

## 7.1 Integration by Substitution

1. Let  $u = x - 8$ . Then  $du = dx$ .

$$\begin{aligned}\int \frac{3}{(x-8)^2} dx &= \int \frac{3}{u^2} du \\ &= \frac{3u^{-1}}{-1} + C \\ &= -\frac{3}{x-8} + C\end{aligned}$$

2. Let  $u = 2 + x$ . Then  $du = dx$ .

$$\begin{aligned}\int \sqrt{2+x} dx &= \int (2+x)^{\frac{1}{2}} dx \\ &= \int u^{\frac{1}{2}} du \\ &= \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C \\ &= \frac{2}{3} u^{\frac{3}{2}} + C \\ &= \frac{2}{3} (2+x)^{\frac{3}{2}} + C\end{aligned}$$

3. Let  $u = 2 + x$ . Then  $du = dx$ .

$$\begin{aligned}\int \frac{1}{\sqrt{2+x}} dx &= \int (2+x)^{-\frac{1}{2}} dx \\ &= \int u^{-\frac{1}{2}} du \\ &= \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= 2u^{\frac{1}{2}} + C \\ &= 2\sqrt{2+x} + C\end{aligned}$$

4.

$$\begin{aligned}\frac{x^2}{x+1} &= x - 1 - \frac{1}{x+1} \\ \int \frac{x^2}{x+1} dx &= \int \left( x - 1 - \frac{1}{x+1} \right) dx \\ &= \frac{x^2}{2} - x - \ln|x+1| + C\end{aligned}$$

5. Let  $u = e^{-x} + 2$ . Then  $du = -e^{-x}dx$ .

$$\begin{aligned}\int \frac{e^{-x}}{e^{-x}+2} dx &= -\int \frac{1}{u} du \\ &= -\ln|u| + C \\ &= \ln|e^{-x} + 2| + C\end{aligned}$$

6.

$$\begin{aligned}\frac{3\sqrt{t}+5}{t} &= \frac{3}{t^{\frac{1}{2}}} + \frac{5}{t} \\ \int \frac{3\sqrt{t}+5}{t} dt &= \int \left( \frac{3}{t^{\frac{1}{2}}} + \frac{5}{t} \right) dt \\ &= \frac{3t^{\frac{1}{2}}}{\frac{1}{2}} + 5\ln|t| + C \\ &= 6\sqrt{t} + 5\ln|t| + C\end{aligned}$$

7. Let  $u = 3x - 1$ . Then  $du = 3 dx$ .

$$\begin{aligned}\int \frac{2}{\sqrt{3x-1}} dx &= 2 \int \frac{1}{3} u^{-\frac{1}{2}} du \\ &= \frac{2}{3} \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= \frac{4}{3} \sqrt{3x-1} + C\end{aligned}$$

8. Let  $u = \sin x$ . Then  $du = \cos x dx$ .

$$\begin{aligned}\int \sin x \cos x dx &= \int u du \\ &= \frac{u^2}{2} + C \\ &= \frac{1}{2} \sin^2 x + C\end{aligned}$$

9. Let  $u = \cos x$ . Then  $du = -\sin x dx$ .

$$\begin{aligned}\int \cos x \sqrt{1 - \cos^2 x} dx &= -\int u du \\ &= -\frac{u^2}{2} + C \\ &= -\frac{1}{2} \cos^2 x + C\end{aligned}$$

10. Let  $u = \sin x$ . Then  $du = \cos x dx$ .

$$\begin{aligned}\int \sin^5 x \cos x \, dx &= \int u^5 \, du \\ &= \frac{u^6}{6} + C \\ &= \frac{1}{6} \sin^6 x + C\end{aligned}$$

11. Let  $u = 4x^4$ . Then  $du = 16x^3 \, dx$ .

$$\begin{aligned}\int x^3 \cos(4x^4) \, dx &= \frac{1}{16} \int \cos u \, du \\ &= \frac{1}{16} \sin u + C \\ &= \frac{1}{16} \sin(4x^4) + c\end{aligned}$$

12. Let  $u = 2x + 4$ . Then  $du = 2 \, dx$ .

$$\begin{aligned}\int \sec^2(2x + 4) \, dx &= \frac{1}{2} \int \sec^2 u \, du \\ &= \frac{1}{2} \tan u + C \\ &= \frac{1}{2} \tan(2x + 4) + C\end{aligned}$$

13. Let  $u = x^2$ . Then  $du = 2x \, dx$ .

Then if  $x = 0$ , then  $u = 0$  and if  $x = 2$ , then  $u = 4$ .

$$\begin{aligned}\int_0^2 x e^{x^2} \, dx &= \frac{1}{2} \int_0^4 e^u \, du \\ &= \frac{1}{2} [e^u]_0^4 \\ &= \frac{1}{2} [e^4 - 1]\end{aligned}$$

14. Let  $u = x^2$ . Then  $du = 2x \, dx$ .

If  $x = 0$  then  $u = 0$ . If  $x = \sqrt{\pi}$ , then  $u = \pi$ .

$$\begin{aligned}\int_0^{\sqrt{\pi}} x \sin(x^2) \, dx &= \frac{1}{2} \int_0^{\pi} \sin u \, du \\ &= -\frac{1}{2} [\cos u]_0^{\pi} \\ &= -\frac{1}{2} (\cos \pi - \cos 0) \\ &= -\frac{1}{2} (-1 - 1) \\ &= 2\end{aligned}$$

15. Let  $u = x + 5$ . Then  $du = dx$  and  $x = u - 5$ .

If  $x = 0$ , then  $u = 5$ . If  $x = 1$  then  $u = 6$ .

$$\begin{aligned}\int_0^1 x(x+5)^4 dx &= \int_5^6 (u-5)u^4 du \\ &= \int_5^6 (u^5 - 5u^4) du \\ &= \left[ \frac{u^6}{6} - 5\frac{u^5}{5} \right]_5^6 \\ &= 7776 - 7776 - \left( \frac{5^6}{6} - 5^5 \right) \\ &= 520\frac{5}{6}\end{aligned}$$