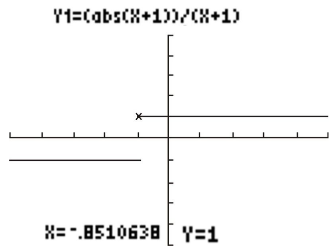
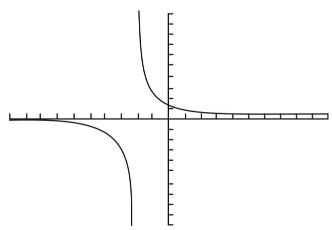
1.7 Continuity

1.



The function has a break at $x = -1$. It is continuous everywhere except for $x = -1$.

2.



The function has a break at $x = -2$. It is continuous everywhere except for $x = -2$.

3.

$$\begin{aligned}
\lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{1+\sqrt{x}}-1} &= \lim_{x\to 0^+} \frac{\sqrt{x}}{\left(\sqrt{1+\sqrt{x}}-1\right)} \times \frac{\left(\sqrt{1+\sqrt{x}}+1\right)}{\left(\sqrt{1+\sqrt{x}}+1\right)} \\
&= \lim_{x\to 0^+} \frac{\sqrt{x}\left(\sqrt{1+\sqrt{x}}+1\right)}{1+\sqrt{x}-1} \\
&= \lim_{x\to 0^+} \frac{\sqrt{x}\left(\sqrt{1+\sqrt{x}}+1\right)}{\sqrt{x}} \\
&= \lim_{x\to 0^+} \sqrt{1+\sqrt{x}}+1 \\
&= \sqrt{1+\sqrt{10}}+1 \\
&= 1+1=2
\end{aligned}$$

4.

$$\begin{aligned}
\lim_{x\to 2^-} \frac{x^3-8}{|x-2|(x-2)} &= \lim_{x\to 2^-} \frac{x^3-8}{(x-2)(x-2)} \\
&= \lim_{x\to 2^-} \frac{(x-2)\left(x^2+2x+4\right)}{-(x-2)(x-2)} \\
&= \lim_{x\to 2^-} \frac{\left(x^2+2x+4\right)}{-x+2}
\end{aligned}$$

x	$y=\dfrac{x^3-8}{\vert x-2\vert(x-2)}$
1.99	1194.08
1.999	$11,994$
1.9999	$119,994$

$$\lim_{x\to 2^-}\frac{x^3-8}{|x-2|(x-2)}=\infty$$

5. For $x>1,\ |x-1|=x-1$. Then $\lim_{x\to 1^+}\frac{2x|x-1|}{(x-1)}=\lim_{x\to 1^+}\frac{2x(x-1)}{(x-1)}=\lim_{x\to 1^+}2x=2.$

6. For $x>2, |x+2|=x+2$. Then

$$\lim_{x\to 2^+}\frac{|x+2|+x+2}{|x+2|-x-2}=\lim_{x\to 2^+}\frac{(x+2)+x+2}{(x+2)-x-2}$$

7. $f(-2)=(-2)^3+2(-2)^2-(-2)+1=-8+8+2+1=3$

$f(-3)=(-3)^3+2(-3)^2-(-3)+1=-27+18+3+1=-5$

By the Intermediate Value Theorem, there is an $x-$value c with $f(c)=0$.

8. $f(9)=\sqrt{9}-\sqrt[3]{9}-1=-0.08$

$f(10)=\sqrt{10}-\sqrt[3]{10}-1=0.008$

By the Intermediate Value Theorem, there is an $x-$value c with $f(c)=0$.

9. The value $x=a$ is considered a maximum because it is a high point in the graph and the graph turns back down. Because it is not the highest point in the interval, $x=a$ is called a relative maximum. The value $x=c$ is a maximum of the interval and is the absolute maximum because it is the highest maximum The value $x=b$ is a minimum of the interval and is, in fact, an absolute minimum of the interval because it is the lowest value of the interval. The value $x=d$ is neither a maximum nor a minimum.

10. [Note: I think that this question should be replaced with an easier question - see comments on pdf file.]