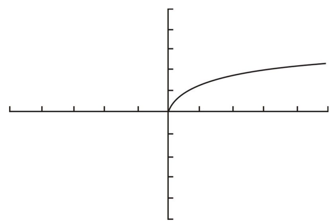
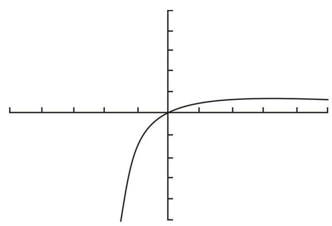
# 3.5 Limits at Infinity

1.



$\lim_{x\to+\infty}[x[\ln(x+3)-\ln(x)]]=3$

2.



$\lim_{x\to+\infty}\frac{x}{\ln(1+2e^x)}=1$

3. Since $\lim_{x\to3}\left(x^2-9\right)=\lim_{x\to3}(x-3)=0$, L'Hospital's Rule applies.

$\lim_{x\to3}\frac{x^2-9}{x-3}=\lim_{x\to3}\frac{2x}{1}=2(3)=6$

4. Since $\lim_{x\to0}\left(\sqrt{1+x}-\sqrt{1-x}\right)=\lim_{x\to0}=0$ , L'Hospital's Rule applies.

$$\begin{aligned}\lim_{x\to0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}&=\lim_{x\to0}\frac{(1+x)^{\frac{1}{2}}-(1-x)^{\frac{1}{2}}}{x}\\&=\lim_{x\to0}\frac{\frac{1}{2}(1+x)^{-\frac{1}{2}}-\frac{1}{2}(1-x)^{-\frac{1}{2}}(-1)}{1}\\&=\lim_{x\to0}\left(\frac{1}{2\sqrt{1+x}}+\frac{1}{2\sqrt{1-x}}\right)\\&=\frac{1}{2}+\frac{1}{2}=1\end{aligned}$$

5. Since $\lim_{x\to+\infty}\ln(x)=\lim_{x\to+\infty}\sqrt{x}=+\infty$, L'Hospital's Rule applies.

$\lim_{x\to+\infty}\frac{\ln(x)}{\sqrt{x}}=\lim_{x\to+\infty}\frac{\ln(x)}{x^{\frac{1}{2}}}=\lim_{x\to+\infty}\frac{\frac{1}{x}}{\frac{1}{2}x^{-\frac{1}{2}}}=\lim_{x\to+\infty}\frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}}=\lim_{x\to+\infty}\frac{2\sqrt{x}}{x}=\lim_{x\to+\infty}\frac{2}{\sqrt{x}}=0$

6. Since $\lim_{x\to+\infty}x^2=+\infty$ and $\lim_{x\to+\infty}e^{-2x}=0$, L'Hospital's Rule does not apply. Rewrite the limit as $\lim_{x\to+\infty}\frac{x^2}{e^{2x}}$ so that we can apply L'Hospital's Rule.

Then $\lim_{x\to+\infty}\frac{x^2}{e^{2x}}=\lim_{x\to+\infty}\frac{2x}{2e^{2x}}=\lim_{x\to+\infty}\frac{2}{4e^{2x}}=0$.

7. $\lim_{x\to0}(1-x)^{\frac{1}{x}}=\lim_{x\to0}e^{\ln(1-x)^{\frac{1}{x}}}=e^{\lim_{x\to0}\ln(1-x)^{\frac{1}{x}}}=e^{\lim_{x\to0}\frac{1}{x}\ln(1-x)}=e^{\lim_{x\to0}\frac{\ln(1-x)}{x}}$

Then we can apply L'Hospital's Rule because $\lim_{x\to 0}\ln(1-x)=\lim_{x\to 0}x=0$.

$$e^{\lim_{x\to 0}\frac{\ln(1-x)}{x}}=e^{\lim_{x\to 0}\frac{\frac{1}{1-x}(-1)}{1}}=e^{-1}=\frac{1}{e}$$

Thus, $\lim_{x\to 0}(1-x)^{\frac{1}{x}}=\frac{1}{e}$.

8. Since $\lim_{x\to 0}(e^x-1-x)=\lim_{x\to 0}x^2=0$, we can apply L'Hospital's Rule. We actually apply it two times.

$$\lim_{x\to 0}\frac{e^x-1-x}{x^2}=\lim_{x\to 0}\frac{e^x-1}{2x}=\lim_{x\to 0}\frac{e^x}{2}=\frac{1}{2}.$$

9. Since $\lim_{x\to -\infty}(e^x-1-x)=\lim_{x\to -\infty}x^2=\infty$, we can apply L'Hospital's Rule.

$$\lim_{x\to -\infty}\frac{e^x-1-x}{x^2}=\lim_{x\to -\infty}\frac{e^x-1}{2x}=\lim_{x\to -\infty}\frac{e^x}{2}=0.$$

10. $\lim_{x\to -\infty}x^{\frac{1}{4}}\ln(x)=\infty$