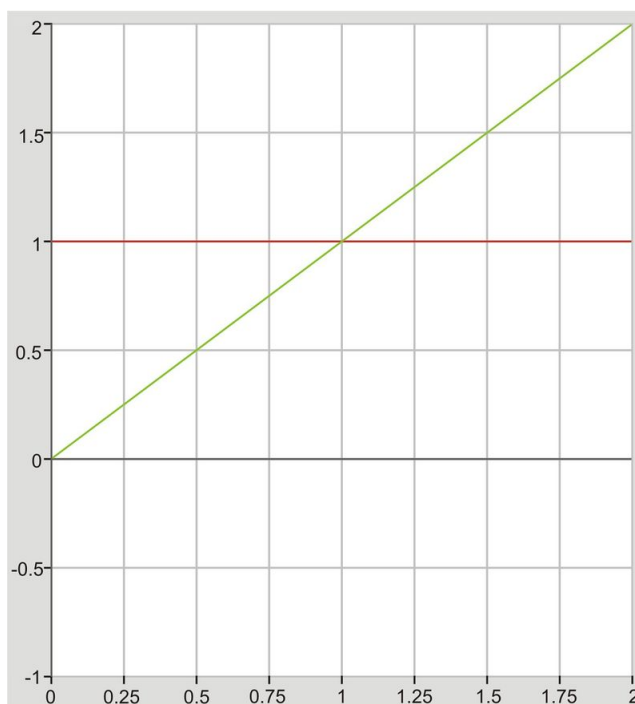
$$\begin{aligned}
 V &= \int_{-1}^2 \pi[(3+x)^2 - (1+x^2)^2] dx \\
 &= \int_{-1}^2 \pi[9 + 6x + x^2 - 1 - 2x^2 - x^4] dx \\
 &= \pi \left[9x + 3x^2 + \frac{x^3}{3} - x - \frac{2x^3}{3} - \frac{x^5}{5} \right] \bigg|_{-1}^2 \\
 &= \pi \left[18 + 12 + \frac{8}{3} - 2 - \frac{16}{3} - \frac{32}{5} + 9 - 3 + \frac{1}{3} - 1 - \frac{2}{3} - \frac{1}{5} \right] \\
 &= \frac{117\pi}{5}
 \end{aligned}$$

3.



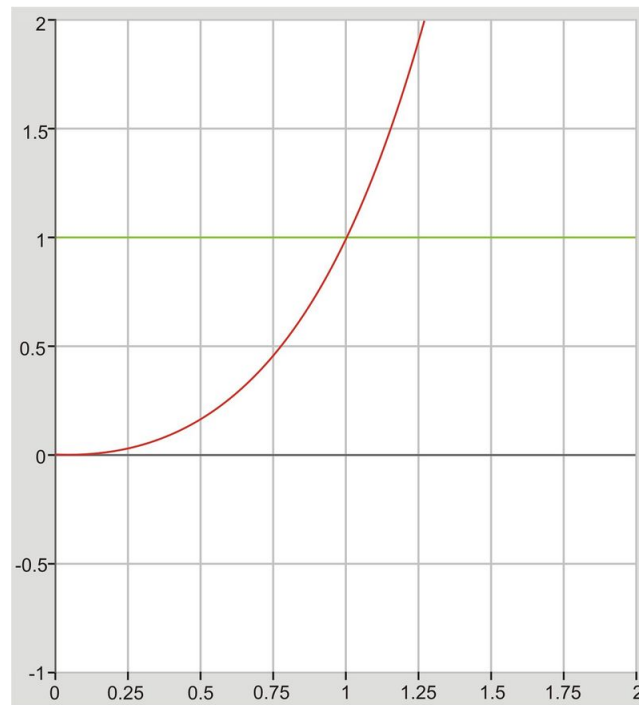
$$\begin{aligned}
 V &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \pi \left[(\sqrt{2})^2 - \sec^2 x \right] dx \\
 &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \pi [2 - \sec^2 x] dx \\
 &= [2\pi x - \pi \tan x] \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \\
 &= 2\pi \left(\frac{\pi}{4} \right) - \pi \tan \frac{\pi}{4} - \left(2\pi \left(-\frac{\pi}{4} \right) - \pi \tan \left(-\frac{\pi}{4} \right) \right) \\
 &= \pi^2 - \pi - \pi \\
 &= \pi^2 - 2\pi
 \end{aligned}$$

4.



$$\begin{aligned}
 V &= \int_0^1 \pi [(1)^2 - x^2] dx \\
 &= \left[\pi x - \frac{\pi x^3}{3} \right]_0^1 \\
 &= \pi - \frac{\pi}{3} \\
 &= \frac{2\pi}{3}
 \end{aligned}$$

5.



$$\begin{aligned}
 V &= \int_0^1 \pi [\sqrt[3]{y}]^2 dy \\
 &= \pi \frac{y^{\frac{5}{3}}}{\frac{5}{3}} \bigg|_0^1 \\
 &= \frac{3\pi}{5} y^{\frac{5}{3}} \bigg|_0^1 \\
 &= \frac{3\pi}{5}
 \end{aligned}$$

6.

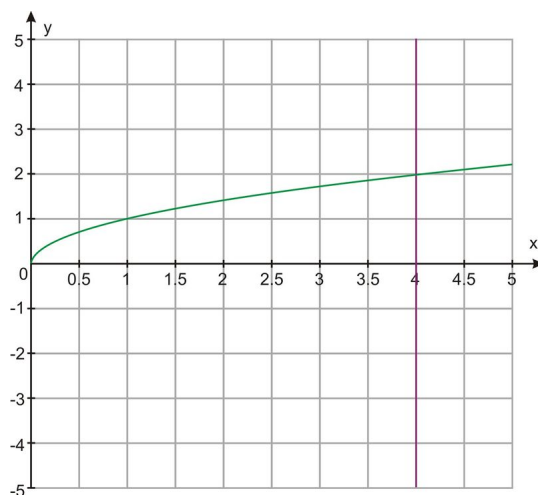


$$\begin{aligned}
 V &= \int_{-1}^2 \pi ([y+2]^2 - [y^2]^2) dy \\
 &= \int_{-1}^2 \pi (y^2 + 4y + 4 - y^4) dy \\
 &= \pi \left(\frac{y^3}{3} + \frac{4y^2}{2} + 4y - \frac{y^5}{5} \right) \bigg|_{-1}^2 \\
 &= \pi \left[\frac{8}{3} + 8 + \frac{32}{5} - \left(-\frac{1}{3} + 2 - 4 + \frac{1}{5} \right) \right] \\
 &= \pi \left(\frac{72}{5} \right) \\
 &= \frac{72\pi}{5}
 \end{aligned}$$

7.

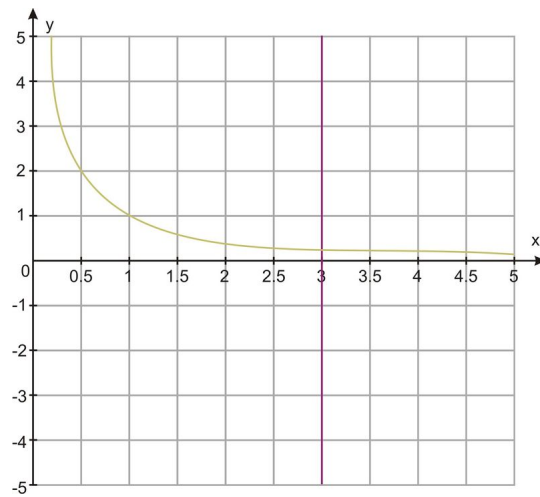
$$\begin{aligned}
 V &= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \pi (\csc^2 y - 0^2) dy \\
 &= -\pi \cot y \bigg|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \\
 &= -\pi \cot \left(\frac{3\pi}{4} \right) - \left[-\pi \cot \left(\frac{\pi}{4} \right) \right] \\
 &= \pi + \pi \\
 &= 2\pi
 \end{aligned}$$

8.



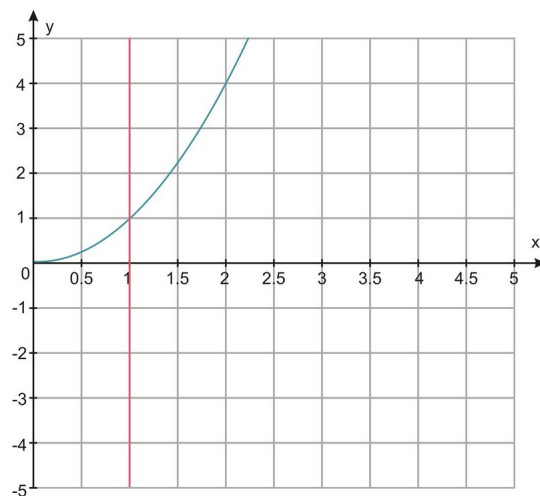
$$\begin{aligned}
 V &= \int_0^2 \pi (4^2 - (y^2)^2) dy \\
 &= \int_0^2 \pi (16 - y^4) dy \\
 &= \pi \left(16y - \frac{y^5}{5} \right) \Big|_0^2 \\
 &= \pi \left(32 - \frac{32}{5} \right) \\
 &= \frac{128\pi}{5}
 \end{aligned}$$

9.



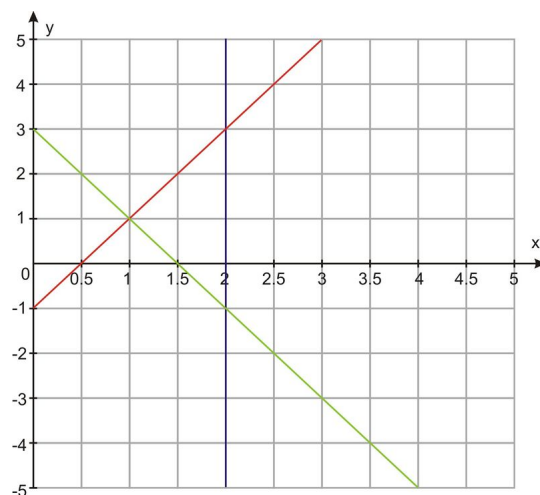
$$\begin{aligned}
 V &= \int_1^3 2\pi y \left(\frac{1}{y} \right) dy \\
 &= 2\pi y \Big|_1^3 \\
 &= 2\pi(3) - 2\pi \\
 &= 4\pi
 \end{aligned}$$

10.



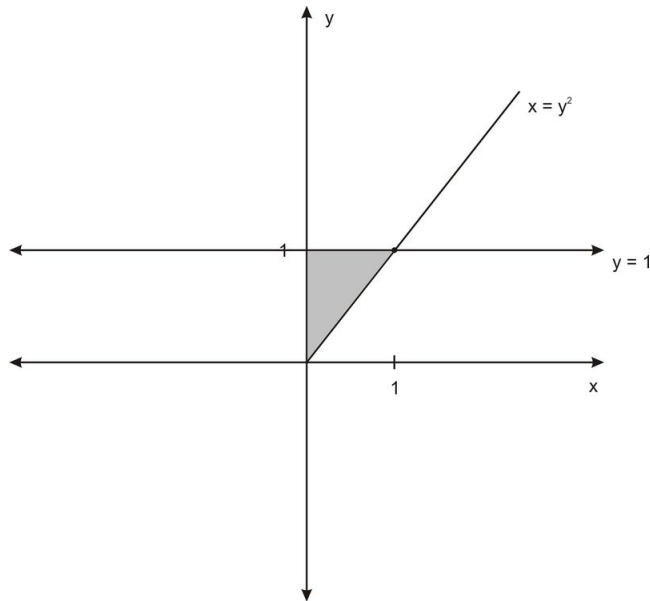
$$\begin{aligned}
 V &= \int_0^1 2\pi y (\sqrt{y}) dy \\
 &= \int_1^3 2\pi y^{\frac{3}{2}} dy \\
 &= 2\pi \frac{2}{5} y^{\frac{5}{2}} \Big|_0^1 \\
 &= \frac{4}{5} \pi (1) \\
 &= \frac{4\pi}{5}
 \end{aligned}$$

11.



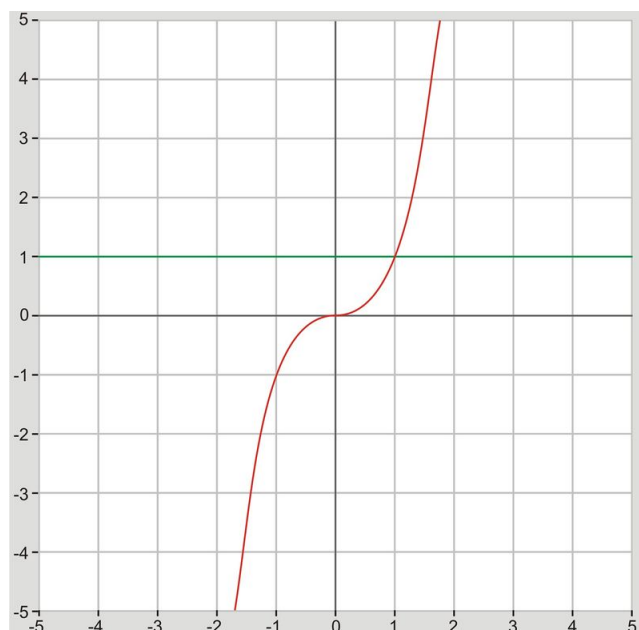
$$\begin{aligned}
 V &= \int_1^2 2\pi x(4x-4)dx \\
 &= \int_1^3 2\pi(4x^2-4x)dy \\
 &= 2\pi \times \left(\frac{4x^3}{3} - \frac{4x^2}{2} \right) \bigg|_1^2 \\
 &= 2\pi \left(\frac{32}{3} - 8 - \frac{4}{3} + 2 \right) \\
 &= 2\pi \left(\frac{10}{3} \right) \\
 &= \frac{20\pi}{3}
 \end{aligned}$$

12.



$$\begin{aligned}
 V &= \int_0^1 2\pi y(y^2)dy \\
 &= \int_0^1 2\pi y^3 dy \\
 &= 2\pi \times \frac{y^4}{4} \bigg|_0^1 \\
 &= 2 \times \frac{\pi}{4} \times (1) \\
 &= \frac{\pi}{2}
 \end{aligned}$$

13.



$$\begin{aligned} V &= 2\pi \int_0^1 (1-y)y^{\frac{1}{3}} dy \\ &= 2\pi \int_0^1 \left(y^{\frac{1}{3}} - y^{\frac{4}{3}}\right) dy \\ &= 2\pi \left[\frac{3}{4}y^{\frac{4}{3}} - \frac{3}{7}y^{\frac{7}{3}}\right]_0^1 \\ &= 2\pi \left(\frac{3}{4} - \frac{3}{7}\right) \\ &= \frac{18\pi}{28} \\ &= \frac{9\pi}{14} \end{aligned}$$