Exponents

Objective 1

Understand the Relationship between Multiplication and Exponents

Recall that $3 \cdot 4$ represents an addition problem.

$$3 \cdot 4 = 3+3+3+3.$$

But what about the expression

$$3 \cdot 3 \cdot 3 \cdot 3?$$

This is where the exponent is used!

> Exponents are used to represent repetitive multiplication of a quantity.

Using an exponent of 4, we write $3 \cdot 3 \cdot 3 \cdot 3$ as 3^4 where 3 is called the base and 4 is the exponent.

Base → 3^4 ← Exponent

Suppose we are given 2^3. This represents $2 \cdot 2 \cdot 2$ which is equal to 8.

If we are given x^3, this represents $x \cdot x \cdot x$.

So what do you think $x^3 \cdot x^4$ is equal to?

Well since $x^3 = x \cdot x \cdot x$ and $x^4 = x \cdot x \cdot x \cdot x$ we get the following result.

$$x^3 \cdot x^4 = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = x^7$$

Notice how we can just add the exponents.

$$x^3 \cdot x^4 = x^{3+4} = x^7$$

When multiplying two quantities that have the same base, we can add the exponents.

$$x^a \cdot x^b = x^{a+b}$$

Suppose we are given $x+x+x+x$. Notice that we have four x's being added together.

Recall that multiplication is used to represent repetitive addition of the same quantity.

Using the **Commutative Property** for multiplication we can now show that

$$4 \cdot x = x \cdot 4 = x+x+x+x = 4x$$

Notice that $4x = x+x+x+x$ and $x^4 = x \cdot x \cdot x \cdot x$.

Note: x^3 is said "x raised to the third power" or "x cubed".
x^2 is said "x raised to the second power" or "x squared".

Answer the following homework questions.

In Exercises 1 – 9, write each quantity in expanded form.

Recall: $4x = x+x+x+x$ and $x^4 = x \cdot x \cdot x \cdot x$.

1) 5^2
2) y^2
3) $4y$
4) a^4
5) $3a$
6) $2x$
7) ab^2
8) x^2y^2
9) p^3q^4

In Exercises 10 – 18, add or multiply as indicated, if possible.

10) $3x + 4x$
11) $y^3 \cdot y^3$
12) $3y + 3y$
13) $a^4 \cdot a^3 \cdot a^2$
14) $w \cdot w^2$
15) $w + 2w$
16) $2h + 2b$
17) $h^2 + b^2$
18) $2c \cdot c^2$

Objective 2

Understand the Zero Exponent

What is the value of 2^0?

We can arrive at a general conclusion by noticing the following pattern.

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

$2^3 = 2 \cdot 2 \cdot 2 = 8$

$2^2 = 2 \cdot 2 = 4$

$2^1 = 2 = 2$

$2^0 = 1 = 1$

Notice that the exponents are decreasing by 1!

Here the numbers are being divided by the base 2 as you move downward!

Let's now try this with a base of 3.

$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

$3^3 = 3 \cdot 3 \cdot 3 = 27$

$3^2 = 3 \cdot 3 = 9$

$3^1 = 3 = 3$

$3^0 = 1 = 1$

Notice that the exponents are decreasing by 1!

Here the numbers are being divided by the base 3 as you move downward!

Regardless of what base you use (except for 0), you will always get the zero power to equal 1! A base of 0 does not work because we can never divide by zero! 0^0 is undefined!

Answer the following homework questions.

In Exercises 19 – 27, find the value of each expression!

Note: In every problem you must first evaluate the quantity with the exponent before performing the arithmetic operation!

19) $2^3 + 3^2$

20) $3^2 - 12^0$

21) $4^3 \div 2^3$

22) $3^4 \cdot 1^{12}$

23) $0^{10} \div 7^2$

24) $10^2 \div 0^3$

25) $2^3 \cdot 3^2$

26) $11^2 - 8^2$

27) $5^0 + 4^0$