# Calculus/Differentiation/Basics of Differentiation/Exercises

## Find the Derivative by Definition

Find the derivative of the following functions using the limit definition of the derivative.

1. 
$$f(x) = x^2$$

2. 
$$f(x) = 2x + 2$$

3. 
$$f(x) = \frac{x^2}{2}$$

4. 
$$f(x) = 2x^2 + 4x + 4$$

5. 
$$f(x) = \sqrt{x+2}$$

6. 
$$f(x) = \frac{1}{x}$$

7. 
$$f(x)=rac{3}{x+1}$$

8. 
$$f(x) = \frac{1}{\sqrt{x+1}}$$

9. 
$$f(x) = \frac{x}{x+2}$$

## **Prove the Constant Rule**

10. Use the definition of the derivative to prove that for any fixed real number c ,  $\dfrac{d}{dx}[c\cdot f(x)]=c\cdot \dfrac{d}{dx}[f(x)]$ 

## Find the Derivative by Rules

Find the derivative of the following functions:

#### **Power Rule**

11. 
$$f(x) = 2x^2 + 4$$

12. 
$$f(x) = 3\sqrt[3]{x}$$

13. 
$$f(x) = 2x^5 + 8x^2 + x - 78$$

14. 
$$f(x) = 7x^7 + 8x^5 + x^3 + x^2 - x$$

15. 
$$f(x) = \frac{1}{x^2} + 3x^{\frac{1}{3}}$$

16. 
$$f(x) = 3x^{15} + \frac{x^2}{17} + \frac{2}{\sqrt{x}}$$

17. 
$$f(x) = \frac{3}{x^4} - \sqrt[4]{x} + x$$

18. 
$$f(x) = 6x^{1/3} - x^{0.4} + \frac{9}{x^2}$$

19. 
$$f(x) = \frac{1}{\sqrt[3]{x}} + \sqrt{x}$$

#### **Product Rule**

20. 
$$f(x) = (x^4 + 4x + 2)(2x + 3)$$

21. 
$$f(x) = (2x-1)(3x^2+2)$$

22. 
$$f(x) = (x^3 - 12x)(3x^2 + 2x)$$

23. 
$$f(x) = (2x^5 - x)(3x + 1)$$

24. 
$$f(x) = (5x^2 + 3)(2x + 7)$$

25. 
$$f(x) = 3x^2(5x^2+1)^4$$

26. 
$$f(x) = x^3(2x^2 - x + 4)^4$$

27. 
$$f(x) = 5x^2(x^3 - x + 1)^3$$

28. 
$$f(x) = (2-x)^6 (5+2x)^4$$

### **Quotient Rule**

24. 
$$f(x) = \frac{2x+1}{x+5}$$

25. 
$$f(x) = \frac{3x^4 + 2x + 2}{3x^2 + 1}$$

26. 
$$f(x) = \frac{x^{\frac{3}{2}} + 1}{x + 2}$$

27. 
$$d(u) = \frac{u^3 + 2}{u^3}$$

28. 
$$f(x) = \frac{x^2 + x}{2x - 1}$$

29. 
$$f(x) = \frac{x+1}{2x^2+2x+3}$$

30. 
$$f(x) = \frac{16x^4 + 2x^2}{x}$$

$$f(x)=\frac{8x^3+2}{5x+5}$$

$$f(x) = \frac{(3x-2)^2}{\sqrt{x}}$$

$$f(x)=rac{\sqrt{x}}{2x-1}$$

$$f(x)=\frac{4x-3}{x+2}$$

$$f(x)=\frac{4x+3}{2x-1}$$

$$f(x) = \frac{x^2}{x+3}$$

$$f(x) = \frac{x^5}{3-x}$$

#### **Chain Rule**

31. 
$$f(x) = (x+5)^2$$

32. 
$$g(x) = (x^3 - 2x + 5)^2$$

33. 
$$f(x) = \sqrt{1 - x^2}$$

34. 
$$f(x) = \frac{(2x+4)^3}{4x^3+1}$$

35. 
$$f(x) = (2x+1)\sqrt{2x+2}$$

36. 
$$f(x) = \frac{2x+1}{\sqrt{2x+2}}$$

37. 
$$f(x) = \sqrt{2x^2 + 1}(3x^4 + 2x)^2$$

38. 
$$f(x) = \frac{2x+3}{(x^4+4x+2)^2}$$

39. 
$$f(x) = \sqrt{x^3 + 1}(x^2 - 1)$$

40. 
$$f(x) = ((2x+3)^4 + 4(2x+3) + 2)^2$$

41. 
$$f(x) = \sqrt{1+x^2}$$

## **Exponentials**

42. 
$$f(x) = (3x^2 + e)e^{2x}$$

43. 
$$f(x) = e^{2x^2 + 3x}$$

44. 
$$f(x) = e^{e^{2x^2+1}}$$

45. 
$$f(x) = 4^x$$

### Logarithms

46. 
$$f(x) = 2^{x-3} \cdot 3\sqrt{x^3 - 2} + \ln(x)$$

47. 
$$f(x) = \ln(x) - 2e^x + \sqrt{x}$$

48. 
$$f(x) = \ln(\ln(x^3(x+1)))$$

49. 
$$f(x) = \ln(2x^2 + 3x)$$

50. 
$$f(x) = \log_4(x) + 2\ln(x)$$

## **Trigonometric functions**

51. 
$$f(x) = 3e^x - 4\cos(x) - \frac{\ln(x)}{4}$$

52. 
$$f(x) = \sin(x) + \cos(x)$$

# **More Differentiation**

53. 
$$\frac{d}{dx}[(x^3+5)^{10}]$$

$$54. \ \frac{d}{dx}[x^3+3x]$$

55. 
$$\frac{d}{dx}[(x+4)(x+2)(x-3)]$$

$$56. \ \frac{d}{dx} \left[ \frac{x+1}{3x^2} \right]$$

57. 
$$\frac{d}{dx}[3x^3]$$

$$58. \ \frac{d}{dx}[x^4\sin(x)]$$

59. 
$$\frac{d}{dx}[2^x]$$

60. 
$$\frac{d}{dx}[e^{x^2}]$$

61. 
$$\frac{d}{dx}[e^{2^x}]$$

## **Implicit Differentiation**

Use implicit differentiation to find y'

62. 
$$x^3 + y^3 = xy$$

63. 
$$(2x+y)^4 + 3x^2 + 3y^2 = \frac{x}{y} + 1$$

## **Logarithmic Differentiation**

Use logarithmic differentiation to find  $rac{dy}{dx}$ 

64. 
$$y = x(\sqrt[4]{1-x^3})$$

65. 
$$y = \sqrt{\frac{x+1}{1-x}}$$

66. 
$$y = (2x)^{2x}$$

67. 
$$y = (x^3 + 4x)^{3x+1}$$

68. 
$$y = (6x)^{\cos(x)+1}$$

# **Equation of Tangent Line**

For each function, f, (a) determine for what values of x the tangent line to f is horizontal and (b) find an equation of the tangent line to f at the given point.

69. 
$$f(x) = \frac{x^3}{3} + x^2 + 5,$$
 (3,23)

70. 
$$f(x) = x^3 - 3x + 1,$$
  $(1,-1)$ 

71. 
$$f(x) = \frac{2x^3}{3} + x^2 - 12x + 6,$$
  $(0,6)$ 

72. 
$$f(x) = 2x + \frac{1}{\sqrt{x}},$$
 (1,3)

73. 
$$f(x) = (x^2 + 1)(2 - x),$$
 (2,0)

74. 
$$f(x) = rac{2x^3}{3} + rac{5x^2}{2} + 2x + 1,$$
  $(3, rac{95}{2})$ 

- 75. Find an equation of the tangent line to the graph defined by  $(x-y-1)^3=x$  at the point (1,-1).
- 76. Find an equation of the tangent line to the graph defined by  $e^{xy} + x^2 = y^2$  at the point (1,0).

## **Higher Order Derivatives**

- 77. What is the second derivative of  $3x^4 + 3x^2 + 2x$ ?
- 78. Use induction to prove that the (n+1)th derivative of a n-th order polynomial is 0.

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