Avogadro's number and the Mole

Introduction to the mole concept in chemistry

The chemical changes we observe always involve *discrete numbers of atoms* that rearrange themselves into new configurations. These numbers are HUGE— far too large in magnitude for us to count or even visualize, but they are still *numbers*, and we need to have a way to deal with them. We also need a bridge between these numbers, which we are unable to measure directly, and the weights of substances, which we do measure and observe. The *mole concept* provides this bridge, and is central to all of quantitative chemistry.

1 Counting atoms: Avogadro's number

Owing to their tiny size, atoms and molecules cannot be counted by direct observation. But much as we do when "counting" beans in a jar, we can estimate the number of particles in a sample of an element or compound if we have some idea of the volume occupied by each particle and the volume of the container.

Once this has been done, we know the number of formula units (to use the most general term for any combination of atoms we wish to define) in any arbitrary weight of the substance. The number will of course depend both on the formula of the substance and on the weight of the sample. But if we consider a weight of substance *that is the same as its formula (molecular) weight expressed in grams, we have only one number to know*: ***Avogadro's number***, $6.022141527 \times 10^{23}$, usually designated by N_A.

Amadeo Avogadro (1766-1856) never knew his own number! Avogadro only originated the *concept* of this number, whose actual value was first estimated by Josef Loschmidt, an Austrian chemistry teacher, in 1895.

You should know it to three significant figures:
$N_A = 6.02 \times 10^{23}$

6.02×10^{23} of *what*? Well, of anything you like: apples, stars in the sky, burritos. But the only *practical* use for N_A is to have a more convenient way of expressing the huge numbers of the tiny particles such as atoms or molecules that we deal with in chemistry. Avogadro's number is a *collective number*, just like a dozen.

Think of 6.02×10^{23} as the "chemist's dozen".

YouTube **videos on Avogadro's Number and the mole**

Before we get into the use of Avogadro's number in problems, take a moment to convince yourself of the reasoning embodied in the following examples.

Problem Example 1: mass ratio from atomic weights
The atomic weights of oxygen and of carbon are 16.0 and 12.0, respectively. How much heavier is the oxygen atom in relation to carbon?

Solution: Atomic weights represent the relative masses of different kinds of atoms. This means that the atom of oxygen has a mass that is 16/12 = **4/3** ≈ 1.33 as great as the mass of a carbon atom.

Problem Example 2: Mass of a single atom
The absolute mass of a carbon atom is 12.0 unified atomic mass units (What are these?). How many grams will a single oxygen atom weigh?

Solution: The absolute mass of the carbon atom is 12.0 **u**,

or $12 \times 1.6605 \times 10^{-27}$ g = 19.9×10^{-27} kg. The mass of the oxygen atom will be 4/3 greater, or $\mathbf{2.66 \times 10^{-26}}$ **kg**.

Alternatively: (12 g/mol) ÷ (6.022×10^{23} mol^{-1}) × (4/3) = $\mathbf{2.66 \times 10^{-23}}$ **g**.

Problem Example 3: Relative masses from atomic weights
Suppose that we have *N* carbon atoms, where *N* is a number large enough to give us a pile of carbon atoms whose mass is 12.0 grams. How much would the same number, *N*, of oxygen atoms weigh?

Solution: The mass of an oxygen atom (16 u) is 16/12 = 4/3 that of a carbon atom (12 u), so the collection of *N* oxygen atoms would have a mass of
4/3 × 12 g = **16.0 g**.

(If the foregoing problems don't make sense, you should review the previous lesson.)

Things to understand about Avogadro's number N_A

• It is a *number*, just as is "dozen", and thus is *dimensionless*.

• It is a *huge* number, far greater in magnitude than we can visualize; see here for some interesting comparisons with other huge numbers.

• Its practical use is limited to counting tiny things like atoms, molecules, "formula units", electrons, or photons.

• The value of N_A can be known only to the precision that the number of atoms in a measurable weight of a substance can be estimated. Because large numbers of atoms cannot be counted directly, a variety of ingenious indirect measurements have been made involving such things as Brownian motion and X-ray scattering.

• The current value was determined by measuring the distances between the atoms of silicon in an ultrapure crystal of this element that was shaped into a perfect sphere. (The measurement was made by X-ray scattering.) When combined with the measured mass of this sphere, it yields Avogadro's number. But there are two problems with this: 1) The silicon sphere is an artifact, rather than being something that occurs in nature, and thus may not be perfectly reproducible. 2) The standard of mass, the kilogram, is not precisely known, and its value appears to be changing. For these reasons, there are proposals to revise the definitions of both N_A and the kilogram. See here for more, and stay tuned!

2 Moles and their uses

The ***mole*** (abbreviated mol) is the the SI measure of *quantity of a "chemical entity"*, which can be an atom, molecule, formula unit, electron or photon. One mol of anything is just Avogadro's number of that something. Or, if you think like a lawyer, you might prefer the official SI definition:

> The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12

Avogadro's number $N_A = 6.02 \times 10^{23}$, like any pure number, is dimensionless. However, it also defines the mole, so we can also express N_A as

6.02×10^{23} mol^{-1}; in this form, it is properly known as ***Avogadro's constant***. This construction emphasizes the role of Avogadro's number as a *conversion factor* between number of moles and number of "entities".

Problem Example 3: number of moles in *N* particles

How many moles of nickel atoms are there in 80 nickel atoms?

Solution: (80 atoms) / (6.02E23 atoms mol^{-1}) = **1.33E-22 mol**

Is this answer reasonable? Yes, because 80 is an extremely small fraction of N_A.

Molar mass (atomic weight) of a formula unit

YouTube (IsaacsTeach, 6½ m) ****

The atomic weight, molecular weight, or formula weight of one mole of the fundamental units (atoms, molecules, or groups of atoms that correspond to the formula of a pure substance) is the ratio of its mass to 1/12 the mass of one mole of C^{12} atoms, and being a ratio, is dimensionless. But at the same time, this ***molar mass*** (as many now prefer to call it) is also the observable mass of one mole (N_A) of the substance, so we frequently emphasize this by stating it explicitly as so many grams (or kilograms) per mole: g mol^{-1}.

> Don't let this confuse you; it is very important always to bear in mind that the mole is a *number* and not a *mass*. But each individual particle has a mass of its own, so a mole of any specific substance will always *correspond to* a certain mass of that substance.

Problem Example 4: Boron content of borax

Borax is the common name of sodium tetraborate, $Na_2B_4O_7$. In 20.0 g of borax,

(a) how many moles of boron are present?

(b) how many grams of boron are present?

Solution: The formula weight of $Na_2B_4O_7$ is (2 × 23.0) + (4 × 10.8) + (7 × 16.0) = 201.2.

a) 20 g of borax contains (20.0 g) ÷ (201 g mol^{-1}) = 0.10 mol of borax, and thus **0.40 mol** of B.

b) 0.40 mol of boron has a mass of (0.40 mol) × (10.8 g mol^{-1}) = **4.3 g**.

Problem Example 5: Magnesium in chlorophyll

The plant photosynthetic pigment chlorophyll contains 2.68 percent magnesium by weight. How many atoms of Mg will there be in 1.00 g of chlorophyll?

Solution: Each gram of chlorophyll contains 0.0268 g of Mg, atomic weight 24.3.

Number of moles in this weight of Mg: (.0268 g) / (24.2 g mol^{-1}) = 0.00110 mol

Number of atoms: (.00110 mol) × (6.02E23 mol^{-1}) = **6.64E20**

Is this answer reasonable? (Always be suspicious of huge-number answers!) Yes, because we would expect to have huge numbers of atoms in any observable quantity of a substance.

Molar volume of a pure substance

This is the volume occupied by one mole of a pure substance. Molar volume depends on the density of a substance and, like density, varies with temperature owing to thermal expansion, and also with the pressure. For solids and liquids, these variables ordinarily have little practical effect, so the values quoted for 1 atm pressure and 25°C are generally useful over a fairly wide range of conditions. This is definitely not the case with gases, whose molar volumes must be calculated for a specific temperature and pressure.

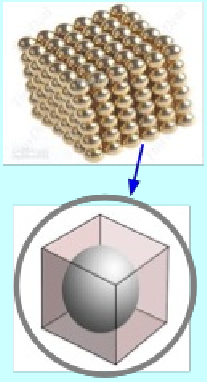
Problem Example 6 : Molar volume of a liquid

Methanol, CH_3OH, is a liquid having a density of 0.79 g per milliliter. Calculate the molar volume of methanol.

Solution: The molar volume will be the volume occupied by one molar mass (32 g) of the liquid. Expressing the density in liters instead of mL, we have

$$V_M = (32 \text{ g mol}^{-1}) / (790 \text{ g L}^{-1}) = \mathbf{0.0405 \text{ L mol}^{-1}}$$

The molar volume of a metallic element allows one to estimate the size of the



atom. The idea is to mentally divide a piece of the metal into as many little cubic boxes as there are atoms, and then calculate the length of each box. Assuming that an atom sits in the center of each box and that each atom is in direct contact with its six neighbors (two along each dimension), this gives the diameter of the atom. The manner in which atoms pack together in actual metallic crystals is usually more complicated than this and it varies from metal to metal, so this calculation only provides an approximate value.

Problem Example 7: Radius of a strontium atom

The density of metallic strontium is 2.60 g cm^{-3}. Use this value to estimate the radius of the atom of Sr, whose atomic weight is 87.6.

Solution: The molar volume of Sr is (87.6 g mol^{-1}) / (2.60 g cm^{-3}) = 33.7 cm^3 mol^{-1}

The volume of each "box" is (33.7 cm^3 mol^{-1}) / (6.02E23 mol^{-1}) = 5.48E–23 cm^3
The side length of each box will be the cube root of this value, 3.79E–8 cm. The atomic radius will be half this value, or 1.9E–8 cm = 1.9E–10 m = **190 pm**.

Note: Your calculator probably has no cube root button, but you *are* expected to be able to find cube roots; you can usually use the x^y button with y=0.333. You should also be able estimate the magnitude of this value for checking. The easiest way is to express the number so that the exponent is a multiple of 3. Take 54.8E–24, for example. Since 3^3=27 and 4^3 = 64, you know that the cube root of 55 will be between 3 and 4, so the cube root should be a bit less than 4×10^{-8}.

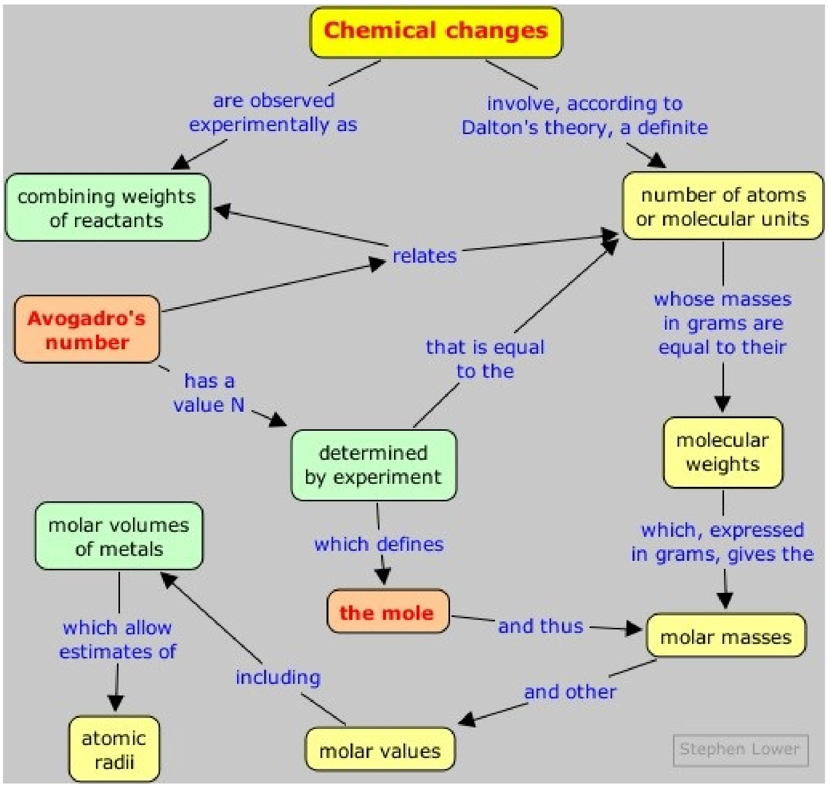
So how good is our atomic radius? Standard tables give the atomic radius of strontium is in the range 192-220 pm, depending on how it is defined.

What you should be able to do

Make sure you thoroughly understand the following essential ideas which have been presented above. It is especially important that you know the precise meanings of all the highlighted terms in the context of this topic.

- Define ***Avogadro's number*** and explain why it is important to know.
- Define the ***mole***. Be able to calculate the number of moles in a given mass of a substance, or the mass corresponding to a given number of moles.
- Define ***molecular weight***, ***formula weight***, and ***molar mass***; explain how the latter differs from the first two.
- Be able to find the number of atoms or molecules in a given weight of a substance.
- Find the ***molar volume*** of a solid or liquid, given its density and molar mass.
- Explain how the molar volume of a metallic solid can lead to an estimate of atomic diameter.

Concept Map


Chemical changes
are observed experimentally as
involve, according to Dalton's theory, a definite
combining weights of reactants
number of atoms or molecular units
relates
Avogadro's number
whose masses in grams are equal to their
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Stephen Lower