Calculus/Differentiation/Basics of Differentiation/Exercises

Find the Derivative by Definition

Find the derivative of the following functions using the limit definition of the derivative.

1. $f(x) = x^2$
2. $f(x) = 2x + 2$
3. $f(x) = \frac{x^2}{2}$
4. $f(x) = 2x^2 + 4x + 4$
5. $f(x) = \sqrt{x+2}$
6. $f(x) = \frac{1}{x}$
7. $f(x) = \frac{3}{x+1}$
8. $f(x) = \frac{1}{\sqrt{x+1}}$
9. $f(x) = \frac{x}{x+2}$

Prove the Constant Rule

10. Use the definition of the derivative to prove that for any fixed real number c, $\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}[f(x)]$

Find the Derivative by Rules

Find the derivative of the following functions:

Power Rule

11. $f(x) = 2x^2 + 4$
12. $f(x) = 3\sqrt[3]{x}$
13. $f(x) = 2x^5 + 8x^2 + x - 78$
14. $f(x) = 7x^7 + 8x^5 + x^3 + x^2 - x$
15. $f(x) = \frac{1}{x^2} + 3x^{\frac{1}{3}}$

16. $f(x) = 3x^{15} + \frac{x^2}{17} + \frac{2}{\sqrt{x}}$

17. $f(x) = \frac{3}{x^4} - \sqrt[4]{x} + x$

18. $f(x) = 6x^{1/3} - x^{0.4} + \frac{9}{x^2}$

19. $f(x) = \frac{1}{\sqrt[3]{x}} + \sqrt{x}$

Product Rule

20. $f(x) = (x^4 + 4x + 2)(2x + 3)$

21. $f(x) = (2x - 1)(3x^2 + 2)$

22. $f(x) = (x^3 - 12x)(3x^2 + 2x)$

23. $f(x) = (2x^5 - x)(3x + 1)$

24. $f(x) = (5x^2 + 3)(2x + 7)$

25. $f(x) = 3x^2(5x^2 + 1)^4$

26. $f(x) = x^3(2x^2 - x + 4)^4$

27. $f(x) = 5x^2(x^3 - x + 1)^3$

28. $f(x) = (2 - x)^6(5 + 2x)^4$

Quotient Rule

24. $f(x) = \frac{2x + 1}{x + 5}$

25. $f(x) = \frac{3x^4 + 2x + 2}{3x^2 + 1}$

26. $f(x) = \frac{x^{\frac{3}{2}} + 1}{x + 2}$

27. $d(u) = \frac{u^3 + 2}{u^3}$

28. $f(x) = \frac{x^2 + x}{2x - 1}$

29. $f(x) = \dfrac{x+1}{2x^2+2x+3}$

30. $f(x) = \dfrac{16x^4+2x^2}{x}$

$f(x) = \dfrac{8x^3+2}{5x+5}$

$f(x) = \dfrac{(3x-2)^2}{\sqrt{x}}$

$f(x) = \dfrac{\sqrt{x}}{2x-1}$

$f(x) = \dfrac{4x-3}{x+2}$

$f(x) = \dfrac{4x+3}{2x-1}$

$f(x) = \dfrac{x^2}{x+3}$

$f(x) = \dfrac{x^5}{3-x}$

Chain Rule

31. $f(x) = (x+5)^2$

32. $g(x) = (x^3-2x+5)^2$

33. $f(x) = \sqrt{1-x^2}$

34. $f(x) = \dfrac{(2x+4)^3}{4x^3+1}$

35. $f(x) = (2x+1)\sqrt{2x+2}$

36. $f(x) = \dfrac{2x+1}{\sqrt{2x+2}}$

37. $f(x) = \sqrt{2x^2+1}(3x^4+2x)^2$

38. $f(x) = \dfrac{2x+3}{(x^4+4x+2)^2}$

39. $f(x) = \sqrt{x^3 + 1}(x^2 - 1)$

40. $f(x) = ((2x + 3)^4 + 4(2x + 3) + 2)^2$

41. $f(x) = \sqrt{1 + x^2}$

Exponentials

42. $f(x) = (3x^2 + e)e^{2x}$

43. $f(x) = e^{2x^2+3x}$

44. $f(x) = e^{e^{2x^2+1}}$

45. $f(x) = 4^x$

Logarithms

46. $f(x) = 2^{x-3} \cdot 3\sqrt{x^3 - 2} + \ln(x)$

47. $f(x) = \ln(x) - 2e^x + \sqrt{x}$

48. $f(x) = \ln(\ln(x^3(x + 1)))$

49. $f(x) = \ln(2x^2 + 3x)$

50. $f(x) = \log_4(x) + 2\ln(x)$

Trigonometric functions

51. $f(x) = 3e^x - 4\cos(x) - \dfrac{\ln(x)}{4}$

52. $f(x) = \sin(x) + \cos(x)$

More Differentiation

53. $\dfrac{d}{dx}[(x^3 + 5)^{10}]$

54. $\dfrac{d}{dx}[x^3 + 3x]$

55. $\dfrac{d}{dx}[(x + 4)(x + 2)(x - 3)]$

56. $\dfrac{d}{dx}[\dfrac{x + 1}{3x^2}]$

57. $\frac{d}{dx}[3x^3]$

58. $\frac{d}{dx}[x^4 \sin(x)]$

59. $\frac{d}{dx}[2^x]$

60. $\frac{d}{dx}[e^{x^2}]$

61. $\frac{d}{dx}[e^{2^x}]$

Implicit Differentiation

Use implicit differentiation to find y'

62. $x^3 + y^3 = xy$

63. $(2x + y)^4 + 3x^2 + 3y^2 = \frac{x}{y} + 1$

Logarithmic Differentiation

Use logarithmic differentiation to find $\frac{dy}{dx}$:

64. $y = x(\sqrt[4]{1 - x^3})$

65. $y = \sqrt{\frac{x+1}{1-x}}$

66. $y = (2x)^{2x}$

67. $y = (x^3 + 4x)^{3x+1}$

68. $y = (6x)^{\cos(x)+1}$

Equation of Tangent Line

For each function, f , (a) determine for what values of x the tangent line to f is horizontal and (b) find an equation of the tangent line to f at the given point.

69. $f(x) = \frac{x^3}{3} + x^2 + 5, \qquad (3, 23)$

70. $f(x) = x^3 - 3x + 1, \qquad (1, -1)$

71. $f(x) = \frac{2x^3}{3} + x^2 - 12x + 6, \qquad (0, 6)$

72. $f(x) = 2x + \frac{1}{\sqrt{x}}, \qquad (1,3)$

73. $f(x) = (x^2+1)(2-x), \qquad (2,0)$

74. $f(x) = \frac{2x^3}{3} + \frac{5x^2}{2} + 2x + 1, \qquad (3, \frac{95}{2})$

75. Find an equation of the tangent line to the graph defined by $(x-y-1)^3 = x$ at the point (1,-1).

76. Find an equation of the tangent line to the graph defined by $e^{xy} + x^2 = y^2$ at the point (1,0).

Higher Order Derivatives

77. What is the second derivative of $3x^4 + 3x^2 + 2x$?

78. Use induction to prove that the *(n+1)th* derivative of a *n-th* order polynomial is 0.