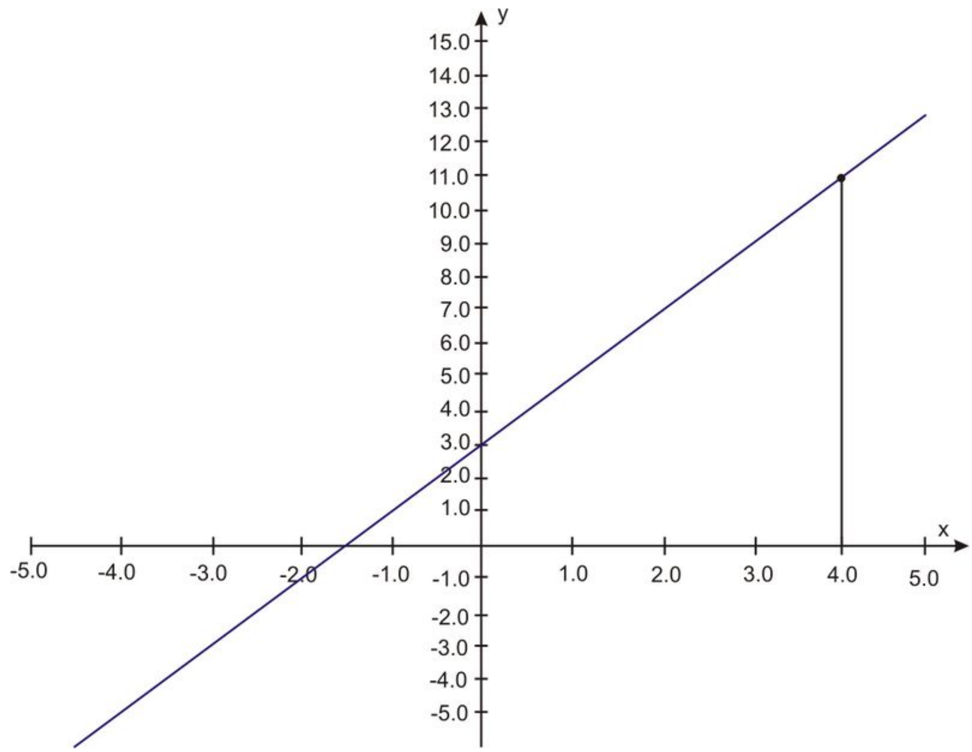
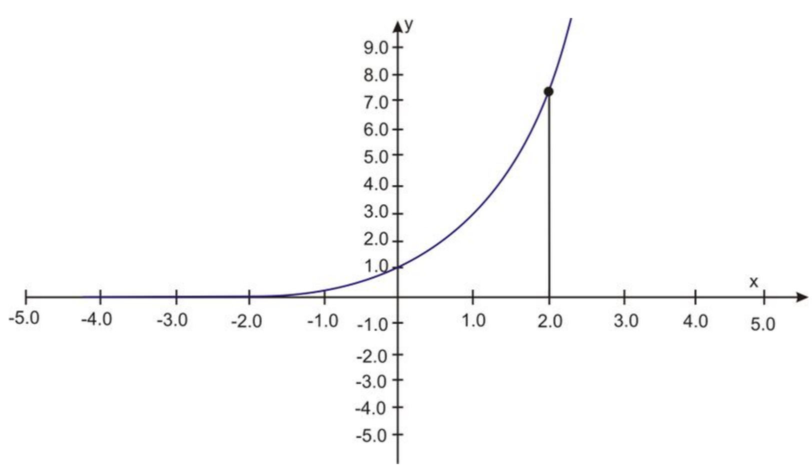
4.6 The Fundamental Theorem of Calculus

1.



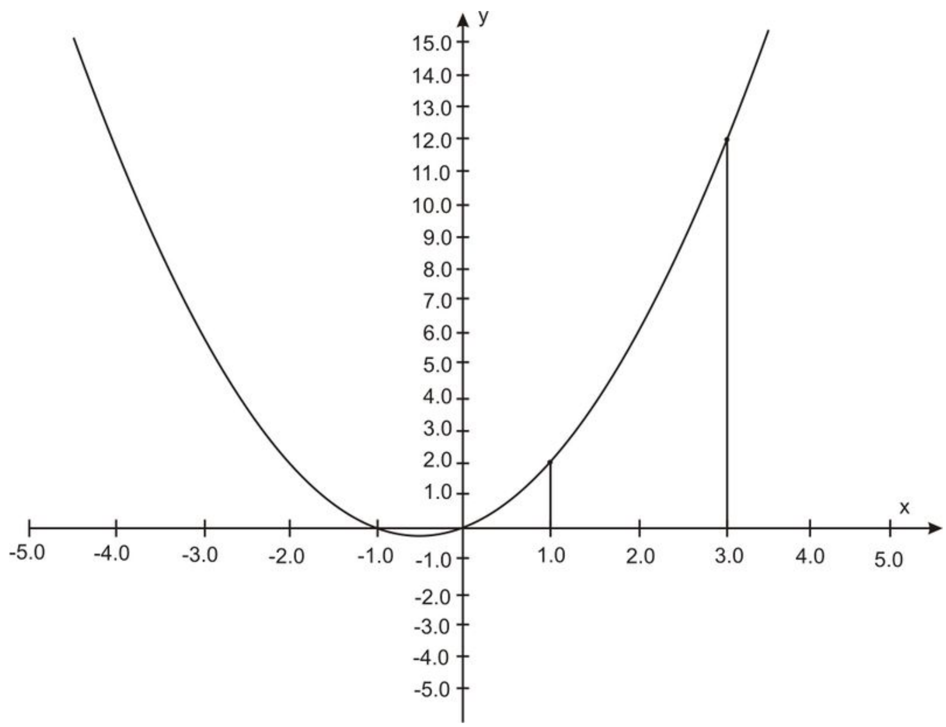
$$\begin{aligned}\int_0^4 (2x+3)\,dx &= \left(\frac{2x^2}{2}+3x\right)\Bigg|_0^4 \\ &= \frac{2(4)^2}{2}+3(4)-0 \\ &= 28\end{aligned}$$

2.



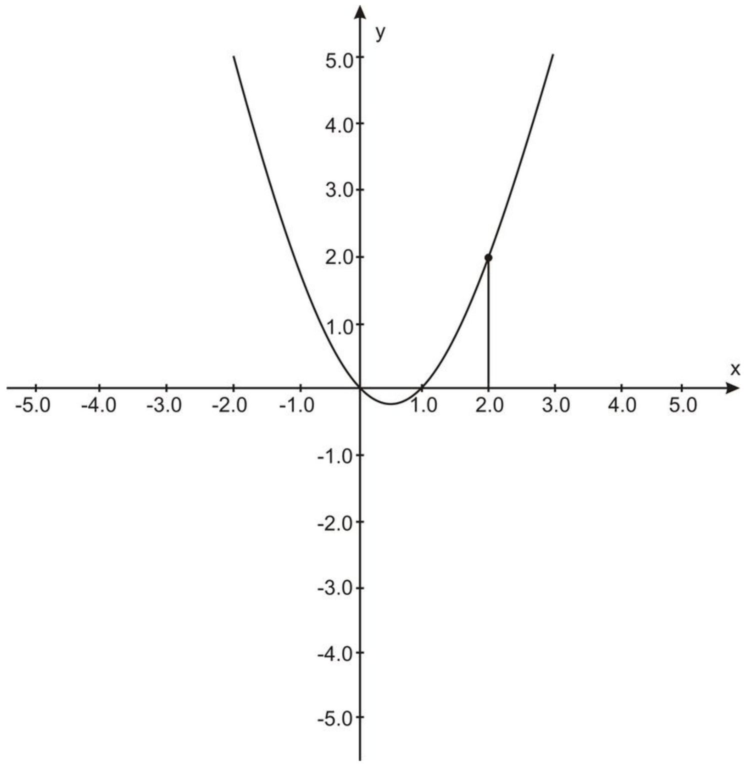
$$\int_0^2 e^x dx = e^x|_0^2 = e^2 - e^0 = e^2 - 1$$

3.



$$\begin{aligned}\int_1^3 \left(x^2+x\right) dx &= \left(\frac{x^3}{3}+\frac{x^2}{2}\right)\Big|_1^3 \\ &= \frac{27}{3}+\frac{9}{2}-\frac{1}{1}-\frac{1}{2} \\ &= \frac{26}{3}+4 \\ &= \frac{38}{3}\end{aligned}$$

4.



$$\begin{aligned}\int_{0}^{2}\left(x^2-x\right)dx &= \left(\frac{x^3}{3}+\frac{x^2}{2}\right)\Bigg|_0^2\\ &= \frac{8}{3}-\frac{4}{2}-0\\ &= \frac{8}{3}-2\\ &= \frac{2}{3}\end{aligned}$$

5.

$$\begin{aligned}\int_{-1}^{+1}|x|dx &= \int_{-1}^{0}-x\,dx+\int_{0}^{1}x\,dx\\ &= -\frac{x^2}{2}\Bigg|_{-1}^{0}+\frac{x^2}{2}\Bigg|_{0}^{1}\\ &= 0-\left(-\frac{1}{2}\right)+\frac{1}{2}-0\\ &= 1\end{aligned}$$

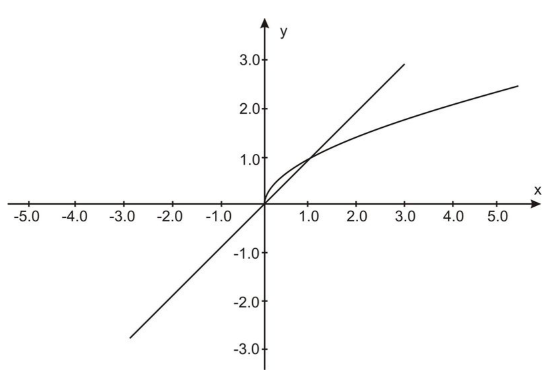
6. The graph has a zero at $x=\sqrt[3]{2}$. The absolute value of the function can be split at that zero.

$$\begin{aligned}
\int_{0}^{3} |x^3-2|dx &= \int_{0}^{\sqrt[3]{2}} \left(-x^3+2\right)dx + \int_{\sqrt[3]{2}}^{3} \left(x^3-2\right)dx \\
&= \left(-\frac{x^4}{4}+2x\right)\Bigg|_{0}^{\sqrt[3]{2}} + \left(-\frac{x^4}{4}+2x\right)\Bigg|_{\sqrt[3]{2}}^{3} \\
&= \left[-\frac{\left(\sqrt[3]{2}\right)^4}{4}+2\left(\sqrt[3]{2}\right)-0\right] + \left[\left(\frac{3^4}{4}\right)-2(3)-\left(\frac{\left(\sqrt[3]{2}\right)^4}{4}-2\left(\sqrt[3]{2}\right)\right)\right] \\
&= -\frac{2\sqrt[3]{2}}{4}+2\sqrt[3]{2}+\frac{81}{4}-\frac{24}{4}-\frac{2\sqrt[3]{2}}{4}+2\sqrt[3]{2} \\
&= \sqrt[3]{2}+42\sqrt[3]{2}+\frac{57}{4} \\
&= 3\sqrt[3]{2}+\frac{57}{4}
\end{aligned}$$

7.

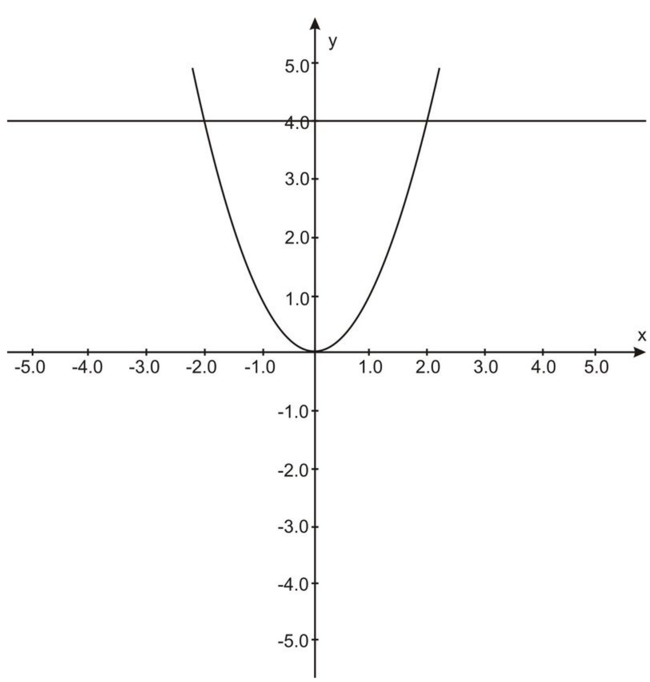
$$\begin{aligned}
\int_{-2}^{+4} [|x-1|+|x+1|]\,dx &= \int_{-2}^{+4} |x-1|dx + \int_{-2}^{+4} |x+1|dx \\
&= \int_{-2}^{1} (1-x)\,dx + \int_{1}^{4} (x-1)\,dx + \int_{-2}^{-1} (-x-1)\,dx + \int_{-1}^{4} (x+1)\,dx \\
&= \left(x-\frac{x^2}{2}\right)\Bigg|_{-2}^{1} + \left(\frac{x^2}{2}-x\right)\Bigg|_{1}^{4} + \left(-\frac{x^2}{2}-x\right)\Bigg|_{-2}^{-1} + \left(\frac{x^2}{2}+x\right)\Bigg|_{-1}^{4} \\
&= \left(1-\frac{1}{2}\right)-\left(-2-\frac{(-2)^2}{2}\right)+\left(\frac{(4)^2}{2}-4\right)-\left(\frac{1^2}{2}-1\right)+\left(-\frac{(-1)^2}{2}-(-1)\right)-\left(-\frac{(-2)^2}{2}-(-2)\right) \\
&= \frac{1}{2}-(4)+(8-4)-\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)-(0)+(12)-\left(-\frac{1}{2}\right) \\
&= 22
\end{aligned}$$

8. Graph the functions.



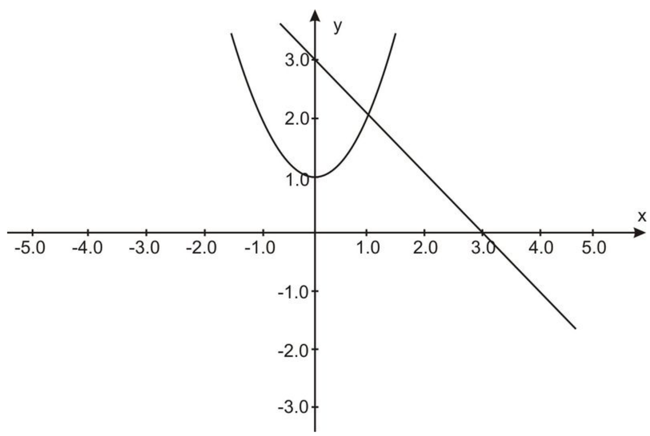
$$
\begin{aligned}
\text{Area} &= \int_0^2 \left(\sqrt{x} - x\right) dx \\
&= \int_0^1 \left(\sqrt{x} - x\right) dx + \int_1^2 \left(x - \sqrt{x}\right) dx \\
&= \int_0^1 x^{\frac{1}{2}} dx - \int_0^1 x\, dx + \int_0^2 x\, dx - \int_1^2 x^{\frac{1}{2}} dx \\
&= \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_0^1 - \frac{x^2}{2}\Bigg|_0^1 + \frac{x^2}{2}\Bigg|_1^2 - \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_1^2 \\
&= \frac{2}{3}(1)^{\frac{2}{3}} - \frac{1}{2} + \left(2 - \frac{1}{2}\right) - \frac{2}{3}\left(2^{\frac{3}{2}} - 1\right) \\
&= \frac{14}{6} - \frac{2}{3}\left(2\sqrt[3]{2}\right) \\
&= \frac{7 - 4\sqrt[3]{2}}{3}
\end{aligned}
$$

9.



$$\begin{aligned}\text{Area} &= \int_0^2 \left(4-x^2\right) dx \\ &= 4x|_0^2 - \left.\frac{x^3}{3}\right|_0^2 \\ &= 8-0-\frac{8}{3}-0 \\ &= \frac{16}{3}\end{aligned}$$

10.



$$\begin{aligned}\text{Area} &= \int_0^3 \left[\left(x^2+1\right)-(3-x)\right] dx \\ &= \int_0^1 \left(3-x-\left(x^2+1\right)\right) dx + \int_1^3 \left(x^2-1-(3-x)\right) dx \\ &= \int_0^1 \left(2-x-x^2\right) dx + \int_1^3 \left(x^2-2+x\right) dx \\ &= 2x|_0^1 - \left.\frac{x^2}{2}\right|_0^1 - \left.\frac{x^3}{3}\right|_0^1 + \left.\frac{x^3}{3}\right|_1^3 - 2x|_1^3 + \left.\frac{x^2}{2}\right|_1^3 \\ &= 2-\frac{1}{2}-\frac{1}{3}+\left(\frac{27}{3}-\frac{1}{3}\right)-(6-2)+\left(\frac{9}{2}-\frac{1}{2}\right) \\ &= \frac{59}{6}\end{aligned}$$