More with Fractions

Objective 1 | Add and Subtract Fractions with Different Denominators

> **Remember:** In order to add or subtract fractions the denominators must be the same.

Let's begin by working the problem $\frac{2}{3} + \frac{1}{4} - \frac{5}{6}$.
In order to perform the indicated operations of addition and subtraction, we must rewrite each fraction as equivalent fractions having the same denominator.

We begin by first finding the **Least Common Denominator (LCD)** of all three fractions. The LCD can simply be thought of as the smallest number that all your denominators divide evenly into.

$$\frac{2}{3} + \frac{1}{4} - \frac{5}{6}$$

Here our denominators our **3**, **4**, and **6**. The smallest number that **3**, **4**, and **6** divide evenly into is **12**. Therefore **12** is the LCD.

> **Note:** The LCD is never smaller than the largest denominator. In fact, it is always a multiple of the largest denominator.

Another method of finding the LCD is to find the **Least Common Multiple (LCM)** of the denominators. A simple way of doing this is to make a list of multiples of the denominators to find the lowest common multiple. This quantity will be the **LCD**.

For the problem $\frac{2}{3} + \frac{1}{4} - \frac{5}{6}$, we will make list of multiples for **3**, **4**, and **6** starting with the largest denominator.

6: 6, **12**, 18, 24, 30, 36, …

4: 4, 8, **12**, 16, 20, 24, 30, …

3: 3, 6, 9, **12**, 15, 18, 21, 24, …

Notice that **12** is the lowest common multiple and therefore **12** is the LCD.

Note: When the denominators involve very large numbers, making a list of common multiples can be very time consuming. In these cases, using prime factorization to find the LCD may be a better approach. This method will be covered in a later section.

For the problem $\frac{2}{3} + \frac{1}{4} - \frac{5}{6}$ we have the LCD=12. To rewrite each fraction as an equivalent fraction with a denominator of 12, we must multiply each fraction by an appropriate factor of 1.

$$\frac{2}{3} + \frac{1}{4} - \frac{5}{6}$$

$$\frac{2}{3}\left(-\right) + \frac{1}{4}\left(-\right) - \frac{5}{6}\left(-\right)$$

$$\frac{8}{12} + \frac{3}{12} - \frac{10}{12}$$

$$\frac{+ \quad -}{12}$$

$$\overline{12}$$

Example 1: Perform the indicated operations.

a) $-\dfrac{5}{6} + \dfrac{3}{10} - \dfrac{4}{5}$

Multiples of Denominators
10: 10, 20, 30, 40, 50, 60, 70, ...
6: 6, 12, 18, 24, 30, 36, 42, ...
5: 5, 10, 15, 20, 25, 30, 35, ...

$$LCD = 30$$

$$-\dfrac{5}{6}\left(\dfrac{5}{5}\right) + \dfrac{3}{10}\left(\dfrac{3}{3}\right) - \dfrac{4}{3}\left(\dfrac{10}{10}\right)$$

$$-\dfrac{25}{30} + \dfrac{9}{30} - \dfrac{40}{30}$$

$$\dfrac{-25 + 9 - 40}{30}$$

$$\dfrac{-56}{30}$$

$$-\dfrac{\overset{28}{\cancel{56}}}{\underset{15}{\cancel{30}}}$$

$$\boxed{-\dfrac{28}{15}}$$

b) $-\dfrac{3}{8} + \left(-\dfrac{1}{4}\right)^2 - \dfrac{5}{32}$

$$-\dfrac{3}{8} + \left(-\dfrac{1}{4}\right)\left(-\dfrac{1}{4}\right) - \dfrac{5}{32}$$

$$-\dfrac{3}{8} + \dfrac{1}{16} - \dfrac{5}{32}$$

Multiples of Denominators.
32: 32, 64, 96, 128, ...
16: 16, 32, 48, 64, 80, 96, ...
8: 8, 16, 24, 32, 40, 48, 56, 64, ...

$$LCD = 32$$

$$-\dfrac{3}{8}\left(\dfrac{4}{4}\right) + \dfrac{1}{16}\left(\dfrac{2}{2}\right) - \dfrac{5}{32}$$

$$-\dfrac{12}{32} + \dfrac{2}{32} - \dfrac{5}{32}$$

$$\dfrac{-12 + 2 - 5}{32}$$

$$\dfrac{-15}{32}$$

$$\boxed{-\dfrac{15}{32}}$$

Answer the following homework questions.

In Exercises 1 – 12, perform the indicated operations.

1) $\dfrac{3}{4} + \dfrac{2}{5}$

2) $\dfrac{3}{4} + \dfrac{2}{5} - \dfrac{1}{10}$

3) $\dfrac{5}{9} - \left(-\dfrac{1}{6}\right)$

4) $\dfrac{4}{3} + \dfrac{1}{t}$
   (LCD=3t)

5) $\dfrac{3}{12} - \left(-\dfrac{1}{2}\right)^3$

6) $\dfrac{1}{8} - \left(-\dfrac{3}{4}\right)^2$

7) $-\dfrac{3}{8} - \dfrac{2}{6} + \dfrac{1}{3}$

8) $2\dfrac{3}{5} + \dfrac{1}{2h}$
   (LCD=10h)

9) $\dfrac{2}{7} - \dfrac{2}{9} - \dfrac{2}{21}$

10) $\left(-\dfrac{2}{3}\right)^2 - \left(-\dfrac{2}{3}\right)^3$

11) $\dfrac{6}{25} - \dfrac{2}{15}$

12) $\dfrac{3}{40} + \dfrac{5}{36}$