Multiplication with Negative Numbers

Objective 1

Understand why a "Negative times a Positive" or "Positive times a Negative" is Negative

Remember that multiplication represents repetitive addition of a number.

Recall: $3 \cdot 4 = 3 + 3 + 3 + 3 =$

A "**positive times a positive**" will always represent a positive number since we are summing positive quantities.

But a "**negative times a positive**" implies that we are summing negative quantities. We therefore will always get a negative result in these cases.

$-3 \cdot 4 = (-3) + (-3) + (-3) + (-3) =$

$-5 \cdot 3 = (-5) + (-5) + (-5) =$

$-1 \cdot 6 = (-1) + (-1) + (-1) + (-1) + (-1) + (-1) =$

By the Commutative Property we can state

$$-3 \cdot 4 = 4 \cdot (-3).$$

So we can conclude that a "**positive times a negative**" is also negative!

Example 1: Rewrite the following multiplication problems as equivalent addition problems. Next, find the value of the sum.

a) $5 \cdot 4 = 5+5+5+5=$

b) $-3 \cdot 6 =$

c) $-6 \cdot 5 =$

d) $4 \cdot (-2) =$

But what about a **negative times a negative**? Notice that multiplying -1 to a number always results in the opposite of the number!

$-1 \cdot 2 = (-1)+(-1) = -2$

$-1 \cdot 3 = (-1)+(-1)+(-1) = -3$

$-1 \cdot 4 = (-1)+(-1)+(-1)+(-1) = -4$

So what happens if we multiply -1 to a negative number?

Since the opposite of any negative number is always positive, the result must be positive.

$-1 \cdot (-2) = 2$

$-1 \cdot (-3) = 3$

$-1 \cdot (-4) = 4$

We can now make a general conclusion that **negative times a negative** will be positive!

To summarize things, we will look at a pattern that occurs in the columns below.

$1 \cdot 2 = 2$	$(-1) \cdot 2 = -2$	$1 \cdot (-2) = -2$	$(-1) \cdot (-2) = 2$
$2 \cdot 2 = 4$	$(-2) \cdot 2 = -4$	$2 \cdot (-2) = -4$	$(-2) \cdot (-2) = 4$
$3 \cdot 2 = 6$	$(-3) \cdot 2 = -6$	$3 \cdot (-2) = -6$	$(-3) \cdot (-2) = 6$
$4 \cdot 2 = 8$	$(-4) \cdot 2 = -8$	$4 \cdot (-2) = -8$	$(-4) \cdot (-2) = 8$

When multiplying two numbers with the same sign, the product will be positive.
When multiplying two numbers with different signs, the product will be negative.

Now let's think about the product of three negative numbers.

$$(-2) \cdot (-2) \cdot (-2)$$

Working left to right, we get the following:

$$(-2) \cdot (-2) \cdot (-2) = 4 \cdot (-2) = -8$$

Now let's think about the product of four negative numbers.

$$(-2)\cdot(-2)\cdot(-2)\cdot(-2)$$

Working left to right, we get the following:

$(-2)\cdot(-2)\cdot(-2)\cdot(-2) = 4\cdot(-2)\cdot(-2) = -8\cdot(-2) = 16$

We can now state the following conclusion.

When multiplying an <u>odd</u> number of negative quantities, the product will be negative.

When multiplying an <u>even</u> number of negative quantities, the product will be positive.

Answer the following homework questions.

In Exercises 1 – 15, find each product.

1) $-8\cdot(-7)$
2) $5\cdot(-9)$
3) $-11\cdot 12$
4) $0\cdot(-5)$
5) $6\cdot(-3)$
6) $12\cdot(-8)$
7) $-2\cdot 14$
8) $-2\cdot(-16)$
9) $6\cdot(-6)$
10) $-1\cdot 0$
11) $-5\cdot(-4)\cdot(-3)$
12) $-2\cdot(-3)\cdot 8$
13) $4\cdot(-8)\cdot 10$
14) $2\cdot(-3)\cdot(-1)\cdot(-4)$
15) $-5\cdot(-2)\cdot(-3)\cdot(-6)$

Objective 2

Understand Negative Numbers with Exponents

It is important to understand the difference between the two expressions -3^2 and $(-3)^2$.

The expression -3^2 is read "**negative one times three squared**".

Therefore -3^2 is equivalent to $-1 \cdot 3^2$. Following order of operations and evaluating the exponent first before multiplication, we find that -3^2 is equal to $-1 \cdot 9$ or -9.

However, the expression $(-3)^2$ is read "**negative three squared**". Notice how a set of parenthesis is used to define the negative base.

Therefore $(-3)^2$ is equivalent to $(-3)(-3)$. In this case, we see that two negatives are being multiplied together. Therefore $(-3)^2$ is equal to $(-3)(-3)$ or 9.

Answer the following homework questions.

In Exercises 16 – 30, find the value of each expression.

Note: Be sure to follow the rules of Order of Operations!

16) 2^3

17) -2^3

18) $(-2)^3$

19) -2^4

20) $(-2)^4$

21) $(-3)^4$

22) $(-3)^3$

23) -3^4

24) -3^3

25) $(-1)^{99}$

26) $1-2^2$

27) $4-(-3)^2$

28) $4-3^2$

29) $-10^2-(-4)^2$

30) $-(-2)^2-(-3)^3$

In Exercises 31 – 36, find the value of each expression.

Note: Be sure to follow the rules of Order of Operations!

31) $-|-2|^2$

32) $-3^2-|-2|^3$

33) $|-2^2-3^2|$

34) $-(-4)^2-|-2^3|$

35) $|-4^2|-|-4^2|$

36) $|-6^2|-|6^2|$