

1.5 Finding Limits

1. a.

x	$y = \frac{x^2 - 4}{x + 2}$
-1.9	$\frac{(-1.9)^2 - 4}{-1.9 + 2} = -3.9$
-1.99	-3.99
-1.999	-3.999
-2.1	-4.1
-2.099	-4.099
-2.0099	-4.0099

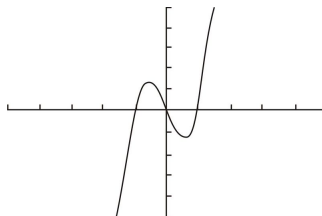
b. The table shows that the sequence of values approach -4 . Thus, $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x + 2} = -4$.

2. a.

x	$y = \frac{2x - 1}{2x^2 + 3x - 2}$
0.49	$\frac{2(0.49) - 1}{2(0.49)^2 + 3(0.49) - 2} = 0.4016$
0.495	0.4008
0.49999	0.4000
0.51	0.3984
0.5099	0.3984
0.500001	0.3999

b. $\lim_{x \rightarrow \frac{1}{2}} \frac{2x - 1}{2x^2 + 3x - 2} = 0.4$, or $\frac{2}{5}$.

3. This is a graph of $p(x) = 3x^3 - 3x$:



a. Use the calculator to generate tables to determine the limit.

X	Y1
4.04	185.7
4.05	187.14
4.06	188.59
4.07	190.05
4.08	191.51
4.09	192.98
4.1	194.46

X=4.04

X	Y1
3.95	173.04
3.96	174.42
3.97	175.8
3.98	177.19
3.99	178.59
4	180
4.01	181.41

X=4.01

$$\lim_{x \rightarrow 4} (3x^3 - 3x) = 180b.$$

X	Y1
-4.04	-185.7
-4.03	-184.3
-4.02	-182.8
-4.01	-181.4
-4	-180
-3.99	-178.6
-3.98	-177.2

X=-4.04

$$\lim_{x \rightarrow -4} (3x^3 - 3x) = -180$$

c.

X	Y1
-0.003	.009
-0.002	.006
-0.001	.003
0	0
.001	-.003
.002	-.006
.003	-.009

X=-.003

X	Y1
-0.997	.01792
-0.998	.01196
-0.999	.00599
-1	0
-1.001	-.006
-1.002	-.012
-1.003	-.0181

X=-.997

$$\lim_{x \rightarrow 0} (3x^3 - 3x) = 0$$

d.

X	Y1
4	180
-4	-180
0	0

X=

The function values are the same as the limits because the function is defined at those values.

4. a. $\lim_{x \rightarrow 3} f(x) = 1.5$ because $f(3) = 1.5$.

b. $\lim_{x \rightarrow 2} f(x) = 0$ because $f(2) = 0$.

c. $\lim_{x \rightarrow 1} f(x) = 2$ because $f(1) = 2$.

- d. $\lim_{x \rightarrow 4} f(x)$ does not exist because the right-hand limit and the left-hand limit are not the same.
5. a. $\lim_{x \rightarrow 2} f(x) = 0$ because $f(2) = 0$.
- b. $\lim_{x \rightarrow 0} f(x)$ because the function is not defined at $x = 0$.
- c. $\lim_{x \rightarrow 4} f(x)$ is a number close to 1 but less than 1 because of the horizontal asymptote of $y = 1$.
- d. $\lim_{x \rightarrow 50} f(x)$ is a number close to 1 but less than 1 because of the horizontal asymptote of $y = 1$.
6. Use a graphing calculator to make a table of values to find the limit.

X	Y1
1.96	6.8416
1.97	6.8809
1.98	6.9204
1.99	6.9601
2	7
2.01	7.0401
2.02	7.0804

X=1.96

The limit exists and $\lim_{x \rightarrow 2} (x^2 + 3) = 7$

7.

X	Y1
-0.98	-0.5051
-0.99	-0.5025
-1	ERROR
-1.01	-0.4975
-1.02	-0.495
-1.03	-0.4926
-1.04	-0.4902

X=-1.04

The limit exists and $\lim_{x \rightarrow -1} \frac{x+1}{x^2-1} = -0.5$. (Note: you will get an error if you try to find $f(-1)$ directly. The limit of values of both sides are approaching -0.5 , but the function is not defined at $x = -1$.)

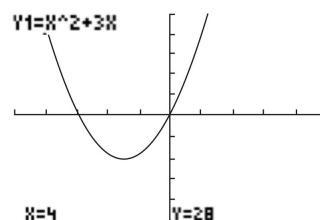
8.

X	Y1
1.98	1.0198
1.99	1.01
2	1
2.01	.98995
2.02	.9798
2.03	.96954
2.04	.95917

X=2.04

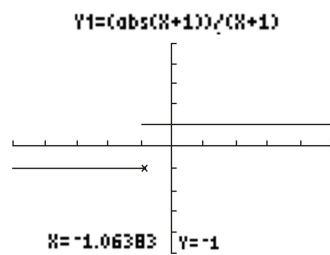
The limit exists and $\lim_{x \rightarrow 2} \sqrt{-2x+5} = 1$.

9.



The limit exists and $\lim_{x \rightarrow 2} (x^2 + 3x) = 28$.

10.



The limit does not exist. Note that there is a break in the graph when $x = -1$.