1.1 Equations and Graphs

1. Let $x = 1$. Then find the corresponding y.

$$\begin{aligned} 2x - 3y &= 5 \\ 2(1) - 3y &= 5 \\ 2 - 3y &= 5 \\ -3y &= 5 - 2 \\ -3y &= 3 \\ y &= -1 \end{aligned}$$

$(1, -1)$ is one solution.

Let $x = 4$. Then find the corresponding y.

$$\begin{aligned} 2x - 3y &= 5 \\ 2(4) - 3y &= 5 \\ 8 - 3y &= 5 \\ -3y &= -3 \\ y &= 1 \end{aligned}$$

$(4, 1)$ is another solution.

To find the $x-$intercept, let $y = 0$ and solve for x.

$$\begin{aligned} 2x - 3y &= 5 \\ 2x - 3(0) &= 5 \\ 2x &= 5 \\ x &= \frac{5}{2} \end{aligned}$$

The $x-$intercept is $\left(\frac{5}{2}, 0\right)$.

To find the $x-$intercept, let $x = 0$ and solve for y.

$$\begin{aligned} 2x - 3y &= 5 \\ 2(0) - 3y &= 5 \\ -3y &= 5 \\ y &= -\frac{5}{3} \end{aligned}$$

The $y-$intercept is $\left(0, -\frac{5}{3}\right)$.

The equation gives a linear relationship between x and y. Its graph can be sketched through any two solutions. There is no symmetry.

2. Solve for y:

$$\begin{aligned} 3x^2 - y &= 5 \\ -y &= -3x^2 + 5 \\ y &= 3x^2 - 5 \end{aligned}$$

If $x = 1$, then $y = 3(1)^2 - 5 = 3 - 5 = -2$. One solution is $(1, -2)$.

If $x = -1$, then $y = 3(-1)^2 - 5 = 3 - 5 = -2$. Another solution is $(-1, -2)$.

To find the $x-$intercepts, let $y = 0$.

$$\begin{aligned} y &= 3x^2 - 5 \\ 0 &= 3x^2 - 5 \\ 5 &= 3x^2 \\ \frac{5}{3} &= x^2 \\ \pm\sqrt{\frac{5}{3}} &= x \end{aligned}$$

The $x-$intercepts are $\left(\pm\sqrt{\frac{5}{3}}, 0\right)$.

To find the $y-$intercept, let $x = 0$.

$$\begin{aligned} y &= 3(0)^2 - 5 \\ y &= -5 \end{aligned}$$

The $y-$intercept is $(0, -5)$.

The graph is a parabola with $a = 3, b = 0$, and $c = -5$. It is symmetric with respect to $x = \frac{-b}{2a} = 0$, which is the $y-$axis.

3. Use a graphing calculator. Enter the relationship on the $Y =$ menu. Look at the table of points. There are many solutions, such as $(2,6)$ and $(-2,-6)$. The $x-$intercepts are $(0,0), (1,0), (-1,0)$. The $y-$intercept is $(0,0)$. By inspection, the graph is symmetric about the origin.

4. Use a graphing calculator. Enter the relationship on the $Y =$ menu. Look at the table of points. There are many solutions, such as $(2,0)$ and $(-1,6)$. The $x-$intercepts are $(0,0), (-3,0), (2,0)$. The $y-$intercept is $(0,0)$. By inspection, the graph has no symmetry.

5. The best answer is b. Even though the values of both cars are falling, the value of the BMW is always greater than that of the Chevy for any value of t.

6. Graph c is the best representation because you would expect a decline as soon as you bought the car and you would expect that the value would decline more gradually after the initial drop.

7. a. Let ℓ represent the length of the pool and let w represent the width of the pool. Then $A(w) = l \times w = (w + 25)w = w^2 + 25w$.

b.

$$
\begin{aligned}
264 &= w^2 + 25w \\
0 &= w^2 + 25w + 264 \\
0 &= (w+33)(w-8) \\
0 &= w+33 \quad \text{or} \quad 0 = w-8 \\
-33 &= w \qquad\quad \text{or} \quad 8 = w
\end{aligned}
$$

The width is 8 yards. The length is $8+25=33$ yards.

8. The rate of change will be $\frac{8{,}500-18{,}000}{2008-2004}=\frac{-9{,}500}{4}=-2,375$. Since the rate of depreciation is constant, the formula for the changing value of the car is linear. Because at time 0, the value of the car is $\$18,000$, the $y-$intercept of the formula is $18,000$. The formula is $y=-2,375x+18,000$.

9. The formula for the changing value of the car is $y=-2,375x+18,000$. When $x=7, y=-2,375(7)+18,000=-16,625+18,000=\$1,375$.

10. A linear model may not be the best function to model depreciation because the graph of the function decreases as time increases; hence at some point the value will take on negative real number values, an impossible situation for the value of real goods and products.