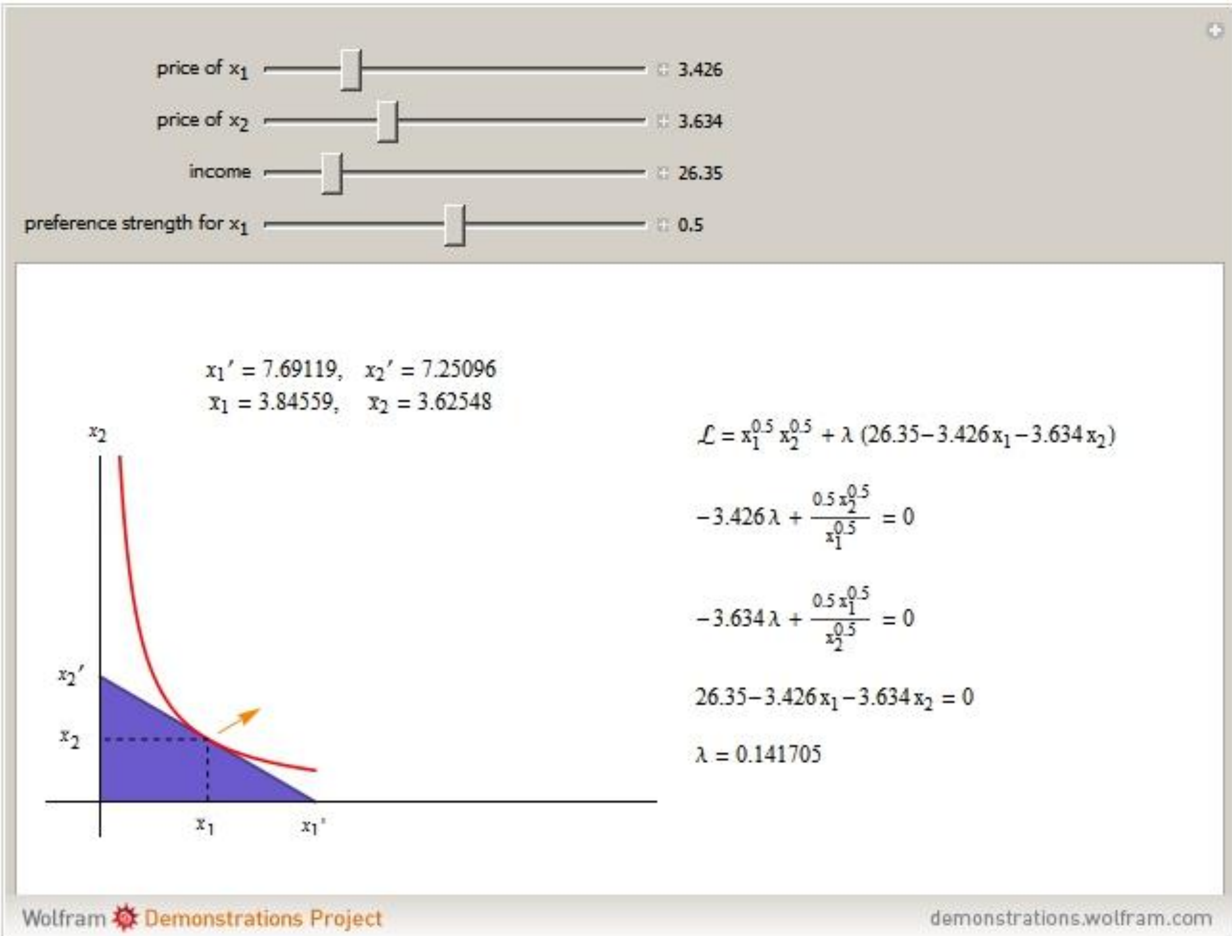
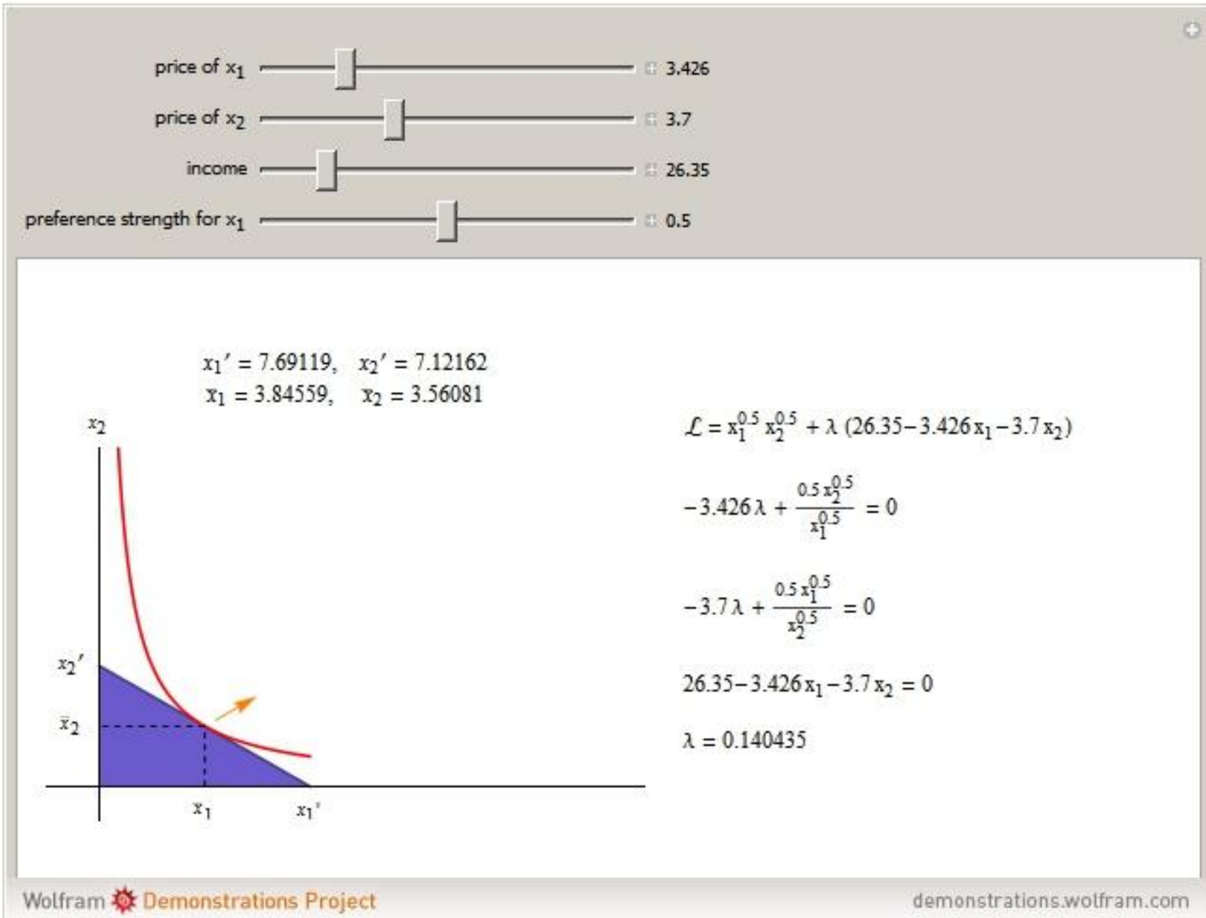
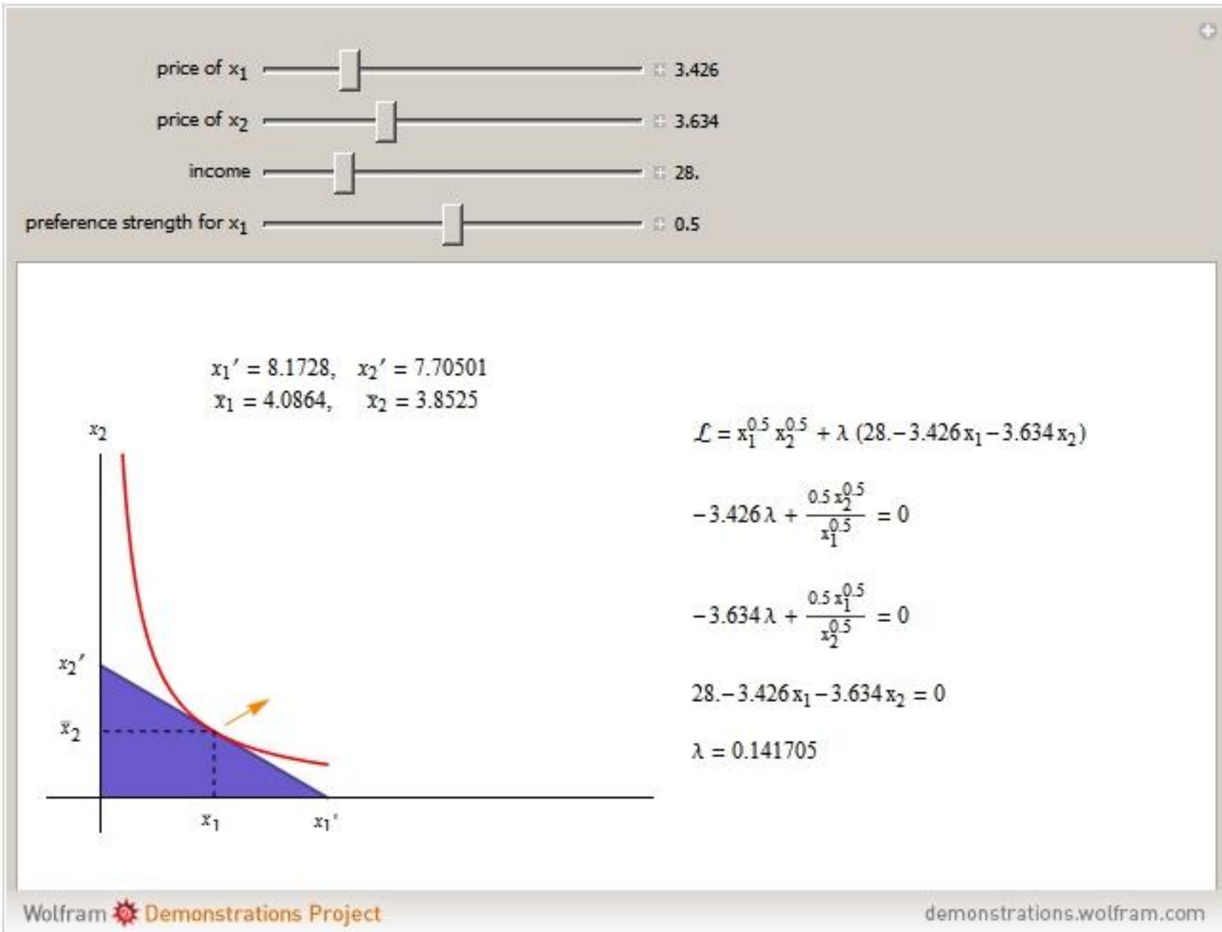
Constrained Optimization: Cobb-Douglas Utility and Interior Solutions Using a Lagrangian




price of x1 3.426
price of x2 3.7
income 26.35
preference strength for x1 0.5
x1′ = 7.69119, x2′ = 7.12162
x̄1 = 3.84559, x̄2 = 3.56081
x2
x2′
x̄2
x̄1
x1′
$\mathcal{L} = x_1^{0.5} x_2^{0.5} + \lambda\,(26.35 - 3.426\,x_1 - 3.7\,x_2)$
$-3.426\,\lambda + \frac{0.5\,x_2^{0.5}}{x_1^{0.5}} = 0$
$-3.7\,\lambda + \frac{0.5\,x_1^{0.5}}{x_2^{0.5}} = 0$
$26.35 - 3.426\,x_1 - 3.7\,x_2 = 0$
$\lambda = 0.140435$
Wolfram Demonstrations Project
demonstrations.wolfram.com



DETAILS

The individual's optimal choice is a result of the tension between what is feasible, to the southwest, and what is desirable, to the northeast. Graphically, as long as the individual's indifference curve and budget constraint are not tangent, the individual could improve by trading off one good for the other. Thus, the optimal choice is on the highest indifference curve that has a feasible bundle, at the tangency.

Mathematically, the Lagrangian shows this by equating the marginal utility of increasing x_1 with its marginal cost and equating the marginal utility of increasing x_2 with its marginal cost. These are the first two first-order conditions. The interpretation of the Lagrange multiplier follows from this. The third first-order condition is the budget constraint.

Suggested exercise: Adjust the values of p_1, p_2, y, and α one at a time, anticipating how the graph will change, and rewriting the Lagrangian and re-solving for the optimal bundle, the value of the Lagrange multiplier, and the resulting optimal utility level; in particular, increase y by 1 and note the change in the resulting utility levels.