14.01 Principles of Microeconomics, Fall 2007
Chia-Hui Chen
November 7, 2007

Lecture 22

Monopoly

Outline

1. Chap 10: *Monopoly*
2. Chap 10: *Shift in Demand and Effect of Tax*

1 Monopoly

The monopolist is the single supply-side of the market and has complete control over the amount offered for sale; the monopolist controls price but must operate along consumer demand.

1.1 Revenue in Monopoly

Review the revenue in perfect competition:

$$R = PQ \tag{1.1}$$

$$AR = MR = P. \tag{1.2}$$

Revenue of monopolist is also

$$R = P(Q)Q,$$

but P changes with Q because the monopolist faces the whole market demand and his quantity supplied affects the market price. Then the average revenue is

$$AR = \frac{R}{Q} = P(Q);$$

and the marginal revenue is

$$MR = \frac{dR}{dQ} = \frac{d(PQ)}{dQ} = P(Q) + Q\frac{dP}{dQ}.$$

The relation between P and Q is determined by the demand curve (see Figure 1). Since

$$\frac{dP}{dQ} < 0,$$

$$MR < P(Q).$$

Example (A Demand Function). Suppose the price is

$$P = 10 - Q_D,$$

where Q_D is the quantity demanded. Calculate the average revenue and the marginal revenue:

$$AR = P = 10 - Q;$$

$$MR = p + Q\frac{dP}{dQ} = 10 - 2Q.$$

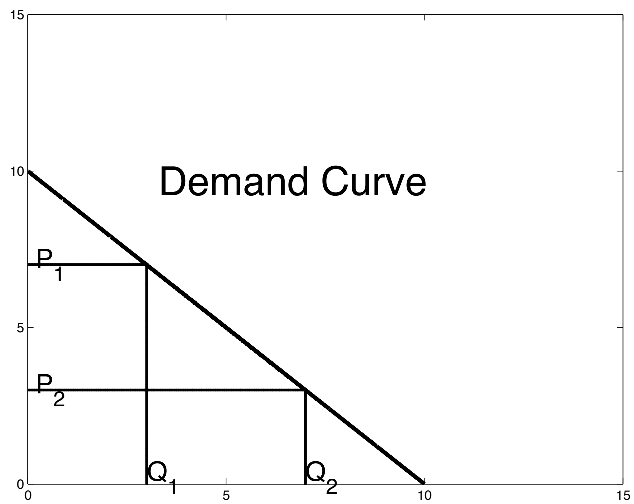

Figure 1: Demand and Supply of Monopolist.

1.2 Output Decision in Monopoly

The monopolist will maximize its profit

$$\pi(Q) = R(Q) - C(Q),$$

which is the difference of revenue and cost. When maximized,

$$\frac{d\pi}{dQ} = \frac{dR}{dQ} - \frac{dC}{dQ} = 0,$$

namely,

$$MR = MC,$$

so the monopolist would choose this point to produce; because

$$P > MR,$$

$$P > MC.$$

The profit equals to

$$(AR - AC)Q = (P - AC)Q$$

(see Figure 2).

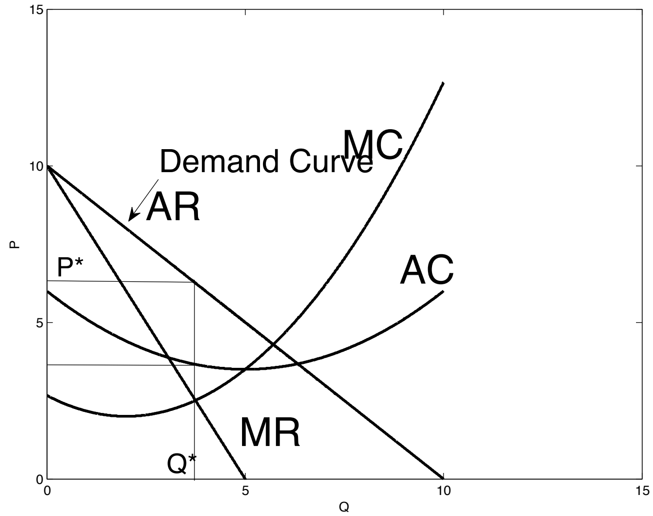

Figure 2: Output Decision of Monopolist.

1.3 Lerner's Index

Rewrite the marginal revenue:

$$MR = P + Q\frac{dP}{dQ} = P + P(\frac{Q}{P}\frac{dP}{dQ}) = P + P\frac{1}{E_D}.$$

The monopolist chooses to produce the quantity where

$$MC = MR = P + P\frac{1}{E_D}.$$

Thus,

$$\frac{1}{|E_D|} = \frac{P - MC}{P}, \tag{1.3}$$

which is the makeup over MC as a percentage of price; this fraction is less than 1. $L = \frac{P-MC}{P}$ measures the monopoly power of a firm and is called Lerner's index.

- In a competitive market,

$$MC = P,$$

and the makeup is zero.

- In a monopolistic market,

$$MC < P,$$

and the makeup is larger than zero.

Comments:

1. The makeup increases with the inverse of demand elasticity.
2. The larger the demand elasticity, the less profitable it is to be a monopolist (see Figure 3 and 4).
3. A monopolist never produces a quantity at the inelastic portion of demand curve, since the makeup right hand side of Equation 1.3 is less than one.

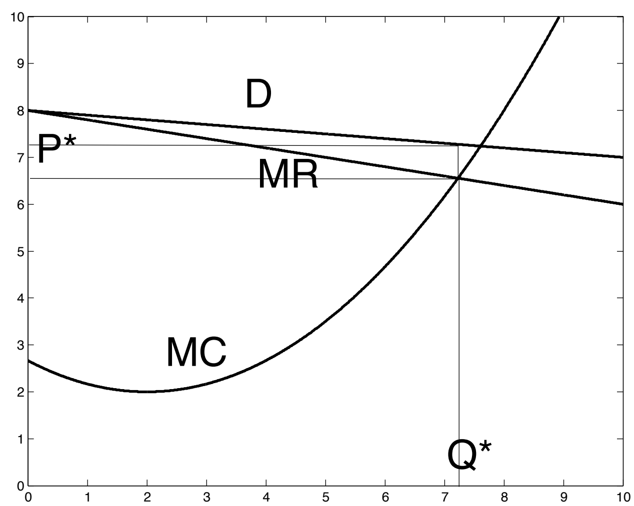

Figure 3: Inelastic Demand.

2 Shift in Demand and Effect of Tax

Compare the competitive market and the monopolistic markets.

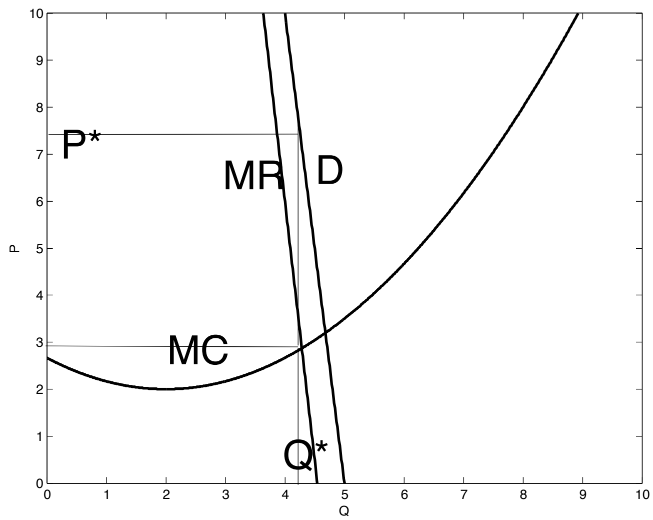

Figure 4: Elastic Demand.

2.1 Supply Curve of Competitive Market and Monopolistic Markets

The supply curve in competitive markets is determined by MC, and there is no supply curve for monopolistic markets.

2.2 Shift in Demand

In competitive markets, when demand shifts, the changes in price and quantity has a positive relation, namely, if the price raises, the quantity increases. In monopolistic markets, when the demand shifts, it may be the case that only price changes (see Figure 5), only quantity changes (see Figure 6), or both change.

2.3 Effect of Tax

In competitive marketes, buyer's prices raise less than the tax, and the burden is shared by Producers and Consumers; in monopolistic markets, the price might raise more than tax (see Figure 7).

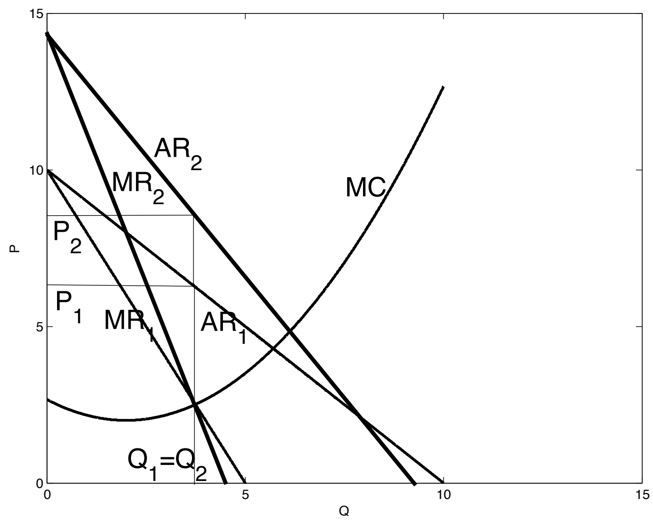

Figure 5: Only Price Change in Monopoly.

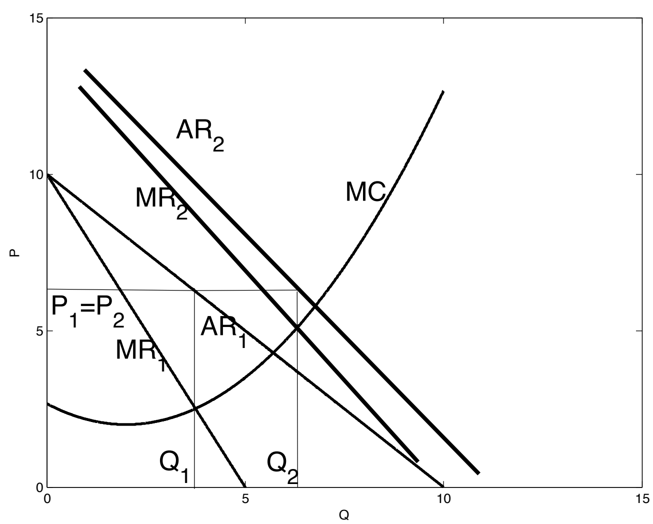

Figure 6: Only Quantity Change in Monopoly.

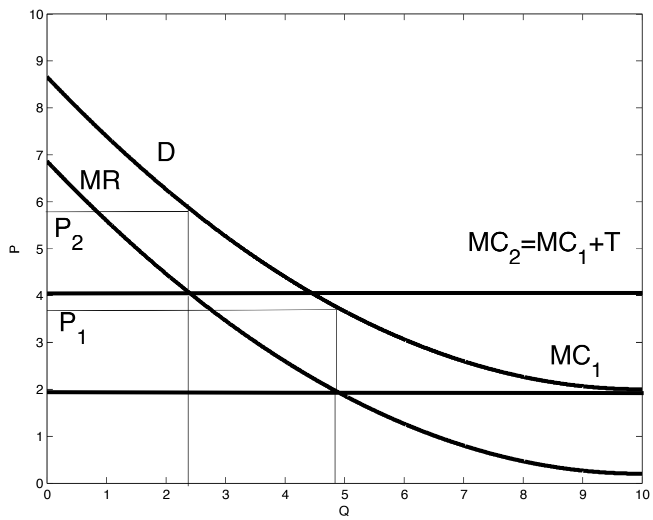

Figure 7: Price Might Raise More than Tax.

14.01 Principles of Microeconomics, Fall 2007
Chia-Hui Chen
November 9, 2007

Lecture 23

Monopoly and Monopsony

Outline

1. Chap 10: *Multi-Plant Firm*
2. Chap 10: *Social Cost of Monopoly Power*
3. Chap 10: *Price Regulation*
4. Chap 10: *Monopsony*

1 Multi-Plant Firm

How does a monopolist allocate production between plants?
Suppose the firm produces quantity Q_1 with cost $C_1(Q_1)$ for plant 1, and quantity Q_2 with cost $C_2(Q_2)$ for plant 2. The total quantity is

$$Q_T = Q_1 + Q_2.$$

And the profit is

$$\pi = Q_T P(Q_T) - C_1(Q_1) - C_2(Q_2) = (Q_1 + Q_2)P(Q_1 + Q_2) - C_1(Q_1) - C_2(Q_2).$$

To solve, use the first order constraint:

$$\frac{d\pi}{dQ_1} = P(Q_1 + Q_2) + (Q_1 + Q_2)\frac{dP(Q_1 + Q_2)}{dQ_1} - \frac{dC_1}{dQ_1} = 0,$$

Since

$$P(Q_T) + Q_T\frac{dP(Q_T)}{dQ_1} = P(Q_T) + Q_T\frac{dP(Q_T)}{dQ_T} = MR(Q_T),$$

$$MR(Q_T) = MC_1(Q_1).$$

Similarly,

$$MR(Q_T) = MC_2(Q_2).$$

Thus,

$$MR(Q_T) = MC_1(Q_1) = MC_2(Q_2).$$

2 Social Cost of Monopoly Power

Firstly, compare the producer and consumer surplus in a competitive market and a monopolistic market. In the competitive market, the quantity is determined by

$$MC = AR,$$

while in the monopolistic market, the quantity is determined by

$$MC = MR$$

(see Figure 1). Therefore, in going from a perfectly competitive market to a

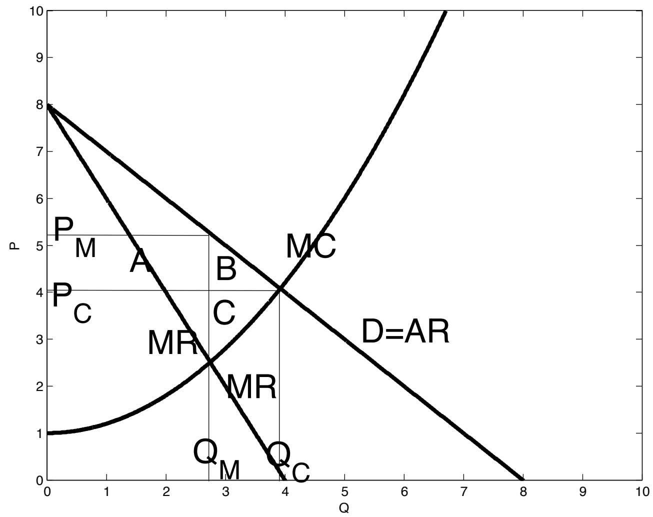

Figure 1: Consumer and Producer Surplus in Monopolist Market.

monopolistic market, the change of consumer surplus and producer surplus are, respectively,

$$\Delta CS = -(A + B),$$

and

$$\Delta PS = A - C.$$

The deadweight loss is

$$DWL = B + C.$$

In fact, social cost should not only include the deadweight loss but also rent seeking. The firm might spend to gain monopoly power by lobbying the government and building excess capacity to threaten opponents.

3 Price Regulation

In perfectly competitive markets, price regulation causes deadweight loss, but in monopoly, price regulation might improve efficiently. Now we discuss four possible price regulations in monopolistic markets. P_1, P_2, P_3, P_4 are:

- $$P_1 \in (P_C, P_M);$$
- $$P_2 = P_C;$$
- $$P_3 \in (P_0, P_C);$$
- $$P_4 < P_0.$$

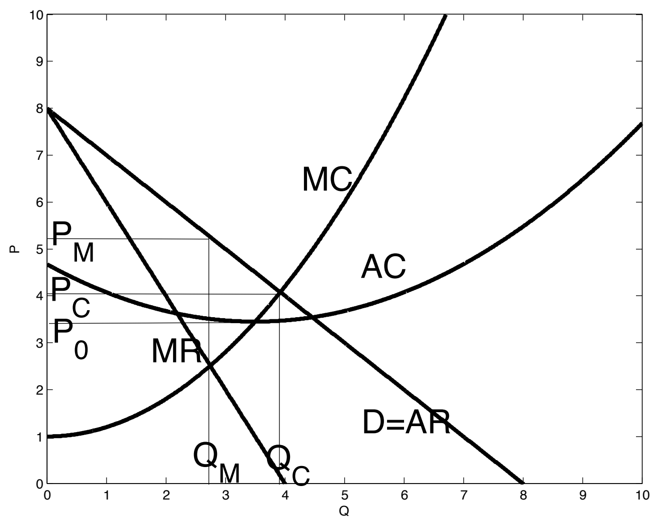

Figure 2: Comparing Competitive and Monopolist Market.

Price between the competitive market price and monopolist market price. Suppose the price ceiling is P_1. The new corresponding AR and MR curves are shown in Figure 3. Given the new MR curve, the equilibrium quantity will be Q_1.

$$Q_1 \in (Q_M, Q_C).$$

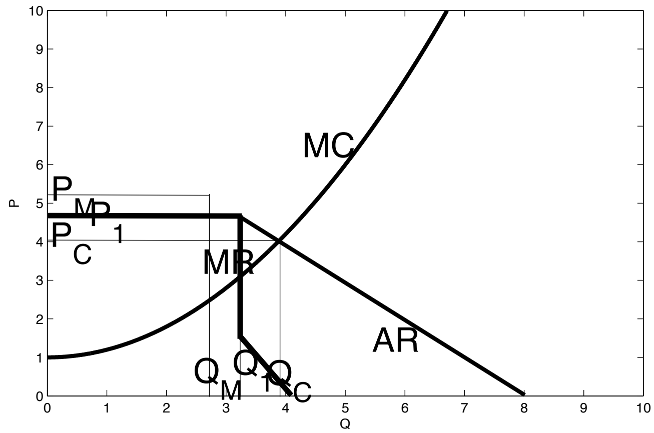

Figure 3: Price between the Competitive Market Price and Monopolist Market Price.

Price equal to the competitive market price. The new corresponding MR and AR curves are shown in Figure 4. In this case the equlibrium price and quantity are as same as those of the competitive market.

Price between the competitive market price and the lowest average cost. Suppose the price ceiling is P_3. The new corresponding MR and AR curves are shown in Figure 5. The equilibrium quantity will be Q_3.

$$Q_3 \in (Q_C, Q_0).$$

The new bold line describes the relation between price and quantity.

Price lower than the lowest average cost. The firm's revenue is not enough for the cost, thus it will quit the market. There is no production.

The analysis shows that if the government sets the price ceiling equal to P_2, the outcome is the same as in a competitive market, and there is no deadweight loss.

Natural monopoly. In a natural monopoly, a firm can produce the entire output of the industry and the cost is lower than what it would be if there were other firms. Natural monopoly arises when there are large economies of scale (see Figure 6). If the market is unregulated, the price will be P_M and the quantity will be Q_M. To improve efficiency, the government can regulate the price. If the price is regulated to be P_C, the firm cannot cover the average cost and will go out of business. P_R is the lowest price that the government can set so that the monopolist will stay in the market.

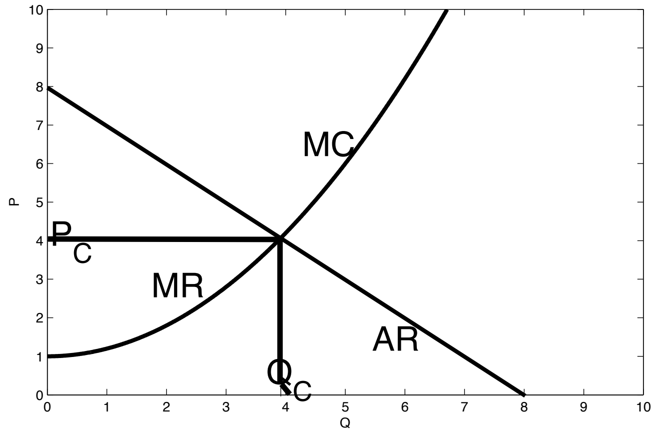

Figure 4: Price Equal to the Competitive Market Price.

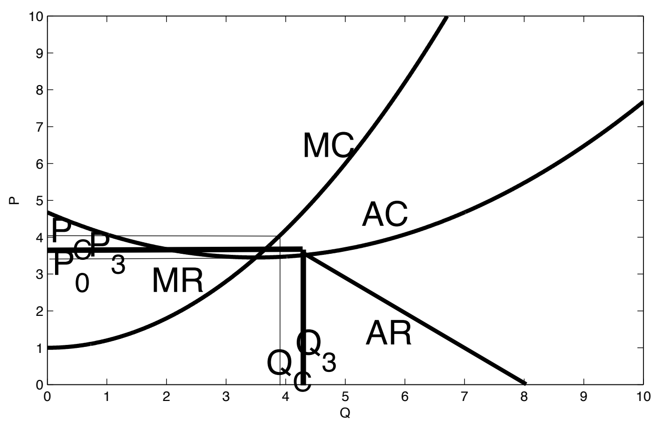

Figure 5: Price between the Competitive Market Price and the lowest Average Cost.

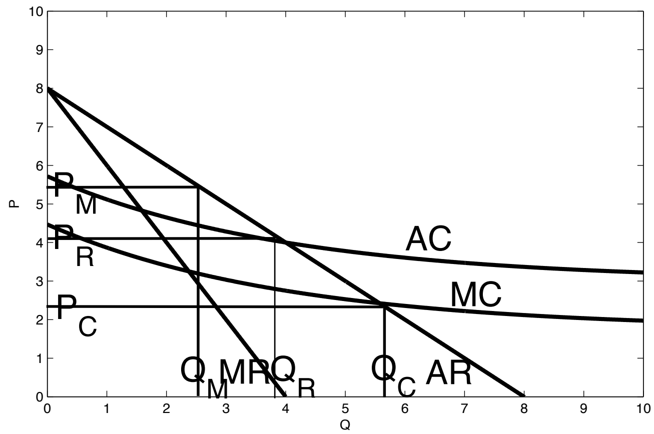

Figure 6: Regulating the Price of a Natural Monopoly.

4 Monopsony

Monopsony refers to a market with only one buyer. In this market, the buyer will maximize its profit, which is the difference of value and expenditure:

$$\max \Pi(Q) = V(Q) - E(Q).$$

When the profit is maximized,

$$\frac{d}{dQ}(V(Q) - E(Q) = 0.$$

Thus

$$MV = ME,$$

namely, the marginal value (additional benefit form buying one more unit of goods) is equal to the marginal expenditure (addtional cost of buying one more unit of goods).

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Chia-Hui Chen
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Lecture 24

Monopoly and Monopsony

Outline

1. Chap 10: *Monopsony*
2. Chap 10: *Monopoly Power*
3. Chap 11: *Price Discrimination*

1 Monopsony

A monopsony is a market in which there is a single buyer. Typically, a monopsonist chooses to maximize the total value derived from buying the goods minus the total expenditure on the goods: $V(Q) - E(Q)$.
Marginal value is the additional benefit derived from purchasing one more unit of a good; since the demand curve shows the buyer's additional willingness to pay for an additional unit, marginal value and the demand curve coincide.

Marginal expenditure is the additional cost of buying one more unit of a good. Average expenditure is the market price paid for each unit, which is determined by the market supply (see Figure 1). Now compare the competitive and monopsony market.

- Competitive buying firms are price takers: The price P^* is fixed; therefore,

$$E = P^* \times Q.$$

  And then

$$AE = ME = P^*$$

  (see Figure 2).

- Monopsonist:

$$E = P_S^*(Q) \times Q.$$

  By definition,

$$AE = \frac{E}{Q} = P_S(Q);$$

  and

$$ME = \frac{dE}{dQ} = P_S(Q) + Q^* \times \frac{dP_S(Q)}{dQ}.$$

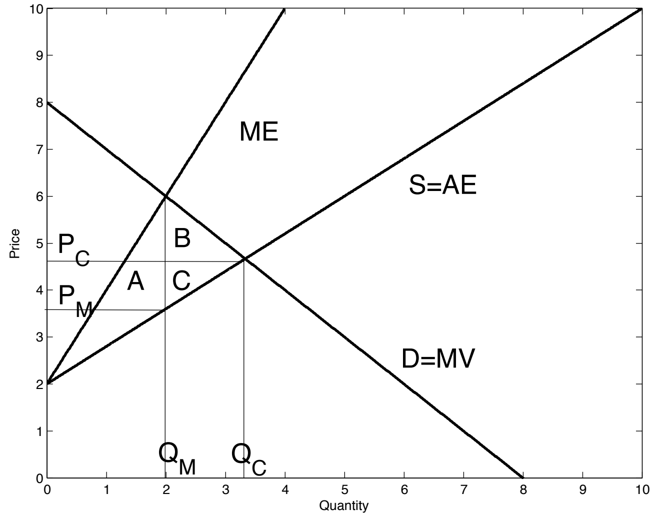

Figure 1: Monopsony Market.

Since the supply curve is upward sloping,

$$ME > P_S(Q) = AE.$$

To maximize

$$V(Q) - E(Q),$$

we have

$$MV(Q) = ME(Q).$$

Buyers gain $A-B$ from monopsony power, while sellers lose $A+C$ (see Figure 1); the deadweight loss is $B+C$.

2 Monopoly Power

There usually is more than one firm in the market, and they have similar but different goods. The Lerner's index is

$$L = \frac{P - MC}{P} = \frac{1}{|E_d|},$$

in which $|E_d|$ is the elasticity of demand for a firm, as oppose to market demand elasticity.
There are several factors that affect monopoly power.

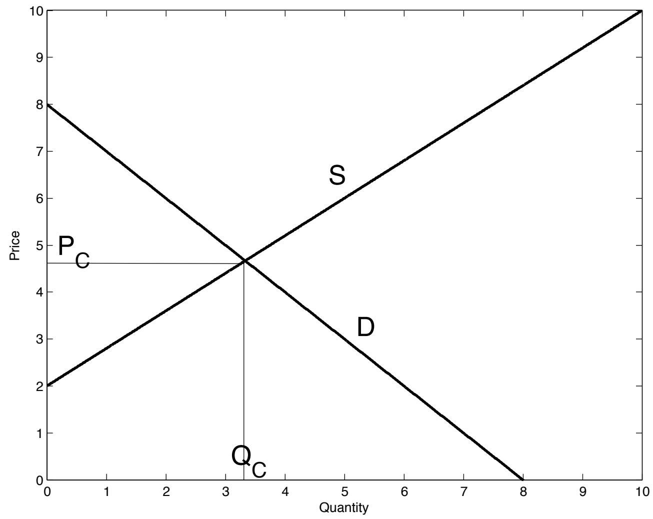

Figure 2: Competitive Buying Market.

- Elasticity of Market Demand: If the market demand is more elastic, the firm's demand is also more elastic. In a competitive market, elasticity of demand for a firm is infinite. With more than one firm, a single firm's demand is more elastic than market demand.
- Number of Firms in Market: With more firms, the firm's demand elasticity is higher, namely, the market power is less.
- Interaction among Firms: If competitors are more aggressive, firms have less market power; if firms collude, they thus have more market power.

3 Price Discrimination

Without market power, the producer would focus on managing production; with market power, the producer not only manages production, but also sets price to capture consumer surplus.

First Degree Price Discrimination

Knowing each consumer's identity and willingness to pay, the producer charges a separate price to each customer.

- $$MR(Q) = P_D(Q).$$

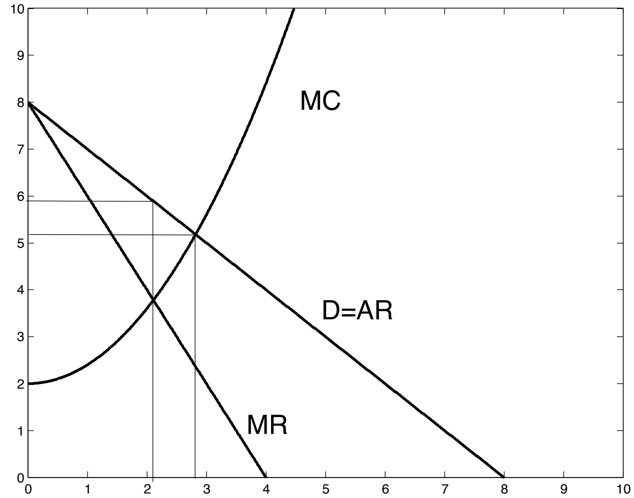

Figure 3: First Degree Price Discrimination.

- Choose Q^* such that
$$MR(Q^*) = MC(Q^*)$$
  Q^* is efficient.
- When the consumer surplus is zero, the producer surplus is maximized.

This kind of price discrimination is not usually encountered in real world.

Second Degree Price Discrimination

The producer charges different unit prices for different quantity purchased. It applies to the situation when consumers are heterogeneous and the seller cannot tell their identity, and consumers have multiple unit demand.

Third Degree Price Discrimination

Refer to next lecture.

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Chia-Hui Chen
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Lecture 25

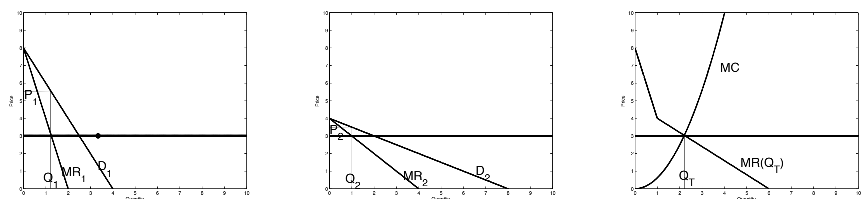
Pricing with Market Power

Outline

1. Chap 11: *Third Degree Price Discrimination*
2. Chap 11: *Peak-Load Pricing*
3. Chap 11: *Two-Part Tariff*

1 Third Degree Price Discrimination

Third degree price discrimination is the practice of dividing consumers into two or more groups with separate demand curves and charging different prices to each group (see Figure 1). Now maximize the profit:



(a) Group 1. (b) Group 2. (c) Total Market.

Figure 1: Third Degree Price Discrimination.

$$\pi(Q_1, Q_2) = P_1(Q_1)Q_1 + P_2(Q_2)Q_2 - C(Q_1 + Q_2);$$

first order conditions

$$\frac{\partial \pi}{\partial Q_1} = 0$$

and

$$\frac{\partial \pi}{\partial Q_2} = 0$$

give

$$MR_1(Q_1) = MC(Q_1 + Q_2),$$

and

$$MR_2(Q_2) = MC(Q_1 + Q_2);$$

finally,

$$MR_1(Q_1) = MR_2(Q_2) = MC(Q_1 + Q_2).$$

Because

$$MR_1 = P_1(1 - \frac{1}{|E_1|}),$$

and

$$MR_2 = P_2(1 - \frac{1}{|E_2|}),$$

we have

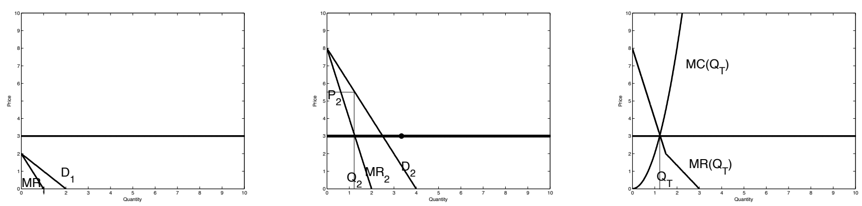
$$\frac{P_1}{P_2} = \frac{1 - 1/|E_1|}{1 - 1/|E_2|};$$

since

$$|E_1| < |E_2|,$$

$$P_1 > P_2.$$

Sometimes a small group might not be served (see Figure 2). The producer only



(a) Group 1. (b) Group 2. (c) Total Market.

Figure 2: Third Degree Price Discrimination with a Small Group.

serves the second group, because the willingness to pay of the first group is too low.

2 Peak-Load Pricing

Producers charge higher prices during peak periods when capacity constraints cause higher MC.

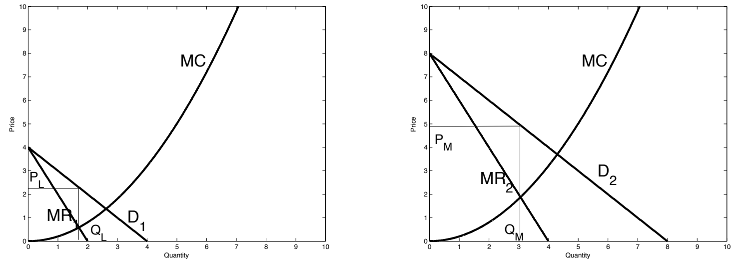
Example (Movie Ticket). Movie ticket is more expensive in the evenings.

Example (Electricity). Price is higher during summer afternoons.

For each time period,

$$MC = MR$$

(see Figure 3).



(a) Period 1. (b) Period 2.

Figure 3: Peak-Load Pricing.

3 Two-Part Tariff

The consumers are charged both an entry (T) and usage (P) fee, that is to say, a fee is charged upfront for right to use/buy the product, and an additional fee is charged for each unit that the consumer wishes to consume. Assume that the firm knows consumer's demand and sets same price for each unit purchased.

Example (Telephone Service, Amusement Park.).

When there is only one consumer. If the firm sets usage fee

$$P = MC,$$

consumer consumes Q^* units (see Figure 4), and the firm can set entry fee

$$T = A,$$

and extract all the consumer surplus.

- If setting
$$P_1 > MC,$$
total revenue is
$$R_1 = A_1 + P_1 \times Q_1,$$
and cost is
$$C_1 = MC \times Q_1,$$
then the profit is
$$\pi_1 = A - B_1.$$
- If setting
$$P_2 < MC,$$
total revenue is
$$R_2 = A_2 + P_2 \times Q_2,$$

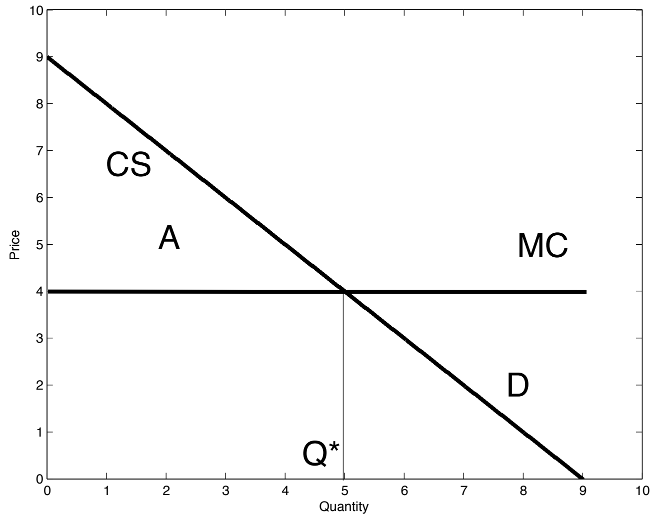
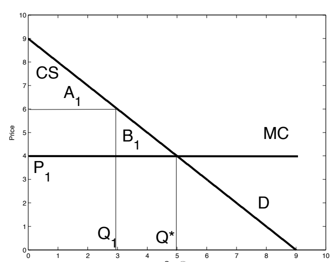
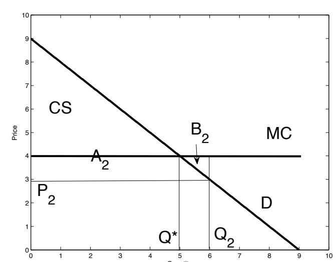


Figure 4: Entry Fee of One Consumer.



(a) Price Higher than Marginal Cost.



(b) Price Lower than Marginal Cost.

Figure 5: Two-Part Tariff.

and cost is

$$C_2 = MC \times Q_2,$$

then the profit is

$$\pi_2 = A - B_2.$$

Either B_1 or B_2 is positive, so the best unit price that maximized the producer surplus is exactly MC.

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Lecture 26

Pricing and Monopolistic Competition

Outline

1. Chap 11: *Two-Part Tariff*
2. Chap 11: *Bundling*
3. Chap 12: *Monopolistic Competition*

1 Two-Part Tariff

When there are two consumers. Consumer 1 has higher demand than consumer 2. If setting $P = MC$, consumer 1 consumes Q_1 units and consumer 2 consumer Q_2 units. A_1 is consumer 1's consumer surplus, and A_2 is consumer 2's consumer surplus. Assume that $2A_2 > A_1$. Then the maximum entry fee the firm can charge is A_2. If more than A_2 is charged, consumer 2 would not consume.

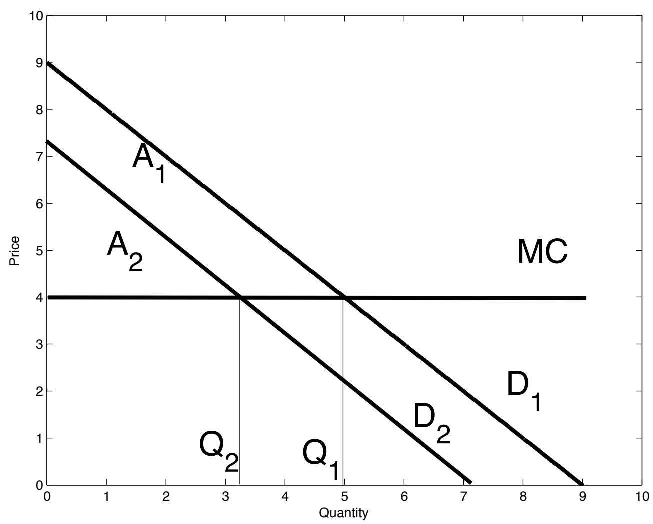

Figure 1: Entry Fee of Two Consumers.

Now consider the case that price is higher or lower than the marginal cost.

- If setting
$$P > MC,\ T = A_2',$$
we have
$$\pi_1 = A_2' + Q_1' \times (P - MC) = A_2 + C,$$
and
$$\pi_2 = A_2' + Q_2' \times (P - MC) = A_2 - B,$$
thus
$$\pi = \pi_1 + \pi_2 = 2A_2 + C - B.$$
Because
$$C > B$$
(see Figure 2),
$$\pi > 2A_2.$$

- If setting
$$P < MC,\ T = A_2''$$
we have
$$\pi_1 = A_2'' - Q_1'' \times (MC - P) = A_2 - D,$$
and
$$\pi_2 = A_2'' - Q_2'' \times (MC - P) = A_2 - E,$$
thus
$$\pi = \pi_1 + \pi_2 = 2A_2 - D - E.$$
Always
$$\pi < 2A_2.$$

Summary: the firm should set

- usage fee
$$P > MC,$$
namely, larger than the marginal cost;

- entry fee
$$T = A_2,$$
namely, equal to the remaining consumer surplus of the consumer with the smaller demand.

Summary: If the demands of two consumers are more similar, the firm should set usage fee close to MC and higher entry fee; if the demands of two consumers are less similar, the firm should set higher usage fee and lower entry fee.

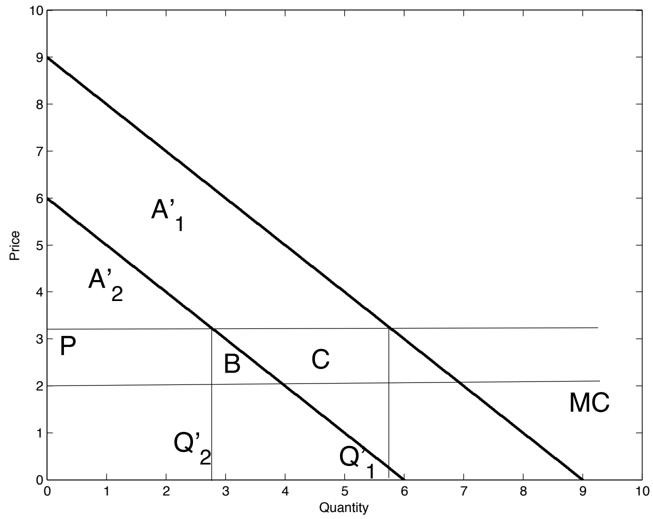

Figure 2: Two-Part Tariff: Price Higher than Marginal Cost

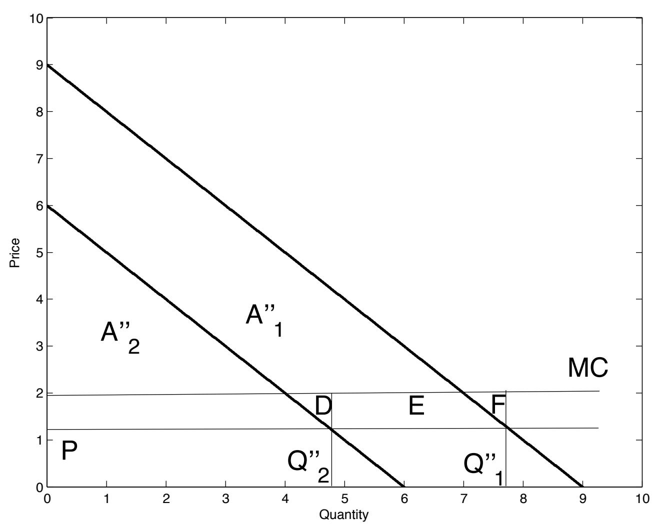

Figure 3: Two-Part Tariff: Price Lower than Marginal Cost

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2 Bundling

Bundling means packaging two or more products, for example, vacation travel usually has a packaging of hotel, airfare, car rental, etc.
Assume there are two goods and many consumers in the market, and the consumers have different reservation prices (willingness to pay).
See Figure 4 and 5. The coordinates are the reservation prices of the two goods respectively.
If the firm sells the goods separately with prices P_1 and P_2 (see Figure 4),

- when
$$r_1 > P_1,$$
and
$$r_2 > P_2,$$
the consumer will buy both good 1 and 2;

- when
$$r_1 > P_1,$$
but
$$r_2 < P_2,$$
the consumer will only buy good 1;

- when
$$r_2 > P_2,$$
but
$$r_1 < P_1,$$
the consumer will only buy good 2;

- when
$$r_1 < P < 1,$$
and
$$r_2 < P < 2,$$
the consumer will buy neither good 1 nor 2.

If the firm sells the two goods in a bundle and charges price P_B,

- if
$$r_1 + r_2 > P_B,$$
the consumer will buy the bundle;

- if
$$r_1 + r_2 < P_B,$$
the consumer will not buy the bundle.

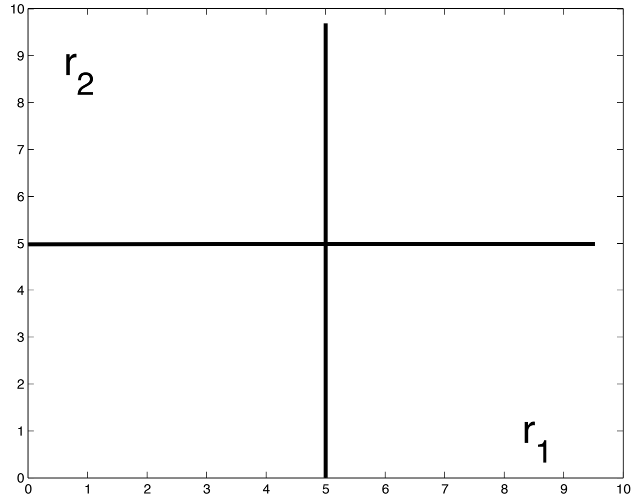


Figure 4: Price without Packaging.

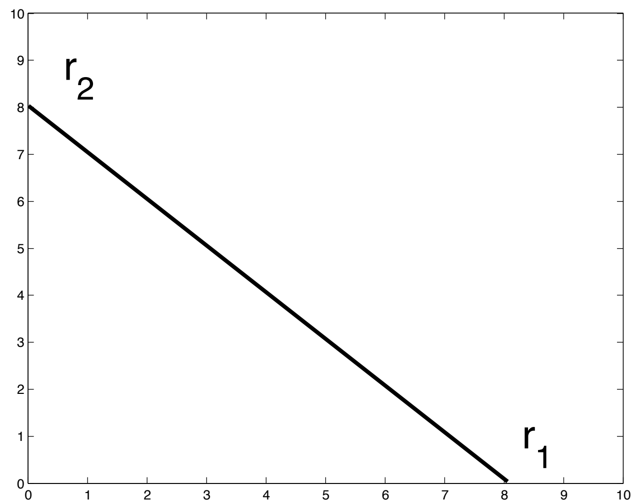


Figure 5: Price with Packaging.

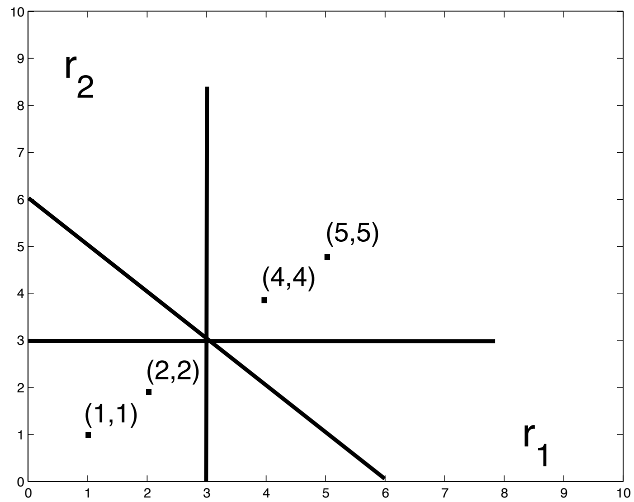


Figure 6: Bundling Example 1.

Bundling Example 1: the four points in Figure 6 represent the four consumers' reservation values. Consider two pricing strategies – one is that the two goods are sold separately with prices $P_1 = 3$ and $P_2 = 3$, and the other is that the two goods are sold in a bundle with price $P_B = 6$. Without bundling, the revenue is

$$R = 12,$$

and with bundling, the revenue is

$$R = 12;$$

bundling does not do better.

Bundling Example 2: Consider the other four consumers shown in Figure 7 and the firm chooses between the two pricing strategies mentioned before. Without bundling, the revenue is

$$R = 12,$$

and with bundling, the revenue is

$$R = 24;$$

obviously, bundling strategy benefits the producer in this case

Conclusion: bundling works well when

- the consumers are heterogeneous;
- price discrimination is not possible;
- the demand for different goods are negatively correlated.

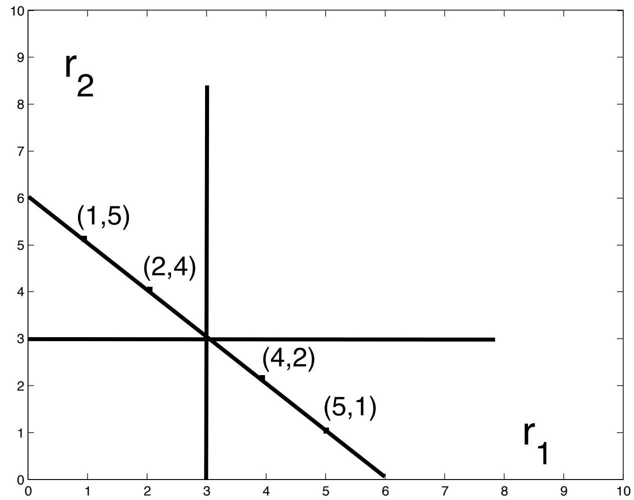


Figure 7: Bundling Example 2.

3 Monopolistic Competition

In monopolistic competition,

- there are many firms;
- there is free entry and exit;
- products are differentiated but close substitutes.

Thus

- each firm faces a distinct demand, which is downward sloping and elastic;
- there is no profit in long run (see Figure 8 and 9);
- price is higher than marginal cost because firms have some monopoly power, and thus there is some deadweight loss.

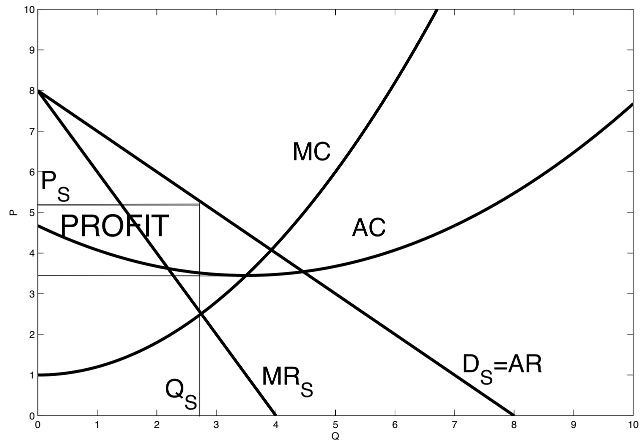


Figure 8: Short Run in Monopolistic Competition Market.

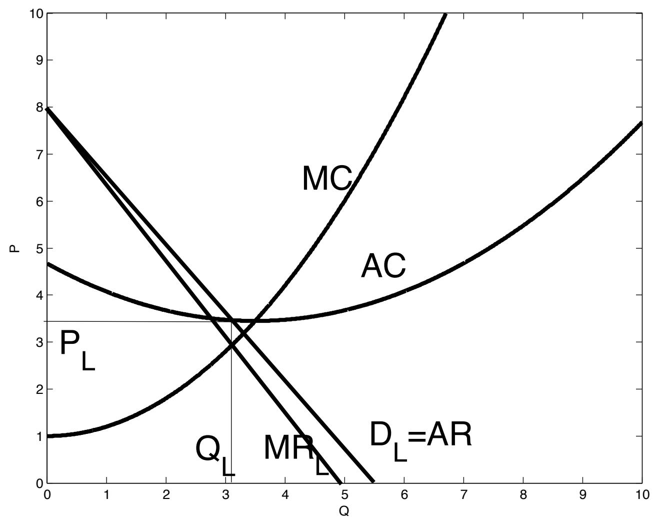


Figure 9: Long Run in Monopolistic Competition Market.

Lecture 27

Game Theory and Oligopoly

Outline

1. Chap 12, 13: *Game Theory*
2. Chap 12, 13: *Oligopoly*

1 Game Theory

In monopolistic competition market, there are many sellers, and the sellers do not consider their opponents' strategies; nonetheless, in oligopoly market, there are a few sellers, and the sellers must consider their opponents' strategies. The tool to analyze the strategies is game theory.

Game theory includes the discussion of noncooperative game and cooperative game. The former refers to a game in which negotiation and enforcement of binding contracts between players is not possible; the latter refers to a game in which players negotiate binding contracts that allow them to plan joint strategies.

A game consists of players, strategies, and payoffs.

Now assume that in a game, there are two players, firm A and firm B; their strategies are whether to advertise or not; consequently, their payoffs can be written as

$$\pi_A(A's\ strategy, B's\ strategy)$$

and

$$\pi_B(A's\ strategy, B's\ strategy)$$

respectively.

Now let's represent the game with a matrix (see Table 1). The first row is the situation that A advertises, and the second row is the situation that A does not advertise; the first column is the situation that B advertises, and the second column is the situation that B does not advertise. The cells provide the payoffs under each situation. The first number in a cell is firm A's payoff, and the second number is firm B's payoff.

Dominant strategy is the optimal strategy no matter what the opponent does. If we change the element $(20, 2)$ to $(10, 2)$, no matter what the other firm does, advertising is always better for firm A (and firm B). Therefore, both firms have a dominant strategy.

		Firm B	
		Advertise	Not Advertise
Firm A	Advertise	10,5	15,0
	Not Advertise	6,8	20,2

Table 1: Payoffs of Firm A and B.

When all players play dominant strategies, we call it equilibrium in dominant strategy.

Now back to original case, B has dominant strategy, but A does not, because

- if B advertises, A had better advertise;
- if B does not advertise, A had better not advertise.

So we see that not all games have dominant strategy. However, since B has dominant strategy and would always advertise, A would choose to advertise in this case.

Now consider another example. Two firms, firm 1 and firm 2, can produce crispy or sweet. If they both produce crispy or sweet, the payoffs are $(-5, -5)$; if one of them produces crispy while the other produces sweet, the payoffs are $(10, 10)$.

		Firm 2	
		Crispy	Sweet
Firm 1	Crispy	-5,-5	10,10
	Sweet	10,10	-5,-5

Table 2: Payoffs of Firm 1 and 2.

There is no dominant strategy for both firms. We define another equilibrium concept – Nash equilibrium.

Nash equilibrium is a set of strategies such that each player is doing the best given the actions of its opponents.

In this case, there are two Nash equilibriums, $(sweet, crispy)$ and $(crispy, sweet)$.

2 Oligopoly

Small number of firms, and production differentiation may exist.

Different Oligopoly Models

1. Cournot Model: firms produce the same good, and they choose the production quantity simultaneously.
2. Stackelberg Model: firms produce the same
3. Bertrand Model: firms produce the same good, and they choose the price.

2.1 Cournot Model

Example. Market has demand

$$P = 30 - Q,$$

with two firms, so

$$Q = Q_1 + Q_2,$$

and assume that there is no fixed cost and marginal cost,

$$MC_1 = MC_2 = 0.$$

Firm 1 would like to maximize its profit

$$P \times Q_1,$$

or

$$(30 - Q_1 - \overline{Q}_2) \times Q_1;$$

from the

$$\frac{d}{dQ_1}((30 - Q_1 - \overline{Q}_2) \times Q_1) = 0,$$

we have firm 1's reaction function

$$Q_1 = 15 - \frac{\overline{Q}_2}{2},$$

in which the $\overline{Q}_2$ is the estimation of firm 2's production by firm 1.
In the same way, firm 2's reaction function is

$$Q_2 = 15 - \frac{\overline{Q}_1}{2},$$

in which the $\overline{Q}_1$ is the expectation of firm 1's production by firm 2.

At equilibrium, firm 1 and firm 2 have correct expectation about the other's production, that is,

$$\overline{Q}_1 = Q_1,$$
$$\overline{Q}_2 = Q_2.$$

Thus, at equilibrium,

$$Q_1 = 10,$$

and

$$Q_2 = 10.$$

14.01 Principles of Microeconomics, Fall 2007
Chia-Hui Chen
November 26, 2007

Lecture 28

Oligopoly

Outline

1. Chap 12, 13: *Stackelberg*
2. Chap 12, 13: *Bertrand*
3. Chap 12, 13: *Prisoner's Dilemma*

In the discussion that follows, all of the games are played only once.
and

1 Stackelberg

Stackelberg model is an oligopoly model in which firms choose quantities sequentially.

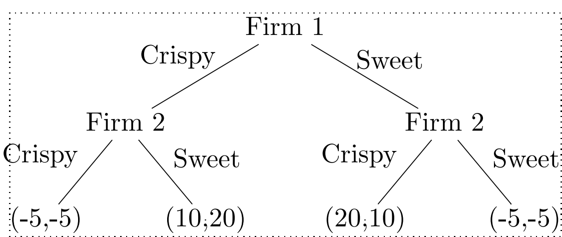
Now change the example discussed in last lecture as follows: if firm 1 produces crispy and firm 2 produces sweet, the payoff is $(10, 20)$; if firm 1 produces sweet and firm 2 produces crispy, the payoff is $(20, 10)$ (see Table 1).

		Firm 2	
		Crispy	Sweet
Firm 1	Crispy	-5,-5	10,20
	Sweet	20,10	-5,-5

Table 1: Payoffs of Firm 1 and 2.

$$\begin{pmatrix} -5,-5 & 10,20 \\ 20,10 & -5,-5 \end{pmatrix}$$

This is an extensive form game; we use a tree structure to describe it.



Start from the bottom using backward induction, namely, solve firm 2's decision problem first, and then firm 1's. If firm 1 chooses crispy, firm 2 will choose sweet to get a higher payoff. If firm 2 chooses sweet, firm 2 will choose crispy. Knowing this, firm 1 will choose sweet in the first place. In this case, going first gives firm 1 the advantage. Now consider the case we discussed for the Cournot model, but firm 1 chooses Q_1 first, and firm 2 choose Q_2 later. For firm 2, the first order condition

$$\frac{d}{dQ_2}(30 - Q_1 - Q_2) \times Q_2 = 0$$

gives that

$$Q_2(Q_1) = 15 - \frac{Q_1}{2}.$$

For firm 1,

$$\frac{d}{dQ_1}(30 - Q_1 - Q_2(Q_1) \times Q_1 = 0$$

gives that

$$Q_1 = 15.$$

Thus, the result will be

$$Q_1 = 15,$$

$$\pi_1 = 112.5;$$

$$Q_2 = 7.5,$$

$$\pi_2 = 56.25.$$

In this case, firm 1 also has advantage to go first.

2 Bertrand

The Bertrand model is the oligopoly model in which firms compete in price. First assume that two firms produce homogeneous goods and choose the prices simultaneously. Assume two firms have the same marginal cost

$$MC_1 = MC_2 = 3;$$

consumers buy goods from the firm with lower price. If

$$P_1 = P_2 = 4,$$

the two firms share the market equally, but this is not the equilibrium. The reason is that one firm can get whole demand by lowering the price a little; therefore, the equilibrium will be

$$P_1 = P_2 = 3,$$

when the price is equal to the marginal cost. Now we check if

$$P_1 = 3$$

is the best choice for firm 1 given

$$P_2 = 3.$$

When

$$P_1 = 3,$$

$$\pi_1 = 0;$$

if

$$P_1 > 3,$$

consumers will not buy firm 1's goods, thus

$$\pi_1 = 0;$$

if

$$P_1 < 3,$$

the price is lower than the marginal cost, thus

$$\pi_1 < 0.$$

It follows that

$$P_1 = 3$$

is optimal for firm 1; by analogy, we can get the same conclusion for firm 2. Therefore,

$$P_1 = P_2 = 3 = MC$$

in a Bertrand game with homogeneous goods. This is like the competitive market.

Suppose the goods from the two firms are heterogeneous, but substitutes. Firm 1 and firm 2 face the following demands:

$$Q_1 = 12 - 2P_1 + P_2,$$

and

$$Q_2 = 12 - 2P_2 + P_1.$$

Firm 1's and firm 2's reaction functions are

$$P_1 = 3 + \frac{\overline{P}_2}{4},$$

and

$$P_2 = 3 + \frac{\overline{P}_1}{4}.$$

At equilibrium,

$$P_1 = \overline{P}_1,$$

and

$$P_2 = \overline{P}_2;$$

so

$$P_1 = P_2 = 4,$$

$$Q_1 = Q_2 = 8,$$

and

$$\pi_1 = \pi_2 = 32.$$

Consider the case when the firms choose prices sequentially. Supposing firm 2's first order condition

$$\frac{d}{dQ_2}(12 - P_2 + P_1) \times P_2 = 0$$

and firm 1's first order condition

$$\frac{d}{dQ_1}(12 - 2P_1 + P_2(P_1)) \times P_1 = 0.$$

From the first equation

$$P_2(P_1) = 3 + \frac{P_1}{4},$$

and then substitute it into the second equation, we obtain

$$P_1 = 4\frac{2}{7}.$$

Therefore,

$$\pi_1 = 32\frac{1}{4};$$

$$P_2 = 4\frac{1}{14},$$

and

$$\pi_2 = 33\frac{15}{98}.$$

In this case, we can see that the firm who goes first has disadvantage, when competing in price.

3 Prisoner's Dilemma

Criminals A and B cooperated, and then got caught. However, the police have no evidence; so they have to interrogate A and B separately, trying to make them tell the truth.

		Firm B	
		Betray	Silent
Firm A	Betray	-3,-3	0,6
	Silent	-6,0	-1,-1

Table 2: Payoffs of Firm A and B.

The above matrix shows A and B's payoffs. Given the payoffs, A and B choose to tell the truth (betray) or keep silent. We can see that, if they both keep silence, the result $(-1, -1)$ is best for them; nonetheless, if one of them betrays another, he will be free but his companion will have payoff -6; moreover, if both of them betray, they will face the result $(-3, -3)$.

Consider what A thinks. Whether B keeps silence or betrays him, A will always be better off if he betrays; so will B. Therefore, the result of this problem is $(-3, -3)$, namely, both prisoners betray.

14.01 Principles of Microeconomics, Fall 2007
Chia-Hui Chen
November 28, 2007

Lecture 29

Strategic Games

Outline

1. Chap 12, 13: *Collusion – Prisoners' Dilemma*
2. Chap 12, 13: *Repeated Games*
3. Chap 12, 13: *Threat, Credibility, Commitment*
4. Chap 14: *Maximin Strategy*

1 Collusion – Prisoners' Dilemma

Last time we talked about the prisoners' dilemma. The conclusion is that they will betray the other.

Now apply it to the cases of Cournot and Bertrand models.

In the Cournot model, the demand is

$$P = 30 - Q_1 - Q_2.$$

The equilibrium will be

$$Q_1 = Q_2 = 10,$$

with

$$\pi_1 = \pi_2 = 100.$$

However, to maximize their total profits, they should choose a total quantity Q so that

$$\frac{d}{dQ}(Q(30 - Q)) = 0,$$

which follows that

$$Q = 15.$$

If they share profit equally,

$$Q_1 = Q_2 = 7.5,$$

and

$$\pi_1 = \pi_2 = 112.5.$$

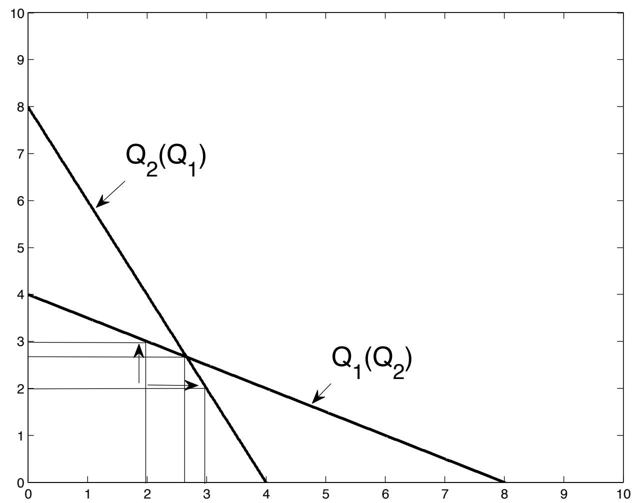


Figure 1: Reaction Curves in Cournot Model.

Obviously, the latter case will make both of them better off. But given the opponent produces 7.5, each of them can increase the profit by producing more (see Figure 1).

In Bertrand model, demand functions for firm 1 and firm 2 are

$$Q_1 = 12 - 2P_1 + P_2,$$

and

$$Q_2 = 12 - 2P_2 + P_1.$$

Equilibrium is

$$P_1 = P_2 = 4,$$

with

$$\pi_1 = \pi_2 = 32.$$

However, firms can choose P_1 and P_2 together to maximize the total revenue

$$\pi = P_1(12 - 2P_1 + P_2) + P_2(12 - 2P_2 + P_1).$$

By first order condition, we obtain

$$12 - 4P_1 + 2P_2 = 0,$$

and

$$12 - 4P_2 + 2P_1 = 0.$$

Thus

$$P_1 = P_2 = 6,$$

with

$$\pi_1 = \pi_2 = 36.$$

But in this case, each firm has incentive to lower its price given the other firm's price (see Figure 2).

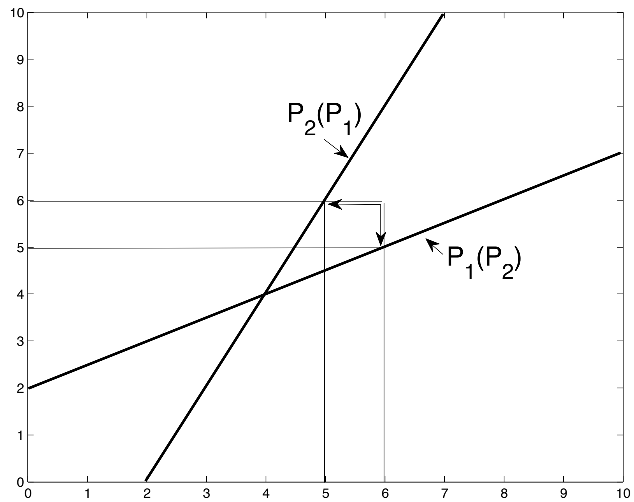


Figure 2: Reaction Curves in Bertrand Model.

2 Repeated Games

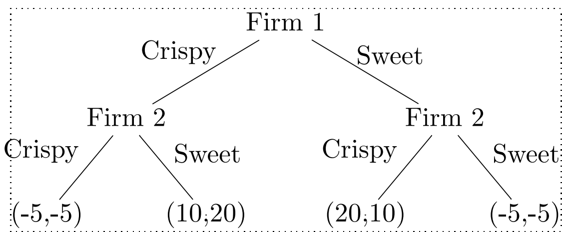
Back to the prisoners' problem. If suspect A and B will cooperate for infinite periods, and they are both patient, they care about future payoffs. Because if one of them betrays this time, the opponent will lose the trust and betray in the future; the payoff changes from −1 to −3 for each time. Therefore, both A and B would like to keep silence. But if they are impatient, and only consider today's payoff, they will still betray. Now move on to the case that A and B will cooperate for finite number times which is fairly large. We deduce from the last time they cooperate; the answer is that they will betray for the last time, so will they for other opportunities. Therefore, the collusion between A and B succeed only if they will be cooperative forever and are patient.

3 Threat, Credibility, Commitment

Back to the crispy-sweet question.

		Firm 2	
		Crispy	Sweet
Firm 1	Crispy	-5,-5	10,20
	Sweet	20,10	-5,-5

Table 1: Payoffs of Firm 1 and 2.

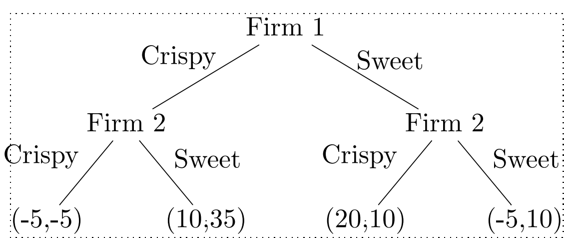


In order to get the largest 20 by producing sweet, firm 2 tries to make firm 1 believe that firm 1 should choose crispy by claiming that it always produces sweet no matter what firm 1 produces. However, firm 1 can ignore firm 2's announcement because once firm 1 choose sweet, firm 2 will produce crispy.

Suppose that firm 2 will advertise and so change the payoffs.

		Firm 2	
		Crispy	Sweet
Firm 1	Crispy	-5,-5	10,35
	Sweet	20,10	-5,10

Table 2: Payoffs of Firm 1 and 2.



In this case, firm 2 feels indifferent between choosing crispy or sweet when firm 1 produces sweet, and chooses sweet when firm 1 produces crispy. So it is credible if firm 2 claims that it always chooses sweet, and then firm 1 had better choose crispy. This example tells us that firm 2 had to do something to make the announcement credible.

4 Maximin Strategy

See Table 3. Firm B has dominant strategy: advertise.

Therefore, the equilibrium should be both A and B advertise.

However, if firm B does not choose the rational option, the minimum payoff of A is 5 if A advertises, and 8 if A does not advertise.

The maximin strategy is the strategy that renders the highest minimum payoff.

When A cannot tell whether B is rational or not, A might use maximin strategy. In this case, the maximin strategy of A is:

		Firm B	
		Advertise	Not Advertise
Firm A	Advertise	10,5	5,0
	Not Advertise	8,8	15,2

Table 3: Payoffs of Firm A and B.